Simulation and Design of Microwave Class-C Amplifiers Through Harmonic Analysis

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Abstract—A method for analyzing microwave class-C amplifiers is proposed which satisfies the requirements of a wide application field, and, at the same time, operates with a fast running time and without convergence problems. It is based on the partitioning of the circuit into linear and nonlinear subnetworks for which, respectively, frequency-domain and time-domain equations are written. Then, taking into account that the time-domain and frequency-domain representations are related by the Fourier series, the circuit behavior is described by means of a system of nonlinear equations whose unknowns are the harmonic components of the incident waves at all the connections. To overcome the numerical problems arising in the search for the solution of this system when strong nonlinearities are involved, a special step-by-step procedure is adopted. The problem is transformed into the search for the solution of a sequence of well-conditioned systems of equations corresponding to a sequence of well-chosen circuits obtained from the original one through progressive changes of the input signal starting from 0 up to the nominal value. The program which implements the method is also described and the results of the analysis relative to a class-C amplifier are compared with measured values.

I. Introduction

FOR MANY nonlinear circuits operating in periodic steady state, such as class-C amplifiers, frequency multipliers, converters, etc., it is interesting to know the waveforms of the voltages and currents at some components in the circuit in order to obtain useful information for the design. The existing general-purpose analysis programs, which perform the analysis throughout the transient interval, require great computing effort and are therefore of no practical use when they must be called as subroutines by optimization programs for CAD. To overcome this drawback two methods have been recently proposed. The first—the "shooting" method [1], [2]—consists of searching, by an optimization process, for those values of the state variables $x(t_0)$ to be taken as starting values at the time t_0 in the integration process, such that after a complete period T the condition $x(t_0+T)=x(t_0)$ is satisfied. The second—the "harmonic balance" method [3]—on the other hand is based on the search for the harmonic components of the voltages and currents at the terminals connecting the linear and nonlinear subnetworks into which the complete circuit is partitioned. This search is made by means of an optimization process whose objective function is the sum of the squares of the differences between the sampled values of voltages or currents at the terminals of the two subnetworks.

Manuscript received April 30, 1979; revised October 12,1979. This work was supported by ESA under Contract 3334.NL/AK of ESTEC Mathematical Analysis Division.

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Although attempts have been made to improve their efficiency, both these methods require considerable computing effort owing to convergence difficulties and the high number of variables in the optimization process. The harmonic balance technique also has the drawback of requiring the explicit form for the equations describing nonlinear components or subnetworks. This property, at the same time, requires a minute partitioning of the circuit and a somewhat difficult choice of the variables to be considered as unknowns.

In this paper a new method is proposed which satisfies the requirements of a wide application field, and, at the same time, operates with a fast running time and without convergence problems. It is based on the partitioning of the circuit into linear and nonlinear subnetworks for which, respectively, frequency-domain and time-domain equations are written. Then, taking into account that the linear and nonlinear subnetworks are interconnected and that the time-domain and frequency-domain representations are related by the Fourier series, the circuit behavior can be described by means of a system of nonlinear equations whose unknowns are the harmonic components of the incident waves at all the connections. As described in Section II, the periodic steady-state analysis of the nonlinear amplifier is thus reduced to the search for the solution of this system of equations from which all the other electrical variables and network functions can easily be determined.

A particular advantage of this method is the possibility of describing the component nonlinearities also with equations in implicit form and the linear subnetwork in terms of wave variables. This feature, since the scattering matrix can exist whatever the composition of the circuit, enables one to avoid the possibly difficult and time-consuming choice of the variables to be assumed as unknowns.

As far as numerical problems are concerned, moreover, the method operates with a special step-by-step procedure which provides fast and reliable convergence. As described in Section III, this iterative procedure transforms the ill-conditioned nonlinear problem, which is difficult to solve, into a series of problems—more numerous but well-conditioned and immediately solvable. In the same section the program structure is also outlined and a description is given of the algorithms which have been implemented to adjust the steps of the iterative procedure and to find the optimum value for the order of harmonics necessary to adequately approximate the waveforms of the electrical variables.

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In Section IV the results of computations relative to a class-C amplifier which has been implemented on microstrip are compared with those obtained by measurements effected on the same amplifier.

II. Description of the Harmonic Analysis Method

In microwave class-C amplifiers the nonlinearities are confined to the active device while all the remaining elements, such as the case parasitics and the matching network components, are linear and constitute the larger part of the circuit. Thus the harmonic analysis applied to the determination of the periodic steady-state response of class-C amplifiers allows advantage to be taken of the easiness of the frequency-domain analysis of linear circuits. It is therefore clearly convenient to partition the circuit so as to obtain a linear subnetwork containing all the linear components, to which other subnetworks made up of nonlinear elements are connected (as shown in Fig. 1).

Although it is somewhat difficult to give general rules for the circuit partitioning which are valid whatever its composition, it is also easy to understand that the nonlinear subnetworks should be simple enough to allow an easy formulation of their equations. On the other hand, for the sake of efficiency, there should be a small enough number of subnetworks on which the number of variables depends.

As far as the nonlinear subnetworks are concerned, the voltages and the currents at the N connections are assumed as variables and must satisfy the system of N nonlinear equations:

$$f_j\left(\boldsymbol{v}, \frac{d\boldsymbol{v}}{dt}, \cdots \frac{d^s\boldsymbol{v}}{dt^s}; \boldsymbol{i}, \frac{d\boldsymbol{i}}{dt}, \cdots \frac{d^r\boldsymbol{i}}{dt^r}\right)=0, \quad \begin{array}{l}\text{for } j=1,2,\cdots,N \\ \forall t, 0 \leqslant t < T\end{array} \tag{1}$$

where $\boldsymbol{v}(t),\cdots,d^s\boldsymbol{v}/dt^s$, $\boldsymbol{i}(t),\cdots,d^r\boldsymbol{i}/dt^r$ are the column vectors of the voltages, currents, and their time derivatives up to the maximum order present in the equations, and T is the period of the circuit response. For the linear subcircuit, on the other hand, the harmonic components of the voltages and currents at the same N connections are assumed as variables and must satisfy the frequency-domain linear equations:

$$\boldsymbol{A}_k \boldsymbol{V}_k + \boldsymbol{B}_k \boldsymbol{I}_k = \boldsymbol{C}_k, \qquad k=0,1,\cdots,M \tag{2}$$

where M is the maximum order of harmonics necessary to adequately approximate the voltage and current waveforms, $\boldsymbol{V}_k, \boldsymbol{I}_k$ are the column vectors of the kth harmonics of the voltages and currents at the connections, $\boldsymbol{A}_k$ and $\boldsymbol{B}_k$ the matrices describing the linear circuit, and $\boldsymbol{C}_k$ is a column vector for taking the independent sources into account.

It follows that the λth elements of the vectors $\boldsymbol{v}(t)$, $\boldsymbol{i}(t)$ and of $\boldsymbol{V}_k$ and $\boldsymbol{I}_k$ which are, respectively, the time-domain and frequency-domain representation of the voltage and the current at the λth connection between the linear and

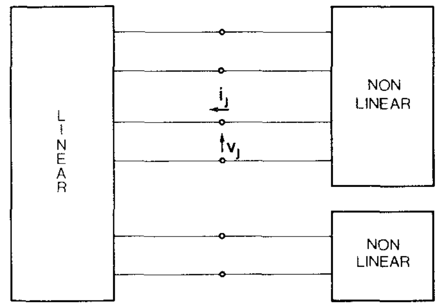

Fig. 1. Circuit partitioning into linear and nonlinear subnetworks linked together through N connections.

nonlinear subnetworks, are related by the Fourier series

$$v_\lambda(t) = \sum_{k=0}^{M} \mathcal{R}_e\left[V_{\lambda k} e^{jk(2\pi/T)t}\right]$$
$$i_\lambda(t) = \sum_{k=0}^{M} \mathcal{R}_e\left[I_{\lambda k} e^{jk(2\pi/T)t}\right] \tag{3}$$

while similar expressions hold for the time derivatives of the sth and rth orders:

$$\frac{d^s v_\lambda(t)}{dt^s} = \sum_{k=0}^{M} \mathcal{R}_e\left[V_{\lambda k}\left(jk\frac{2\pi}{T}\right)^s e^{jk(2\pi/T)t}\right]$$
$$\frac{d^r i_\lambda(t)}{dt^r} = \sum_{k=0}^{M} \mathcal{R}_e\left[I_{\lambda k}\left(jk\frac{2\pi}{T}\right)^r e^{jk(2\pi/T)t}\right], \quad \text{for } \lambda=1,\cdots,N. \tag{4}$$

Under these conditions the periodic steady-state analysis can be effected by searching for the values of the $2N(M+1)$ complex variables $V_{\lambda k}$ and $I_{\lambda k}$ (with $\lambda=1,2,\cdots,N$, and $K=0,1,\cdots,M$) that satisfy the system of equations (1)–(4) for each t in the interval 0 to T.

However, since the linear and nonlinear equations must be solved together, it is not convenient to deal with (1) and (2) at the same time, but preferable to use the linear equations (2) to express half the variables as linear functions of the remaining ones, so that the solution of the system (1), which is the hardest part of the problem, can be treated with half the number of variables. For example, if for the linear subnetwork the impedance matrix $\boldsymbol{Z}$ exists, (2) takes the form

$$\boldsymbol{V}_k = \boldsymbol{Z}_k \boldsymbol{I}_k + \boldsymbol{E}_k, \qquad \text{with } k=0,1,\cdots,M \tag{5}$$

and, using the Fourier series (3) and (4), (1) becomes

$$f_j[\boldsymbol{I}_0, \boldsymbol{I}_1, \cdots, \boldsymbol{I}_M; t]=0, \qquad \begin{array}{l}\text{with } j=1,2,\cdots,N \\ \forall t, 0 \leqslant t < T.\end{array} \tag{6}$$

In this way the analysis is transformed into the search for the $(M+1)$ unknown vectors $\boldsymbol{I}_k$, whose elements represent the kth harmonic components of the currents at the N connections, such that (6) is satisfied for any t in the interval 0 to T.

However, if the linear subnetwork does not admit the Z-matrix description, the method will still be valid, provided a different and convenient choice is made for the variables to be considered as unknowns. In fact, if the admittance matrix exists, the currents have to be computed (by means of the linear equation) in terms of the voltages, which must therefore be assumed as unknowns of the nonlinear problem. In general, for the linear subnetwork, it is necessary to find which kind of matrix representation exists that allows N variables (voltages and/or currents) to be computed in terms of the remaining ones. This search could be time-consuming and difficult and could also require different matrix representations for the various harmonics so that the choice of the independent variables could be quite complicated. In order to overcome this drawback, a transformation of variables could be effected. In fact, by considering the wave variables instead of voltages and currents, the response of the linear subcircuit at the kth harmonic would be described by the system

$$\boldsymbol{b}_k = \boldsymbol{S}_k \boldsymbol{a}_k + \boldsymbol{\gamma}_k \tag{7}$$

$\boldsymbol{S}_k$ being the scattering matrix at the kth harmonic, and $\boldsymbol{a}_k$, $\boldsymbol{b}_k$ $\boldsymbol{\gamma}_k$ the column vectors whose elements in the λth entry are, respectively, the incident, reflected, and impressed waves at the λth connection which are related to the kth harmonics of the voltages and currents at the same connection by

$$\begin{aligned} a_{\lambda k} &= \frac{1}{2}\left(\frac{V_{\lambda k}}{\sqrt{R_0}} + I_{\lambda k}\sqrt{R_0}\right) \\ b_{\lambda k} &= \frac{1}{2}\left(\frac{V_{\lambda k}}{\sqrt{R_0}} - I_{\lambda k}\sqrt{R_0}\right) \end{aligned} \tag{8}$$

or inversely by

$$\begin{aligned} V_{\lambda k} &= (a_{\lambda k} + b_{\lambda k})\sqrt{R_0} \\ I_{\lambda k} &= (a_{\lambda k} - b_{\lambda k})/\sqrt{R_0} \end{aligned} \tag{9}$$

R_0 being the normalization resistance. In this way the necessity for a suitable choice of the independent electrical variables is overcome, since the existence condition for the scattering matrix is, in practice, always satisfied.

Thus by taking (3), (4), (7), and (9) into account, the nonlinear system (1) can be expressed in the form

$$g_i(\boldsymbol{a}_0, \boldsymbol{a}_1, \cdots, \boldsymbol{a}_M; t) = g_i(\boldsymbol{a}, t) = 0 \quad \begin{array}{l} \text{with } i = 1, 2, \cdots N \\ \forall t, 0 < t < T. \end{array} \tag{10}$$

The solution of this system of equations gives the values of the harmonic components $\boldsymbol{a}_0, \boldsymbol{a}_1, \cdots \boldsymbol{a}_M$ of the incident waves at the N connections so that, by means of a simple frequency-domain analysis of the linear subnetwork, all the electrical variables and network functions of interest can be easily computed. Therefore, the number of real unknowns for the nonlinear problem is $N(2M+1)$, since for all the incident waves at the N connections the dc components and real and imaginary parts of the M harmonics must be found so that the *time-dependent* system (10) is identically satisfied for all the t's in the interval 0 to T.

However, since also the functions (10) are periodic with period T, they can be expressed by the Fourier series

$$g_i(\boldsymbol{a}, t) = \sum_{k=0}^{M_g} R_e\left[G_{ik}(\boldsymbol{a}) e^{jk(2\pi/T)t}\right] = 0, \quad \begin{array}{l} \text{for } i = 1, 2, \cdots N \\ \forall t, 0 \leqslant t < T \end{array} \tag{11}$$

whose harmonics $G_{ik}(\boldsymbol{a})$ can be computed by the fast Fourier transform algorithm, M_g being the maximum order of harmonics necessary to approximate functions (10). Letting $M_g = M$, i.e. supposing that each function $g_i(\boldsymbol{a}, t)$ can be approximated by a Fourier series with a number of harmonics equal to that considered for the wave variables at the connections between the linear and nonlinear subnetworks, the system of nonlinear time-dependent equations (10) can be transformed into the system

$$G_{ik}(\boldsymbol{a}_0, \boldsymbol{a}_1, \cdots, \boldsymbol{a}_M) = 0, \quad \begin{array}{l} \text{for } i = 1, 2, \cdots N \\ k = 0, 1, \cdots M \end{array} \tag{12}$$

which consists of $N(M+1)$ nonlinear time-independent equations with an equal number of unknowns, which are the $M+1$ harmonics of the incident waves at the N connections. The periodic steady-state analysis of the amplifier is thus reduced to the search for the unknowns in system (12), in terms of which all the electrical variables and network functions can be determined.

It is to be noted that the maximum order of harmonics M necessary to adequately approximate the waveforms is generally unknown and its value must be chosen taking into account that, if M is greater than necessary, the waveform approximation is certainly good but the number of equations and unknowns is too high and the computing time too long. If, on the other hand, M is smaller than its appropriate value, the solution of (12) is not acceptable or does not exist because the number of harmonics is not sufficient to represent the waveforms in the circuit. Therefore, as it is convenient to keep M as small as possible in order to have a small number of unknowns, and consequently less computing time, an algorithm is needed which, starting with a small tentative value M_0 for M, increases it until the solution of (12) also represents a good approximation of (10). More details on this algorithm are given in the next section.

III. Numerical Solution

The solution of the nonlinear equations (12) describing the circuit behavior can be numerically found using one of the well-known iterative techniques. Then, by ordering the unknown variables (the real and imaginary parts of the harmonics of the incident waves at the N connection) in the column vector $\boldsymbol{x}$ and the real and imaginary parts of all the complex functions G_{ik} in the column vector $\boldsymbol{\Phi}(\boldsymbol{x})$, system (12) becomes

$$\boldsymbol{\Phi}(\boldsymbol{x}) = \boldsymbol{0} \tag{13}$$

where $\boldsymbol{\Phi}(\boldsymbol{x})$ represents a set of real functions in the real variables $\boldsymbol{x}$. Thus by adopting the Newton–Raphson method, the solution is sought by linearizing the nonlinear system (13) so that at every kth iteration the linear system

$$\boldsymbol{\Phi}[\boldsymbol{x}^{(k+1)}] = \boldsymbol{\Phi}[\boldsymbol{x}^{(k)}] + \boldsymbol{J}^{(k)}[\boldsymbol{x}^{(k+1)} - \boldsymbol{x}^{(k)}] = \boldsymbol{0} \quad (14)$$

is obtained, where $\boldsymbol{x}^{(k)}$ is the vector of variable values at the kth iteration and $\boldsymbol{J}^{(k)}$ the Jacobian of $\boldsymbol{\Phi}$ with respect to $\boldsymbol{x}$. This iterative procedure permits the computation, at every step, through the solution of the linear system of (14), of the vector of the variables $x^{(k+1)}$ necessary for the $(k+1)$th step. The procedure ends when, for all variables, the difference between the values relative to two successive steps becomes less than a suitably fixed value.

This method is very efficient only if the starting estimated values $\boldsymbol{x}^{(0)}$ are not too far, in relation to function nonlinearity, from the solution. On the other hand, when the initial values are not well chosen, the method may be very slow and sometimes may not converge. This difficulty of convergence, however, does not depend on the method chosen since it arises, although in a different way, when the solution of the system (12) is sought for through the minimization of an "error function" defined as

$$E(\boldsymbol{x}) = \tilde{\boldsymbol{\Phi}}\boldsymbol{\Phi} \quad (15)$$

$\sim$ denoting transposition. In fact, in this case the minimization algorithm might converge to local minima which do not coincide with the true solution. Therefore, it must be taken into account that the possibility of obtaining the solution of the nonlinear system (13) is conditioned by the initial estimate of the unknown vector and that making a good estimate may in practice prove quite difficult, especially when there is high nonlinearity in the circuit.

This drawback can be overcome if a particular solution, corresponding to particular values of some parameters of the circuit, can be easily found and also if it is possible to define a strategy such that starting from the particular solution, the required solution can be obtained through successive gradual transformations of the circuit. Analytically the concept may be explained by making explicit in (13) its dependence on the circuit parameters such as, for example, the bias conditions, the impressed signals, and the values of some components. These parameters identifying a particular configuration may be ordered in a vector ξ that can be explicitly introduced in the system (13), which takes the form

$$\boldsymbol{\Phi} = \boldsymbol{\Phi}(\boldsymbol{x}, \xi) = \boldsymbol{0}. \quad (16)$$

Thus the circuit transformations described above are operated by changing ξ . In fact, by indicating with ξ^0 the vector of the parameters in the particular configuration to which the known solution $\boldsymbol{x}^0$ corresponds and with ξ^* the vector relative to the final configuration to be analyzed, the circuit transformation may be expressed in terms of a single parameter h, whose value may change from 0 to 1, by letting

$$\xi(h) = \xi^0 + h(\xi^* - \xi^0). \quad (17)$$

In these conditions, increasing with a suitable strategy the parameter h, it is possible to modify the circuit step-by-step in such a way that at every step the search for the solution relative to that circuit configuration, starting from the results of the analyses relative to the previous one, is easy. At every rth step the column vector $\boldsymbol{x}^{(r+1)}$ must be found corresponding to the parameter $h^{(r+1)}$ in such a way that

$$\boldsymbol{\Phi}[\boldsymbol{x}^{(r+1)}, \xi(h^{(r+1)})] = 0. \quad (18)$$

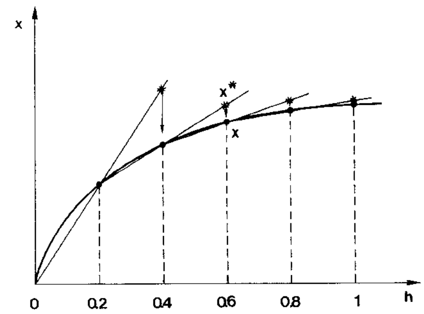

Fig. 2. Extrapolation procedure for estimating, at every step, the starting value x^* on the basis of the previous results (the linear case).

The increment $\Delta h^{(r)} = h^{(r+1)} - h^{(r)}$ must be chosen small enough to obtain a rapid convergence but as large as possible to reach the given circuit configuration, corresponding to $h = 1$, in a small number of steps.

In order to make the search for the solution of (18) efficient also with relatively high values of Δh it is convenient to make, at every rth step, an estimate $\boldsymbol{x}^{(r+1)}$ of the solution on the basis of the results $\boldsymbol{x}^{(r)}, \boldsymbol{x}^{(r-1)}, \cdots, \boldsymbol{x}^{(0)}$ of the analyses already effected through a preliminary extrapolation. In practice, it is convenient to limit the extrapolation to the last points in order to avoid the use of polynomials of too high order. It has been verified that it is sufficient to take only the last three values into account. In Fig. 2 the case of linear extrapolation is shown.

It is also necessary to define an algorithm which finds the most convenient value for Δh in order to satisfy the opposing demands specified above. The flowchart of a very simple algorithm which has been tested with good results is shown in Fig. 3. It operates by halving the value of Δh when the solution of the system (18) is not found and doubling it when in the last three steps the solution has been successfully found with the current value of Δh. Even though many other more sophisticated strategies can be defined to update Δh, it is not worth dwelling on this point here.

It should be noted that the method proposed, besides making the search for the solution more reliable and faster, at the same time gives results which are useful to completely characterize the circuit behavior. For example, in the analysis of a class-C amplifier the particular circuit configuration whose solution is clearly known corresponds to a zero input signal and, therefore, the vector ξ contains only one element: the input signal amplitude which must be increased with h, starting from zero, until the nominal value is reached for $h = 1$. The input signal is zero for $h = 0$ and the solution is known since the transistor is biased in the interdiction region, so that both the emitter and collector current harmonics are zero. Increasing the value of h means increasing the amplitude of the

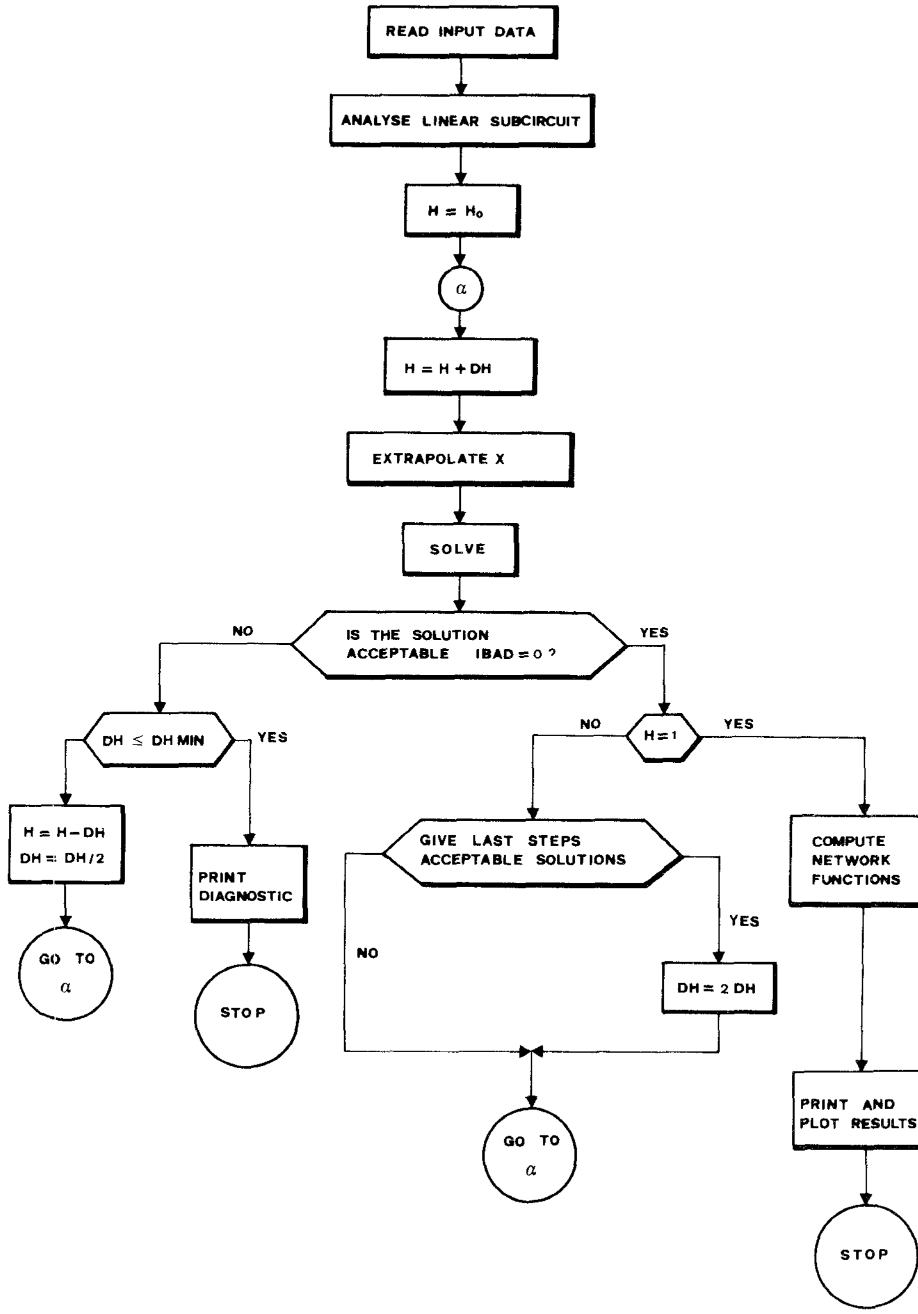

Fig. 3. Flowchart of the step-by-step algorithm for the nonlinear circuit analysis.

input signal which characterizes the various circuit configurations. In this case, therefore, the step-by-step technique proposed here not only gives the solution corresponding to the nominal value of the input signal, but supplies also, as an extra result, the circuit response for a set of input signal values lower than the nominal one.

Moreover, for better computing efficiency, it is also necessary to define the algorithm to find the minimum value for the number M of harmonics needed to adequately approximate the waveforms of the unknown electrical variables taking into account that, as has been stated in the previous paragraph, M must increase as the circuit nonlinearities increase and, therefore, with h, which, in the case of the class-C amplifier, fixes the input signal amplitude. This algorithm has been included in the routine SOLVE whose flowchart is shown in Fig. 4. At every step, after the solution corresponding to the current value of h and M has been found by the Newton–Raphson algorithm, the "accuracy indexes"

$$R_j=\sqrt{\frac{1}{T}\int_0^T g_j^2(\boldsymbol{a},t)\,dt}, \qquad \text{for } j=1,\cdots,N \quad (19)$$

which are the rms values of the residuals of the circuit equation (10), are computed and the tests

$$R_j<\epsilon, \qquad \text{for } j=1,\cdots,N \quad (20)$$

are effected with a suitable value for ϵ. The verification of these tests indicates that, for the current value of M, the solution of the system (18) represents an acceptable approximation to the solution of the circuit equation (10). If

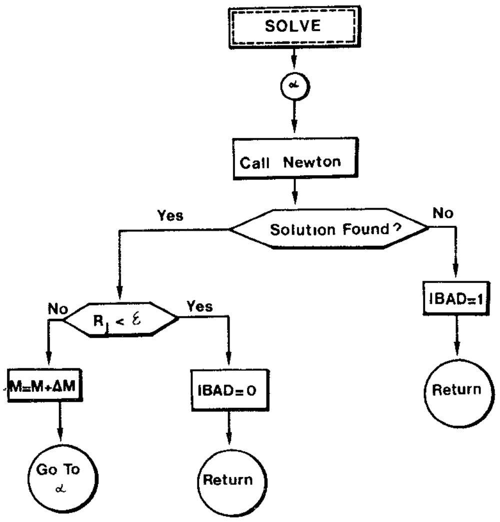

Fig. 4. Flowchart of the routine "SOLVE."

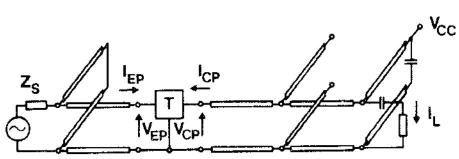

Fig. 5. The analyzed microwave class-C transistor amplifier.

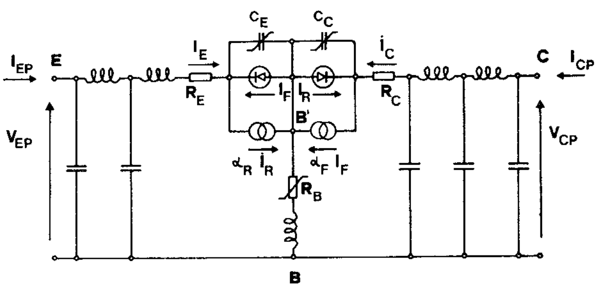

Fig. 6. The MSC 3000 transistor model.

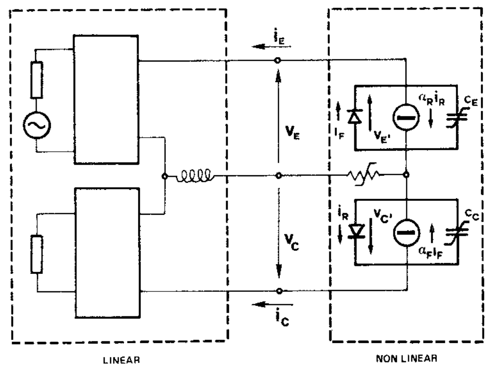

Fig. 7. Amplifier partitioning into linear and nonlinear subnetworks.

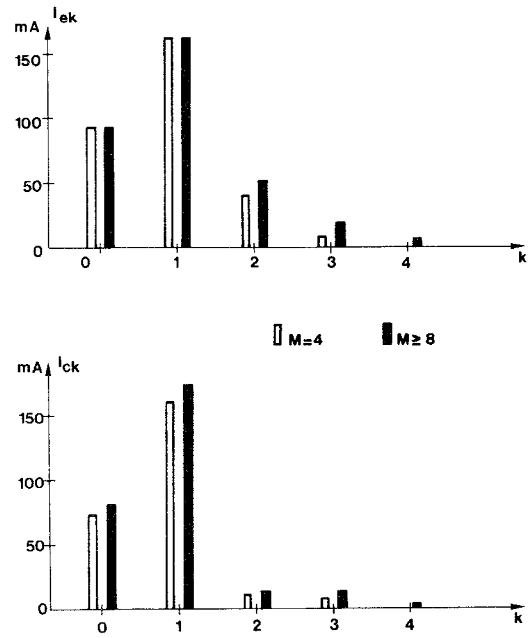

Fig. 8. Amplitudes of the harmonics of the emitter and collector currents computed with $M=4$ and $M=8$.

the tests are not verified, the number M of harmonics is not sufficient to represent the circuit waveforms and must, therefore, be increased by a convenient increment.

The computer program, which has been written to analyze nonlinear circuits on the basis of the method and the algorithms described above, has been structured in such a way that it is easy to prepare the input data describing the circuit. The user first has to partition the circuit into linear and nonlinear subnetworks and describe the linear one with the language of the program adopted to perform the frequency-domain linear analysis [4]. Then the data and the routine for computing the nonlinear functions relative to the nonlinear subnetworks must be provided. This operation is very simple since it consists in writing the Kirchhoff's laws in terms of the connection voltages and currents and the functions relative to every nonlinear component. However, some care must be taken in partitioning the circuit in order to avoid the introduction of a large number of variables from which the computing time strongly depends. A practical example of these operations is given in the following section where a class-C amplifier is considered.

IV. Analysis of a Class-C Amplifier

The program has been used to analyze the response of the class-C transistor amplifier with a center-band frequency of 1.6 GHz shown in Fig. 5 and provided by ESA-ESTEC under a research contract with the Turin Polytechnic. The amplifier implemented on microstrips employs a MSC 3000 transistor which has been modeled with the equivalent circuit [5] shown in Fig. 6. Its parameters have been measured at the Turin Polytechnic under the same contract [6].

For the analysis, the amplifier has been partitioned into two subnetworks, one linear and the other nonlinear, connected through three terminals as shown in Fig. 7. The nonlinear subnetwork behavior is described by

$$f_E = i_E - i_F + C_E \frac{dv_{E'}}{dt} + \alpha_R i_R = 0$$

$$f_C = i_C - i_R + C_C \frac{dv_{C'}}{dt} + \alpha_F i_F = 0 \tag{21}$$

with

$$v_{E'} = v_E + R_B(i_E + i_C)$$

$$v_{C'} = v_C + R_B(i_E + i_C) \tag{22}$$

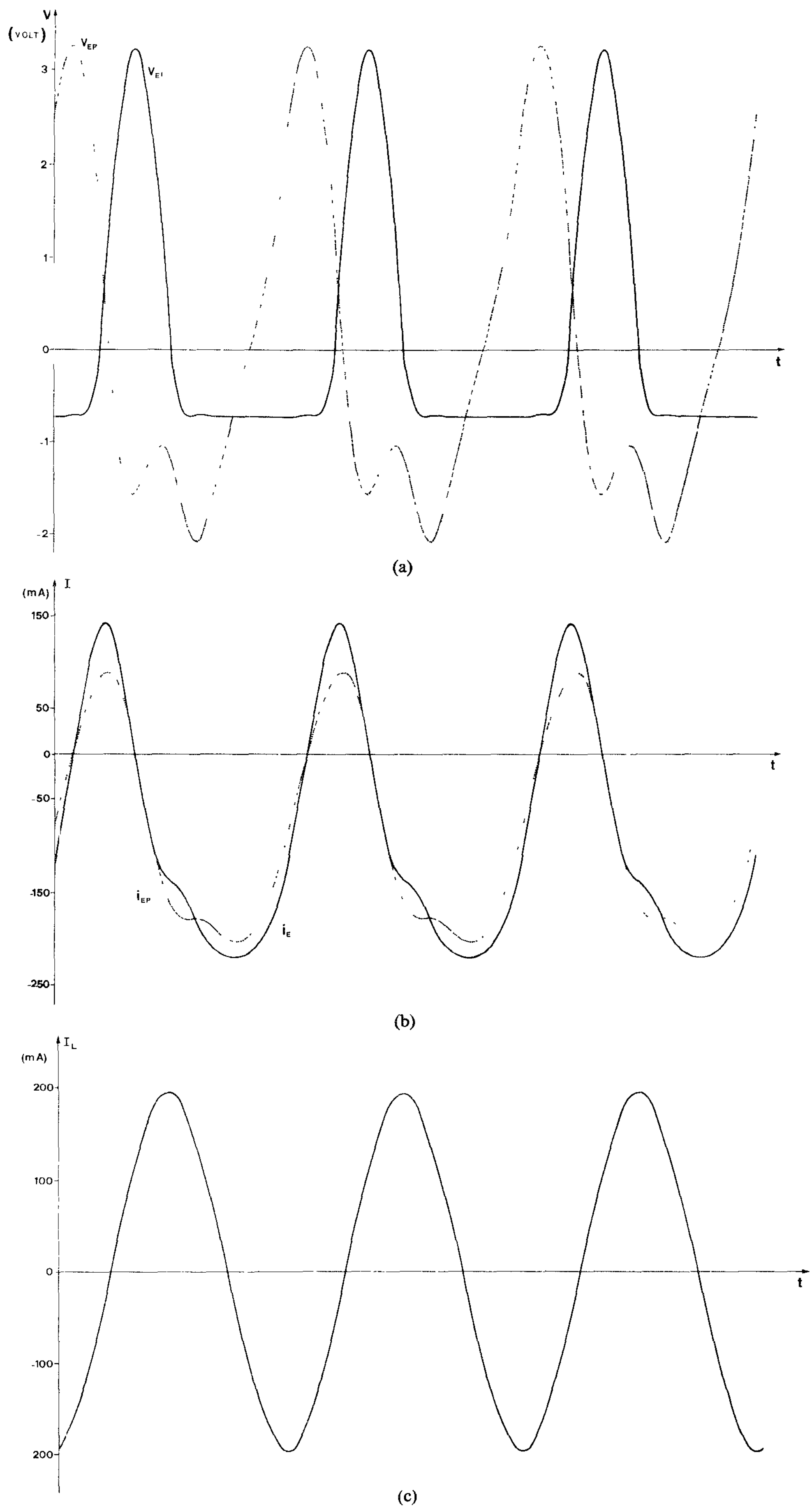

Fig. 9. The computed waveforms of some voltages and currents in the class-C transistor amplifier.

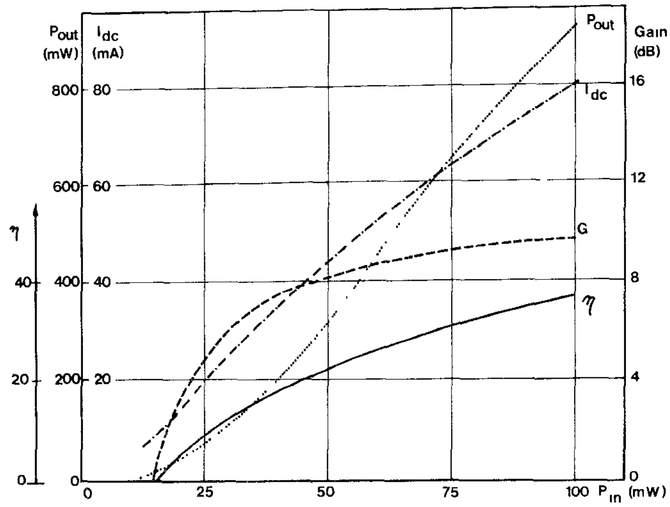

Fig. 10. Plots of the output power P_{out}, dc collector current I_{dc}, efficiency η, and power insertion gain G versus input power P_{IN}.

where i_F, i_R, C_E, C_C, α_F, and α_R are nonlinear functions of $v_{E'}$ and $v_{C'}$, and R_B is a function of $i_B = i_E + i_C$. Thus (21) can be expressed in terms of the voltages and currents at the connections and of their first derivatives

$$f_E = f_E\left(i_E, \frac{di_E}{dt}, i_C, \frac{di_C}{dt}, v_E, \frac{dv_E}{dt}, v_C, \frac{dv_C}{dt}\right) = 0$$

$$f_C = f_C\left(i_E, \frac{di_E}{dt}, i_C, \frac{di_C}{dt}, v_E, \frac{dv_E}{dt}, v_C, \frac{dv_C}{dt}\right) = 0. \quad (23)$$

These functions have been coded in the routine NONLIN which is attached to the main program.

As far as the linear subnetwork is concerned, by means of SCAMAT [10] (a general-purpose program for frequency-domain linear analysis) the impressed waves γ_k and the scattering matrix S_k of the linear subnetwork have been determined for every kth harmonic. By means of these and calling the routine NONLIN, the program computes, according to the technique described, the incident waves at emitter and collector connections, in terms of which all the network functions are also evaluated. In the amplifier analysis the program introduced $M = 8$ harmonics and reached the solution through 14 different values of h which increased the input power from 1 percent to 100 percent of its nominal value. The Newton–Raphson algorithm required an average of 4 iterations/step and a total number of 58 computations of the circuit equations. Moreover, care must be taken not to increase unnecessarily the number M of harmonics since the number of variables, and thus the computing time, depend on it. Therefore, the maximum value for the accuracy index R_j, according to which the analysis algorithm establishes the value of M, must be carefully chosen.

In order to compare the analyses of the amplifier made with different values of M, the amplitudes of the harmonics of the emitter and collector currents computed with $M = 4$ and $M = 8$ are shown in Fig. 8. The waveforms of some voltages and currents, relative to the analysis effected with $M = 8$, are plotted in Fig. 9. The dc collector current, the output power, the efficiency, and the power insertion gain versus the input power are plotted in Fig. 10, by using the partial results relative to the different increasing values of h. Then, to verify the validity of the results obtained, the computed network functions have been compared with the corresponding ones directly measured on the amplifier and the relative values are given in Table I.

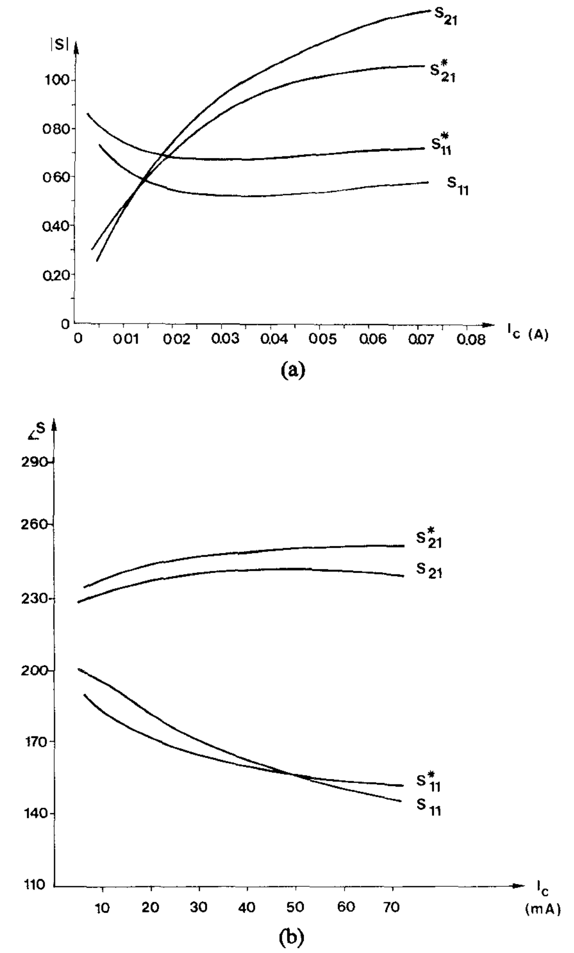

Fig. 11. Plots of the amplitudes (a) and phases (b) of the computed (S^*) and measured (S) values of the large-signal S-parameters versus dc collector current for transistor MSC 3000 at 1.6 GHz.

TABLE I
COMPUTED AND MEASURED VALUES RELATIVE TO THE AMPLIFIER IN FIG. 5 WITH INPUT POWER $P_{IN} = 100$ mW, $V_{CB} = 28$ V, FREQUENCY $f = 1.6$ GHz

Network Functions	Computed Values	Measured Values
dc Collector Current	79.2 mA	82 mA
Output Power	922 mW	990 mW
Gain	9.65 dB	9.96 dB
Efficiency	37 percent	39 percent

In order to validate the transistor model the large-signal S-parameters have also been measured and compared with those computed by the analysis program by simulating the measurement procedure. In particular, as in the actual measurement technique, the incident and reflected waves at the transistor ports have been computed under two operating conditions: one with the output loaded with a 50-Ω resistor and the input connected to a signal generator; the other with the generator connected to the transistor output and the resistive load to the input. The curves of the computed and measured amplitudes and phases of S_{11} and S_{21} as functions of the input power are shown in Fig. 11. The differences between the measured and computed values of the large-signal S-parameters, due to the approximation in the transistor modeling, justify the slight discrepancies between the computed and measured network functions.

V. Conclusions

A method has been described for analyzing nonlinear circuits with periodic response. It is based on the iterative solution of a nonlinear system whose unknowns are the harmonic components of the wave variables at the connections between the linear and nonlinear subnetworks into which the circuit is partitioned. This method, in comparison with previous ones, shows some interesting properties. These make its use preferable especially when, as in the case of class-C amplifiers, the circuit is composed of numerous linear reactive lumped and distributed components and a few nonlinear components with strong nonlinearities. These, according to our formulation, can be expressed in a very general implicit differential form. Such features derive from the particular formulation which has been given to the problem and from the procedure which has been proposed to solve it. The main characteristics of the computer program which implements this technique have also been described. The results of the analysis of a class-C amplifier have also been presented and discussed.

However, the use of this program is not limited to the circuit analysis for verifying the agreement with the given specifications, but the possibility of including it in an optimization program for CAD should also be considered. In this respect, the program seems to be suitable for a more efficient use by taking advantage of the possibilities offered by the proposed step-by-step procedure which performs the analysis by gradually increasing the input signal amplitude. In fact the same procedure can be applied to the one-dimensional searches, along the directions given by the optimization algorithm, for those component values which minimize a user-defined objective function. In this perspective further developments of the proposed method are being studied.

VI. Acknowledgment

The authors wish to thank Dr. J. G. Ferrante of the Mathematical Division of the European Space Research and Technology Centre for useful discussions and suggestions given in the development of the analysis method.

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