Simulated annealing optimization of walls, portal and box reinforced concrete road structures

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Abstract

This paper deals with the economic optimization of reinforced concrete walls, portal and box frame structures typically used in road construction. It shows the efficiency of heuristic optimization by the simulated annealing algorithm. The evaluation of solutions follows the Spanish Code for structural concrete. Stress resultants and envelopes of framed structures are computed by an external finite element program. Design loads are in accordance with the national IAP Code for road bridges. The algorithm is first applied to RC retaining walls with 26 continuous design variables of geometry, materials and reinforcement. Results on this topic show the importance of limiting the deflection of walls. No restriction leads to slender solutions with deflections of up to 1/40 the height of the wall. Such elements are unfeasible and, hence, a limitation of 1/150 of the height is adopted for the design of these structures. The second structure analysed is a 10 m horizontal span RC portal frame. This example has 28 discrete variables, 5 geometrical, 3 types of concrete and 20 types of reinforcement bars of fixed length. The evaluation module includes the limit states that are commonly checked in design: flexure, shear, deflections, etc. Results of this research are again quite slender, i.e. a slab of 0.375 m (1/26.67 slab/span ratio), not complying with the rarely checked fatigue of concrete. The last type of structure analysed is a 13 m horizontal span RC box road frame. This example has 44 discrete variables, 2 geometrical, 2 types of concrete and 40 reinforcement bars and bar lengths. The evaluation module includes fatigue plus other limit states. Results are now reasonably slender, i.e. a slab of 0.65 m (1/20 slab/span ratio). Finally, run times indicate that heuristic optimization is a forthcoming option for the design of real RC structures.
Keywords: economic optimization, heuristics, concrete structures, structural design.

1 Introduction

Present design of economic concrete structures is much conditioned by the experience of structural engineers. Most procedures are based on the adoption of cross-section dimensions and material grades based on sanctioned common practice. Once the structure is defined, it follows the analysis of stress resultants and the computation of passive and active reinforcement that satisfy the limit states prescribed by concrete codes. Should the dimensions or material grades be insufficient, the structure is redefined on a trial and error basis. Such process leads to safe designs, but the economy of the concrete structures is, therefore, very much linked to the experience of the structural designer.

The methods of structural optimization may be classified into two broad groups: exact methods and heuristic methods. The exact methods are the traditional approach. They are based on the calculation of optimal solutions following iterative techniques of linear programming [1,2]. The second main group are the heuristic methods, whose recent development is linked to the evolution of artificial intelligence procedures. This group includes a broad number of search algorithms [3-6], such as genetic algorithms, simulated annealing, threshold accepting, tabu search, ant colonies, etc. These methods have been successful in areas different to structural engineering [7]. They consist of simple algorithms, but require a great computational effort, since they include a large number of iterations in which the objective function is evaluated and the structural restrictions are checked. Among the first works of heuristic optimization applied to structures, the contributions of Jenkins [8-9] and of Rajeev and Krishnamoorthy [10] in 1991-1992 are to be mentioned. Both authors applied genetic algorithms to the optimization of the weight of steel structures. As regards RC structures, early applications in 1997 include the work of Coello et al [11], who applied genetic algorithms to the economic optimization of RC beams. Recently, there have been a number of RC applications [12-15], which optimize RC beams and building frames by genetic algorithms.

The structures which are the object of this work are walls, portal frames and box frames which are usually built of RC in road construction. RC earth retaining walls are generally designed with a thickness at the base of 1/10 of the height of the wall and a footing width of 0.50-0.70 of the height of the wall. Box and portal frames are used with spans between 3.00 and 20.00 m for solving the intersection of transverse hydraulic or traffic courses with the main upper road. Box frames are preferred when there is a low bearing strength terrain or when there is a risk of scour due to flooding. The depth of the top and bottom slab is typically designed between 1/10 to 1/15 of the horizontal free span; and the depth of the walls is typically designed between 1/12 of the vertical free span and the depth of the slabs. Frames are calculated to sustain the traffic and earth loads prescribed by the codes and have to satisfy all of the limit states required as an RC structure. The method followed in this work has consisted first in the development of evaluation computer modules where dimensions, materials and steel reinforcement have been taken as variables. These modules compute the cost of a solution and check all the relevant limit states. Simulated annealing is then used to search the solution space.

2 Simulated annealing optimization procedure

The problem of structural concrete optimization that is put forward in the present work consists of an economic optimization. It deals with the minimization of the objective function F of expression (1), satisfying also the restrictions of expressions (2).

$$F(x_1, x_2, \ldots x_n) = \sum_{i=1,r} p_i * m_i(x_1, x_2, \ldots, x_n) \tag{1}$$

$$g_j(x_1, x_2, \ldots\ldots x_n) \leq 0 \tag{2}$$

Note that the objective function in expression (1) is the sum of unit prizes multiplied by the measurements of construction units (concrete, steel, formwork, etc). And that the restrictions in expression (2) are all the service and ultimate limit states that the structure has to satisfy. Unit prizes considered are given in Table 1.

Table 1: Basic prizes of the cost function of the reported structures.

Unit	Cost (€)
kg of steel (B-500S)	0.583
m^2 of lower slab formwork	18.030
m^2 of wall formwork	18.631
m^2 of upper slab formwork	30.652
m^3 of scaffolding	6.010
m^3 of lower slab concrete (labour)	5.409
m^3 of wall concrete (labour)	9.015
m^3 of upper slab concrete (labour)	7.212
m^3 of concrete pump rent	6.010
m^3 of concrete HA-25	48.244
m^3 of concrete HA-30	49.379
m^3 of concrete HA-35	53.899
m^3 of concrete HA-40	58.995
m^3 of concrete HA-45	63.803
m^3 of concrete HA-50	68.612
m^3 of earth removal	3.005
m^3 of earth fill-in	4.808

The search method used in this work is the simulated annealing (SA henceforth), that was originally proposed by Kirkpatrick et al. [16] for the design of electronic circuits. The SA algorithm is based on the analogy of crystal formation from masses melted at high temperature and let cool slowly. At high temperatures, configurations of greater energy than previous ones may randomly form, but, as the mass cools, the probability of higher energy configurations forming decreases. The process is governed by Boltzmann expression $exp(-\Delta E/T)$, where ΔE is the increment of energy of the new configuration and T is the temperature. The algorithm starts with a feasible solution randomly generated and a high initial temperature. The initial working solution is changed by a small random move of the values of the variables. The new current solution is evaluated in terms of cost. Greater cost solutions are accepted when a 0 to 1 random number is smaller than the expression $exp(-\Delta E/T)$, where ΔE is the cost increment and T is the current temperature. The current solution is then checked against structural restrictions and if it is feasible, it is adopted as the new working solution. The initial temperature is decreased geometrically ($T=kT$) by means of a coefficient of cooling k. A number of iterations called Markov chains are allowed at each step of temperature. The algorithm stops when the temperature is a small percentage of the initial temperature (typically 1%). The SA method is capable of surpassing local optima at high-medium temperatures and gradually converges as the temperature reduces to zero. The SA method requires calibration of the initial temperature, the length of the Markov chains and the cooling coefficient. Adopted values for the three examples of this work will be given below. The initial temperature was adjusted following the method proposed by Medina [17], which consists in choosing an initial value and checking whether the percentage of acceptances of higher energy solutions is between 10-30 percent. If the percentage is greater than 30%, the initial temperature is halved; and if it is smaller than 10%, the initial temperature is doubled. Computer runs were performed 9 times so as to obtain minimum, mean and standard deviation of the random results.

3 Case study 1: earth retaining RC walls

The first example studied relates to earth retaining RC cantilever walls used in road construction [18,19]. Fig.1 shows the 26 variables considered in this analysis. They include 4 geometrical variables (the thickness of the stem and 3 dimensions for the footing), 4 concrete and steel grades (stem and footing) and 18 variables for the definition of steel reinforcement, which includes both areas of reinforcement and bar lengths. Variables are continuous except for material grades which are discrete. A total of 17 parameters are considered for the complete definition of the problem, the most relevant of which are the total height of the wall H (stem plus footing), the top slope of the fill and the acting top uniform distributed load, the internal friction angle of the fill φ, the permissible ground stress and the partial coefficients of safety. Structural restrictions considered followed a standard analysis by Calavera [20], that includes checks against sliding, overturning, ground stresses and service-ultimate limit states of flexural and shear of different cross-sections of the wall and the footing. A vertical inclination of φ degrees of the earth pressure was considered. Additionally, a restriction of deflection at the top of 1/150 of the height of the stem was also considered.

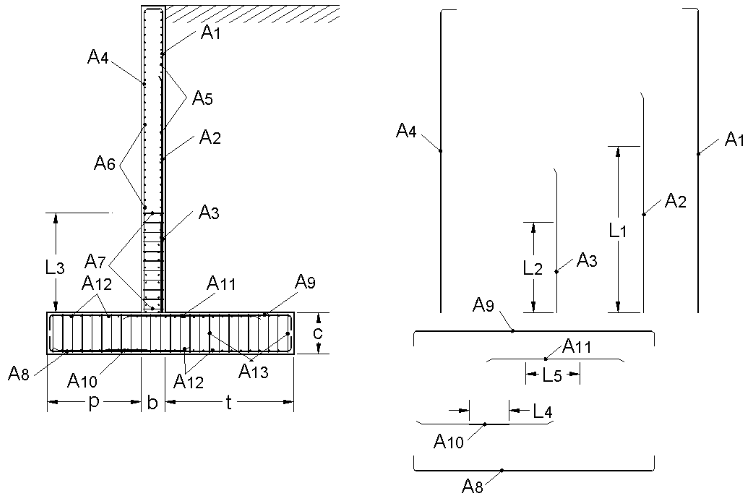

Figure 1: Variables of earth retaining walls for case study 1.

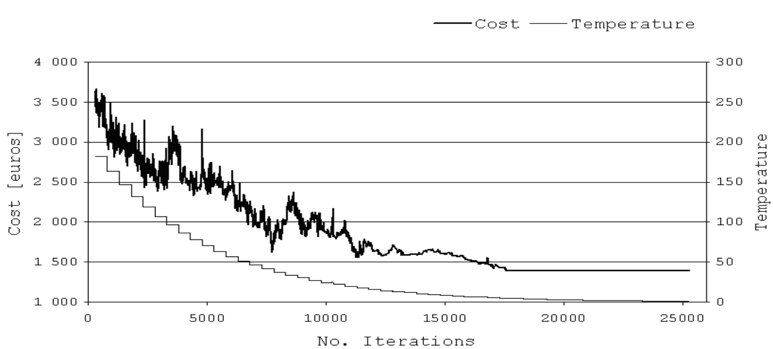

Figure 2: Typical cost evolution of SA algorithm.

The simulated annealing algorithm was programmed in Visual Basic 6.3 with an Excel input/output interface. Typical runs were 21 minutes in a Pentium IV of 2.41 GHz. The calibration of the SA recommended Markov chains of 1000 iterations and a cooling coefficient of 0.80. As regards the type of moves, the most efficient move found consisted of random variation of 16 of the 26 variables. Fig. 2 shows a typical cost evolution by the SA algorithm. Table 2 details the results of the SA analysis for a wall of 7.00 m of total height and a 0.30 MPa permissible stress of the ground (additional parameters are 5.80 m of top to bottom level difference; fill horizontal at the top with no surface load; specific weight of the fill 18 kN/m^3; internal friction angle of the fill of 30 degrees; ground friction resistance 0.577; partial safety coefficients for sliding, overturning and structural of 1.50, 1.80 and 1.50 respectively; partial coefficients of 1.50 and 1.15 for concrete and steel as materials). Table 2 shows results for this wall with and without a check on deflections, and it also lists the reference values taken from the collection of walls by Calavera [20]. Results indicate that the inclusion of a limit on deflections of 1/150 of the height of the stem is crucial, since otherwise the slenderness of the stem goes up to 1/24.27 and deflections are as high as 163 mm (1/40 of the height of the stem). Should the top deflection be limited to 1/150, the slenderness goes down to 1/10.59, which is quite similar to the standard 1/10 adopted in practice by many practitioners. In terms of cost, the SA results improve by 6.06% the cost of the reference wall and it amounts a total cost of 750.42 euros/m.

Table 2: Summary of best wall for case study 1 (total height 7.00 m).

Variable	Permissible stress = 0.3 N/mm^2		
	Reference	Deflections unrestricted	Deflections limited
b	0.25-0.70 m	0.265 m	0.607 m
p	0.75 m	0.833 m	0.770 m
t	1.70 m	1.248 m	0.900 m
c	0.70 m	0.568 m	0.605 m
$f_{ck,ste}$	25	35	30
$f_{ck,foo}$	25	25	25
$f_{yk,ste}$	500	500	500
$f_{yk,foo}$	500	500	500
A_1	7.70 cm^2	6.946 cm^2	11.442 cm^2
A_2	7.70 cm^2	29.602 cm^2	1.431 cm^2
A_3	0	26.730 cm^2	10.332 cm^2
A_4	4.35 cm^2	1.000 cm^2	1.149 cm^2
A_5	3.74 cm^2	3.400 cm^2	6.552 cm^2
A_6	7.73 cm^2	5.661 cm^2	13.120 cm^2
A_7	0	0	0
A_8	13.40 cm^2	16.837 cm^2	16.958 cm^2
A_9	10.05 cm^2	1.000 cm^2	17.013 cm^2
A_{10}	0	19.549 cm^2	1.000 cm^2
A_{11}	0	1.447 cm^2	1.000 cm^2
A_{12}	3.74-1.67 cm^2 (low-up)	3.955 cm^2	8.776 cm^2
A_{13}	0	0	0
L_1	2.18 m	2.954 m	0.849 m
L_2	0	0.852 m	0.834 m
L_3	0	0	0
L_4	0	1.248 m	0.745 m
L_5	0	0.568 m	0.689 m

4 Case study 2: road portal RC frames

The second example studied relates to portal RC frames used in road construction [21,22]. Fig. 3 shows the 28 variables considered in this analysis. Variables include 5 geometrical values: the depth of the walls, the depth of the top slab and the footing, plus 2 dimensions for the size of the base of the footing; 3 different grades of concrete for the 3 types of elements; and 20 types of reinforcement bars following a standard setup. All variables are discrete in this analysis. The total number of parameters is 16, the most important of which are the horizontal free span, the vertical free span, the earth cover, the permissible bearing stress and the partial coefficients of safety. Structural restrictions considered followed standard provisions for Spanish design of this type of structure [23,24], that include checks of the service and ultimate limit states of flexure and shear for the stress envelopes due to the traffic loads and the earth fill. Traffic loads considered are a uniform distributed load of 4 kN/m^2 and a heavy vehicle of 600 kN. Stress resultants and reactions were calculated by an external finite element program using a 2-D mesh with 30 bars and 31 sections (out of plane bending moments had to be assumed as a practical one fifth proportion of in plane bending moments). Deflections were limited to 1/250 of the free span for the quasi-permanent combination. Fatigue of concrete and steel was not considered since this ultimate limit state is rarely checked in road structures.

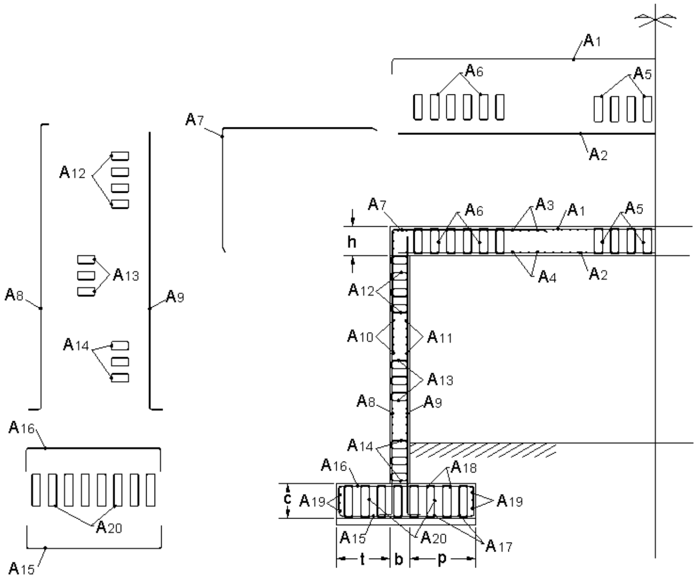

Figure 3: Variables of the RC portal frame for case study 2.

The SA algorithm was programmed in Visual Basic 6.3. Typical runs were 10.76 hours in an AMD Athlon processor of 1.49 GHz. In this case, the calibration recommended Markov chains of 375 iterations and a cooling coefficient of 0.70, the total amount of iterations being about 7500. The most efficient move found consisted of random variation of 4 of the 28 variables of the problem. Table 3 details the main results of the SA analysis for a portal frame of 10.00 m of horizontal free span, 6.00 m of vertical free span and 0.10 m of asphalt cover (additional parameters are 0.25 MPa permissible bearing stress, specific weight of the fill of 20 kN/m^3, 30 degrees internal friction angle of the fill and partial safety coefficients of 1.60 for loading and 1.50-1.15 for concrete-steel as materials). The depth of the top slab is only 0.375 m for the 10.00 m span, which means a very slender span/depth ratio of 26.67. The cost of this solution is 2619 euros/m. This best solution was then checked by hand calculations against fatigue of structural concrete. The loading considered was a 468 kN heavy vehicle prescribed for fatigue by the Spanish loading code for bridges [24]. It was found that the solution did not comply with Eurocode 2 limitations for fatigue [25]. Hence, it was concluded that this rarely checked ULS should be included in future works of optimization dealing with road structures.

Table 3: Summary of best portal frame for case study 2.

Geometric variables	
h	0.375 m
b	0.400 m
c	0.400 m
p	0.950 m
t	0.750 m
Concrete grades	
upper slab	HA-25
wall	HA-25
foundation	HA-25
Reinforcement	
A_1	15ø12/m
A_2	10ø20/m
A_6	12.06 cm^2/m
A_7	15ø12/m
A_8	8ø16/m
A_9	12ø8/m
A_{15}	10ø16/m
A_{16}	12ø10/m
A_{20}	9.05 cm^2/m

5 Case study 3: road box RC frames

The last example studied relates to box RC frames used in road construction [26,27]. Fig. 4 shows the 44 variables considered in this analysis. Variables include 2 geometrical values: the depth of the walls and slabs; 2 different grades of concrete for the 2 types of elements; and 40 types of reinforcement bars and bar lengths following a standard setup. All variables are again discrete in this analysis. The most important parameters are the horizontal free span, the vertical free span, the earth cover, the ballast coefficient of the bearing and the partial coefficients of safety. Structural restrictions considered followed standard provisions similar to those of portal frames. However, this time the ULS of fatigue was included following the conclusions for case study 2. Stress resultants and reactions were calculated by an external finite element program using a 2-D mesh with 40 bars and 40 sections.

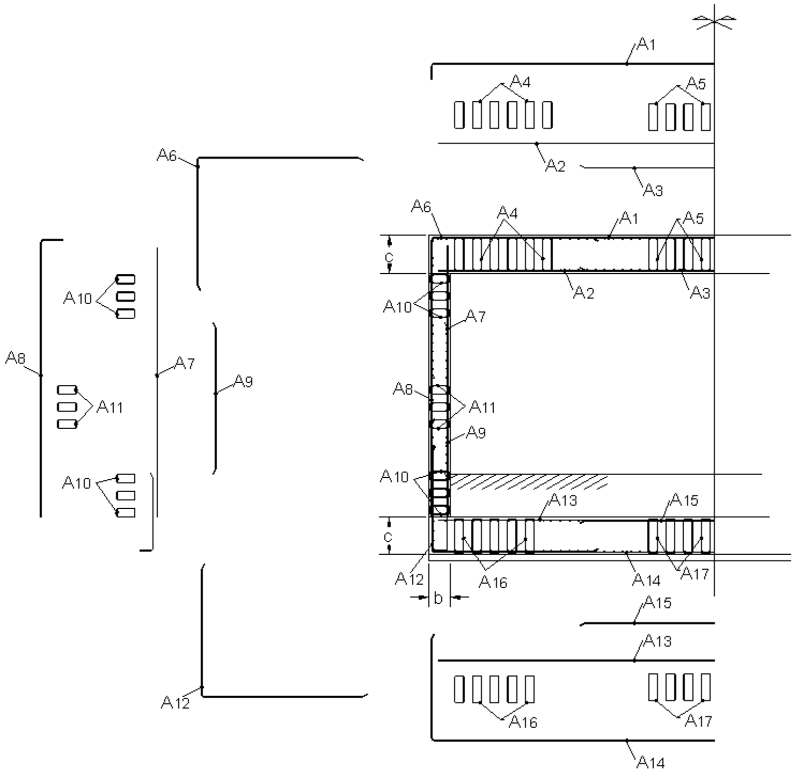

Figure 4: Variables of the RC box frame for case study 3.

The SA algorithm was programmed this time in Compaq Visual Fortran Professional 6.6.0. Typical runs reduced to 47 minutes in a Pentium IV of 2.4 GHz. In this case, the calibration recommended Markov chains of 500 iterations and a cooling coefficient of 0.90. The most efficient move found was random variation of 9 variables of the 44 of the problem. Fig. 5 details the main results of the SA analysis for a box frame of 13.00 m of horizontal free span, 6.17 m of

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vertical free span and 1.50 m of earth cover (additional parameters are 10 MN/m^3 ballast coefficient, specific weight of the fill of 20 kN/m^3, 30 degrees internal friction angle of the fill and partial safety coefficients of 1.50 for loading and 1.50-1.15 for concrete-steel as materials). The cost of this solution is 4478 euros/m. The depth of the slabs is 0.65 m of C30 (30 MPa of characteristic strength), which represents a slender span/depth ratio of 20. And the depth of the wall is 0.50 m in C45, which represents a vertical span/depth ratio of 12.34. The overall ratio of reinforcement in the top slab is 160 kg/m^3**.** It may, hence, be concluded that results of the optimization search tend to slender and highly reinforced structural box frames. As regards deflections and fatigue limit states, their inclusion has shown to be crucial. Neglecting both limit states leads to a 7.9% more economical solution, but obviously unsafe. It is important to note that fatigue checks are usually considered in railways designs but, on the other hand, they are commonly neglected in road structures design and, as it has been shown, this may lead to unsafe designs.

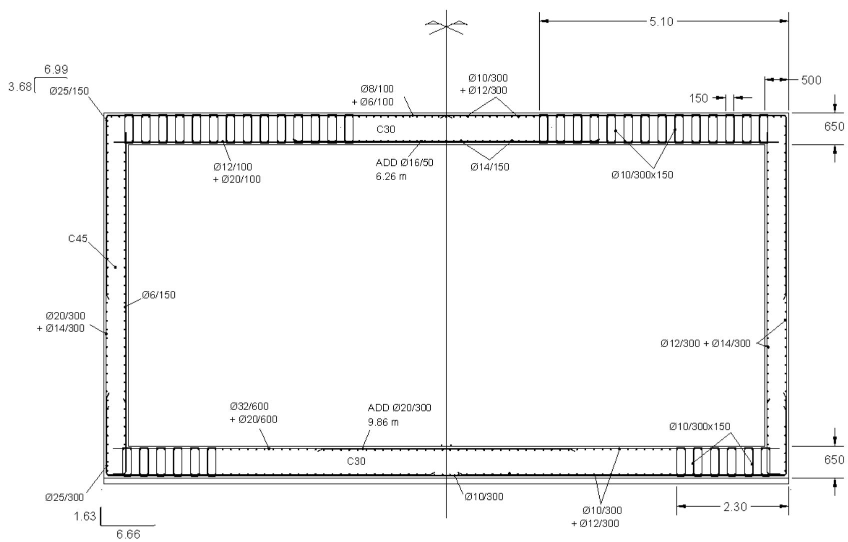

Figure 5: Optimized design of RC box frame for case study 3.

6 Conclusions

From the above work, the following conclusions may be derived:

- The study of earth retaining walls optimization shows that the inclusion of a limit of 1/150 on the deflection of the top of the walls is needed. Otherwise, results of the SA optimization are excessively deformable.
- Results of the optimization of portal road frames indicated the need of including the rarely checked ULS of fatigue in the list of structural restrictions for the optimization of road structures.

- The study of road box frames shows the importance of the inclusion of the SLS of deflections and the ULS of fatigue. The SA optimization of the 13 m free horizontal span box frame results in a slender and highly reinforced top slab.
- As regards the SA procedure, it has proved an efficient search algorithm for the 3 case studies of walls, portal and box frames used in road construction.

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