

Simulation and optimization of one-way car-sharing systems with variant relocation policies

hEART 2014

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1 **ABSTRACT**

2 Car-sharing is a transportation service consisting of vehicles distributed over an urban area that
3 any driver registered to the system can use. This paper focuses on one-way electric car-sharing
4 systems. The success of such systems relies strongly on operations management and attractive
5 rental conditions. Immediate availability and possibility of reservation in advance are key points.
6 This induces strong constraints for the operator especially when some stations attract more trips as
7 a destination than as an origin and vice versa. These imbalances must be corrected by performing
8 vehicle relocations in a smart way to maximize vehicle availability and minimize operator's costs.
9 In order to understand the demand patterns and explore relocation possibilities, an event-based
10 simulator is built in C#.

11 We develop a new relocation strategy to minimize the demand loss due to vehicle unavail-
12 ability. Implemented in parallel to rentals, it relies on the regular update of the relocation plans
13 based on an optimization framework which utilizes the current state of the system and partial
14 knowledge of near-future demand. This strategy is compared to three other strategies on a case
15 study based on real data from Nice, France. We show that it maximizes the number of served
16 demand and succeeds in keeping the system in a balanced state contrary to the other strategies
17 considered.

INTRODUCTION

The emergence of car-sharing as a new intermediate transportation mean between public and private transport is quite recent. People benefitting from this service have to register and then can use vehicles spread all over the operating area for some cost per minute rental. The history of car-sharing companies began in the 1940s. The first car-sharing company, "Selbstfahrengesellschaft", was founded in 1948 in Zurich, Switzerland (1). Sharing transportation means, manpower or tools is a way to maximize use while decreasing the cost imputed to each user. This was historically done for costly items for which everyone cannot afford. Expenses linked to car ownership are nowadays quite high when we consider purchase, gasoline, insurance, maintenance and reparation fees. Avoiding these expenses may be a motivation for a driver to adopt car-sharing as a regular transportation mean. In addition to users' benefits, car-sharing systems are also interesting for cities because they reduce environmental and congestion impacts as well as consumption of urban space and parking needs.

Car-sharing systems can be classified in different categories depending on the rental conditions. Free-floating systems allow people to park the vehicles anywhere in the covered area whereas non-floating impose to users to park them inside stations with limited number of allowed spots. Another differentiating feature is the "one-way/two-way" characteristic: two-way systems force the user to return the car to the location where it was picked-up whereas one-way systems allow drop-off at any station. The type of vehicle (combustion, electric, ...) affects also the system's use. We focus in this paper on non-floating one-way electric car-sharing systems because of their increased flexibility towards the user compared to two-way systems and their eco-friendly characteristics.

In such installations, rental operations naturally induce imbalances in the distribution of vehicles. To maximize the number of demand served, vehicle distribution needs then to be corrected by performing relocations to maximize vehicle availability. There may be a wide variety of demand types: some are made at last minute and others a long time in advance for instance when a vehicle is needed for a scheduled activity. Both introduce constraints for the operator. Thus, the operations management of a car-sharing system is complex because of demand characteristics, imbalance issues and limited information about the future available to the system.

Our goal is to model and simulate system's operations to analyze the way the vehicle distribution and service rate evolves with time. Relocations can improve drastically system's efficiency and we aim at developing innovative operations management policies. The objective is to maximize customers' satisfaction, i.e. minimize unserved demands while offering the most flexible service to the customers. Meanwhile, the operations of the system must be kept sustainable for the operator. For testing purposes, we develop in parallel simulation tools that reproduce the system's behavior and can implement the newly designed policies. Real data from an existing car-sharing system is utilized to test the methodological framework and calibrate the simulator.

BACKGROUND

There exists a wide variety of problems linked to car-sharing. In terms of system design and operations' management, two main level of decisions can be distinguished:

- i. Strategic and tactical decisions concern respectively number, size and location of station and fleet size, vehicle distribution, staff number. These are long term decisions linked to system's conception. They are considered as given in our problem.

- 1 ii. Operational decisions concern the daily management of the system and address espe-
2 cially relocation management.

3 These decision levels are generally treated separately. However, strategic and tactical deci-
4 sions influence directly operational decisions. If operational matters are, even partially, taken into
5 account when considering strategic decisions, this leads to an easier and more efficient manage-
6 ment of the system as shown in Boyacı et al. (2). There are various other papers regarding strategic
7 decisions of depot location (e.g. 3, 4) and, fleet size and distribution (e.g. 5) separately.

8 Vehicle relocation is a technique proposed to improve the system performance of one-way
9 car-sharing systems (e.g. 6, 7, 8). Relocations can be either staff-based or user-based. User-based
10 relocations try to use users' actual trips to balance the system; simulation showed that the need
11 for relocations can be decreased by 42% using such a strategy as investigated in Barth et al. (9).
12 Staff-based relocations strategies were tested through simulation by Kek et al. (6). In their shortest
13 time strategy, the vehicle to be relocated is chosen in a way to minimize the time needed for the
14 personnel to perform the relocation: i.e. to move to the station where the vehicle is and then drive
15 it to the station experiencing vehicle shortage. In their inventory balancing strategy, vehicles are
16 relocated from stations where vehicles are accumulated to stations experiencing vehicle shortage.

17 Fan et al. (10) modeled dynamic allocation of vehicles which maximizes total profit. Fan
18 (11) regarded dynamic vehicle relocation problem as a multi-stage stochastic linear programming
19 problem. Due to computational complexity of his method, they only implement it on small sized
20 instances with short time planning horizons.

21 The simulator initially developed was reused later to test an optimization-trend support
22 system for some strategic decisions presented in Kek et al. (12). The impact of relocation policies is
23 evaluated according to cumulative duration of vehicle shortage in the stations, cumulative duration
24 of empty parking spot shortage in stations and number of relocations.

25 In Barth et al. (13), a queuing based-discrete event model representing a shared vehicle
26 system is developed. No reservations are considered and people are supposed to be queuing at sta-
27 tions when no vehicle is available. Three main relocations strategies are tested using respectively
28 thresholds at stations, historical demand and full knowledge of the future to decide on relocation
29 patterns. System's efficiency is evaluated with average wait times at stations and number of cus-
30 tomers waiting for vehicles. It shows that historical data increases system's efficiency but not as
31 much as full knowledge of future, which results in maximum efficiency.

32 In all these cases, the relocation process is done in parallel of the rentals contrary to some
33 other models which consider doing relocation at the end of the day when the system does not
34 operate such as in Fan et al. (10) or in Chemla et al. (14), which considers bike-sharing systems
35 instead of car-sharing ones. Additional models using similar approaches have been proposed by
36 Raviv et al. (15) and Contardo et al. (16).

37 The level of information is crucial to make efficient operational decisions. Future can be
38 totally ignored or near-future demands can be taken into account as in the chance constrained
39 model developed by Nair and Miller (17). In the extreme case, all demand is known for the rest of
40 the day as in Boyacı (18). In a real system, this supposes to either serve only reservations made
41 on the long-term or that the demand does not fluctuate from one day to the next. The question
42 of information is important when considering reservations since the operator must have a vehicle
43 available when a planned demand is realized: having two types of demand and the specificity of
44 long-term demands was never explored before to our knowledge.

Our approach of the problem is user-oriented: The system has to be kept as flexible as possible, enabling to do reservations in the long-term as well as at last minute. The objective of this work is first to design a versatile simulator that can be used on any system. It will provide tools to analyze its behavior, help understand how any change in the system impacts its efficiency and forecast its performance after any modification. Reservation at origin as well as booking in advance have to be managed. We later integrate a new relocation strategy to further improve the quality of service. Different types of relocations are considered. Relocation plans are regularly updated based on the actual vehicle distribution and known demand with a moving time window. The remainder of the paper is organized as follows: next section describes the characteristics and the development of the event-based simulation, the following section develops the relocation strategies together with an optimization framework for a real-time application, while the last two sections present the results of an application in a case study and discuss future research directions.

EVENT-BASED SIMULATOR FRAMEWORK

The simulator framework is modular: a base structure simulates the most basic features of a car-sharing system (demand request, car pick-up, car drop-off) but several other modules are added to test new strategies or study other aspects of system's behavior.

The simulator is composed of different real physical elements of car-sharing systems such as Stations, Spots, Vehicles and relocation Personnel. During simulation, these elements cannot be modified contrary to their states that describe their current occupation. For instance, a vehicle can be available, occupied (expecting customer's pick-up), under service or under relocation. Other classes describe actions and movements of personnel and vehicles such as Demand (customer's demand for a vehicle), Trip (displacement of a vehicle due to a demand accepted by the system) and Relocation (movement of vehicles performed by personnel to redistribute vehicles).

The simulations are continuous in time and their progression is event-based. Any actions such as modification of the states of the physical elements are ruled by events happening with time. A rental request, the beginning/end of a rental/relocation, a personnel's shift end constitute such events. Other events are designed to retrieve information from the system such as the calculation of statistics to evaluate how the system performs.

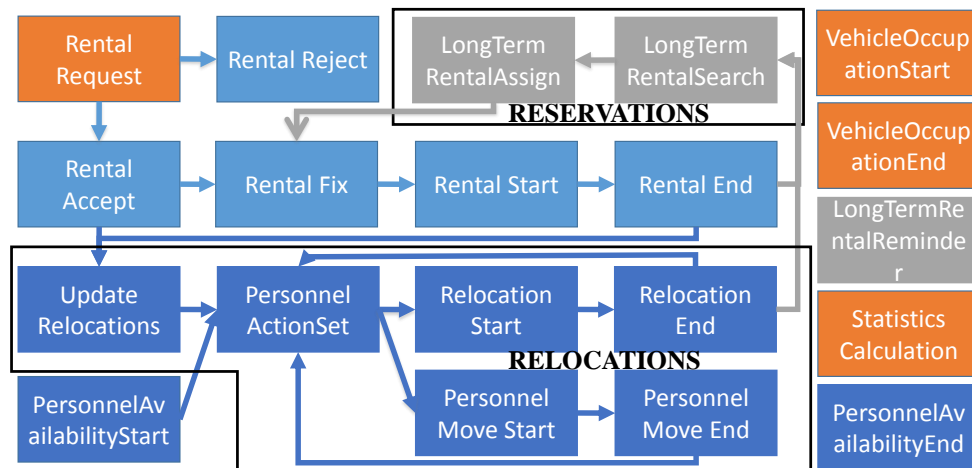


FIGURE 1: Simulator event structure listing all events and how they trigger each other

1 Events are numerous due to the complexity of the system. Their relations are described
2 in figure 1. Evolution of time in the simulator is modeled through an "Event List" where events
3 are sorted by their time of realization: whenever an event occurs, the first element of the list is
4 retrieved and the simulator time is updated. This "Event List" is filled before the beginning of the
5 simulation with rental requests and any event we wish to see happen. It is continuously refilled
6 during the simulation since the happening of an event triggers generally the creation of a new one.
7 Simulation is stopped either when the "Event List" is empty or at a predetermined time.

8 In Figure 1, events in orange are those added at the beginning of the simulation before it
9 starts. In light blue are the events linked to the basic structure, in grey those linked to reservations'
10 management and in dark blue, those linked to relocations' management.

11 The studied systems have some specific features concerning demand, vehicle allocation
12 and rental conditions that are reproduced in the simulator framework.

13 **Request type**

14 Rental request can be of two types: short-term or reservation (also described as long-term).

- 15 • Reservations are automatically accepted by the system. They correspond to a demand
16 made at least a certain amount of time in advance. Reservations are reminded to the
17 system two hours before the rental begins so that the need for a vehicle at a specific
18 location is taken into account when relocating vehicles.
- 19 • Short-term demands are always examined by the system but they can be refused if the
20 system cannot serve them. A demand is considered as short-term if it is done less than an
21 hour before the rental is supposed to start. If a demand is accepted, the assigned vehicle
22 is kept reserved for the customer for a given amount of time set to half an hour. Once this
23 duration has passed, the vehicle is available again.

24 Reservations are only made at origin and not at destination. For reservations at destination, the
25 reader can refer to (2).

26 **Vehicle allocation in rentals**

27 Vehicle allocation depends on demand type. When a reservation arrives, the operator should be
28 able to bring a car at a specific station. When it is a short-term demand, a specific allocation
29 algorithm is used. To decide on serving a short-term demand, the simulator checks at the station
30 closest to demand location if:

- 31 i. a vehicle is available
- 32 ii. the charging level of the battery is over a minimum threshold (which is a parameter
33 that may influence the number of demands served)

34 If vehicles meeting those criteria can be found, then the one with the highest battery level is as-
35 signed to the service of the demand. If there is no such vehicle at the wished station, same check
36 is done at all stations in a nearby range (500 meters, in order from the closest to the farthest). If no
37 adequate vehicle is found, the demand is rejected.

Vehicle allocation in relocations

When a relocation has to begin according to the tasking plan of a personnel, the best vehicle is selected in the station where the personnel is. The choice process is the same as for vehicle allocation for short term rentals except that it is limited to the station from which the relocation must be done and minimum battery level to allow relocation is equal to the minimum value to allow rental plus some additional battery equal to the consumption of the relocation trip. If battery is too low according to this criterion or no vehicle is to be found due to other changes in the simulator, the relocation is not done and the optimization reruns to be able to task this personnel.

Partial floating

Though the systems targeted by this study are non-floating, this characteristic is relaxed to increase user's satisfaction. A vehicle can be dropped-off outside a station in a close range if the station is already full. The vehicle will however not be charged which may be problematic if its battery level is already low. In the simulator, this feature is managed by adding another category of spots, named extra-spots. They do not allow charging of vehicles but link the vehicle to the station.

A FRAMEWORK FOR RELOCATION STRATEGY

The proposed strategy relies on a regular update of the personnel tasks according to system state evolution. It involves an optimization framework which helps to decide on the best relocation plan to follow in the coming hours given actual distribution and reservations in the near-future. The optimization module is aware only of the reservations whose trips should begin soon (in less than two hours). Optimization shall be rerun whenever unexpected events happen in the system, namely a new "RentalRequest" or a "RentalEnd". It decides on which relocations to perform (this is relocation choice) and who will perform them and when (this is personnel task assignment). Performing relocations has two main objectives:

- Relocate vehicles in order to serve a demand which was planned in advance and that the operator has to serve. They must be generally performed in a specific time window so that the relocated vehicle is assigned to the corresponding demand when it is dropped-off at a relocation destination point.
- Balance the system so that demand in the short-term have higher chances to be served without penalizing in advance reservations

Relocation choice and personnel task assignment can be decided in a sequential process or in the same model. At an early building stage of the models, the sequential solution has proved to be much faster than a global optimization, and with small deviations from the optimal solution.

Relocation choice

Performing relocations will transform the actual vehicle distribution in another one that shall be ideal to serve the incoming demand. Choosing relocations corresponds to finding the shortest relocation path between the two distributions, i.e. reaching the ideal distribution of the system with minimum effort and cost from the personnel.. To define what an ideal distribution is, the following criteria apply:

- i. When a rental happens, it is not important if the serving vehicle is at the specific station or at a neighbor station as long as it is not too far by walk. As a consequence, the

1 conditions to define an ideal distribution should concern clusters of neighboring sta-
2 tions and not each station individually. For each station, a cluster is created grouping
3 all the stations within 500 meters.

- 4 ii. Future short-term demand is not known so we cannot define precisely the number of
5 vehicles needed in a station. A range for this number can be set arbitrarily or based
6 on historical data.
- 7 iii. Exceptions can be made to the previous rule for stations where reservations will
8 shortly begin: a minimum value can be defined equal to the number of reservations
9 beginning at station in the two coming hours.
- 10 iv. In order to avoid accumulation of vehicles in successful destination points, vehicles
11 should be equally enough spread among stations inside the same cluster.

12 To introduce the model formulation the following variables and parameters are needed:

13 *Sets and indices*

- 14 • $i \in C$: cluster index
- 15 • j and $k \in N$: station indices

16 *Parameters*

- 17 • $NC_i^{max} = 2NS_i$ and $NC_i^{min} = NS_i - (1 + E(\frac{NS_i}{3}))$: minimum and maximum values
18 of the acceptable range for cluster i where NS_i is the number of stations in the cluster
- 19 • n_j^0 : number of vehicles at station j in the distribution.
- 20 • n_j^{min} : number of reservations expected to begin from station j in the next interval.
- 21 • S_i : number of stations in cluster i .
- 22 • C_{jk} : travel duration by car from station j to station k .
- 23

24 *Variables*

- 25 • NC_i : number of vehicles in all stations of cluster i .
- 26 • n_j : number of vehicles in station j .
- 27 • u_{jk} : number of moves from station j to station k .
- 28 • δ : maximum deviation between the number of vehicles in a station and the mean number
29 of vehicles in each station in the cluster it belongs to.

$$\min \sum_{j,k,i \neq j} C_{jk} u_{jk} \quad (1)$$

$$NC_i = \sum_{j \in i} n_j \quad \forall i \in C \quad (2)$$

$$n_j = n_j^0 + \sum_{k \in N} u_{kj} - u_{jk} \quad \forall j \in N \quad (3)$$

$$n_j \geq n_j^{min} \quad \forall j \in N \quad (4)$$

$$NC_i^{min} \geq NC_i \geq NC_i^{max} \quad \forall i \in C \quad (5)$$

$$-\delta < n_j - \frac{NC_i}{S_i} < +\delta \quad \forall i \in C, \forall j \in i \quad (6)$$

Objective 1 minimizes time needed to perform all relocations. Constraints 2 link the number of vehicles in every cluster to the number of vehicles at each station. Constraints 3 calculate the required number of moves between stations. Constraints 4, 5, 6 bound the number of vehicles at each station (4) and in each cluster (5 and 6).

If the number of vehicles in state "Available" is too small or the number of expected reservations at one station is large, the problem is infeasible because no ideal distribution, according to our criteria, exists. Since optimization is run on-line while the system is operating, it is necessary to have a solution even if it is not ideal. Thus, instead of enforcing constraints 4, 5 and 6, we choose to penalize the objective when they are violated. In that case, any distribution becomes a feasible solution and the model will be looking for the least penalized one. Penalizations have to be added to the parameters with some penalty variables. Penalization is not the same for the violation of different constraints because of their relative importance. The main priority is to guarantee vehicles to reservations and then to guarantee that there are always vehicles in the neighborhood of each station. Finally, less important is to assure a good distribution of vehicles inside clusters. Three parameters are introduced related to this penalization.

- P_{eq} : penalization for violating the clusters' equilibrium constraint. Set to 10.
- P_b : penalization for violating the clusters' boundaries constraint. Set to 100.
- P_{min} : penalization for violating the minimum number of vehicles in station constraint. Set to 10000.

Penalty variables

- ω_j : penalty for minimum number of vehicles in station j constraint, $\omega_j \geq 0$
- α_i and β_i : penalties for cluster i range constraint, $\alpha_i \geq 0, \beta_i \geq 0$

- μ_{ij} and ν_{ij} : penalties for cluster i equilibrium constraint, j being a station included in cluster i , $\mu_{ij} \geq 0, \nu_{ij} \geq 0$

Constraints 5 are transformed to two constraints 12 and 13. Constraints 6 are also divided in two constraints, 14 and 15. Penalizations are added to the objective function.

$$\min \sum_{j,k,k \neq j} C_{jk} u_{jk} + P \quad (7)$$

$$P = \sum_{i \in C} P_b(\alpha_i + \beta_i) + \sum_{i \in C, j \in i} P_{eq}(\mu_{ij} + \nu_{ij}) + \sum_{j \in N} P_{min} \omega_j \quad (8)$$

$$NC_i = \sum_{j \in i} n_j \quad \forall i \in C \quad (9)$$

$$n_j = n_j^0 + \sum_{k \in N} (u_{kj} - u_{jk}) \quad \forall j \in N \quad (10)$$

$$n_j + \omega_j \geq n_j^{min} \quad \forall j \in N \quad (11)$$

$$NC_i^{min} \geq NC_i + \alpha_i \quad \forall i \in C \quad (12)$$

$$NC_i - \beta_i \leq NC_i^{max} \quad \forall i \in C \quad (13)$$

$$-\delta < n_j - \frac{NC_i}{S_i} + \mu_{ij} \quad \forall i \in C, \forall j \in i \quad (14)$$

$$n_j - \frac{NC_i}{S_i} - \nu_{ij} < +\delta \quad \forall i \in C, \forall j \in i \quad (15)$$

Personnel tasking

Personnel tasking takes the set of relocations given by the relocation choice model as an input and assigns it to the personnel available. In the mixed integer linear programming framework developed, two objectives can be considered:

- Minimize the duration needed to perform all relocations in the set
- Maximize the number of relocations done in a given duration

Since optimization performs in a real time framework, planning far in advance is not necessary since the system's state will change shortly after optimization.

Sets and indices

- a : arc index representing relocations and moves
 - A : set of all arcs
 - A_R : set of all relocation arcs
 - A_M : set of all move arcs
 - A_{book} : set of relocation arcs used to move vehicles in order to serve reservations

- n : node index representing stations
 - N : all nodes except the sink node
 - N_s : group of all source nodes where the flows begin
 - n_e : sink node
- k : flow index representing personnel
 - K : set of all flows
 - K_n : set of flows starting at source node $n \in N_s$
 - A_n^{in}/A_n^{out} : set of arcs coming to/leaving from node n
- l : task index of each personnel
 - $l = 0$ is first task of the personnel
 - maximum value for l is $l_{max} = 2 \times \text{numberOfRelocations} + 1$
 - $L = \{0, 1, \dots, l_{max}\}$ and $L^* = \{0, 1, \dots, l_{max} - 1\}$

Parameters

- C_a : travel duration of an arc (i.e. its travel cost)
- T_k^{start}/T_k^{end} : availability start/end time of personnel k (i.e. when he has to begin/finish performing relocations).
- a_n^{STS} the arc leaving from starting node $n \in N_{source}$ and directed to the sink
- *OptimizationHorizon* is the size of the allotted window to do relocations which is set to a few hours, generally two or three, namely the time horizon allowed to perform the relocations.
- t_a^{min} and t_a^{max} : minimum and maximum time to start traveling arc $a \in A_{book}$. This time-window is defined by the parameters of the demand relocation a moves a vehicle for.
- η : a weight parameter to prioritize relocations done to serve reservations. Set to 1000.
- M : a big enough positive number to enforce big-M conditions

Variables

- $u_{akl} \in \{0, 1\}$, $a \in A$, $k \in K$, $l \in L$
1 if arc a is traveled by personnel k during task l , 0 otherwise.
- $T_{kl} \in \mathbf{R}$, $k \in K$, $l \in L$
time at which personnel k begins task l .
- $w_{kl} \geq 0$, $k \in K$, $l \in L$
waiting time for personnel k before beginning task l .

- $\tau_a \geq 0, a \in A_{book}$
time at which relocation $a \in A_{book}$ is traveled.
- $p_k \in \{0, 1\}, k \in K$
1 if the personnel effectively realizes relocations for the system, 0 otherwise

$$\max \sum_{k \in K} \sum_{l \in L^*} \left(\sum_{a \in A_{book}} \eta u_{akl} + \sum_{a \in A_R \setminus A_{book}} u_{akl} \right) \quad (16)$$

$$\sum_{a \in A} u_{akl} \leq 1 \quad \forall k \in K, \forall l \in L \quad (17)$$

$$\sum_{a \in A_n^{out}} u_{ak0} = 1 \quad \forall n \in N_s, \forall k \in K_n \quad (18)$$

$$\sum_{a \in A_{ne}^{in}} \sum_{l \in L} u_{akl} = 1 \quad \forall k \in K \quad (19)$$

$$\sum_{l \in L} \sum_{k \in K} u_{akl} \leq 1 \quad \forall a \in A_R \quad (20)$$

$$\sum_{a \in A_n^{in}} u_{akl} = \sum_{a \in A_n^{out}} u_{akl+1} \quad \forall k \in K, l \in L^* \quad (21)$$

$$T_{k0} = w_{k0} + T_k^{start} p_k \quad \forall k \in K \quad (22)$$

$$T_{kl+1} = T_{kl} + w_{kl+1} + \sum_{a \in A} C_a u_{akl} \quad \forall k \in K, \forall l \in L^* \quad (23)$$

$$T_k^{end} \geq T_{kl_{max}} \quad \forall k \in K \quad (24)$$

$$Optimization Horizon \geq T_{kl_{max}} \quad \forall k \in K \quad (25)$$

$$p_k + u_{a_n^{STS}, k, 0} = 1 \quad \forall n \in N_{source}, \forall k \in K_n \quad (26)$$

$$t_a^{min} \leq \tau_a \leq t_a^{max} \quad \forall a \in A_{book} \quad (27)$$

$$\tau_a \leq T_{kl} + M(1 - u_{akl}) \quad \forall a \in A_{book}, \forall k \in K, \forall l \in L^* \quad (28)$$

$$\tau_a \geq T_{kl} - M(1 - u_{akl}) \quad \forall a \in A_{book}, \forall k \in K, \forall l \in L^* \quad (29)$$

Objective 16 maximizes the number of relocations programmed; relocations done to move vehicles to serve reservations are strongly weighted in order to realize as many of them as possible. Constraints 17 restrict that each personnel has at most one assigned task at a time. Constraints 18 and 19 ensure that each flow begins its journey from its source node and terminates it in the sink node. Constraints 20 guarantee that every relocation arc is traveled not more than once throughout the simulation (because once it is traveled, it cannot be traveled any more). Constraints 21 ensure that what comes in a node gets out of it for all orders except order 0. The same type of condition for task 0 is already guaranteed for each flow by constraints 18.

Constraints 22 sets the initial time where availability of personnel starts depending on effective participation and stated availability start time. For each personnel, constraints 26 link its participation variable to the move arc leaving from its source node directed to the sink node: if this arc is traveled at task 0, it means that the personnel does not do any relocations. Time evolution of each personnel's schedule from one task to the next considering potential waiting times

1 is then handled by Constraints 23. Constraints 24 and 25 limit the final tasking time according to
2 availabilities and optimization horizon. Constraints 27 correspond to the time windows to respect
3 when a relocation is performed to move a vehicle for a specific reservation. Constraints 28 and
4 29 link the travel of the arcs from A_{book} to the task of the staff: if personnel k at task l travels arc
5 $a \in A_{book}$, then $\tau_a = T_{kl}$, namely he should begin traveling it in the authorized time window.

6 **CASE-STUDY RESULTS**

7 The case study in this paper is based on a real system implemented in Nice. There are 59 operating
8 stations distributed in the urban area: station density is higher in the city center. Demand is extrap-
9 olated from real demand collected on the system when it was operating two-way. It is transformed
10 to one-way by a splitting process proposed in (2). Seeds are generated by selecting randomly from
11 the pool of generated one-way demands.

12 **Efficiency indicators**

13 The efficiency of the system is evaluated from two point of views. First, we consider the impact
14 of the system characteristics such as fleet size, number of spots per station, minimum battery level
15 to allow rental and staff size. Second, we consider the impacts of strategies and policies on rental
16 service. Four main indicators are followed through simulation:

- 17 • Global demand rejections (or demand loss rate) since the beginning of the simulation.
- 18 • Counts of the station states describe vehicle distribution and accumulation in stations.
- 19 • Counts of vehicles states give a snapshot of the system state complementary to station
20 states counts.
- 21 • Rejection counts per station highlights the largest vehicle shortages.

22 Simulations always have a duration of 10 days. Three levels of demand are tested: 50, 100, and
23 200 per day. We consider also different percentages of reservations in the demand: 0%, 25%, 50%
24 and 100%.

25 **Impact of system characteristics and tactical decisions**

26 Fleet size and number of spots per station vary together such that in the initial distribution (where
27 vehicles are equally spread), there are always two free spots per station. Three different vehicles
28 distributions were considered:

- 29 • **V1:** Fleet size is equal to the number of stations. There is initially 1 vehicle in each
30 station and 2 free spots.
- 31 • **V2:** Fleet size is equal to the number of stations + 10. There is 1 vehicle per station with
32 2 free spots except in the 10 stations facing the most demand where 2 vehicles are set
33 along with 2 free spots (with a total of 4 possible spots in these stations).
- 34 • **V3:** Fleet size is equal to twice the number of stations. There are 2 vehicles in each
35 station along with 2 free spots.

1 Simulations showed that adding vehicles in the system increases demand acceptance rate
2 by 15% when the number of vehicles is doubled. This increase is proportional to the number
3 of vehicles added. When the vehicles are added at specific stations such as in V2, rejection rate
4 decreases mostly in the reinforced stations.

5 Minimum battery level was tested for values varying from 20% to 60%. Allowing rental
6 with very low battery is indeed not reasonable when we consider the distance traveled with each
7 trip. Setting also a high threshold does not make sense because 70% battery is more than enough
8 to start a trip. The impact of minimum battery level is limited in that range: demand acceptance
9 rate decreases very slowly with the battery threshold increasing.

10 Number of personnel working was also tested, varying from 4 to 19 people distributed in 4
11 shifts over all day. Increasing the number of personnel working during each shift does not influence
12 much the results. Lost demand observed varies by 1 to 5% at most when the number of personnel
13 is doubled. Only the case with 4 personnel is clearly worse than all the other strategies though the
14 difference is very small. For the rest of the paper, we present results with the number of personnel
15 set to 7.

16 **Impact of various management strategies**

17 Battery threshold to allow rental is set to 40%. Chosen initial vehicle distribution is V1. Four
18 strategies were tested:

- 19 • No relocation strategy (noted SNo): no relocations are performed.
- 20 • Reset strategy (noted SRe): vehicles are returned back to their original position every
21 night. This is equivalent to static balancing strategies mentioned in the literature.
- 22 • Only reservations relocation strategy (noted SOR). The model developed for S7 is used
23 but constraints 12, 13, 14 and 15 are deactivated. The ideal distribution depends only on
24 reservations and not anymore on a balanced vehicle distribution.
- 25 • The strategy designed with developed optimization framework (noted S7).

26 *Lost demand*

27 In the different subgraphs of figure 2, TD should be read as total demand, STD should be read as
28 short-term demand and LTD as long-term demand. The graph on top shows lost demand variation
29 for the same demand seed when long-term demand percentage in total demand varies. The two
30 graphs under show the same variations when we consider, from top to bottom, only short-term
31 demand among all demand and only long-term demand among all demand.

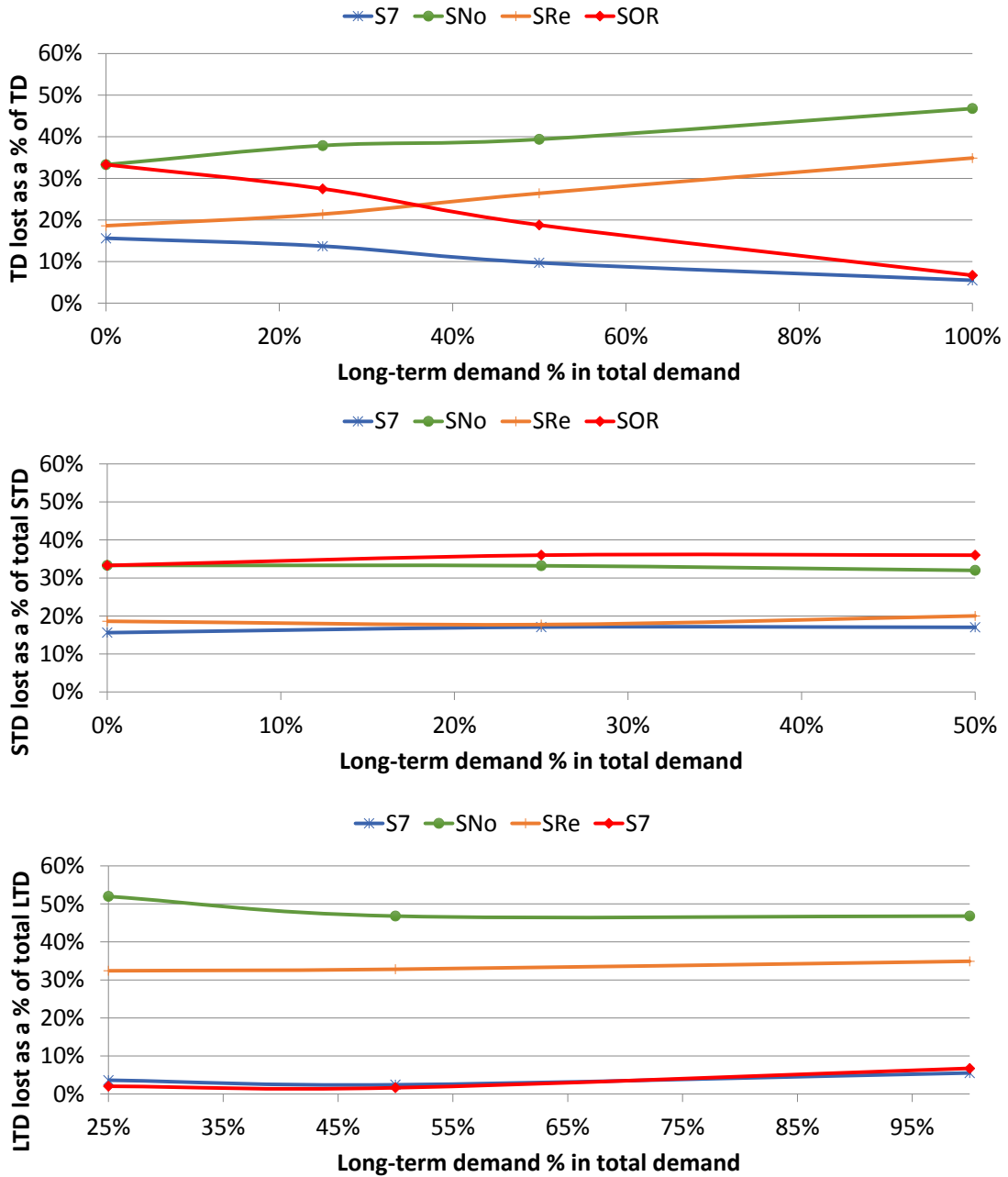


FIGURE 2: Lost demand characteristics for a demand seed of 100 rentals per day

1 S7 strategy is globally more efficient than the three strategies presented. When we consider
 2 only short-term demand however, it performs as good as SRe does, which shows that to handle
 3 short-term demands, continuous relocations with the actual parameters of our model may not be
 4 the most efficient. With demand 200 per day, it is possible, depending on the seed, that SRe
 5 performs better than S7 when reservation percentage is 0%.

6 On the contrary, S7 strategy is much more adapted to handle reservations than SNo and
 7 SRe since the loss rate concerning long-term demands is always less than 10%. It is worth noting
 8 that the same percentage of reservations approximately is served, independently of reservations

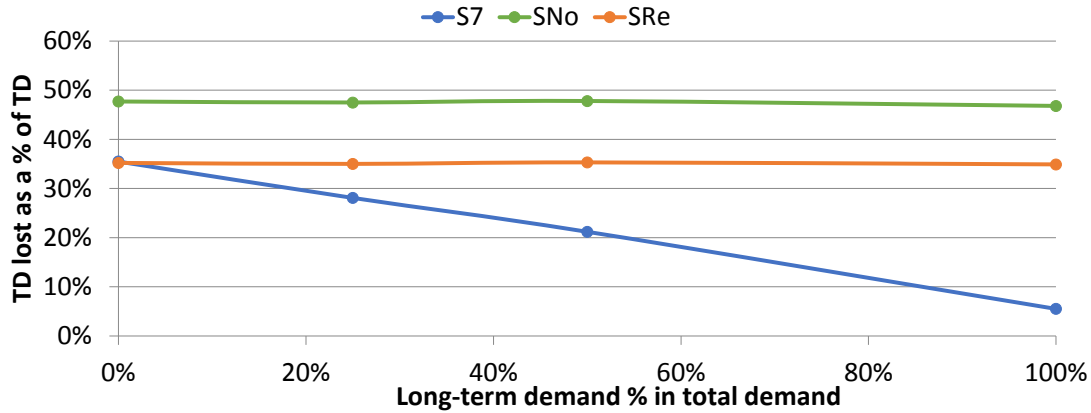


FIGURE 3: Lost demand characteristics for a demand seed of 100 rentals per day with same vehicle allocation for long-term and short-term demands

percentage in total demand.

The comparison with SOR strategy suggests the importance of an equitable distribution of vehicles among the stations. If SOR performs as good as S7 for reservations, the handling of short-term demands is as bad as with SNo. The importance of a good vehicle distribution is then crucial: one could expect an even better service for short-term demand if the clusters' ranges in constraints 12 and 13 were adapted to historical demand trends.

Since the strategy we implemented manages vehicle allocation differently short-term demand and long-term demand, it is possible as we observe it here that the lost demand rate varies with long-term percentage especially for SRe and SNo. If we consider the same type of vehicle allocation as in figure 3, lost demand rate stays almost the same for SNo and SRe independently of reservation percentage in total demand. For S7 and SOR, lost demand rate decreases whereas the chosen seed is the same. The system has more information concerning long-term demands since they are reminded to him some time before they really happen. It can use this time to prepare serving these demands. The more information there is, the better the system performs.

Station states

The different strategies studied lead to different accumulation patterns in the stations of the system. Accumulation outside of the station, due to partial floating authorization, has to be avoided. Partial floating must be used as little as possible and only to ease customer's search for a destination spot. Figure 4 presents the evolution of the global state of the system in terms of accumulation.

Each station has an accumulation state which is noted O_x and E_y , x being the number of vehicles parked in the station's spots at snapshot time and y being the number of vehicles parked outside of the station due to partial floating. The station states count graph, showing accumulation, consists in two superposed graphs. In ordinate, from 0 to 59 are presented the number of stations in each possible O state; from 59 to 118 are presented the number of stations in each E state. The top part refers to E states, the bottom to O states. The 4 different strategies (no relocation, reset, only mandatory relocations and designed strategy) are described in figure 4 for 10 days. These results refer to a total demand of 100 and a short term reservation of 50%.

SRe presents a periodical pattern due to the fact that the system is immediately reset at the

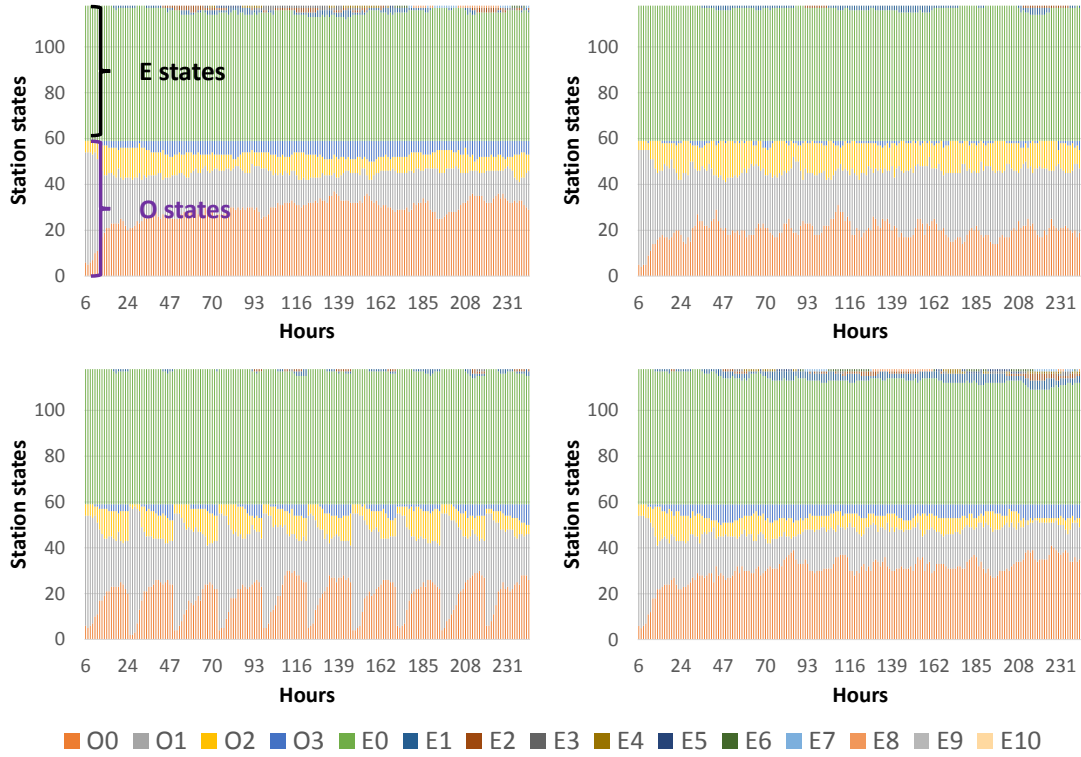


FIGURE 4: Station states graphs for a demand seed of 100 rentals per day with 50% reservations. top left: SNo; bottom left: SRe; top right: S7 and bottom right: SOR

end of each day. The aspects of all three other strategies look similar with big changes in the first days to reach some steady state in terms of O states from the end of the 2nd day or the beginning of the 3rd day.

The number of stations with state higher than O1 is very small in the case of S7 as well as the number of station with high E states, when compared to SNo, SRe and SOR. Accumulation is avoided thanks to the proactive equilibrating strategy represented by the constraints 12 and 13. SRe also has very few high E states but it is due to reset. It shall also be noted that the number of stations with no vehicles in their spots (O0 state) is the smallest with strategy S7. This shows that the designed strategy maximizes vehicle availability in most stations. For all these reasons, S7 is by far the best strategy in terms of vehicle distribution.

CONCLUSIONS AND ON-GOING WORK

Through this work, we propose a versatile simulator than can be easily adapted to different (one-way) (electric) car-sharing systems in order to analyze their response to demand and test any upgrades the operator could implement to increase its efficiency. We focused especially on the design of relocation strategies that allow to handle last-minute demands as well as long-term reservations.

The system settings are such that the experience of the user is the simplest and most flexible. The strategy is a continuous update of the relocation tasking plan of the personnel through optimization to maximize number of demands served. The sequential optimization framework consists first of an algorithm deciding on relocations to perform using the concept of ideal distri-

1 bution and second in an assignment process that maximizes the number of relocations done over a
2 short period of time ahead. This optimization is run whenever an unexpected change occurs in the
3 system: this maximizes the information at system's disposal to decide on relocations. According
4 to simulations, the strategy we propose is efficient since it maximizes vehicle availability all day
5 long, avoids accumulation of vehicles outside stations and serves as many demands as possible
6 though all reservations are not guaranteed. Other effects, especially linked to tactical decisions
7 but that impact operational decisions, were investigated. The importance of information on the
8 demand to handle the system at its best was shown.

9 Some new features shall be added soon to this relocation model. The use of historical data
10 should help increasing the service of short-term demands.

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