Simulation and optimization of supply chains: alternative or complementary approaches?

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Abstract Discrete-event simulation and (mixed-integer) linear programming are widely used for supply chain planning. We present a general framework to support the operational decisions for supply chain networks using a combination of an optimization model and discrete-event simulation. The simulation model includes nonlinear and stochastic elements, whereas the optimization model represents a simplified version. Based on initial simulation runs cost parameters, production, and transportation times are estimated for the optimization model. The solution of the optimization model is translated into decision rules for the discrete-event simulation. This procedure is applied iteratively until the difference between subsequent solutions is small enough. This method is applied successfully to several test examples and is shown to deliver competitive results much faster compared to conventional mixed-integer models in a stochastic environment. It provides the possibility to model and solve more realistic problems (incorporating dynamism and uncertainty) in an acceptable way. The limitations of this approach are given as well.

Keywords Supply chain management · Optimization · Discrete-event simulation · Hybrid method

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1 Introduction

In recent years intra-company supply chains have been growing significantly spanning production and distribution sites all over the world. At the same time global competition has increased, such that there is a strong demand for new decision support tools on strategic, tactical and operational levels. Biswas and Narahari (2004) classified the relevant research on such decision support systems into three categories:

(a) Optimization models mainly for multi-echelon inventory control. In most cases these models are deterministic and used for strategic or tactical decisions.
(b) Analytical performance models, which consider a dynamic and stochastic environment. They are used to investigate design or principal management decisions. Such systems are represented as Markov chains, Petri nets or queuing models.
(c) Simulation and information models, which are used to analyze complex dynamic and stochastic situations and to understand issues of supply chain decision making.

For the first and the second categories, it is often necessary to make several simplifications from the real-world case in order to develop solvable models. Nevertheless the problem size is usually very limited. Although there are promising developments of combinations of these two categories (cf. Sect. 2), many of them remain on a strategic level and the stochastic property is considered by a small number of different scenarios.

In this paper we develop a new solution approach by applying a LP/MIP formulation in the context of a discrete-event simulation. So we are able to combine the advantages of models from all categories mentioned above by considering a detailed representation of a dynamic and stochastic environment and allow the application of optimization methods in this context. Our investigations are based on a general supply chain network model with different facilities (suppliers, manufacturers, distributors) and different transportation modes connecting these facilities. We assume that there is a central planner with perfect information such as for intra-company supply chains or supply chains with a dominant member. This problem setting is motivated by a case study about a global supply chain network in the paper industry (Gronalt et al. 2007). The goal is to reduce costs by simultaneously optimizing the production/transportation schedule and reducing inventory levels. We are aiming for a robust solution, in the sense that a stochastic environment is considered. Comparing our problem to the tasks in the supply chain matrix (cf. Stadtler 2005), the problem is a combination of several operational tasks: production planning, distribution planning and transport planning. In addition to other approaches, we assume a stochastic simulation model for these tasks and combine it with classical optimization approaches.

Our goal is to achieve an optimal operation plan for supply chain networks by combining optimization models and simulation models. We do not use the optimization on top of the simulation, where an optimization algorithm uses the simulation model as a black-box evaluation function (cf. Glover et al. 1999). Instead we include simulation and optimization in an iterative process in order to gain the advantages of optimization (exact solution) and simulation (nonlinearities, complex structure, stochasticity). In the previous research (Almeder and Preusser 2004; Preusser et al. 2005a,b) we

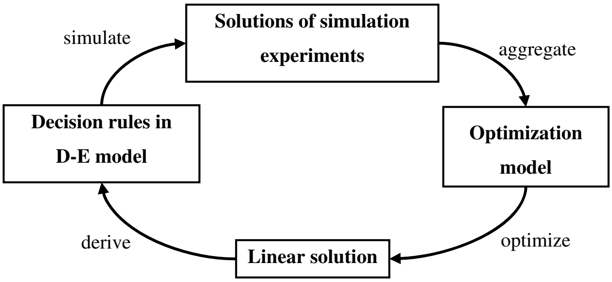

Fig. 1 Interaction between simulation and optimization

developed a rough idea of this concept. In the current paper we extended this concept, such that it is possible to apply it to a wide range of supply chain problems. Furthermore we analyze in detail the advantages and disadvantages of this approach and present results for different test cases.

The supply chain is represented as a discrete-event model (D-E model) and a simplified version is modeled as an optimization model. We start by performing several simulation runs in order to get average values of the parameters (e.g., unit transportation costs) which are then fed into the optimization model. After solving the optimization model the result is transformed into decision rules that are used in the discrete-event model. Then we start again with further simulation experiments (see Fig. 1), and so on.

Our contribution is twofold:

- Development and analysis of a general framework (Fig. 1) and a toolbox for the combination of discrete-event simulation and optimization of supply chains.
- For stochastic supply chains, an iterative combination of simulation and linear programming is empirically shown to be competitive compared to deterministic MIP-models.

The paper is organized as follows: We start with a literature review in Sect. 2, followed by a description of the general model framework in Sect. 3. In Sect. 4 we explain the linkage between the simulation model and its linear version. Finally, we report on different test results in Sect. 5 and give conclusions and an outlook for possible future research in Sect. 6.

2 Literature review

2.1 Supply chains

Aspects of the integration of transport and production planning within supply chains have been investigated in several papers (cf. Erengüc et al. 1999). Combined planning approaches for different decision levels (e.g., tactical and operational decisions) can be found in Meyr (2002) and Schneeweiss (2003). There are numerous papers dealing with linear or mixed-integer programs for supply chain networks and network flows. Yaged (1971) discussed in his paper a static network model which includes nonlinearities. He tries to optimize the product flow by solving a linearized version of the network and to improve the flow in the network. Paraschis (1989) discussed several different possibilities to linearize such networks and Fleischmann (1993) presented several applications of network flow models, which are solved through linearization. Pankaj and Fisher (1994) showed that based on an MIP model the coordination of production and distribution can reduce the operating cost substantially. Dogan and Goetschalckx (1999) showed that larger supply chain design problems can be solved using decomposition. A recent case study about a supply chain of the pulp industry modeled as a MIP is given by Gunnarsson et al. (2007). In general the problems solved with LPs and MIPs usually include several simplifications in order to keep them solvable.

Recent publications also included stochastic elements in the optimization models. Santoso et al. (2005) considered a stochastic programming approach for the supply chain network design. They used a sample average approximation and Benders decomposition to solve design problems for a supply chain while considering future operational costs. For that purpose they developed a linear model with uncertain cost factors and demand. Although they used a fast algorithm, realistic problems with sample sizes of up to 60 scenarios need several hours to be solved. Alonso-Ayuso et al. (2003) considered a similar combined design and operation problem. Their stochastic programming approach was able to solve medium sized problems with about 100 binary decisions within 1 h. Leung et al. (2007) presented a robust optimization model for a simultaneous production planning for several sites in a supply chain under uncertainty. But still they are restricted to rather small models and consider only four different scenarios.

In the field of supply chain simulation Kleijnen (2005) gave a short overview of simulation tools and techniques used for supply chains. He distinguished between four different approaches: spreadsheet simulation, system dynamics, discrete-event dynamic systems simulation, and business games. Clearly, discrete-event simulation is the most powerful tool to consider complex stochastic systems. Numerous software packages for discrete-event simulation are available, both very specialized ones for a specific part of the supply chain and general ones with a high functionality in modeling and visualization of supply chains (cf. Kelton et al. 2002; Kuhn and Rabe 1998). One example is the Supply Net Simulator presented by Stäblein et al. (2007). It allows simulating the behavior of individual members in a supply chain network. They used an agent-based approach, where each member optimizes its own operations in the sense of an advanced planning system. But there is no interaction between simulation and optimization.

2.2 Optimization and simulation

Most of today's simulators include possibilities to do a black-box parameter optimization of a simulation model. Glover et al. (1999) presented the successful development of OptQuest (© OptTek Systems, Inc.[1]), an optimization toolbox containing different

[1] http://www.opttek.com.

algorithms (mainly metaheuristics) designed to optimize configuration decisions in simulation models. The simulation model is used only for the evaluation of the objective value, no further structural information is used. Swisher et al. (2000) and Fu (2002) stated in their papers that there is still a big gap between optimization methods for simulation-based optimization used in commercial software and methods available in research literature.

Truong and Azadivar (2003) developed an environment for solving supply chain design problems, where they combine simulation with genetic algorithms and mixed-integer programs. Strategic decisions regarding facility location and partner selection are considered.

The work by Lee and Kim (2002), possibly the most related work in this context, shows a combination of simulation and optimization for the case of a production-distribution system. They use simulation to check the capacity assumptions used for a simpler linear model in a more realistic environment with stochastic machine breakdowns and to update these capacity parameters for the optimization. After several iterations they end up with a solution of the optimization model which is also within the constraints of the stochastic simulation model. Their method is quite similar to our approach, but they aim for more realistic capacity estimation for the optimization model. In contrast, we try to find a robust plan for production, stocking, and transportation considering stochastic and nonlinear operations and costs by estimating delays and cost factors based on simulation experiments.

3 The supply chain network model

The general description of the supply chain originates from a case study about a supply chain in the paper industry (cf. Gronalt et al. 2007). Several production sites are used to manufacture different paper products, which are delivered either directly or via hubs to customers all over the world. The main task in this case study was to develop a 1-year plan for production quantities and transportation links. In this case study a static model was developed, which was used to get rough estimates quickly.

Inspired by this case study we formulate the following problem setting. The basis for our supply chain model is a predefined network, i.e., the locations of all actors and the connections between them are given. Within the network we differentiate between three types of participants connected by transportation links:

- suppliers providing raw materials;
- customers who demand certain products at a specific time;
- production/warehouse sites where production, stocking, and transshipment takes place.

The whole supply chain is order-driven, that means products are manufactured or transported only if a subsequent member of the supply chain requests it. So the origin for all activities is the predefined deterministic demand of the customers. All activities are based on time periods, which might be days or shorter time periods.

The suppliers are used as source for raw materials, which are sent to production sites if requested. Production/warehouse sites can store incoming products. These products can be used to manufacture new products, or they are simply transferred to the output inventory. From there they are sent to subsequent members of the supply chain.

The simulation model is implemented using AnyLogic (© XJ Technologies), a Java-based simulation tool. The model is constructed as a library including several different modules. These modules represent the four types of participants in the supply chain network plus a general control module necessary for controlling the simulation experiments as well as the communication with the optimization model which was developed using Xpress-MP (© Dash Optimization). This model is a simplified deterministic version of the library modules of the simulation model. In this section we will explain the different modules of our supply chain network.

3.1 Module supplier

Simulation model. This module is used to generate certain products, store them, and deliver them if demanded. It has one input port to receive orders for products and one output port to deliver products. If this module receives an order through the input port, then it sends the requested amount of products via the output port. If the amount exceeds the current inventory level, only the available amount is sent. As soon as new products arrive in the inventory they are delivered until the whole order has been fulfilled. The costs arising in this module are only inventory costs for storing products prior to delivery. These costs may have any user-defined functional form. According to the given parameters in each period, new products are generated and added to the stock.

Optimization model. We also developed a simplified representation as an optimization model. We denote by J_S the set of suppliers within a network, by P the set of products, and by T the number of periods. The representation of the supplier's behavior in the optimization model can be formulated as follows (If p and t are free indices, i.e. not used as a summation index, then the set of equations is meant to be valid $\forall t = 1, \ldots, T,\ p \in P$.):

$$\mathrm{TC}_i^S = \sum_{p \in P} \sum_{t=1}^{T} {}^{\mathrm{out}}H_i^p\left({}^{\mathrm{out}}l_i^p(t)\right) \qquad \forall i \in J_S, \tag{1}$$

$${}^{\mathrm{out}}l_i^p(t) = {}^{\mathrm{out}}l_i^p(t-1) - {}^{\mathrm{out}}f_i^p(t) + S_i^p(t) \qquad \forall i \in J_S, \tag{2}$$

$${}^{\mathrm{out}}l_i^p(t) \geq 0 \qquad \forall i \in J_S. \tag{3}$$

For a complete list of parameters and variables, see Appendix C. The overall cost of supplier i is denoted by TC_i^S, consisting only of the holding cost ${}^{\mathrm{out}}H_i^p(\cdot)$ of the output inventory, expressed by the right-hand side of (1). Equations (2) are the inventory balance equations for the output inventories ${}^{\mathrm{out}}l_i^p(t)$. The stock is diminished by the outflow of materials, ${}^{\mathrm{out}}f_i^p(t)$, and increased by the given supply $S_i^p(t)$. The last set of constraints (3) guarantees that the inventory level cannot be negative.

The simulation and the optimization model are connected via the holding costs in (1) which represent the user-defined cost function in the simulation.

3.2 Module production

Simulation model. This module is the core of the whole model. It represents a production site as well as a transshipment point. It consists of an input and an output storage. Items are either transformed into new items or simply transferred from the input to the output storage. This module has one input port and one output port for orders, as well as one input and one output port for products. The input storage is replenished by ordering products via the output port for orders from a supplier or another production module. The ordering policy may be either autonomous (e.g., an (s,S)-policy or any user-defined policy) or it is determined by the result of the optimization model. Products are received through the product input port and stored in the input inventory. The production of new products or the transfer of products is initiated by an internal order placed by the output inventory (either autonomous or based on the solution of the optimization model). The delay for production and transfer is a user-defined function. It may contain stochastic elements and depend on other parameters (e.g., the current load). Production and transfer have limited capacities and furthermore production is restricted to the availability of raw materials (other products). If these capacities do not allow producing (or transferring) a lot as a whole, it is split into several batches. Through the input order port the module receives orders from other production or customer modules. Products are sent through the output product port according to these orders and based on availability. Costs arise in this model for inventory holding (input and output), for production, and for transfer.

Optimization model. The optimization model for the production node is as follows (we denote by J_I the set of production nodes in the supply chain network):

$$\begin{aligned}
\mathrm{TC}_i^I = & \sum_{p\in P}\sum_{t=1}^{T} W_i^p\left(m_i^p(t)\right) + \sum_{p\in P}\sum_{t=1}^{T} Z_i^p\left(u_i^p(t)\right) \\
& + \sum_{p\in P}\sum_{t=1}^{T} {}^{\mathrm{in}}H_i^p\left({}^{\mathrm{in}}l_i^p(t)\right) + \sum_{p\in P}\sum_{t=1}^{T} {}^{\mathrm{out}}H_i^p\left({}^{\mathrm{out}}l_i^p(t)\right) \qquad \forall i \in J_I, \quad (4)
\end{aligned}$$

$$m_i^p(t) \le {}^{\mathrm{prod}}\mathrm{Cap}_i^p(t), \quad \sum_{p\in P} a_i^p \cdot m_i^p(t) \le {}^{\mathrm{prod}}C_i(t) \qquad \forall i \in J_I, \quad (5)$$

$$u_i^p(t) \le {}^{\mathrm{ta}}\mathrm{Cap}_i^p(t), \quad \sum_{p\in P} d_i^p \cdot u_i^p(t) \le {}^{\mathrm{ta}}C_i(t) \qquad \forall i \in J_I, \quad (6)$$

$$\begin{aligned}
{}^{\mathrm{in}}l_i^p(t) = & \; {}^{\mathrm{in}}l_i^p(t-1) + {}^{\mathrm{in}}f_i^p(t) \\
& - \sum_{p'\in P} \alpha_i^p(p') \cdot m_i^{p'}(t) - u_i^p(t) + r_i^p(t) \qquad \forall i \in J_I, \quad (7)
\end{aligned}$$

$$\begin{aligned}
{}^{\mathrm{out}}l_i^p(t) = & \; {}^{\mathrm{out}}l_i^p(t-1) - {}^{\mathrm{out}}f_i^p(t) + \chi_{t\ge\delta_i^p} \cdot m_i^p(t-\delta_i^p) \\
& + \chi_{t\ge\sigma_i^p} \cdot u_i^p(t-\sigma_i^p) + s_i^p(t) \qquad \forall i \in J_I, \quad (8)
\end{aligned}$$

$${}^{\mathrm{in}}l_i^p(t) \le {}^{\mathrm{invin}}\mathrm{Cap}_i^p(t), \quad \sum_{p\in P} q_i^p \cdot {}^{\mathrm{in}}l_i^p(t) \le {}^{\mathrm{in}}L_i(t) \qquad \forall i \in J_I, \quad (9)$$

$$ {}^{\mathrm{out}}l_i^p(t) \le {}^{\mathrm{invout}}\mathrm{Cap}_i^p(t), \quad \sum_{p\in P} q_i^p \cdot {}^{\mathrm{out}}l_i^p(t) \le {}^{\mathrm{out}}L_i(t) \qquad \forall i \in J_I, \tag{10} $$

$$ m_i^p(t) \ge 0, \quad u_i^p(t) \ge 0, \quad {}^{\mathrm{in}}l_i^p(t) \ge 0, \quad {}^{\mathrm{out}}l_i^p(t) \ge 0 \qquad \forall i \in J_I. \tag{11} $$

The overall cost of a production node i is represented by TC_i^I. These costs consist of production costs (the production amounts are denoted by $m_i^p(t)$) transfer costs (the transfer amounts are denoted by $u_i^p(t)$) and the holding costs of the input and the output inventory. Constraints (5) and (6) restrict the production and the transfer for each product individually, as well as for the total production and transfer. In the latter case the amounts are multiplied by the resource requirements. The distinction between individual capacity constraints for each product and global capacity constraints are necessary to cover general situations where different resources as well as common resources are necessary for production. Equations (7) are the inventory balance equations for the input inventories. The current inventory level is determined by the inventory level of the previous period, the inflow from other nodes, the required raw materials for production, the transfer amount and some external inflow (from outside of the system); $\alpha_i^p(p')$ represents the units of raw material p which is necessary to produce one unit of product p'. The inventory balance equations for the output inventories (8) are similar, but the production and transfer delays (δ_i^p, σ_i^p) have to be considered before a new product arrives in the output inventory. Function $\chi_{t\ge\varepsilon}$ is an indicator function, used in order to avoid the use of production and transfer amounts for negative periods. Equations (9) and (10) are used to restrict the stock of the input and the output inventory (for each product separately and accumulated using the space requirements q_i^p). These two types of restrictions allow modeling a dedicated-storage as well as a random-storage policy.

The simulation model and the optimization model are connected through the cost factors in Eqs. (4) and production and transfer delays (δ_i^p, σ_i^p), which are user-defined functions in the simulation model possibly containing stochastic and nonlinear elements. Furthermore the production and transfer amounts ($m_i^p(t), u_i^p(t)$) of the optimization model are used to determine production plans in the simulation model. For example, if $m_i^p(t) > 0$ for a specific product and period, then in the simulation model the amount given by $m_i^p(t)$ is ordered in period t.

3.3 Module customer

Simulation model. According to a given demand table, the customer places orders at the production sites. Due to stochastic features within the simulation, it is not possible to time deliveries exactly. Therefore the customer has an input inventory, which is used to satisfy the demand. The inventory level can be negative (shortages), as well as positive (oversupply). In both cases penalty costs occur, which are higher for shortages. The module has one output port for sending requests and one input port for receiving products. The orders are sent either according to the demand table (including a standard delay time for transportation) or according to the solution of the optimization model.

Optimization model. The optimization model for the customers' behavior can be written as follows (we denote by J_C the set of customer nodes in the supply chain network):

$$\mathrm{TC}_i^C = \sum_{p\in P}\sum_{t=1}^{T} R_i^p({}^{\mathrm{in}}b_i^p(t)) \qquad \forall i \in J_C, \tag{12}$$

$$\begin{aligned} {}^{\mathrm{in}}l_i^p(t) - {}^{\mathrm{in}}b_i^p(t) &= {}^{\mathrm{in}}l_i^p(t-1) - {}^{\mathrm{in}}b_i^p(t-1) + {}^{\mathrm{in}}f_i^p(t) \\ &\quad - D_i^p(t) + r_i^p(t) \qquad \forall i \in J_C, \end{aligned} \tag{13}$$

$${}^{\mathrm{in}}l_i^p(t) = 0 \qquad \forall i \in J_C. \tag{14}$$

In (12) we calculate the cost at the supplier which consists only of penalty cost for back orders ${}^{\mathrm{in}}b_i^p(t)$. Equations (13) are the inventory balance equations where the customers' demands $D_i^p(t)$ are considered. It is assumed that all customers are just-in-time customers. Therefore, constraints (14) ensure that no oversupply (positive stock level) is possible, i.e. it is not allowed to send more products than demanded by the customers. This JIT assumption may be dropped and holding costs for positive stock may be included. In the simulation model the JIT assumption is weakened, because stochastic transportation times may cause an unwanted early delivery. These early deliveries are penalized.

The differences between the simulation model and its representation as an optimization model are the penalty cost factors in (12) and the JIT assumption expressed in (14).

3.4 Module transport

Simulation model. This module is used to transport products between different modules. It receives products through its single input port and sends it (according to some time delay) through the output port to the next module (*Production* or *Customer*). It has a limited capacity and organizes the transports according to a FIFO rule. It is also possible to split shipments if the available capacity does not allow single shipment. The time delay may be stochastic and may depend on other parameters. User-defined costs arise for transportation and may include transportation time, amounts, and fixed charge parts.

Optimization model. The representation of the transport modules as an optimization model can be formulated as follows. Each transport module is identified by the indices of the nodes, which it connects. Furthermore, we need an additional index $v \in V$ denoting the different transport modes (if v is not used as a summation index, the equations are valid for all $v \in V$):

$$\mathrm{TC}_{ij}^T = \sum_{t=1}^{T}\sum_{v\in V}\sum_{p\in P} {}^{v}C_{ij}^p\left({}^{v}x_{ij}^p(t)\right) \qquad \forall i \in J_S \cup J_I,\ j \in J_I \cup J_C, \tag{15}$$

$$^{v}x_{ij}^{p}(t) \le {}^{v}\mathrm{Cap}_{ij}^{p}(t), \sum_{p\in P} {}^{v}g^{p} \cdot {}^{v}x_{ij}^{p}(t) \le {}^{v}C_{ij}(t) \quad \forall i \in J_S \cup J_I,\ j \in J_I \cup J_C, \tag{16}$$

$$^{\mathrm{in}}f_j^p(t) = \sum_{\substack{i\in J_s\cup J_I \\ {}^{v}\tau_{ij}<t}} \sum_{v\in V} {}^{v}x_{ij}^{p}(t - {}^{v}\tau_{ij}) \qquad \forall j \in J_I \cup J_C, \tag{17}$$

$$^{\mathrm{out}}f_i^p(t) = \sum_{j\in J_I\cup J_C} \sum_{v\in V} {}^{v}x_{ij}^{p}(t) \qquad \forall i \in J_S \cup J_I, \tag{18}$$

$$^{v}x_{ij}^{p}(t) \ge 0 \qquad \forall i \in J_S \cup J_{I,} j \in J_I \cup J_C. \tag{19}$$

The total transportation cost for transports from member i to member j of the supply chain network is denoted by TC_{ij}^{T} and ${}^{v}x_{ij}^{p}(t)$ gives the transportation amount for each period, each product, and each transportation mode. Constraints (16) limit the transportation to a product-specific and an overall capacity limit. Equations (17) and (18) represent the inflow of products to member j and the outflow of products from member i.

The connection between the simulation and the optimization model is established by the transportation cost functions in (15) and the transportation delays ${}^{v}\tau_{ij}$ in (17) on the one hand, and through the transportation amounts ${}^{v}x_{ij}^{p}(t)$ on the other hand. These transportation amounts are used to define ordering schemes for the *Production* and *Customer* modules in the simulation model. That means if ${}^{v}x_{ij}^{p}(t) > 0$, then member j of the supply chain sends a request for ${}^{v}x_{ij}^{p}(t)$ units of product p to member i of the supply chain at time t.

3.5 Supply chain optimization model

The optimization model of the whole supply chain network is defined by minimizing the total cost

$$\min \sum_{i\in J_S} \mathrm{TC}_i^S + \sum_{i\in J_I} \mathrm{TC}_i^I + \sum_{i\in J_C} \mathrm{TC}_i^C + \sum_{i\in J_S\cup J_I} \sum_{j\in J_I\cup J_C} \mathrm{TC}_{ij}^T \tag{20}$$

subject to the constraints (1)–(19). If we assume that all cost functions are linear, i.e. that the objective (20) is a linear function, we can write it as follows:

$$\begin{aligned} \min &\sum_{ij\in J} \sum_{p\in P} \sum_{t=1,..T} \sum_{v\in V} {}^{v}c_{ij}^{p} \cdot {}^{v}x_{ij}^{p}(t) + \sum_{i\in J_I} \sum_{p\in P} \sum_{t=1,..T} w_i^p \cdot m_i^p(t) \\ &+ \sum_{i\in J_I} \sum_{p\in P} \sum_{t=1,..T} z_i^p \cdot u_i^p(t) + \sum_{i\in J_I} \sum_{p\in P} \sum_{t=1,..T} {}^{\mathrm{in}}h_i^p \cdot {}^{\mathrm{in}}l_i^p(t) \\ &+ \sum_{i\in J_S\cup J_I} \sum_{p\in P} \sum_{t=1,..T} {}^{\mathrm{out}}h_i^p \cdot {}^{\mathrm{out}}l_i^p(t) + \sum_{i\in J_C} \sum_{p\in P} \sum_{t=1,..T} \rho_i^p \cdot {}^{\mathrm{in}}b_i^p(t). \end{aligned} \tag{21}$$

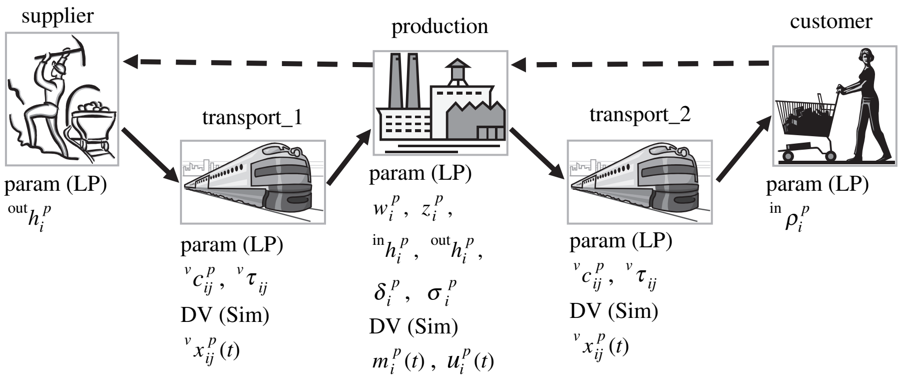

Fig. 2 An example of the module configuration for a simple supply chain consisting of one supplier, one production site, and one customer (dashed lines indicate information flow and solid lines indicate material flow). For the case of a linear program the information below the modules represents the parameters which are calculated during the simulation runs and transferred to the LP, param (LP), and the decision variables of the LP used as decision rules in the simulation model, DV (Sim.)

Hence, we get a linear programming model which we will use in connection with the simulation model as depicted in Figure 1. A detailed description of the linear model can be found in Preusser et al. (2005a,b).

This model is a pure linear program which can be solved easily with any standard LP-solver within very short time. If necessary, it is possible to extend the model formulation to consider more features, e.g., fixed-cost transportation, binary decisions, step functions, etc. These extensions lead to a mixed-integer formulation, thus increasing computational time (cf. Appendix A).

3.6 Supply chain simulation model

Implementing a simulation model in AnyLogic means to arrange the according modules and connect them. In Fig. 2 an illustrative example shows, how these modules can be connected in order to maintain information flow (direct connections between *Supplier, Production*, and *Customer*) and material flow (via *Transport* modules). Furthermore the cost and delay functions for each module must be specified.

4 Connecting the optimization with the simulation

In order to couple the optimization model and the simulation model, we first have to define the required data and the way they should be exchanged. We decided to use an MS Access database to store all necessary information which is:

- General network structure: This includes the number of actors in the supply chain and the according links between them.
- General parameters used in the simulation and optimization models: These sets of parameters include all capacity limitations, resource parameters, bill-of-materials, predefined supply at the suppliers, and predefined demand at the customers.

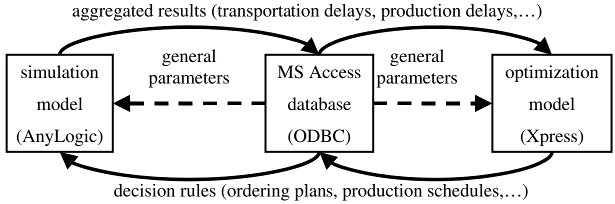

Fig. 3 This scheme shows the data exchange between the simulation and the optimization model via the MS Access database in the middle

- Results of the optimization model (= parameters for the simulation): The results of the optimization used as decision rules in the simulation are production and transfer quantities, as well as transportation amounts $(m_i^p(t), u_i^p(t), {}^v x_{ij}^p(t))$.
- Results of the simulation model (= parameters for the optimization model): The main results of the simulation experiments used in the optimization model are the cost parameters and the delays for production, transfer and transport.

The simulation model is designed as the master process, which controls the data communication and the LP/MIP-solver. The simulation model and the optimization model retrieve and store values from and to the database using the Open Database Connectivity (ODBC) interface (see Fig. 3).

To initiate the optimization process in our system, a few simulation runs are performed using the data from the database. Missing decision rules which, in later iterations, are generated using the results of the optimization model, are substituted by autonomous decision rules (like an (s,S)-policy for the replenishment). These first simulation runs are only necessary to generate initial parameter values for the optimization model, but their results will be ignored in further iterations in order to avoid biasing effects caused by the autonomous decision rules. The results of the initial runs (delays, per unit costs, etc.) are aggregated and according means and variances are stored in the database (see Sect. 4.1). Afterwards Xpress-MP is executed. It loads the general data and the simulation results from the database, computes the solution of the optimization model and stores the results (ordering and delivery plans, production and transfer schedules, etc.) in the database. Then we start again with five simulation experiments using now the newly computed decision rules (see Sect. 4.2), based on the solution of the optimization model. Further on we will denote this algorithm by SimOpt (or SimLP, if a pure LP model is used for the optimization part, and SimMIP, if a mixed-integer formulation is used). In Table 1 a pseudo code of this SimOpt algorithm is given.

4.1 Aggregating simulation results

Since the simulation model may contain stochastic and nonlinear elements, it is necessary to perform several simulation runs and combine the results. For the cost parameters, necessary for a linear optimization model, we calculate average per unit cost. That means, e.g., for the production costs we accumulate the total cost for the

Table 1 Pseudo code for the combined simulation optimization approach SimOpt

```
SimOpt:
Load necessary simulation parameters from the database
Perform a few simulation runs using autonomous decision rules
Aggregate results and store them in the database
while stopping criteria are not met
   Load aggregated parameters into LP/MIP-Solver
   Solve the optimization model
   Write new decision rules to the database
   Load new decision rules into simulation model
   Perform simulation runs using these decision rules
   Aggregate results and store them in the database
end-while
```

whole planning horizon for a certain product and divide these costs by the number of products produced. Other parameters, called critical parameters, have a direct influence on the material flow (e.g., transportation delay). The use of average values for those parameters would most probably lead to bad results. In about half of the cases the delay would be longer than assumed and would cause additional delays in subsequent operations. Therefore, it seems reasonable to use, e.g., a 90%-quantile (based on a normal distribution with estimated mean and variance calculated from the different simulation runs) for such delay parameters. This results in an overestimation of the delays for the optimization model, because the time is determined such that 90% of the observed delays will be shorter, but it ensures that a smooth material flow through the network is possible.

For the critical parameters it is useful to combine results from the previous iterations with current ones, in order to enlarge the sample size and to get better estimates of the mean and the variance.

4.2 Decision rules based on the solution of the optimization model

There are several possible ways to use the solution of the LP-model within the simulation model. One method, which we apply here, is to use the transportation, production and transfer results $({}^v x_{ij}^p(t), m_i^p(t), u_i^p(t))$ as a given ordering plan. According to the transportation values ${}^v x_{ij}^p(t)$ module j sends a request to module i at time t for the given amount ${}^v x_{ij}^p(t)$ of products of type p using the transportation mode v. Similarly, production and transfer results can be used. In some cases, due to the stochastic features of the simulation model, it may happen that some of the modules are out-of-stock for a specific product. Since unfulfilled orders are backlogged, these requests are fulfilled as soon as the products are available.

More complex procedures would be, e.g., to use results of the sensitivity analysis (dual variables, reduced costs) to determine the critical parameters, to observe these parameters during the simulation runs, and to adapt decision rules (use a different

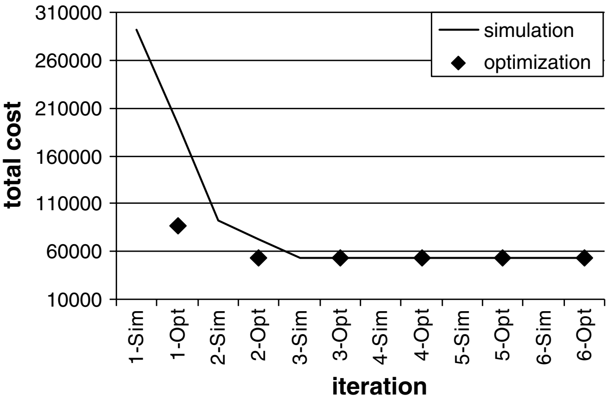

Fig. 4 Objective values of the optimization model and the simulation model for each iteration for a deterministic model considering fixed costs for production, transfer and transport

solution of the optimization model) if the observed parameters reach a certain threshold.

For our examples we use the first approach for translating the solution of the optimization model into decision rules for the simulation model (cf. Sect. 3). In this paper we wish to investigate the direct interactions between the solution of the optimization model and the simulation results. The analysis of more complex decision rules goes beyond the scope of this paper and might by a subject for further research.

5 Tests and results

We wish to investigate the following research questions with empirical tests using a set of test instances:

- Does this method converge in practice for realistic test cases?
- If we can observe convergence, is the result optimal or at least a good approximation?
- Is this method advantageous compared with traditional planning methods?

Although it is not possible to prove general convergence for all our test instances, we observe fast convergence of the objective values of the simulation and optimization model. Figure 4 shows a typical situation using the results of the deterministic test instance D1-L described in Sect. 5.1.

We start with the simulation model using an autonomous rule for replenishing the inventories. Since we start with all inventories empty, it takes a long time, until the orders are fulfilled. This causes high costs and an overestimation of transportation and production delays. Therefore, the first solution of the linear model leads also to a high objective value mainly consisting of penalty costs for late (or even no) deliveries. Consequently, the simulation model leads to a similar objective function in iteration 2, because it uses the delivery plans of the solution of the linear model. Due to the fact that the solution of the linear model causes a somehow synchronized material flow, the measured delays are much lower now. Therefore, the cost of the solution of the linear model in the second iteration decreases. After three iterations the simulation and the linear model have converged to the same solution.

5.1 Deterministic problems with fixed costs

In order to verify the quality of the solutions, we create a set of 12 examples. For these test instances we consider a simple supply chain consisting of three actors (a supplier, a producer and a customer) and a time horizon of 30 periods. For transportation of products two transport modules are used, which connect the supplier and the producer as well as the producer and the customer. Two types of products are demanded by the customer: product 1 which is provided by the supplier and sent via the producer to the customer and product 2 which is manufactured by the producer using product 1 as a raw material. The cost structure is as follows:[2]

- The transportation costs consist of fixed costs per delivery, which are subject to a step function. A transport costs 100, 200 or 300 monetary units, depending on the amount delivered.
- The costs of production and transfer are separated into variable costs and fixed costs. The variable production costs are set to 30 and the fixed part is 50 monetary units. Transferring products costs 15 per product unit plus a fixed part of 10.
- Delayed deliveries are penalized by 100 monetary units per product unit and period.

Concerning the demand at the customer we distinguish between instances with high demand and others with low demand. The difference lies in the frequency of orders sent off by the customer. In high demand cases the occurring orders in each period are around the maximum possible quantities which could be delivered considering the capacities of the supplier and the producer. In low demand models the ordered amounts cover approximately 70% of the possible deliveries in each period. Instances D1-L to D5-L (see Table 2) represent five different realizations for low demand models. Accordingly, D6-H to D10-H correspond to five different realizations of high demand models. The last two instances, D1a-L and D6a-H, are modifications of instances D1-L and D6-H, respectively. The former ones consider exactly the same ordering amounts as D1-L and D6-H but the fixed costs for production and transfer at the intermediate node are increased to 1000 and 500, respectively.

For examples of this size it is possible to formulate an exact mixed-integer model and determine the optimal solution. The corresponding MIP formulation consists of 1,342 constraints, 1,080 continuous and 300 binary decision variables. For the simulation approach the nonlinear parts are only considered in the simulation model itself, the connected linear (non-integer) model does not include any of them. See Table 2 for the resulting total costs of the simulation and the optimal solution (MIP).

The gap between our SimLP approach the optimal solutions gained by solving the exact MIP formulation varies between 0.44 and 3.46% and averages in 1.87%. As we would expect, for the two test instances with high fixed cost the gap increases.

[2] All datasets are available at http://www.univie.ac.at/bwl/prod/download/SCM-Data.

Table 2 Comparison of total costs between our simulation-based optimization approach SimLP and the exact MIP-model for deterministic test cases classified by the occurrence of customer demand (H—high demand, L—low demand)

Instance	SimLP	Exact MIP	Difference (%)
D1-L	53640	52947	1.31
D2-L	55032	53860	2.18
D3-L	52626	52394	0.44
D4-L	54442	53600	1.57
D5-L	55198	54057	2.11
D6-H	59885	58830	1.79
D7-H	61257	60129	1.88
D8-H	59028	58347	1.17
D9-H	60403	59501	1.52
D10-H	61436	60365	1.77
D1a-L	63720	61587	3.46
D6a-H	76165	73761*	3.26
Average	**77165**	**73760**	**1.87**

MIP solutions marked with (*) are best solutions found after one hour calculation time

For test instances with low demand the variation of the gap seems higher than for the test instances with high demand. But on average there seems no significant difference between the results of those two groups of instances.

Based on the above results we may conclude that the error caused by neglecting fixed cost is low as long as the fixed costs are low compared with other costs. If the fixed costs increase (relative to the other costs), the nonlinear properties should be considered in the optimization model used for the SimOpt approach, i.e. a SimMIP should be used instead of the SimLP (see also the following subsection).

5.2 Test instances with stochastic transport delays

In order to measure the quality of our solutions in a stochastic environment we prepare a set of test examples including stochastic transportation times. We compare our Sim-LP approach using a simplified linear model without binary variables with an exact MIP-model. This MIP-model does not cover stochastic features and we have to provide estimated values of the transportation times. Within the simulation we consider uniformly distributed transportation delays between 1 and 9 for transportations from the supplier to the producer and between 1 and 5 for transportations from the producer to the customer. For estimating the delay parameters we perform runs using 90%-, 70%-, and 50%-quantiles. The corresponding transportation delays for the MIP-model are set according to the used quantile. For the small test cases there would probably be no noticeable difference between the results of a 99%- and a 90%- quantile. Therefore, we test a high quantile (90%, risk averse), the average value (50%), and some intermediate value (70%). The maximum runtime is set to 30 minutes for the MIP-model, i.e. we report the best solution found after this time limit, while the simulation approach converges after a few seconds. For the simulation approach we again process 8 iterations, each consisting of 5 simulation runs and one LP computation. Finally the solution of the MIP-model and the solution of the SimLP are evaluated by performing

Table 3 Difference of the mean total costs of 20 runs between the SimLP and SimMIP method and the solution found by a deterministic MIP model classified by the occurrence of customer demand (H—high demand, L—low demand)

Instance	MIP		SimLP			SimMIP		
	Cost	Quant. (%)	Cost	Quant. (%)	Diff. (%)	Cost	Quant. (%)	Diff. (%)
S1-L	66400	90	62601	90	−5.72	61637	90	−7.17
S2-L	61338	90	60635	90	−1.15	60282	90	−1.72
S3-L	63323	90	63566	70	0.38	63618	70	0.47
S4-L	63122	90	64067	90	1.50	64060	90	1.49
S5-L	60954	90	62399	90	2.37	62229	90	2.09
S6-H	72485	90	72342	90	−0.20	70871	90	−2.23
S7-H	70928	90	70751	90	−0.25	71040	90	0.16
S8-H	73257	90	77537	70	5.84	74999	70	2.38
S9-H	73501	90	74637	70	1.55	72845	90	−0.89
S10-H	71606	90	70230	90	−1.92	73934	90	3.25
S1a-L	71511	90	71686	90	0.25	70350	90	−1.62
S6a-H	88442	70	90582	90	2.42	88442	70	0.00
Av.-L	**64441**		**64159**		**−0.39**	**63696**		**−1.08**
Av.-H	**75037**		**76013**		**1.24**	**75355**		**0.44**
Average	**69739**		**70086**		**0.42**	**69526**		**−0.32**

The total costs are reported in the columns *Cost*. The quantile which lead to the best results is reported in the column *Quant*. The difference with respect to the solution of the deterministic MIP is denoted in column *Diff*.

20 independent simulation runs. Furthermore, we replace the simplified linear model with the MIP model (SimMIP approach) and perform the same tests. The results for all three methods are displayed in Table 3, where negative percentage values imply that the simulation achieves a better result than the exact MIP-model.

The results of the SimLP approach, where we combine the pure LP model with the simulation model, are on average slightly worse compared with the deterministic MIP approach. For the low demand cases alone we can observe a small improvement. Considering the variation of the results these differences are not significant.

Furthermore, we test a second approach, the SimMIP approach, where all nonlinear features of the model are also considered in the optimization part which is represented by a MIP model. So the difference between the SimMIP and the exact approach is only, that in the first case the parameters are estimated based on simulation experiments and in the latter case the parameters are determined using the known distribution functions. Here we see that the solution quality can be raised for 9 out of 12 test instances and on average the result is slightly better than the deterministic MIP approach.

Even if for some instances the 70%-quantile yields the best results, the 90%-quantile leads to only slightly higher costs. Using the 50%-quantile, i.e. the expected value, always caused much higher costs.

Additionally we analyze the variation of the 20 final simulation runs for each method. There is no significant difference of the variation for all methods. The coefficient of variation for the total costs is always around 0.07.

For larger test instances it would not be possible to solve the MIP or to apply the SimMIP method. In a preliminary study (cf. Mitrovic 2006) we focused on that issue and tried to find the approximate limits of solving MIP formulations of supply chain problems by the means of three state-of-the-art LP/MIP-solvers. The tests were conducted on a PC (Intel P4 2.4 GHz, 1 GB RAM) using Windows 2000. We used a set of different sized supply chain network problems considering fixed costs in transportation, production and transfer. The best performing solver succeeded in solving problems with 20 supply chain actors, 8 products, 5 periods and 2 transportation modes, considering 3,360 binary variables before and 250 binary variables after presolving. The next in size, which included 10 products instead of 8, could not be solved within a time limit of one hour. Thus, there is a trade-off between a good approximation resulting from MIP-models or fast computational times. Definitely, important decisions involving high fixed costs should be considered within the optimization model of our SimOpt method.

5.3 Quantile tests on larger instances

In addition to the small instances used in the previous subsections we generate a set of 12 instances representing larger supply chain networks. Using these test instances we analyze the influence of the quantile on the results if the uncertainty is concentrated in a specific part of the supply chain. The size and structure of these test cases is shown in Fig. 5.

This fictitious supply chain network consists of 10 actors: 3 suppliers, 4 production nodes, and 3 customers. The intermediate nodes are separated into two layers and all of them are authorized to produce and also transfer products. All actors are connected

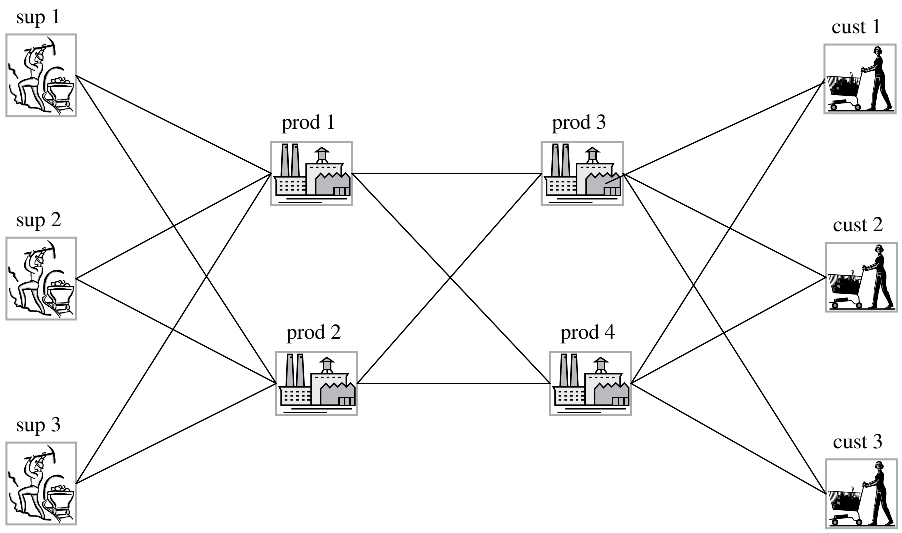

Fig. 5 Exemplary supply chain network. For simplicity the transport modules have been omitted

Table 4 Total costs of SimLP for test example with ten supply chain actors and stochastic transportation delays at the beginning of the supply chain

Instances	Quantile		
	90%	70%	50%
L1-L-S	274239	−0.19%	11.20%
L2-L-S	274241	9.98%	6.00%
L3-L-S	274995	5.76%	10.85%
L4-L-S	275366	5.83%	11.51%
L5-L-S	273214	1.33%	9.80%
L6-H-S	270286	2.72%	10.10%
L7-H-S	270491	2.23%	4.51%
L8-H-S	270438	−0.59%	10.92%
L9-H-S	267766	2.71%	8.65%
L10-H-S	270155	−0.57%	10.12%
L1a-L-S	333772	2.77%	4.04%
L6a-H-S	346953	1.42%	4.52%
Total Avg.	**283493**	**2.78%**	**8.52%**

The results for the 90%-quantile are taken as basic values. For the remaining quantiles the difference to the corresponding basic value is given. The *S* indicates that there is more stochasticity close to the supplier

by one transportation mode. The customers request 4 different products. Products 1 and 2 are on the one hand final products, which have to be delivered to the customers and on the other hand raw materials used to produce products 3 and 4.

First we consider the case with the stochasticity concentrated at the beginning of the supply chain. Hence, for the connection between the suppliers and the first layer of production sites we assume stochastic transportation times, which are uniformly distributed between 1 and 5. The transportation times between the two layers of production nodes are uniformly distributed between 1 and 3. For the remaining links we assume deterministic transportation times of 1. The costs functions for production and transport consist of fixed costs and variable costs. If all binary decisions would be considered in a MIP-model, this would lead to more than 3,800 binary variables, which is beyond the size of problems we could solve with the best MIP/LP-solvers within several hours. In comparison, our SimLP algorithm takes about 12 min for one test instance to converge to a solution. We evaluate three different values for the quantile used for the estimation of the delay parameters: 90, 70, and 50%. See Table 4 for the results.

In this case it seems that the 90%-quantile is most robust choice, although the 70%-quantile delivers only slightly worse results on average and in some cases even better ones, whereas the 50%-quantile leads to the worst results for all instances. Due to the fact that there is less stochastic near to the customer, it is possible to reduce the safety factors for the delays to some extent without increasing the costs too much, because lost time at the beginning of the supply chain can be made up at the end.

We also conduct experiments where the transportation delays at the end of the supply chain are stochastic, i.e. the connection between production nodes and customers are uniformly distributed between 1 and 5. Transportation times between the two layers of production nodes are again uniformly distributed between 1 and 3. Remaining transportation times are set to 1. The corresponding results are summarized in Table 5.

Table 5 Total costs of SimLP for test example with ten supply chain actors and stochasticity concentrated near the end of the supply chain

Instance	Quantile		
	90%	70%	50%
L1-L-C	263957	11.70%	43.96%
L2-L-C	263252	16.81%	45.02%
L3-L-C	263948	13.79%	48.81%
L4-L-C	263114	12.65%	50.60%
L5-L-C	263707	13.98%	51.41%
L6-H-C	258440	18.11%	52.09%
L7-H-C	257963	14.60%	59.29%
L8-H-C	260606	12.67%	41.68%
L9-H-C	258762	24.74%	52.44%
L10-H-C	258936	17.52%	43.16%
L1a-L-C	335837	7.15%	32.56%
L6a-H-C	335973	9.61%	32.95%
Total Avg.	**273708**	**14.45%**	**46.16%**

The results for the 90%-quantile are taken as basic values. For the remaining quantiles the percentage difference to the corresponding basic value is given. The *C* indicates that there is more stochastic at the customer

For these instances the best choice would be to use the highest safety factor (90%), because if there are delays at the end of the supply chain, there is no chance to catch up.

For the test instances L1a-H-C and L6a-H-C with high fixed cost, we also apply a SimMIP method where we include only the high fixed-charge production costs. The low fixed-charge transportation costs are still neglected. For all quantiles the SimMIP method delivers slightly lower costs (L1a-L-C: $-4.48\%/-10.86\%/-1.19\%$ for 90%/70%/50%-quantiles; L6a-H-C: $-3.47\%/-2.96\%/-0.37\%$ for 90%/70%/50%-quantiles) but the calculation times are more than five times longer.

If we assume stochastic transportation times for the whole supply chain (all transportation delays are uniformly distributed between 1 and 5), the results are similar to those in Table 5, i.e. the 90%-quantile is always the best choice.

6 Conclusions

In this paper we have presented a new approach that combines the advantages of complex simulation models and abstract optimization models. We have shown that our method is able to generate competitive solutions quickly, even compared with traditional planning approaches that are much more time consuming. Our investigations can be summarized as follows:

- In many cases the SimLP method seems to be a good trade-off between solution quality and computational time. If the nonlinear elements in the model are dominating it is better to apply the SimMIP approach and consider these nonlinearities in the optimization model as along as the computational time for solving the optimization model is acceptable.

- Furthermore, we investigated the impact of safety times for delays on the solution quality. If we use the 90%-quantile, we can generate robust plans, but for specific situations we might get better results with less safety time. Only for the case if stochastic is near the customer, then the 90%-quantile is clearly the best. Nevertheless, the choice of the quantile depends on the structure of the supply chain and has to be fine-tuned in each case.
- Using the 50%-quantile, i.e. the expected values for the delays, always leads to pure results. If the uncertainty is concentrated far away from the customer, the cost increase by using the expectation value is about 10% whereas the increase is almost 50% if the uncertainty occurs close to the customer.

Further research for different aspects of this method is still possible and necessary. The aggregation step and the generation of new decision rules is an open field. One possibility is to interpret the solution of the optimization model only as a target strategy and use adaptive decision rules to approximate this target strategy in an uncertain environment. The use of sensitivity results of the optimization model might lead to improved decision rules. Further investigations are possible for the boundaries between the simulation and the optimization model. The question, which aspects should be included in the optimization model, is not completely answered yet. If more complex models are used, other fast solution methods (e.g., heuristics, metaheuristics, etc.) should be taken into consideration.

We conclude by answering the question posed in the title of this paper: simulation and optimization are complementary approaches and it is worthwhile combining them.

Acknowledgment We wish to thank Martin Grunow and two anonymous referees for their valuable comments on this manuscript.

Appendix A: MIP formulation for fixed-charge transportation cost

The objective (20) of the optimization model can be transformed into a mixed-integer program considering for example fixed transportation costs. In case, the transportation cost functions ${}^{v}C_{ij}^{p}\left({}^{v}x_{ij}^{p}(t)\right)$ can be written as follows:

$$ {}^{v}C_{ij}^{p}\left({}^{v}x_{ij}^{p}(t)\right) = \begin{cases} {}^{v}c_{ij}^{p} & \text{if } {}^{v}x_{ij}^{p}(t) > 0 \\ 0 & \text{otherwise} \end{cases} \qquad \forall i, p, t, v. \tag{22} $$

In order to capture this situation, it is necessary to introduce binary decision variables ${}^{v}\Gamma_{ij}^{p}(t)$ which indicate if there is positive transportation. So by adding the following constraints:

$$ {}^{v}x_{ij}^{p}(t) \leq G \cdot {}^{v}\Gamma_{ij}^{p}(t) \qquad \forall i, p, t, v \tag{23} $$

it is possible to formulate the transportation costs as the linear functions

$$^{v}C_{ij}^{p}(t) = {}^{v}c_{ij}^{p}\,{}^{v}\Gamma_{ij}^{p}(t) \qquad \forall i, p, t, v. \tag{24}$$

The resulting mixed-integer linear program includes now $|J_I| \times |P| \times T \times |V|$ binary decision variables.

A similar approach can be used for modeling step functions like

$$^{v}C_{ij}^{p}\left({}^{v}x_{ij}^{p}(t)\right) = \begin{cases} {}^{v}c_{ij}^{p} & \text{if } {}^{v}x_{ij}^{p}(t) \in ({}^{v}X_{ij}^{p}, {}^{v}Y_{ij}^{p}] \\ {}^{v}d_{ij}^{p} & \text{if } {}^{v}x_{ij}^{p}(t) \in (0, {}^{v}X_{ij}^{p}] \\ 0 & \text{otherwise} \end{cases} \qquad \forall i, p, t, v. \tag{25}$$

Here we need 2 different binary decision variables ${}^{v}\Gamma_{ij}^{p}(t)$ and ${}^{v}\Delta_{ij}^{p}(t)$ to represent this situation. If we add two additional constraints

$$^{v}x_{ij}^{p}(t) \leq G \cdot {}^{v}\Gamma_{ij}^{p}(t) \qquad \forall i, p, t, v, \tag{26}$$

$$^{v}x_{ij}^{p}(t) \leq G \cdot {}^{v}\Delta_{ij}^{p}(t) + {}^{v}X_{ij}^{p}(t) \qquad \forall i, p, t, v, \tag{27}$$

the cost functions can be written as

$$^{v}C_{ij}^{p}(t) = {}^{v}d_{ij}^{p}\,{}^{v}\Gamma_{ij}^{p}(t) + \left({}^{v}c_{ij}^{p} - {}^{v}d_{ij}^{p}\right){}^{v}\Delta_{ij}^{p}(t) \qquad \forall i, p, t, v. \tag{28}$$

The resulting mixed-integer linear program includes now $2 \times |J_I| \times |P| \times T \times |V|$ binary decision variables.

Appendix B: Notation

Notation used for the optimization model

J	set of locations $J = J_S \cup J_I \cup J_C$
$j \in J_S$	raw-material supplier (starting nodes)
$j \in J_C$	customer (end nodes)
$j \in J_I$	nodes between supplier and customer
P	set of products
V	set of transportation modes
T	number of periods

Decision variables

$m_i^p(t)$	amount of product p (product p is the end product of the production process at location i) that starts to be produced at location i in period t
$u_i^p(t)$	amount of product p that starts to be transacted in location i in time period t
${}^{v}x_{ij}^{p}(t)$	flow of product p from location i to location j with transportation mode v (sent away in period t)

Costs, delays, and general parameters

a_i^p	factor indicating the amount of capacity units required to produce one unit of product p at location i
$\alpha_i^p(p')$	amount of product p' required to produce one unit of product p at location i
${}^{\mathrm{in}}b_i^p(t)$	amount of backorders of product p at customer i in period t
${}^{v}C_{ij}^p(\cdot)$	transportation cost function of product p transported from location i to location j with transportation mode v
${}^{v}C_{ij}(t)$	maximum transportation capacity of transportation mode v on the way from location i to location j
${}^{\mathrm{prod}}C_i(t)$	maximum production capacity at location i in period t
${}^{\mathrm{ta}}C_i(t)$	maximum transaction capacity at location i in period t
${}^{\mathrm{invin}}Cap_i^p(t)$	maximum amount of product p that can be held in the inbound inventory of intermediate i in period t
${}^{\mathrm{invout}}Cap_i^p(t)$	maximum amount of product p that can be held in the outbound inventory of intermediate i in period t
${}^{v}Cap_{ij}^p(t)$	amount of product p that transportation mode v can transport from location i to location j in period t
${}^{\mathrm{prod}}Cap_i^p(t)$	amount of product p that can be produced at location i in period t
${}^{\mathrm{ta}}Cap_i^p(t)$	amount of product p that can be transacted at location i in period t
${}^{v}c_{ij}^p$	cost factor used in case of linear transportation costs for deliveries of product p between location i and location j with transportation mode v
$D_i^p(t)$	demand for product p at location i in period t
d_i^p	factor indicating the amount of capacity units required to transact one unit of product p at location i
δ_i^p	amount of periods required to produce product p at location i
${}^{\mathrm{in}}f_j^p(t)$	amount of product p arriving at location j in period t
${}^{\mathrm{out}}f_j^p(t)$	amount of product p sent away at location j in period t
${}^{v}g^p$	factor indicating the amount of capacity units required to transport one unit of product p with transportation mode v
${}^{\mathrm{in}}H_i^p(\cdot)$	inbound inventory cost function for product p at location i
${}^{\mathrm{out}}H_i^p(\cdot)$	outbound inventory cost function for product p at location i
${}^{\mathrm{in}}h_i^p$	cost factor used in case of linear inventory costs for the inbound inventory of actor i and for product p
${}^{\mathrm{out}}h_i^p$	cost factor used in case of linear inventory costs for the outbound inventory of actor i and for product p
${}^{\mathrm{in}}L_i(t)$	maximum capacity of inbound inventory at location i in period t
${}^{\mathrm{out}}L_i(t)$	maximum capacity of outbound inventory at location i in period t
${}^{\mathrm{in}}l_i^p(t)$	inbound inventory level of product p at location i in period t
${}^{\mathrm{out}}l_i^p(t)$	outbound inventory level of product p at location i in period t
q_i^p	factor indicating the amount of capacity units required to hold one unit of product p at the inventory of location i
$R_i^p(\cdot)$	penalty cost function at location i for product p

$r_i^p(t)$ amount of product p which is already transported at period 0 and will arrive at location i in period t (or external increase of inventory)
ρ_i^p cost factor used in case of linear penalty costs at customer c and for product p
$S_j^p(t)$ supply of product p at location j in period t
$s_i^p(t)$ amount of product p which is already in production process in period 0 and will be finished in period t (or external increase of inventory)
σ_i^p amount of periods required to transact product p at location i
${}^v\tau_{ij}$ amount of periods transportation mode v requires to go from location i to location j
$W_i^p(\cdot)$ production cost function of product p at location i
w_i^p cost factor used in case of linear production costs at site i and for product p
$Z_i^p(\cdot)$ transaction cost function of product p at location i
z_i^p cost factor used in case of linear transaction costs at site i and for product p

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