Simplified probabilistic non-linear assessment of existing railway bridges

Dawid F. Wisniewski (†)
Joan R. Casas (‡)
Michel Ghosn (§)

(†) Bridge Engineer, M.Sc., Ph.D.
Operation Management, Rehabilitation & Systems
Bridge, Tunnel and Marine Structures
COWI A/S
Parallelvej 2
DK-2800 Kongens Lyngby, Denmark
(formerly at the University of Minho, Guimarães, Portugal)
e-mail: dawi@cowi.dk

(‡) Professor *(corresponding author)*
Department of Civil Engineering
Technical University of Catalunya
Campus Nord, edifici C2,
C. Jordi Girona, 1-3. 08034 Barcelona, Spain
e-mail: joan.ramon.casas@upc.edu

(§) Professor
Department of Civil Engineering
The City College of New York/CUNY
Steinmann Hall, T-101
Convent Ave. @ 140th Street
New York, NY 10031, USA
e-mail: ghosn@ce.ccny.cuny.edu

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Simplified probabilistic non-linear assessment of existing railway bridges

D.F. WISNIEWSKI†, J.R. CASAS*‡ and M. GHOSN§

†COWI A/S, Denmark *(formerly at University of Minho, Portugal)*
‡Technical University of Catalunya, Spain
§The City College of New York/CUNY, US

An objective of the European Commission's 6th Framework Research Project, *Sustainable Bridges*, is to advance our understanding of the behaviour of existing railway bridges and develop tools to assess their ability to safely handle future traffic demands and extend their service lives. This paper presents the findings of a study that reviews structural safety models applicable to the assessment of existing bridges. The study proposes the use of simplified probabilistic non-linear structural analysis methods to provide more accurate assessments of the load capacity of bridge systems than traditional methods. The simplified methods use the results of a limited number of deterministic non-linear structural analyses and apply these results into a reliability framework. The application of the proposed methods is illustrated by assessing the safety of an existing bridge. The accuracy and efficiency of the simplified methods are verified by comparing the results of the simplified methods to those obtained from full probabilistic non-linear analysis procedures.

Keywords: Existing bridges; Capacity assessment; System capacity; Safety formats; Non-linear analysis.

1 Introduction

The vast majority of Europe's railway bridges were built more than 50 years ago and 35% of the bridge stock is older than 100 years (Bell 2004). Hence, many bridges are subjected to loads far higher than those envisaged during design. Also, due to insufficient investment in bridge maintenance, many of the existing railway bridges have significantly deteriorated over their years of service. The enlargement of the European Community and the continuous growth of its economy, have led to an increase in traffic loads and speeds on its railway lines, a trend that is expected to continue into the foreseeable future. Therefore, it is of vital importance to ensure that the existing railway network and its bridges, which form its critical links, can still provide adequate levels of safety under increased loads and higher speeds.

The safety assessment of railway bridge structures can in many cases be addressed using traditional bridge load capacity evaluation methods. However, current load evaluation procedures for existing structures are usually adopted from design codes, which are meant for new bridges, and may not be adequate for the assessment of certain types of existing bridges. Most current methods of safety assessment are based on a linear elastic structural analysis and a deterministic evaluation of individual member strengths. In reality, a bridge consists of a system of interconnected members where the failure of any single member may not necessarily cause the collapse of the whole structure. Therefore, the reliability of the member may not be representative of the reliability of the whole bridge. Furthermore, most of the variables describing the bridge's geometry, material properties, structural response, and applied loads are not deterministic parameters and their design or characteristic values, which are often also used during the assessment of existing bridges, do not always properly reflect the in-situ conditions. Even when such parameters are measured on site, the inherent uncertainties in estimating their values are not adequately considered. Due to all the simplifications and conservative assumptions usually made during the design process, using the same standards for the assessment of existing bridges may lead to having many bridges that are in reality completely safe be rated as unsafe. For these reasons, many researchers have recommended the application of advanced probabilistic analysis methods for assessing the safety of existing structures (Schneider 1997, BRIME 2001, Enevoldsen 2001, COST354 2004, Lauridsen 2004).

The benefits gained by performing a full-fledged structural system reliability analysis during the process of designing new ordinary bridges are usually quite low. This is due to the fact that the more advanced analysis, in most cases, will only lead to a small decrease in member sizes and a negligible reduction in amount of the materials used for construction which contribute to a small fraction of bridge construction costs. Therefore, the significant computational effort necessary to perform system reliability analysis is not usually justified at the design level. Alternatively, the explicit consideration of structural redundancy and that of the uncertainty in estimating the most important parameters can be significant and could lead to considerable economical benefits when assessing the safety of existing bridges. This is especially the case when decisions have to be made regarding what appropriate maintenance actions to undertake such as rehabilitation, strengthening or replacement of bridges that may not satisfy the design member-based safety criteria but are known to have significant levels of reserve strength. For this reason, the use of probability-based safety assessment methods for existing bridges is increasing in practical applications (Casas 1999, 2000, Enevoldsen 2001, Lauridsen 2004). To avoid the need to perform a probabilistic analysis for all bridges, several research studies (BA79/98 1998, BRIME 2001, COST354 2004, SAMARIS 2006) have recommended that structural assessment strategies be based on different analysis levels with increasing degrees of complexity. The recommendation to go forward to the next analysis level is made only if the bridge fails to pass the previous assessment level.

This “step-level” philosophy has been also proposed in the soon to appear European Guideline for the load capacity and safety assessment of existing railway bridges (SB-LRA 2007), currently under preparation within the *Sustainable Bridges* project, where three levels of assessment are proposed. The most advanced assessment method recommended in this Guideline combines a load redistribution analysis (non-linear analysis) with a probabilistic analysis. This level can be applied as a last resort to save a bridge from unnecessary repair/strengthening or replacement.

The quantification of bridge system reliability with the required accuracy is possible thanks to available non-linear probabilistic analysis methods, the most commonly used of which are summarized in this paper. The problem with the practical application of these methods is that they require excessive computational effort, which in many cases can be difficult to accommodate, without necessarily providing a commensurate level of accuracy. For this reason, two simplified probabilistic methods are also presented in this paper. These simplified models require much less computational effort while providing a sufficient level of accuracy. The benefits of these simplified methods lie in the ease of their use by bridge evaluators with standard computational tools.

2 Probabilistic non-linear analysis

2.1 *General formulation*

The probabilistic safety assessment of existing bridges using non-linear analysis can be formulated using a limit state function g(X), where X represents the vector of random variables. In the simplest case, the limit state function g(X) can be defined as the difference between the generalized structural resistance *R* and generalized action effect *S*, so that the probability of failure (or probability of limit state violation) p_f is expressed as:

$$p_f = Pr(g(X) < 0) = Pr((R - S) < 0) \tag{1}$$

In many bridge applications, the reliability index, ß, defined by equation (2) is a widely used measure of structural reliability:

$$b = -\Phi^{-1}(p_f) \tag{2}$$

where F^{-1} is the inverse standard normal probability distribution function.

2.2 *Computational methods*

Over the last few decades, several computational methods that can be linked to non-linear Finite Element Method algorithms, FEM, have been proposed to obtain probabilistic evaluations of the safety of structural systems. These methods can be broadly divided into three categories (Haldar and Mahadevan 2000): 1) Monte Carlo Simulations, MCS (including efficient sampling methods and variance reduction techniques), 2) the Response Surface Method, RSM, and 3) sensitivity based analyses (including Stochastic Finite Element Methods, SFEM).

Direct MCS (Melchers 1999, Haldar and Mahadevan 2000, Nowak and Collins 2000) and more advanced simulation techniques such as the importance sampling (IS) method (Melchers 1999), the Latin Hypercube Sampling (LHS) (Nowak and Collins 2000) or the directional sampling technique (DS) (Melchers 1999) give good overall results and have been applied for nearly all types of structural reliability problems (Eamon *et al.* 2005). However, they require significant numbers of runs especially for problems with high numbers of random variables and low probabilities of failure.

The RSM (Haldar and Mahadevan 2000, Nowak and Collins 2000) in its various forms has been often adopted as the method of choice for structural applications. It is quite practical and effective in most common situations even though it is designed to obtain the reliability index directly rather than calculate the probability of failure. The application of RSM for highly non-linear limit state functions or for problems with several modes of failure could be inefficient and may lead to divergence or inaccurate results, even when using its more advanced variants such as the adaptive method (Rajashekhar and Ellingwood 1993) or DARS, Directional Adaptive Response Surface sampling method (Waarts 2000).

Sensitivity methods and SFEM (Haldar and Mahadevan 2000) offer solutions that are more mathematically elegant than MCS or RSM. These methods however require specialized programs that are not yet widely available or easily adaptable for practical applications.

Although the methods described above are the most commonly used probabilistic non-linear analysis tools, modifications and refinements to these methods are continuously being introduced to improve their efficiency or accuracy by using advanced interpolation techniques and neural networks to approximate the limit state function (Kaymaz 2005, Schueremans and Van Gemert 2005) or Genetic Algorithms that replace gradient-based optimization techniques during the search for the reliability index (Deng *et al.* 2005, Schueremans and Van Gemert 2005, Wang and Ghosn 2005).

3 Simplified models

3.1 *Background*

The application of the advanced methods of probabilistic non-linear analysis requires advanced knowledge of the structural reliability theory as well as significant computational effort. For these reasons, simplified probabilistic non-linear analysis methods, which only require a single non-linear analysis performed with widely available non-linear FEM packages can provide adequate alternatives when evaluating the safety of common type bridges. This section describes two simplified probabilistic non-linear analysis methods that will be shown to be sufficiently accurate for the purpose of assessing the safety of existing common type railway bridges.

3.2 *Method of Ghosn and Moses*

According to the simplified method proposed by Ghosn and Moses (1998), a bridge may be considered safe if it provides a reasonable safety level against first member failure, it does not produce large deformations under high loads, it does not reach its ultimate system capacity under extreme loading conditions and it is able to carry some traffic loads after damage or the loss of a main load-carrying member. Hence, system safety is not only related to the ultimate system capacity, but also to the deformation, and post-damage capacity. This implies that four limit sates should be checked to insure adequate bridge system safety. The first is a member failure limit state (this is the traditional check of individual member safety). The remaining three system limit states are the functionality limit state (this is defined to limit maximum live load displacements accounting for the non-linear behaviour of the bridge system to ensure that the bridge remains functional after high load crossings); ultimate limit state (this is the ultimate capacity of the bridge system or the formation of a collapse mechanism) and damaged condition limit state (this is defined as the ultimate capacity of the bridge system after the complete removal of one main load carrying component from the structural model). This latter limit state is often referred to as structural robustness.

The incorporation of system behaviour during the safety assessment is done using the relative reliability indices Δb_i. For each of the three system limit states defined above, Δb_i gives the difference between the safety indices for the system b_i and the safety index for the member b_{memb}. In order to guarantee bridge safety, each of the relative reliability indices must be greater than an appropriate target value Δb_i^{targ} while at the same time member safety has to be assured by requiring that the member's reliability index remains above an acceptable level defined by b_{memb}^{targ}. This method was proposed for the design of new structures where the bridge members can be designed with appropriate level of safety. In existing structures, where in some cases individual members may not meet the safety requirements, global system safety should be exclusively used as criteria as proposed by Casas *et al.* (2007). In this case, the proposed safety format would take the form:

$$\Delta b_{ultim} + b_{memb} = b_{ultim} \geq \Delta b_{ultim}^{targ} + b_{memb}^{targ} = b_{ultim}^{targ} \quad (3a)$$

$$\Delta b_{func} + b_{memb} = b_{func} \geq \Delta b_{func}^{targ} + b_{memb}^{targ} = b_{func}^{targ} \tag{3b}$$

$$\Delta b_{damage} + b_{memb} = b_{damage} \geq \Delta b_{damage}^{targ} + b_{memb}^{targ} = b_{damage}^{targ} \tag{3c}$$

where b_i^{targ} is target system reliability index. The subscripts 'ultim', 'func' and 'damage' correspond to ultimate, functionality and damaged condition limit states respectively.

The target values for the relative reliability indices proposed for highway bridge design by Ghosn and Moses (1998) and adopted for certain situations to railway bridges assessment by Casas *et al.* (2007) are presented in Table 1.

The target value for member reliability should be defined for every specific situation based on a cost-benefit analysis (JCSS 2001a). However, in the cases when a cost-benefit analysis cannot be performed, the target value for member reliability can be set to match that used for the calibration of traditional bridge assessment codes or conservatively those used in design codes. Usually, the target for the reliability index for the verification of bridge members takes a value between 2.5 and 4.7 (Casas *et al.* 2007). As an example, Table 2 shows target reliability values for a one-year exposure period as proposed in JCSS (2001b).

In this simplified approach, the reliability indices for individual members as well as for the system can be calculated using two different formats (normal or log-normal), depending on the assumed probability distribution types of random variables *R* and *S*. In this paper, the calculation of the reliability index ß is performed using the normal format:

$$b = \frac{\overline{R} - \overline{S}}{\sqrt{s_R^2 + s_S^2}} \tag{4}$$

where $\overline{R}$ and $\overline{S}$ are the mean values of R and S respectively. s_R and s_S are the standard deviations of R and S.

Normalizing the resistance and load effects R and S in the equation (4), i.e. assuming that $\overline{R} = \overline{LF_1} \times L_{TRAIN}$ and $\overline{S} = \overline{LL_{TRAIN}} \times L_{TRAIN}$, the following equation for the member reliability index for a railway bridge can be obtained (Casas *et al.* 2007):

$$b_{member} = \frac{\overline{LF_1} - \overline{LL_{TRAIN}}}{\sqrt{s_{LF}^2 + s_{LL}^2}} \qquad (5)$$

In equation (5) $\overline{LF_1}$ is the mean value of LF_1 which is the design load multiplier where the design load is given as L_{TRAIN}. L_{TRAIN} is the effect of the design train load (e.g. characteristic UIC train load) which is the original load that is incremented during the non-linear analysis. $\overline{LL_{TRAIN}}$ is the mean value of the maximum expected lifetime live load (e.g. UIC train load) including dynamic allowance effect expressed as a function of the design train load L_{TRAIN}. s_{LF} is the standard deviation of LF_1 while s_{LL} is the standard deviation of the maximum expected live load LL_{TRAIN}.

The mean value of the load factor LF_1 can be calculated using the following expression:

$$\overline{LF_1} = \frac{\overline{R} - \overline{D}}{L_{TRAIN}} \qquad (6)$$

where $\overline{R}$ is the mean member resistance, $\overline{D}$ is the mean dead load effect and L_{TRAIN} is as defined above. The nominal value of LF_1 can be obtained using the same equation (6) but considering nominal values of R and D instead of the mean values.

The standard deviation s_{LF} of the load factor LF_1 is expressed by:

$$s_{LF} = \frac{\sqrt{s_R^2 + s_D^2}}{L_{TRAIN}} \tag{7}$$

where s_R is the standard deviation of R and s_D is the standard deviation of D.

Similarly, the system reliability indexes for the functionality, ultimate and damaged limit states are defined by:

$$b_{func} = \frac{\overline{LF_f} - \overline{LL_{TRAIN}}}{\sqrt{s_{LF}^2 + s_{LL}^2}} \tag{8a}$$

$$b_{ult} = \frac{\overline{LF_u} - \overline{LL_{TRAIN}}}{\sqrt{s_{LF}^2 + s_{LL}^2}} \tag{8b}$$

$$b_{damage} = \frac{\overline{LF_d} - \overline{LL_{train}}}{\sqrt{s_{LF}^2 + s_{LL}^2}} \tag{8c}$$

where $\overline{LF_f}$ (see Figure 1) is the mean value of the load factor corresponding to the functionality limit state. This is the load factor by which the design load has to be multiplied to reach the functionality limit state, normally represented by a maximum deflection allowance. $\overline{LF_u}$ (see Figure 1) is the mean value of the load factor corresponding to the ultimate limit state and $\overline{LF_d}$ (see Figure 1) is the mean value of the load factor corresponding to the damaged condition limit state. For this purpose, the bridge model is modified to simulate a severe damage scenario. $\overline{LL_{train}}$ is the mean value of the maximum expected load (including dynamic allowance effect) corresponding to a low return period usually selected to correspond to the period of routine inspection. The exposure period is made to coincide with the routine inspection period to reflect the fact that severe damage to the bridge would be detected during the inspection and necessary repairs are made at that point. The remaining parameters are the same as those of equation (5).

Because of lack of data on the coefficients of variation (COV's) associated with estimating the capacity of bridge systems, it is herein assumed that the load factors LF_u, LF_f and LF_d have the same COV, V_{LF}, as that of the load factor LF_1 which can be expresses as:

$$V_{LF} = \frac{s_{LF}}{\overline{LF_1}} \tag{9}$$

where s_{LF} is as in equation (7) and mean value of LF_1 is determined by equation (6).

Also, the bias factor which relates the mean values to the nominal values of LF_u, LF_f and LF_d is assumed to be equal to the bias associated with LF_1. The bias factor ?$_{LF}$ can be calculated according to the expression:

$$l_{LF} = \frac{\overline{LF_1}}{LF_1} \tag{10}$$

It is noted that, generally speaking, system reliability analyses will lead to lower COV's when evaluating system uncertainties as compared to individual member uncertainties. This assumes that the system analysis process and the system analysis tools and models are highly accurate. However, in this paper the assumption that the COV of the member and the system are equal is made to account for the high level of modelling uncertainties associated with the finite element analysis of non-linear bridge structures in the as-built conditions.

The calculation of LF_1, LF_u, LF_f and LF_d requires the development of the structural model of the railway bridge being assessed and the use of a finite element package that can perform a static non-linear analysis of the structure. The input used for defining the structural model includes the best estimates of material properties, geometry and dead loads, identification of the bridge's most critical members and the identification of the loading positions and the most critical loading patterns for the critical members under consideration.

3.3 *Method of Sobrino and Casas*

Another simplified procedure for the reliability-based assessment of existing bridges at the structural system level was proposed by Sobrino and Casas (1994) and Casas *et al.* (2007). The method, which takes into account the redundancy in bending about the longitudinal direction, is most appropriate for continuous bridges. The proposed method requires the calculation of the probability of failure of the system (or safety index) and compares the calculated value with a target index for the system. The method defines the Limit State function g(X) in bending for each critical section i situated over intermediate supports or at mid-span as:

$$g(X) = M_R^i - l^{i}\left(M_G^e + M_{IQ}^e\right) \tag{11}$$

where M^i_R is the ultimate resistance moment of the i-th section of the continuous beam, M^e_G is the bending moment due to dead loads calculated for the equivalent simply supported beam and M^e_{IQ} is the maximum bending moment due to traffic loads including impact, calculated also for the equivalent simply supported beam. The equivalent simply supported beam is defined as the simply supported beam with span-length equal to the length of the span where section i is located (see Figure 2).

In equation (11), $?^i$ is the moment redistribution factor for the i-th section defined as:

$$l^{\cdot i} = \frac{M_{nla}^{i}}{\frac{M_{nla}^{1} + M_{nla}^{3}}{2} + M_{nla}^{2}} \quad (12)$$

where M^i_{nla}, M^1_{nla}, M^3_{nla} and M^2_{nla} are the bending moments at failure obtained in the non-linear analysis for the critical i-th section under consideration and the sections over the supports and at mid-span respectively, for the span where i-th section is located (see Figure 3).

Sobrino and Casas (1994) verified that the variability in the mechanical properties and geometrical uncertainties do not change the failure mode of common type continuous bridge structures. Also, the COV of the moment response for each section remains practically constant after yielding. Therefore, because at failure the values of M^i_{nla}, M^1_{nla}, M^3_{nla} and M^2_{nla} will be close to their ultimate values, it can be assumed that the COV of these variables is the same as the COV of the corresponding ultimate member bending capacity. The latter can be easily obtained for each section by simulation taking into consideration the random nature of the basic variables that control the bending capacity which are known to be the section's dimensions, as well as the concrete and steel strengths. The mean values of variables the M^i_{nla}, M^1_{nla}, M^3_{nla} and M^2_{nla} can be approximated by executing a non-linear analysis of the bridge members using as input the mean values of the basic variables.

As with the previous method, the procedure to determine the reliability index associated with Equation (11) requires the development of a structural model of the bridge and the use of a finite element package allowing for the static non-linear analysis of structures. The best estimates of material properties, geometry and dead loads are used as input. The analysis requires the identification of the bridge critical sections and the identification of the loading position and the most critical loading patterns for the critical section under consideration. Only one non-linear analysis per failure mode or critical section is required.

4 Application example – Brunna Bridge

4.1 *General information*

The Brunna Bridge is a four-span continuous reinforced concrete structure constructed in 1969. The bridge has the dimensions, cross section and reinforcing details presented in Figures 4 and 5. The values of the most important variables describing the geometry and mechanical properties of the bridge are presented in Table 3. The COV's given in the table are collected from the work of various researchers and presented in Casas (2007). Other material properties such as the elastic modulus of concrete and the concrete tensile strength are considered to be functions of the compression strength of concrete as defined in EC2 (2003). All mechanical properties required as FEM input that are not presented in the table are taken as defined in EC2 (2003).

The following loads were considered in the analysis : G_s - Self-weight of the structure; G_a - Additional permanent loads; Q - Live load on the railway track (UIC train load model) as presented in Table 3 along with the COV of each load.

The values of railway traffic loads are obtained from the UIC train load model considering that the combined effect of the characteristic axle load (250kN) and distributed load (80kN/m) corresponds to the 98-th percentile of the PDF of the railway load assuming normal distribution. Considering this assumption, the mean value for the axle loads and distributed load are calculated to be respectively 207kN and 63.4kN/m. The values of the railway traffic load presented in Table 3 are obtained by equally distributing the load to the two beam lines and distributing the concentrated load from the axles through the ballast (the distribution length was considered equal to 6.4m).

The analysis is performed for a single loading scheme which causes the failure of the mid-span section of the first span (variable loads are applied on the first and the third span). Furthermore, the analysis is performed for two condition states of the bridge. The first analysis is carried out for the original bridge where it is assumed that the structure is in perfect condition. The second analysis is performed assuming a serious level of deterioration where 50% of the bottom reinforcement of the section in the middle of the first span is assumed to be corroded and is removed from the model. The situation, where only one section of the bridge is subjected to such high level of deterioration while other sections remain intact is hypothetical and is only considered to illustrate the benefits of the proposed method for the safety assessment of existing bridges in the case where the standard member level assessment technique recommended by existing codes fails. It also has to be stressed that the corrosion in this example is not considered as a stochastic process. But, for the purpose of illustrating the procedure, it is assumed that the corrosion is a cause of damage of the reinforcement and that the extent of this damage has been identified with high level of certainty of the same order as the level associated with determining the properties of non-deteriorated structural members.

4.2 *Finite element model*

As an approximation, the girder is modelled as two equal and parallel longitudinal beams coinciding with the webs. Only one of the beams is analysed assuming that no transverse redistribution of loads between the two webs is allowed and that the effect of the skew is negligible. Thus, the load is equally distributed to each beam, which ignores the random eccentricity in the transverse location of the load. Furthermore, the bending moments due to dead loads are time invariant and the concrete behaviour throughout the design or effective life of the bridge remains as that obtained for the concrete at 28 days.

The special structural analysis software Plastd90 , which accounts for material non-linearity of structural steel and concrete, is used for modelling the bridge (Henriques 1998). The boundary conditions between the main girder and the end piers (A and E of Figure 5) are assumed to be pinned supports since the connections were designed to only transfer the vertical reactions. Due to the fact that the interior reinforced concrete circular columns (B, C and D in Figure 5) were rigidly connected to the superstructure and to the footing, the model assumes a rigid frame connection between the columns and the longitudinal beams. The connections of the columns to the foundation are considered as fixed.

4.3 *Structural analysis*

4.3.1 Linear elastic analysis

The bending moments for the middle span section (Sect.2) and the sections over the piers (Sect.1, Sect.3) of the first span, are listed in Table 4 for each of the loads obtained from the linear elastic analysis.

Table 5 presents the results of the bending moments in the mid-span section (Sect.2) obtained for the equivalent simply supported beam, with a length of 13.5 m. The results in the first two rows correspond to the bending moments due to the mean values of the permanent loads. The last row corresponds to the bending moment due to the mean value of the railway traffic load without impact.

4.3.2 Non-linear analysis for Ghosn and Moses method

The load factors for the functionality, LF_f, ultimate, LF_u, and damaged condition limit states, LF_d, obtained from the non-linear analysis and using UIC characteristic train load model are presented in Table 6. Two major damage scenarios are assumed in this example. The first damage scenario consists of the formation of a hinge in the mid-span section. The second scenario assumes a hinge in the section over the pier B of the main girder (see Figure 5).

4.3.3 Non-linear analysis for Sobrino and Casas method

The bending moments prior to the failure of the sections, Sect.2, Sect.1, and Sect.3 of the first span are obtained from a non-linear analysis and presented in Table 7. The results are obtained considering that all the variables describing structure geometry and material behaviour take their mean values (see Table 3). The dead loads and the railway traffic loads were also considered at their mean values.

4.4 *Section resistances*

The bending resistance of each of the bridge's critical sections is obtained from the ultimate analysis of reinforced concrete sections. The two previously defined condition states are considered, original and the deteriorated. At first, the sectional analyses are carried out for the characteristic values of the concrete compressive strength and steel yielding strength as defined in Table 3. These results are presented in Table 8. Subsequently, simulations using the LHS method were performed, as reported in Casas *et al.* (2007), to obtain the probabilistic resistance model of the sections subjected to bending. The results of the simulations sections Sec.2, Sec.1 and Sec.3, for both the original and deteriorated conditions are summarized in Table 8.

4.5 *Safety assessment*

4.5.1 Partial safety factor method

In order to compare the deterministic standard assessment method with the probabilistic methods proposed in this paper, as a first step, the safety of the main girder of the Brunna Bridge in bending was checked using the standard partial safety factor method and linear elastic analysis. The safety check is performed for the middle span section of the first span considering the original and the deteriorated bridge condition.

For bending, the checking equation with the typical Eurocode load and resistance factors can be expressed by:

$$0.86 M_{Rk} \geq 1.35 M_{Gsk} + 1.35 M_{Gak} + 1.5 I M_{Qk} \tag{13}$$

Using the values of the load effects as presented in Table 4, the moment capacity as presented in Table 8 and impact factor as presented in Table 3 the checking equations for the original bridge verifies that the safety of the mid-span section of the bridge in its original condition is satisfied with a high margin $fM_R = 4441 \geq \sum g_i Q_i = 2992 \quad [kNm]$. Performing the same calculation for the deteriorated bridge and taking into account that the ultimate capacity of the mid-span section is reduced down to 2742 kNm, the safety check is not satisfied since

$fM_R = 2358 < \sum g_i Q_i = 2992 \quad [kNm]$.

Thus, due to damage, the bending capacity of the sections becomes lower than the applied design moments and the bridge should be declared as unsafe. The difference between the required and available member capacity is quite significant on the order of 25%.

4.5.2 Latin Hypercube Sampling (LHS) method

To perform the risk analysis using the LHS method, a set of sample values is generated for each of the 9 random variables that control the bridge strength listed in Table 3 (i.e. without the live loads and impact). A non-linear structural analysis is performed for each combination of random variables for a total of 100 simulations. The loads applied correspond to the mean live load augmented by the mean impact factor. The strength capacity is thus expressed by the load factor by which the original mean load should be multiplied to cause the failure of the system. The means and standard deviations of the strength capacities of the original bridge system and the deteriorated bridge system are calculated and the histograms from the simulation's results are approximated by Normal distributions.

The reliability index ß for both the original and deteriorated states are calculated using the normal model defined by Equation (4), where the resistance R is modelled by the load factor by which the applied mean loads should be multiplied to cause the failure of the system. Thus, the mean value of the applied loads S in this case takes unit value. The standard deviation of S is considered to be equal to 0.14. This value is the effect of the multiplication of the railway load with a COV equal to 10% by the impact factor with a COV equal to 50%. Table 9 summarizes the results of the reliability index ß for both condition states.

4.5.3 Response Surface Method (RSM)

At first, a deterministic analysis is performed considering the mean values of all the structure-related independent random variables (9 variables), then 18 analyses are performed taking one of the 9 variables at its mean plus or minus 10% while the remaining variables are kept at their mean values. The results of the 19 analyses are subsequently fitted in a linear polynomial function using a regression analysis. A first estimate of the reliability index ß is calculated using FORM for the limit state function defined as the response function obtained in the previous step minus the live load, Q. The FORM algorithm also gives the coordinates of the design point for the calculated reliability index. The next step follows the same process, however, instead of the central (mean) values of the parameters, the coordinates of the design point obtained are used to define the linear regression fit. The process is repeated until convergence while the perturbation of the values is progressively reduced from 10% to 2.5%. During the iterative process special care is taken to ensure that in each iteration the design point remains within a reasonable range from the previously estimated value to avoid that the algorithm converges to the less important failure modes.

Equations (14) and (15) show the limit state functions obtained following the iterative procedure for the original and deteriorated condition states respectively.

$$\begin{aligned} g(X) = & -3.9718007335 + 0.0000052326 \cdot f_c + 1.5205547862 \cdot h_g + 0.1484513209 \cdot h_s + \\ & + 0.0000092929 \cdot f_y + 0.7040553589 \cdot A_{Sb} + 0.9844101575 \cdot A_{St} + \\ & - 0.0101214575 \cdot G_S - 0.0092131489 \cdot G_{Ab} - 0.0098242475 \cdot G_{At} - Q \end{aligned} \tag{14}$$

$$\begin{aligned} g(X) = & -3.4776301070 + 0.0000057766 \cdot f_c + 1.1547095388 \cdot h_g - 1.3725490196 \cdot h_s + \\ & + 0.0000147243 \cdot f_y + 0.6819640565 \cdot A_{Sb} + 1.2471336042 \cdot A_{St} + \\ & - 0.0050335570 \cdot G_S - 0.0101619433 \cdot G_{Ab} - 0.0100000000 \cdot G_{At} - Q \end{aligned} \tag{15}$$

where the variables are defined a shown in Table 3.

The reliability indices calculated by FORM for the ultimate limit state functions for the intact and deteriorated bridge defined by equations (14) and (15) are ß=7.37 and ß=6.17 respectively. The reliability indices were calculated assuming all the parameters have the values defined in Table 3. However, due to the fact that during the simulations the railway loads were applied as their mean value including impact and later incremented (by multiplying the mean load by the load factor) to reach the structure failure, the mean value of Q is considered as unity. Furthermore, as already explained, the standard deviation of the Q was considered to be equal to 0.14. Similarly, the variability in the areas of the different reinforcement layers was considered to be fully dependent on the two random variables A_{sb} and A_{st} (for bottom and top reinforcement respectively). Thus, the variability of the reinforcement areas was accounted for by multiplying the characteristic area of each layer by a random variable with a mean value equal to unity and a COV of 2%.

4.5.4 Method of Ghosn and Moses

The analyses necessary to obtain the load factors for functionality LF_f, ultimate LF_u and damaged condition LF_d limit states (see Table 6) were performed according to the methodology presented in section 3.2. After the determination of the load factors LF_i, the parameters necessary for the reliability analysis (bias factor and COV) are determined by comparing the results of the analysis performed at the mean values of the input parameters and the results when the input parameters are taken at their nominal or characteristic values. Thus, the nominal and mean load factors for first member failure are obtained from equation (6) considering that: M_R, is the section resistance, *R*; the sum of M_{Gs} and M_{Ga} is the dead load effect, *D*; and M_{Qk}, is the effect of design train load, L_{TRAIN}. Considering the moment capacity of the section as defined in Table 8 and considering the bending moments due to the dead loads and railway traffic load as defined in Table 4, the following values are obtained for the original bridge: $\mathrm{LF}_1 = 3.92$, $\overline{LF_1} = 4.45$. The nominal and mean load factors for the deteriorated bridge are found to be $LF_1 = 1.84$ and $\overline{LF_1} = 2.12$. Thus, the bias values for the original bridge and deteriorated bridge obtained from Equation (10) are respectively $\lambda_{LF} = 1.135$ and $\lambda_{LF} = 1.152$.

The COV of the member capacity is obtained from Equations (9) and (7). In the analysed example M_{Qk} is the effect of design train load, L_{TRAIN}, and $\sqrt{s_R^2 + s_D^2}$ takes the form $\sqrt{s_{MR}^2 + s_{MGs}^2 + s_{MGa}^2}$. Considering the values listed in Table 8, Table 4 and Table 3, the COV for the load capacity of the original bridge is obtained as $V_{LF} = 0.112$ and for the deteriorated bridge as $V_{LF} = 0.125$.

The member reliability index, b_{member}, is calculated from Equation (5). The calculations are performed considering the mean value of the member capacity and its COV as defined above, the mean value of the maximum expected lifetime live load as the product of the impact factor (see Table 3) and the live load bias factor (the factor equal to 0.82 relating characteristic value of the railway traffic load effects to the mean value of the railway traffic load effects as presented in Table 4). The COV for the live load with impact is calculated to be equal to 0.14. The calculated values for b_{member} for the original and deteriorated bridge respectively are presented in Table 10.

The calculations of the system reliability index for the functionality, ultimate, and damaged condition limit state (b_{func}, b_{ult} and b_{damage}) are performed according to Equations (8a), (8b) and (8c). The system reliability index for the functionality limit state is defined as the allowable deformation equal to span length/500. The calculated values of b_{func}, b_{ult} and b_{damage} for the original and deteriorated bridge respectively are listed in Table 10. It is clearly observed that all the system reliability indices are higher than the target values defined in Casas *et al.* (2007). Thus, the structure can be considered to be safe.

To check the level of inherent redundancy of the bridge, the relative reliability indices of equations (3a, 3b and 3c) for the original bridge are calculated to be: $\Delta b_{ult} = 0.79$; $\Delta b_{func} = -0.01$; $\Delta b_{damage} = -3.24$ and for the deteriorated bridge: $\Delta b_{ult} = 2.05$, $\Delta b_{func} = 1.55$; $\Delta b_{damage} = -1.79$. Comparing the relative reliability indices with the target values defined in Table 1 it is concluded, that the bridge in its original condition is not sufficiently redundant. Nevertheless, the bridge is still considered to be safe due to the fact that the member safety is high. For the deteriorated condition the redundancy is already sufficiently high allowing us to consider the bridge safe even though the member safety is violated.

Comparing the values of the reliability indices for the ultimate limit state calculated above (ß=7.40 and ß=5.69 for the original bridge and deteriorated bridge respectively) with the values of the reliability indices obtained with the LHS method (ß=9.65 and ß=6.48 for the original bridge and deteriorated bridge) or those of the RSM (ß=7.37 and ß=6.17) it can be concluded that the presented method is sufficiently accurate for practical applications.

4.5.5 Method of Sobrino and Casas

According to the procedure presented in section 3.3 the limit state function g(X) is defined according to equation (11) and (12) considering that the i-th section in the analysed example is a middle span section (sect. 2) of the first span of the bridge. Furthermore, in the analysed example M^e_G is the sum of the self weight and additional dead load effects M^e_{Gs} and M^e_{Ga} defined in Table 5. However, M^e_{IQ} is the product of the static live load moment M^e_Q , defined in Table 5, and the impact factor I.

The reliability analysis is executed using FORM. The distribution types and statistical parameters of the impact factor I are provided in Table 3. The mean value of M^e_{Gs}, M^e_{Ga} and M^e_Q are presented in Table 5. The distribution types and COV's of M^e_{Gs}, M^e_{Ga} and M^e_Q are considered equal to those of the corresponding loads (see Table 3). The statistical definition of M^2_R is provided in Table 8. The mean values of M^1_{nla}, M^3_{nla} and M^2_{nla} are presented in Table 7. Their distribution types are assumed to be normal and the COV's are taken equal to the COV's of the moment capacity of the corresponding sections presented in Table 8.

For the defined limit state function, the reliability indices obtained using FORM are found to be ß=6.61 and ß=4.67 for original bridge and deteriorated bridge respectively. The FORM analyses were performed assuming statistical independence between all the variables. When M^2_{nla} and M^2_R are assumed to be correlated, the reliability index increases to ß=7.16 and ß=9.21 for the correlation coefficients C=0.5 and C=0.99 respectively when analysing the original bridge. When analysing the deteriorated bridge, the reliability index increases to the values ß=5.28 and ß=6.84 for the correlation coefficients C=0.5 and C=0.99 respectively. The effect of the statistical correlation between these two variables was studied due to the significant likelihood of their mutual dependency since they essentially represent the moments at the same section.

Due to the fact that the calculated values of the reliability index ß are higher than the target values defined in Casas *et al.* (2007), the structure can be rated as safe. Comparing the calculated reliability indices with the values obtained from the LHS method (ß=9.65 and ß=6.48 for the original bridge and deteriorated bridge) or from the

RSM (ß=7.37 and ß=6.17) it can be concluded that this method is also sufficiently accurate for practical purposes.

4.6 *Analysis of results*

The safety assessment of this bridge performed using the simplified methods of Ghosn and Moses and Sobrino and Casas as well as the LHS and RSM advanced probabilistic non-linear analyses shows that the Brunna Bridge is sufficiently safe (see Table 11). This is found to be true for both the original (as constructed) and the deteriorated conditions. The latter assumes that 50% of the mid-span reinforcement has corroded which would have meant that the bridge would have failed the standard safety check using code-specified partial safety factors and linear elastic analysis.

The results in Table 11 show that the reliability index values obtained from the RSM and LHS are somewhat different. This lack of conformity can be explained by the fact, that the LHS is not sufficiently accurate for such high reliability levels. In the case of LHS, the reliability index is calculated based on the results of sampling performed in the region relatively close to the mean values of all the variables. However, for high reliability levels such as those observed in this analysis, the design point (failure region) is located far away from the region of mean values and the sampling performed there may not describe the failure region appropriately.

In the case of the RSM used in this study, sampling is performed iteratively close to the design point (failure region). Due to the iterative procedure, the polynomial function determined by the regression analysis gives a fairly accurate representation of the tangent to the failure surface near the most likely failure point and the reliability index obtained by means of this method is more exact. It is noted however, that the analysis performed using the RSM was designed to concentrate on the ultimate moment capacity and ignored other numerically possible failure modes. In the case of the deteriorated bridge, the reliability level is significantly lower than that of the original bridge and the reliability indexes obtained by the LHS and RSM for the deteriorated bridge are closer to each other than those of the original intact bridge.

The reliability index values calculated using the simplified method of Ghosn and Moses are generally lower than those obtained from the more advanced methods (except for the results obtained by RSM for the original bridge where the results are very close). This is because the simplified method of Ghosn and Moses implicitly assumes full correlation between all the members' strengths and that the COV's for the system limit states are equal to the COV of the most critical member. On the other hand, although the fully probabilistic non-linear analysis approaches (RSM and LHS) lead to some level of correlation between the member strengths since the basic parameters that control each member's strength are the same (see Table 3), the various sizes, shapes and reinforcing details of each member would lead to slightly lower correlations in the member strengths, leading to slightly higher overall system reliability levels. Furthermore, the simplified reliability analysis of Ghosn and Moses assumes that the overall COV is the same as that of the most critical member. This assumption would lead to a higher overall COV than that obtained from the full probabilistic non-linear system analysis. The justification for using the higher COV in the Ghosn and Moses method is that the uncertainties in the modelling of the non-linear system effects must be at least as high as those for the modelling of the individual member effects. It is well known that the non-linear finite element analysis of structural systems is not exactly accurate due to difficulties in modelling the material behaviour at high loads as well as the variations in the bridge boundary conditions and secondary member effects. Thus, the method of Ghosn and Moses is generally more conservative than the more exact simulation methods when the latter do not explicitly consider the modelling uncertainties associated with the reliability analysis of the structural system.

In the case of the Sobrino and Casas method, the reliability index calculated assuming no correlation between the bending moment at failure in the mid-span section M^2_{nla} and moment capacity of the mid-span section M^2_R, is significantly lower than that obtained from the more advanced methods. When assuming partial correlation between these two parameters the reliability index obtained is closer to the exact values. It is herein recommended to include, a partial correlation between the bending moment at failure in the mid-span section M^2_{nla} and the moment capacity of the mid-span section M^2_R since the response of the same section at ultimate and the response very close to ultimate are expected to be correlated.

5 Conclusions

Two methodologies for the safety assessment of railway bridge systems are presented based on simplified probabilistic non-linear analysis procedures. The detailed safety analysis of a typical railway bridge demonstrate that despite the simplifying assumptions, the proposed methods are still sufficiently accurate when compared to the results of full probabilistic non-linear analysis procedures. The combination of accuracy and simplicity would make these proposed methods more likely to be used by engineering practitioners for the safety assessment of existing railway bridges. Furthermore, the analysis of an example bridge shows how a bridge that would have been rated as deficient using traditional member safety analysis methods may in actuality have sufficiently high reliability levels to eliminate the need for its replacement or rehabilitation.

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Table 1: Target values of relative reliability indices.

Bridge part	Relative reliability indices - target values		
	? $ß_{ult}$	? $ß_{func}$	? $ß_{damage}$
Superstructure	+0.85	+0.25	-2.70
Substructure	+0.50	+0.50	-2.00

Table 2: Target reliability indices related to 1 year reference period and ultimate limit states (JCSS 2001b).

Relative cost of safety measure	Consequences of failure		
	Minor	Moderate	Large
Large (A)	3.1	3.3	3.7
Normal (B)	3.7	4.2	4.4
Small (C)	4.2	4.4	4.7

Table 3: Random variables considered in the analysis

Random variable	Symbol	Unit	Char. value	Mean value	COV	PDF
Concrete compressive strength	f_c	MPa	28.00	34.00	0.15	normal
Reinforcement yield strength	f_y	MPa	400.00	454.00	0.10	normal
Height of the girder	h_g	m	1.50	1.50	0.02	normal
Height of the slab	h_s	m	0.40	0.40	0.07	normal
Top Reinforcement area	A_{St}	m	nominal	nominal	0.02	normal
Bottom Reinforcement area	A_{Sb}	m	nominal	nominal	0.02	normal
Self weight of the structure	G_S	kN/m	47.53	47.53	0.08	normal
Additional dead loads (ballast)	G_{Ab}	kN/m	19.07	19.07	0.10	normal
Additional dead loads (track)	G_{At}	kN/m	2.00	2.00	0.10	normal
Railway traffic load (conc.)	Q_c	kN/m	78.13	64.69	0.10	normal
Railway traffic load (distr.)	Q_d	kN/m	40.00	31.70	0.10	normal
Impact factor	I	-	1.25	1.25	0.50	normal

Table 4: Bending moments in the critical sections of the first span (13.5 m).

Load	Symbol	Unit	Bending moment		
			Sect.1	Sect.2	Sect.3
Self weight of the structure	M_{Gsk}	kNm	-481.19	415.60	-853.10
Additional dead loads	M_{Gak}	kNm	-213.35	184.24	-378.25
Railway traffic load	M_{Qk}	kNm	0	1163.58	-705.72
Railway traffic load (mean)	M_{Qk}	kNm	0	955.89	-579.91

Table 5: Bending moments in the equivalent simple supported beam (13.5 m).

Load	Symbol	Unit	Bending moment – Sect. 2
Self weight of the structure (mean)	M^e_{Gs}	kNm	1082.75
Additional dead loads (mean)	M^e_{Ga}	kNm	480.04
Railway traffic load (mean)	M^e_Q	kNm	1245.85

Table 6: Load factors for functionality, ultimate and damaged condition limit state.

Condition state	Symbol	Unit	Load Factor		
			LF_f	LF_u	LF_d [a]
Original bridge	LF	-	3.93	5.80	1.66 (2.00)
Deteriorated bridge	LF	-	2.85	3.43	1.66 (1.26)

[a] The first value corresponds to the situation where a hinge is assumed in the mid-span section. The second value (in parenthesis) corresponds to the situation where a hinge is assumed over pier B.

Table 7: Bending moments in the critical sections of the first span (13.5 m).

Condition state	Symbol	Unit	Bending moment		
			Sect.1	Sect.2	Sect.3
Original bridge	M_{nla}	kNm	-639.76	5751.30	-8073.67
Deteriorated bridge	M_{nla}	kNm	-657.30	3010.65	-6321.26

Table 8: Probabilistic ultimate response of critical sections

Section	Symbol	Unit	Char. Value	Mean value	COV	PDF
Section over the pier A	M^1_R	kNm		2228	0.10	normal
Mid-span section (original)	M^2_R	kNm	5164	5772	0.10	normal
Mid-span section (deteriorated)	M^2_R	kNm	2742	3063	0.10	normal
Section over the pier B	M^3_R	kNm		8606	0.10	normal

Table 9: Calculation of the reliability index

Condition state	Resistance R		Action S		Safety margin R-S		Reliability index ß
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.	
Original	5.576	0.453	1	0.14	4.576	0.474	9.65
Deteriorated	3.286	0.324	1	0.14	2.286	0.353	6.48

Table 10: Reliability indices in Ghosn and Moses method.

Condition state	Reliability indices			
	$ß_{member}$	$ß_{func}$	$ß_{ult}$	$ß_{damage}$
Original bridge	6.61	6.60	7.40	3.37
Deteriorated bridge	3.64	5.19	5.69	1.85

Table 11: Results of the assessment of the Brunna Bridge

Safety format		Result of the safety assessment	
		Original bridge	Deteriorated bridge
Member	Partial safety factor method	safe	unsafe
	Mean Load Method	ß = 6.61	ß = 3.64
System	Method of Sobrino and Casas	ß=6.61 (7.16;9.21)[a]	ß=4.67 (5.28;6.84)[a]
	Method of Ghosn and Moses	ß = 7.40	ß = 5.69
	Latin Hypercube method (LHS)	ß = 9.65	ß = 6.48
	Response Surface method (RSM)	ß = 7.37	ß = 6.17

[a] Values in brackets were obtained considering correlations between M^2_{nla} and u M^2_R C=0.5 and C=0.99 respectively.

Figure 1. Load factor versus deflection curves obtained due to non-linear analysis for original structure and for the structure with some hypothetical damage.

Figure 2. Bending moments in the equivalent simple supported beam obtained due to linear analysis.

Figure 3. Bending moments at the failure state obtained due to non-linear analysis.

Figure 4. Brunna Bridge – Cross Section.

Figure 5. Brunna Bridge – Outline of the longitudinal reinforcement (values in parenthesis are vertical location of reinforcement measured from the bottom of the girder).

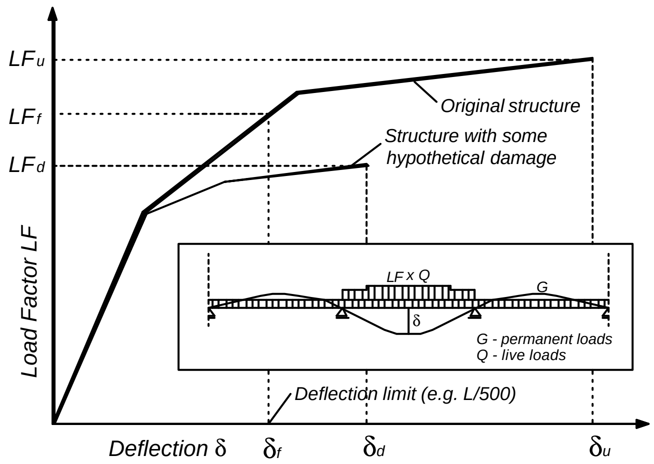

Figure 1. Load factor versus deflection curves obtained due to non-linear analysis for original structure and for the structure with some hypothetical damage.

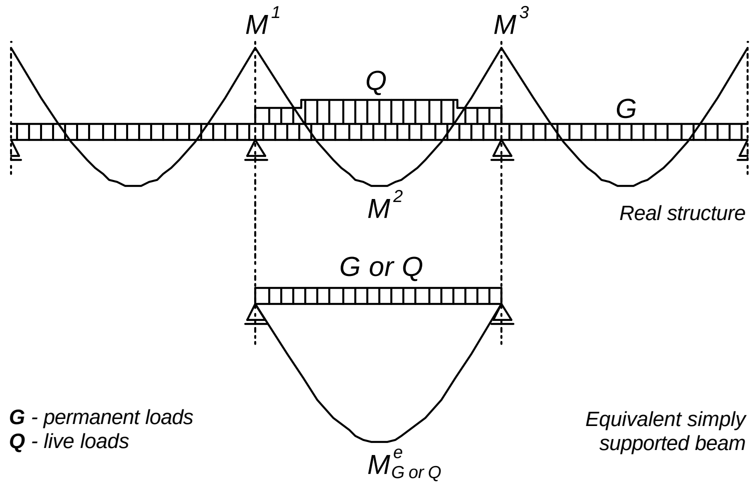

Figure 2. Bending moments in the equivalent simple supported beam obtained due to linear analysis.

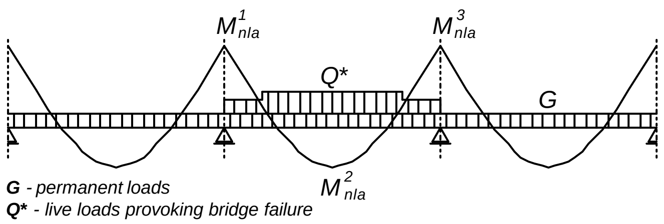

Figure 3. Bending moments at the failure state obtained due to non-linear analysis.

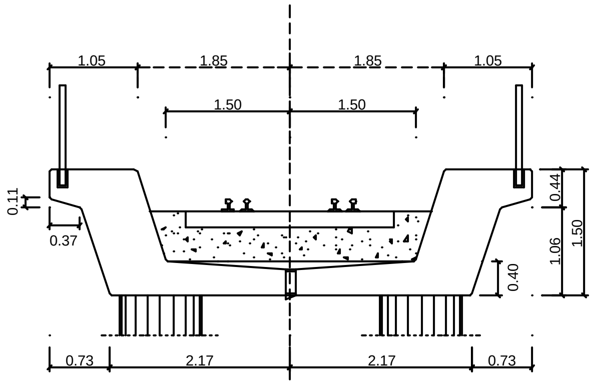

Figure 4. Brunna Bridge – Cross Section.

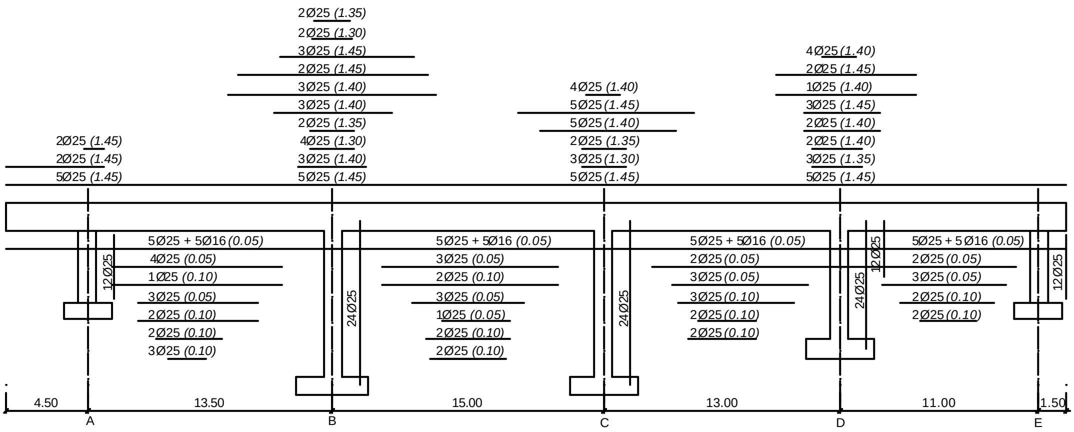

Figure 5. Brunna Bridge – Outline of the longitudinal reinforcement (values in parenthesis are vertical location of reinforcement measured from the bottom of the girder).