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SIMPLIFIED SETTLING VELOCITY FORMULA FOR SEDIMENT PARTICLE

By Nian-Sheng Cheng[1]

ABSTRACT: A new and simplified formula for predicting the settling velocity of natural sediment particles is developed. The formula proposes an explicit relationship between the particle Reynolds number and a dimensionless particle parameter. It is applicable to a wide range of Reynolds numbers from the Stokes flow to the turbulent regime. The proposed formula has the highest degree of prediction accuracy when compared with other published formulas. It also agrees well with the widely used diagrams and tables proposed by the U.S. Inter-Agency Committee in 1957.

INTRODUCTION

A prerequisite to certain quantitative analysis in sediment transport is a knowledge of the settling velocity of sediment particles. Many attempts have been made for its prediction but most of the relevant researches apply only to spherical particles. Basically, there are two types of prediction methods for settling velocity of either spherical or nonspherical particles. One is the analytical solution of Stokes that is applicable only for particle Reynolds number, $R = wd/v \leq 1$, where w = settling velocity of a particle; d = particle diameter; and v = kinematic viscosity. The other includes tabulated data and diagrams consisting of families of curves based on the experimental data, e.g. Schiller and Naumann (1933) and U.S. Inter-Agency Committee (1957). They are suited to a wide range of Reynolds numbers but inconvenient to use in practice.

The objective of this note is to derive a simple expression for the determination of the settling velocity of natural sediment particles over a wide range R.

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DRAG COEFFICIENT FOR SETTLING OF INDIVIDUAL PARTICLE

In 1851, Stokes obtained the solution for the drag resistance of flow past a sphere by expressing the simplified Navier-Stokes equation together with the continuity equation in polar coordinates. Using his solution, the following expression for the settling velocity of spherical particles can be derived as

$$w = \frac{1}{18}\frac{\Delta g d^2}{\nu} \tag{1}$$

where, $\Delta = (\rho_s - \rho)/\rho$, ρ = density of fluid; p_s = density of particles; g = gravitational acceleration. Unfortunately, (1) is only valid for R $\leq$ 1.

Generally, by equating the effective weight force to the Newtonian expression of drag resistance, i.e.

$$(\rho_s - \rho)g\frac{\pi}{6}d^3 = C_D\frac{\pi}{4}d^2\frac{\rho w^2}{2} \tag{2}$$

the drag coefficient C_D can be expressed as

$$C_D = \frac{4}{3}\frac{\Delta g d}{w^2} \tag{3}$$

By substituting the Stokes' solution in (1) into (3), C_D may be related to the Reynolds number

$$C_D = \frac{A}{\mathrm{R}} \tag{4}$$

where A = a constant, which is dependent on the shape factor of the particle. For Stokes' solution, A = 24 for spherical particles. The effect of particle shape on the drag coefficient varies, being small at low R and more appreciable at high R (Schulz et al. 1954). Usually the shape factor of sediment particle is less than unity and for natural sand particles, the shape factor is about 0.7. Table 1 shows the value of A to be about 32 based on the work of various investigators.

Under the condition of high Reynolds numbers, say, $10^3 \sim 10^5$, the drag coefficient of spheres has an average value of about 0.4. For natural sediment particles, C_D lies between 1.0 ~ 1.2 as shown in Table 1.

As the Stokes-type equation is restricted to R ≤ 1, efforts have been made to develop a method for extending (4) to a much wider range of Reynolds numbers. Some quasi-theoretical formulas or empirical correlations for evaluating the settling velocity of individual particle can be found in the literature, e.g. Oseen (1927), Sha (1956), Zanke (1977), and Raudkivi (1990).

In light of all these studies, the following relation between C_D and R is assumed for natural sediment perticles

$$C_D = \left[\left(\frac{A}{\mathrm{R}} \right)^{(1/n)} + B^{(1/n)} \right]^n \tag{5}$$

where A and B = constants; and n = an exponent. Eq. (5) automatically satisfies the two extreme conditions at low and high Reynolds numbers, that is, C_D is inversely proportional to R at low Reynolds numbers and becomes a constant at high Reynolds numbers. According to Table 1, A can be taken as 32, as most researchers did, and B = 1, being the lowest limit of the drag coefficient for sediment particles.

As the relationship between C_D and R at the extreme Reynolds numbers is unaffected appreciably by n in (5), the latter may be estimated by fitting (5) with the experimental data in the intermediate Reynolds number range, i.e., 1 < R < 1,000. Based on the experimental data of Concharov (Ibad-zade 1992), Zegzhda (1934), Arkhangel' skii (1935), and Sarkisyan (1958) for quartz sand particles, the average n-value was found to be 1.5. Therefore, with the foregoing values proposed for A and B, (5) can be rewritten as

$$C_D = \left[\left(\frac{32}{\mathrm{R}} \right)^{(1/1.5)} + 1 \right]^{1.5} \tag{6}$$

Eq. (6) is a general relationship between the drag coefficient and particle Reynolds number for natural sediment particles.

Using a dimensionless particle parameter d_* defined as

$$d_* = \left(\frac{\Delta g}{\nu^2} \right)^{(1/3)} d \tag{7}$$

together with (3), we have

$$C_D = \frac{4}{3} \frac{d_*^3}{\mathrm{R}^2} \tag{8}$$

Substituting (8) into (6) yields

$$\frac{wd}{\nu} = (\sqrt{25 + 1.2d_*^2} - 5)^{1.5} \tag{9}$$

Eq. (9) can be used to evaluate the settling velocity of natural sand particles explicitly.

COMPARISON WITH PREVIOUS STUDIES

There are numerous settling velocity formulas developed by different investigators for spherical and nonspherical particles. The settling velocity formulas for sediment particles used for comparison in this note are as follows.

From Sha (1954)

$$w = \frac{1}{24}\frac{\Delta g d^2}{\nu} \quad \text{for } d < 0.01 \text{ cm} \tag{10a}$$

$$w = 1.14\sqrt{\Delta g d} \quad \text{for } d > 0.2 \text{ cm} \tag{10b}$$

$$\left(\log \frac{R}{d_*} + 3.790\right)^2 + \log d_* - 5.777)^2 = 39$$

$$\text{for } d = 0.01 \sim 0.2 \text{ cm} \tag{10c}$$

From Concharov (Ibad-zade 1992)

$$w = \frac{1}{24}\frac{\Delta g d^2}{\nu} \quad \text{for } d < 0.015 \text{ cm} \tag{11a}$$

$$w = 1.068\sqrt{\Delta g d} \quad \text{for } d > 0.15 \text{ cm} \tag{11b}$$

$$w = 67.6\Delta d + 0.52\Delta\left(\frac{T}{26} - 1\right) \quad \text{for } d = 0.015 \sim 0.15 \text{ cm} \tag{11c}$$

In (11*c*), the temperature T is in ℃, d in cm and w in cm/s.

From Zhang (1989)

$$w = \sqrt{\left(13.95\frac{\nu}{d}\right)^2 + 1.09\Delta g d} - 13.95\frac{\nu}{d} \tag{12}$$

From Van Rijn (1989)

$$w = \frac{1}{18}\frac{\Delta g d^2}{\nu} \quad \text{for } d < 0.01 \text{ cm} \tag{13a}$$

$$w = 1.1\sqrt{\Delta g d} \quad \text{for } d > 0.1 \text{ cm} \tag{13b}$$

$$w = 10\frac{\nu}{d}\left(\sqrt{1 + 0.01 d_*^3} - 1\right) \text{ (Zanke 1977)}$$

$$\text{for } d = 0.01 \sim 0.1 \text{ cm} \tag{13c}$$

From Zhu and Cheng (1993)

$$\frac{wd}{\nu} = \frac{-24\cos^3\alpha + \sqrt{576\cos^6\alpha + (18\cos^3\alpha + 3.6\sin^2\alpha)d_*^3}}{9\cos^3\alpha + 1.8\sin^2\alpha} \tag{14}$$

where $\alpha = 0$ for $d_* \leq 1$ and $\alpha = \pi/[2 + 2.5(\log d_*)^{-3}]$ for $d_* > 1$.

To test the accuracy of (9) and the others from (10) to (14), three data sets for sand particles were used. The first is the relations and table in Raudkivi (1990), on the average settling velocity of quartz sand particles in water at 20 ℃. Table 2 shows the 13 sets data points that are computed using the method outlined in Raudkivi (1990). The second is the experimental data of Zegzhda (1934), Arkhangel'skii (1935), and Sarkisyan (1958), which were compiled in the order of decreasing particle diameter by Zhu and Cheng (1993), as shown in Table 3. The last is the tabulated data given by the U.S. Inter-Agency Committee (1957) (also see: Raudkivi, 1990) for settling velocity of natural sediment particles with a shape factor of 0.7, and specific gravity ranging from 2.0 - 4.3. The basic parameter used for the determination of accuracy of a formula is the average value of the relative error defined as

$$\text{error} = \frac{|\text{calculated} - \text{given}|}{\text{given}} \times 100 \tag{15}$$

Table 2 presents the comparison of the calculated settling velocity using (9) together with those using the other five methods, with the average values reproduced from Raudkivi (1990). It can be seen that (9) has the smallest relative error when compared with the other formulas. The comparison given in Table 3 is between the various computed results and the experimental data of Zegzhda (1934), Arkhangel' skii (1935) and Sarkisyan (1958). It shows that the average relative error of (9) is 6.1%, which is very close to the 5.8% achieved by Zhu and Cheng's (1993) formula and the degree of accuracy is better than all the other formulas. The present formula is also simpler to use than that proposed earlier by Zhu and Cheng (1993). Fig. 1 displays the relationship of R and d_* derived from (9) and it can be seen that the computed data also agree very well with the tabulated ones given by the U.S. Inter-Agency Committee (1957).

CONCLUSIONS

An explicit and simple formula was developed for evaluating the settling velocity of individual natural sediment particles. The formula is applicable to the different regimes ranging from the Stokes flow to the high Reynolds number. Comparison with published experimental data shows that the proposed formula has a high degree of prediction accuracy.

ACKNOWLEDGMENTS

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APPENDIX I. REFERENCES

[1] Arkhangel' skii, B. V. (1935). "Experimental study of accuracy of hydraulic coarseness scale of particles." *Izv. NIIG*, 15, Moscow, Russia (in Russian).

[2] Ibad-zade, Y. A. (1992). *Movement of sediment in open channels.* S. P. Ghosh, translator, *Russian translations series,* Vol. 49, A. A. Balkema, Rotterdam, The Netherlands.

[3] Oseen, C. W. (1927). "Neuere methoden and ergebnisse in der hydro-dynamik." Akademische Verlagsgesellschaft, Leipzig, Germany.

[4] Raudkivi, A. J. (1990). *Loose boundary hydraulics*, 3rd Ed., Pergamon Press, Inc., Tarrytown, N.Y

[5] Sarkisyan, A. A. (1958). "Deposition of sediment in a turbulent stream." *Izd, AN SSSR*, Moscow, Russia (in Russian).

[6] Schiller, L., and Naumann, A. (1933). "Uber die grundlegenden Berechnungen bei der Schwekraftaubereitung." *Zeitschrift des Vereines Deutscher Ingenieure*, 77(12), 318-320.

[7] Schulz, S. F., Wilde, R. H., and Albertson, M. L. (1954). "Influence of shape on the fall velocity of sedimentary particles." *M.R.D. Sediment Ser., No. 5*, Missouri River Div., U.S. Army Corps of Engrs.

[8] Sha, Y. Q. (1956). "Basic principles of sediment transport." *J. Sediment Res.*, Beijing, China, 1(2), 1-54 (in Chinese).

[9] U.S. Inter-Agency Committee. (1957). "Some fundamentals of particle size analysis. A study of methods used in measurement and analysis of sediment loads in streams." *Rep. No. 12,* Sub-com. on Sedimentation, U.S. Inter-Agency Com. on Water Resour., St. Anthony Falls Hydr. Lab., Minneapolis, Minn.

[10] Van Rijn, L. C. (1989). "Handbook: sediment transport by currents and waves." *Rep. H461,* Delft Hydraulics, Delft, The Netherlands.

[11] Zanke, U. (1977). "Berechnung der Sinkgeschwindigkeiten von Sedimenten." Mitt. Des Franzius-Instituts fur Wasserbau, Technical Univ., Hannover, Germany.

[12] Zegzhda, A. P. (1934). "Settlement of sand gravel particles in still water." *Izv. NIIG*, No. 12, Moscow, Russia (in Russian).

[13] Zhang, R. J. (1989). *Sediment dynamics in rivers.* Water Resources Press (in Chinese).

[14] Zhu, L. J., and Cheng, N. S. (1993). "Settlement of sediment particles." *Res. Rep.*, Dept. of River and Harbor Engrg., Nanjing Hydr. Res. Inst., Nanjing, China (in Chinese).

APPENDIX II. NOTATION

The following symbols are used in this paper:

A, B = constants;

C_D = drag coefficient;

d = diameter of a particle;

d_* = dimensionless particle parameter;

g = gravitational acceleration;

n = exponent;

R = wd/v = particle Reynolds number;

T = temperature;

w = settling velocity of a particle;

α = parameter;

$\Delta = (\rho_s - \rho)/\rho$;

v = kinematic viscosity of fluid;

ρ = density of fluid; and

ρ_s = density of sediment particles.

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Author (1)	C_D (low R) (2)	C_D (high R) (3)
Sha (1956)	32/R	1.0
Concharov (1962)	32/R	1.2
Zhang (1989)	34/R	1.2
Van Rijn (1989)	24/R	1.1
Raudkivi (1990)	32/R	1.2
Zhu and Cheng (1993)	32/R	1.2

Table 1

Number (1)	*d* (cm) (2)	*T* (°C) (3)	Given settling velocity (cm/s) (4)	R (5)	Calculated Settling Velocity (cm/s) Present study[a] (6)	Sha (1956)[b] (7)	Concharov (1962)[c] (8)	Zhang (1989)[d] (9)	Van Rijn (1989)[e] (10)	Zhu et al. (1993)[f] (11)
1	0.001	20	0.00663	6.583E-4	0.00667	0.00669	0.00669	0.00627	0.00892	0.00669
2	0.0025	20	0.0414	1.029E-2	0.0414	0.0418	0.0418	0.0392	0.0557	0.0417
3	0.005	20	0.166	8.229E-2	0.162	0.167	0.167	0.156	0.223	0.165
4	0.0075	20	0.373	0.2777	0.353	0.376	0.376	0.350	0.502	0.359
5	0.01	20	0.663	0.6583	0.602	0.619	0.669	0.614	0.773	0.606
6	0.025	20	2.86	7.099	2.65	2.42	2.59	3.08	3.50	2.56
7	0.05	20	6.40	31.77	6.05	5.65	5.38	6.99	7.20	6.05
8	0.075	20	9.38	69.85	8.82	8.77	8.17	9.78	9.75	9.12
9	0.1	20	12.10	120.1	11.12	11.73	10.96	11.95	11.75	11.64
10	0.25	20	21.66	537.6	20.68	22.92	21.47	20.44	22.12	20.89
11	0.5	20	31.06	1542.0	30.87	32.41	30.37	29.41	31.28	30.01
12	0.75	20	38.35	2856.0	38.51	39.70	37.19	36.17	38.31	36.81
13	1	20	44.54	4422.0	44.87	45.84	42.95	41.84	44.23	42.51

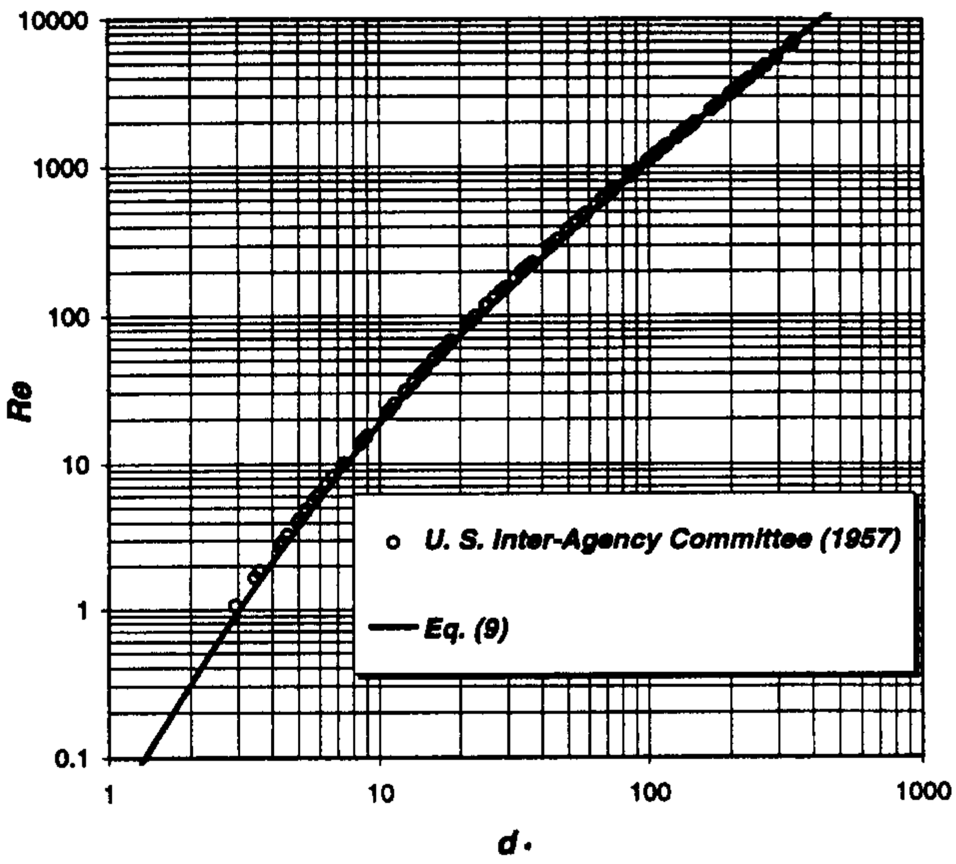
Note: The average relative errors for calculated settling velocity are as follows: [a]3.9; [b]4.9; [c]4.8; [d]5.8; [e]15.4; and [f]4.0.

Table 2

Number (1)	*d* (cm) (2)	*T* (°C) (3)	Measured settling velocity (cm/s) (4)	R (5)	Calculated Settling Velocity (cm/s) Present study[a] (6)	Sha (1956)[b] (7)	Concharov (1962)[c] (8)	Zhang (1989)[d] (9)	Van Rijn (1989)[e] (10)	Zhu et al. (1993)[f] (11)
1	0.0001	15	0.000057	4.994E-7	0.000059	0.000059	0.000059	0.000055	0.000079	0.000059
2	0.0005	15	0.00141	6.177E-5	0.00147	0.00148	0.00148	0.00138	0.00197	0.00148
3	0.001	15	0.00565	4.950E-4	0.00588	0.00590	0.00590	0.00554	0.00787	0.00590
4	0.002	15	0.0223	3.901E-3	0.0235	0.0236	0.0236	0.0221	0.0315	0.0236
5	0.005	15	0.141	6.170E-2	0.144	0.148	0.148	0.138	0.197	0.146
6	0.0061	15	0.235	0.1256	0.212	0.220	0.220	0.205	0.293	0.215
7	0.00687	15	0.292	0.1758	0.266	0.279	0.279	0.260	0.371	0.270
8	0.00804	15	0.340	0.2395	0.359	0.382	0.382	0.355	0.509	0.364
9	0.00936	15	0.406	0.3330	0.477	0.517	0.517	0.478	0.690	0.483
10	0.01	15	0.515	0.4512	0.539	0.559	0.590	0.544	0.688	0.545
11	0.0123	15	0.775	0.8352	0.784	0.792	0.893	0.812	1.02	0.785
12	0.015	15	1.14	1.496	1.11	1.08	1.31	1.18	1.45	1.09
13	0.02	15	1.70	2.979	1.76	1.66	1.87	1.97	2.35	1.72
14	0.0225	15	1.73	3.414	2.11	1.96	2.15	2.40	2.81	2.04
15	0.025	15	2.29	5.016	2.46	2.26	2.43	2.83	3.26	2.38
16	0.0275	15	2.99	7.204	2.81	2.57	2.70	3.27	3.70	2.71
17	0.03	15	3.21	8.438	3.16	2.88	2.98	3.70	4.13	3.05
18	0.0325	15	3.08	8.771	3.51	3.19	3.26	4.12	4.54	3.39
19	0.0375	15	4.18	13.73	4.19	3.81	3.82	4.93	5.32	4.07
20	0.04	15	4.61	16.16	4.52	4.12	4.10	5.31	5.68	4.41
21	0.05	15	5.67	24.84	5.79	5.36	5.21	6.73	6.99	5.74
22	0.05	15	6.20	27.16	5.79	5.36	5.21	6.73	6.99	5.74
23	0.0556	18	8.46	44.47	6.62	6.24	5.94	7.60	7.77	6.66
24	0.0556	7.5	7.82	30.99	6.00	5.53	5.59	6.99	7.29	5.93
25	0.06	15	7.28	38.27	6.96	6.58	6.33	7.97	8.13	7.02
26	0.07	15	8.04	49.31	8.04	7.78	7.44	9.06	9.13	8.22
27	0.075	15	8.58	56.38	8.55	8.38	8.00	9.57	9.60	8.80
28	0.0775	15	10.09	68.52	8.80	8.67	8.28	9.81	9.82	9.07
29	0.085	8.2	10.85	67.09	9.11	8.92	8.89	10.19	10.22	9.37
30	0.1	15	11.15	97.69	10.87	11.26	10.79	11.78	11.63	11.35
31	0.1	15	12.17	106.6	10.87	11.26	13.58	11.78	11.63	11.35
32	0.15	12	15.00	182.1	14.51	16.24	16.63	15.15	17.13	15.16
33	0.15	10	15.62	179.4	14.40	15.98	16.63	15.09	17.13	15.06
34	0.2	15	19.25	337.3	17.79	20.50	19.21	18.00	19.78	18.31
35	0.25	18	20.70	489.3	20.60	22.92	21.47	20.41	22.12	20.86
36	0.25	15	21.60	473.1	20.48	22.92	21.47	20.36	22.12	20.80
37	0.25	12	20.80	420.8	20.35	22.92	21.47	20.31	22.12	20.74
38	0.25	10	20.10	383.8	20.26	22.92	21.47	20.28	22.12	20.69
39	0.3	15	23.00	604.6	22.88	25.11	23.52	22.47	24.23	22.98
40	0.35	18.5	25.00	837.6	25.19	27.12	25.41	24.42	26.17	24.98
41	0.35	15	25.38	778.3	25.07	27.12	25.41	24.39	26.17	24.94
42	0.35	7.5	25.90	646.1	24.76	27.12	25.41	24.28	26.17	24.84
43	0.45	8.2	28.10	919.8	28.71	30.75	28.81	27.74	29.67	28.36

Note: Measured settling velocity from Zegzhda (1934), Arkhangel'skii (1935), and Sarkisyan (1958). The average relative errors for calculated settling velocity are as follows: [a]6.1; [b]8.9; [c]9.3; [d]8.7; [e]21.7; [f]5.8.

Table 3



10000
1000
100
10
1
0.1
Re
1
10
100
1000
d*
U. S. Inter-Agency Committee (1957)
Eq. (9)

Fig.1