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# Tilted Rotation and Backbending in an Odd-Proton Nucleus 

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#### Abstract

Tilted-axis rotation, arising from Fermi-aligned configurations, has been observed for the first time to cause backbending in an odd-proton nucleus. In ${ }^{181} \mathrm{Re}$, two $t$-bands are found to be energetically favored relative to the usual rotation-aligned $s$-bands, presenting an alternative form of cold nuclear rotation. Interactions between the bands are weak, and unambiguous comparisons with tilted-axiscranking calculations can be made. [S0031-9007(97)03669-7]


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Rotational excitations in atomic nuclei have long been a source of information about the underlying nucleonic structure. A dramatic increase in apparent moment of inertia, known as backbending, was first discovered [1] in ${ }^{160}$ Dy at an angular momentum of $I \approx 16 \hbar$, and is now well established as a general feature of nuclear rotation. In well deformed, axially symmetric nuclei, backbending has been understood [2] to be due to the alignment of the angular momentum of a pair of high- $j$ nucleons along the rotation axis, so that the mean angular-momentum component along the symmetry axis (perpendicular to the rotation axis) is $\langle K\rangle \approx 0$. The aligned structure ( $s$-band) becomes favored in energy at high spin and crosses the nonaligned structure ( $g$ band).

In the $A \approx 180$ mass region, where backbending is due to a pair of $i_{13 / 2}$ neutrons, it has been proposed $[3,4]$ that a new structure arises due to the Fermi-aligned [4,5] coupling scheme which is active in the middle of the neutron shell ( $N \approx 104$ ). In this scheme the nucleon angular-momentum precesses about an axis lying between the rotation and symmetry axes. The projections of the angular momentum onto the symmetry axis, $K$, and the rotation axis, $i$, are both localized at a nonzero value. This differs from the strong-coupling scheme in which the nucleon angular-momentum precesses about the nuclear symmetry axis resulting in good $K$, with $\langle i\rangle \approx 0$. States formed from a pair of Fermi-aligned nucleons have $K \approx$ $\left|K_{1} \pm K_{2}\right|$ and $i \approx i_{1}+i_{2}$, resulting in the usual $s$ states, having $K \approx 0$, and also $t$ states having large $K$ and a total angular momentum tilted between the rotation and symmetry axes. The $s$ - and $t$-bands compete for "yrast" status (the lowest energy for a given angular momentum) corresponding to cold nuclear rotation. The nature and extent of this competition is not yet understood, and the possibility of there being large-amplitude high- $K$ components in the yrast bands of nuclei with $N \approx 104$
is contrary to the usual interpretation of backbending as intrinsically a low- $K$ phenomenon.

Evidence for the role of $t$-band structures in backbending has been observed in ${ }^{179,180} \mathrm{~W}[3,6]$ and ${ }^{181,182} \mathrm{Os}$ [7] (with proton numbers $Z=74,76$, respectively). However, only in ${ }^{179} \mathrm{~W}$ are the band interactions sufficiently weak to enable a clear signature to be obtained of the high$K$ band crossing, and in this unique case a chance near degeneracy at the band crossing could be responsible for the special features. Furthermore, rotational models [4] have failed to reproduce the band crossings in ${ }^{179} \mathrm{~W}$. Consequently, the generality of $t$-bands being a significant structural feature in the $A \approx 180$ region has been in doubt.

As yet no evidence for $t$-bands has been found in odd$Z$ nuclei. Apart from the need for additional information about $t$-bands, the odd proton can be expected to enhance $M 1$ transitions, and allow $M 1 / E 2$ branching ratios to be measured for rotational states with the potential of providing new structural information. Odd-Z $t$-bands would also have different pairing and residual spin-spin interactions compared to the odd- $N$ and even-even cases, testing the concepts that have so far been proposed [3,4].
An experiment to investigate the $Z=75$ nucleus ${ }^{181} \mathrm{Re}$, an isotone of ${ }^{180} \mathrm{~W}$ and ${ }^{182} \mathrm{Os}$, with $N=106$, was carried out at the Australian National University 14UD tandem accelerator. An ${ }^{11} \mathrm{~B}$ beam of energy 77 MeV was incident on a self-supporting $5 \mathrm{mg} / \mathrm{cm}^{2}{ }^{176} \mathrm{Yb}$ target. The beam energy was chosen to maximize production of high angular-momentum states in the ${ }^{176} \mathrm{Yb}\left({ }^{11} \mathrm{~B}, 6 n\right){ }^{181}$ Re reaction. Gamma rays were detected using the CAESAR array of six Compton-suppressed germanium detectors and two unsuppressed planar LEPS detectors. The beam was bunched and chopped to give 1 ns wide pulses separated by $1.7 \mu \mathrm{~s}$. This allowed measurement of $\gamma$-ray coincidences across isomers with half-lives of up to $10 \mu \mathrm{~s}$ in strongly populated cases. A total of $3 \times 10^{8} \gamma \gamma$ coincidences was
recorded-including energy and time relative to the beam pulse for each $\gamma$ ray. In addition, singles- $\gamma$-ray energy and time measurements were performed in which a beam with $1 \mu \mathrm{~s}$ pulses separated by $20 \mu \mathrm{~s}$ was used, allowing the observation of longer half-lives. The beam-pulsing and time information allowed highly sensitive measurements of $\gamma$ rays above and below isomers, which proved vital in deducing the level structure. ${ }^{181}$ Re has a complex decay scheme with two strongly populated long-lived isomers with half-lives of 1.4 and $11 \mu \mathrm{~s}$, three isomers having half-lives of $\sim 100 \mathrm{~ns}$, and one short-lived isomer with a half-life of $\sim 20 \mathrm{~ns}$.

A complementary data set was obtained at the Niels Bohr Institute Tandem Accelerator Laboratory, for prompt (in-beam) events. The ${ }^{150} \mathrm{Nd}\left({ }^{36} \mathrm{~S}, p 4 n\right)^{181}$ Re reaction was employed at a beam energy of 166 MeV , with stacked targets of approximately $1 \mathrm{Mg} / \mathrm{cm}^{2}$ total thickness. The Nordball array, equipped with 20 germanium detectors, and a "Si-ball" charged-particle array, consisting of $21 \Delta E$ detectors of $170 \mu \mathrm{~m}$ thickness, was used to collect approximately $3 \times 10^{7} \gamma \gamma$-particle coincidences. The results are consistent with the CAESAR data. A detailed comparison of the two data sets will be made in a later full report on this work.

Fourteen sequences of rotational states were identified based on one, three, and five quasiparticle structures, of which three were known [8] previously. The partial decay scheme relevant to this Letter, shown in Fig. 1, focuses on two pairs of interacting one- and three-quasiparticle bands associated with the $5 / 2^{+}$[402] and $9 / 2^{-}$[514] Nilsson orbitals which are close to the proton Fermi surface. The $5 / 2^{+}[402]$ band was known up to spin $\frac{25}{2} \hbar[8]$ but is now observed up to spin $\frac{45}{2} \hbar$. A new band based on an $I^{\pi}=$ $21 / 2^{+}$state at 1858 keV is observed up to spin $\frac{53}{2} \hbar$. It has a well defined bandhead which decays via a 164 keV transition to a previously identified [8] $I^{\pi}=\left(17 / 2^{+}\right)$ state at 1694 keV . The band based on the 1858 keV state crosses the $K^{\pi}=5 / 2^{+}$band at $I=25 / 2$, where out-ofband transitions to the $K^{\pi}=5 / 2^{+}$band compete with in-band transitions.

A second pair of interacting one- and three-quasiparticle bands is observed. The band based on the $9 / 2^{-}$[514] Nilsson orbital is now identified up to spin $\frac{43}{2} \hbar$ compared to the spin $\frac{23}{2} \hbar$ known from previous work [8]. The new band based on the $I^{\pi}=25 / 2^{-}$state at 2225 keV is observed up to spin $\frac{55}{2} \hbar$. It has a well defined bandhead which decays via a 345 keV transition to a known [8] $11 \mu \mathrm{~s}$ isomer at 1881 keV . The band based on the


FIG. 1. Partial decay scheme for ${ }^{181} \mathrm{Re}$, showing the rotational bands based on the $9 / 2^{-}[514]$ and $5 / 2^{+}$[402] one-quasiparticle states, the associated $t$-bands, and their decay paths. Energies are in keV .

2225 keV state crosses the $K^{\pi}=9 / 2^{-}$band at $I=$ $\frac{27}{2} \hbar$, where out-of-band transitions to the $K^{\pi}=9 / 2^{-}$ band compete with in-band transitions. Strong $\Delta I=1$ transitions are seen through each band.

Given the previous one-quasiparticle band assignments [8], the present spin and parity assignments of the higher-spin levels are determined by the regularity of the rotational sequences, the systematically competing $\Delta I=1$ and $\Delta I=2$ transitions, and a consistent set of $\gamma-\gamma$ angular correlation (DCO) ratios. In particular, the $514 \mathrm{keV}\left(25 / 2^{+} \rightarrow 21 / 2^{+}\right)$and $544 \mathrm{keV}\left(27 / 2^{-} \rightarrow\right.$ $23 / 2^{-}$) interband transitions have stretched quadrupole character. The assignments are also consistent with the band-crossing interpretation (see below) whereby closelying states of equal spin and parity mix sufficiently to enable interband transitions to compete with collective intraband transitions.

The two pairs of bands behave in a similar way, as illustrated in Fig. 2 (top). In each case the onequasiparticle band is crossed by what appears to be a band of the same parity and with an additional eight units of $K$ (assuming that the $K$ value is equal to the spin of the bandhead). Since the Fermi surface at $N=106$


FIG. 2. Energies and $B(M 1) / B(E 2)$ ratios corresponding to the bands shown in Fig. 1. The upper panels show experimental energies, with an arbitrary rigid-rotor reference subtracted in order to highlight the band-crossing features. The middle panels show the corresponding energies from TAC calculations. The lower panels show the experimental $B(M 1) / B(E 2)$ ratios (symbols) for the energetically favored states, together with values from TAC calculations (full and dotted lines).
is between the $7 / 2^{+}$[633] and $9 / 2^{+}$[624] Nilsson orbits, each of $i_{13 / 2}$ character, it is reasonable to attribute the increase in $K$ to the excitation of this neutron pair coupled to $K^{\pi}=8^{+}$. At each band crossing a moment-of-inertia increase is indicated by the change in slope [Fig. 2 (top)]. This corresponds to an alignment increase of $\sim 6 \hbar$, due to the Fermi alignment of the high- $j$ neutrons. The crossing bands are therefore interpreted as $t$-bands, composed of two $i_{13 / 2}$ neutrons and a spectator proton, analogous to the $t$-bands found in the neighboring odd$N$ nuclei, ${ }^{179} \mathrm{~W}$ [3], and ${ }^{181}$ Os [7], where the spectator is a $7 / 2^{-}$[514] neutron. A significant difference is that in ${ }^{181} \mathrm{Re}$ the band crossings involve almost negligible perturbations of the level energies, with interaction matrix elements less than 8 keV . At first sight it appears surprising that, with such a small interaction, each high$K$ band has strong $\gamma$-ray branches to its partner onequasiparticle band at the crossing. This is, however, consistent with the $K$ values discussed above, with band interactions of only 4 keV (c.f. [3] for detailed discussion of the methods used to determine the interactions) and arises because the high- $K$ intraband transition strengths are strongly retarded by their small angular-momentumcoupling (Clebsch-Gordan) coefficients.

A consistent picture emerges of high- $K t$-bands that become yrast relative to their one-quasiparticle $g$-band partners. In each case the structural change at the band crossing is the excitation of two $i_{13 / 2}$ neutrons coupled to $K \approx 8$, where the approximation sign is used to acknowledge that the Fermi alignment leads to some spreading of the $K$ distribution. This is precisely the situation that has been modeled by tilted-axis-cranking (TAC) calculations [4] for odd- $N{ }^{179} \mathrm{~W}$. The TAC model is in contrast to the usual principal-axis-cranking (PAC) model [9] in which the high-spin states are generated by cranking about an axis perpendicular to the nuclear symmetry axis. The PAC model could not be expected to give a consistent description of high- $K$ rotational bands, since these have a significant fraction of the angular momentum aligned with the symmetry axis. However, the TAC model permits the total angular-momentum vector to be at an angle intermediate between the symmetry axis and the rotation axis. This additional degree of freedom is required if a cranking model is to be able to describe high- $K$ structures. Nevertheless, in the application to the odd- $N$ case, ${ }^{179} \mathrm{~W}$ [4], strong interactions between the calculated bands resulted in ambiguities in the comparison with experiment, and the calculated $s$-band appears to be lower in energy than the $t$-band at high spin, contrary to experiment.

TAC calculations have now been performed for odd$Z{ }^{181} \mathrm{Re}$, using standard deformation and pair-gap parameters $\left(\varepsilon_{2}=0.225, \varepsilon_{4}=0.046, \Delta_{p}=0.87 \mathrm{MeV}, \Delta_{n}=\right.$ $0.67 \mathrm{MeV})$. The calculated and experimental ${ }^{181}$ Re level energies are compared in Fig. 2. Although no fitting has been attempted and the gradients are not well reproduced,
other qualitative features are in good accord with the data. An unambiguous comparison between observed and calculated $t$-band crossings can be made for the first time. The $t$-bands are calculated to become energetically favored, as observed, and the gradual gradient decrease in the unfavored $g$-band extensions can be identified with $s$-band crossings. The comparison between calculation and experiment shows a considerable improvement over the ${ }^{179} \mathrm{~W}$ situation [4], which appears to be related to the interactions between the ${ }^{181} \mathrm{Re}$ bands being weaker (most likely due to less blocking of the neutron pairing in the odd-Z nucleus). The calculated $t$-band wave functions are also of interest. For example, close to the band crossing, the $t$-band that crosses the $9 / 2^{-}$[514] band has $\langle K\rangle=12.2$, which is close to the strong-coupling limit of $K=12.5$ (for two $i_{13 / 2}$ neutrons coupled to $K=8$, together with the $K=9 / 2$ proton).

The present data contain many $\Delta I=1$ transitions from which estimates of the magnetic dipole strength can be made. It is therefore possible for the first time to make comparisons with the TAC model calculation of the $M 1$ transition rates [4,10], relative to the collective $E 2$ transitions, through the $t$-band crossings. This is illustrated in Fig. 2 (bottom), for the energetically favored sequences, i.e., for the states having the lowest energy for a given spin, above the $9 / 2^{-}$[514] and $5 / 2^{+}$[402] one-quasiparticle states. The TAC calculations (full and dotted lines) give a reasonable description of the data, though in the absence of detailed level-energy agreement, the quality of the transition-rate comparisons is difficult to judge. The poorer agreement for the $5 / 2^{+}[402]$ $t$-band may be due to interactions with another (in this case unidentified) positive-parity sequence, possibly associated with the observed 1694 keV intrinsic state. The rapid decrease with spin of the $B(M 1) / B(E 2)$ ratio close to each bandhead can be understood as arising from the geometrical factors (angular-momentum coupling coefficients) associated with the $K$ value, or, in the TAC calculations, from the increasing tilt angle, $\theta$, where $B(M 1) / B(E 2) \propto 1 / \sin ^{2} \theta$ in the strong-coupling limit [10]. [Note that the experimental $B(M 1)$ values have been obtained assuming no $E 2$ component in the $\Delta I=1$ transitions; these $E 2$ components are estimated to be less than $10 \%$, which is supported by measured $\gamma$-ray angular distributions where available.] The lack of detailed quantitative agreement for the $t$-bands may be, at least in part, a consequence of the simplified assumption of equal neutron pairing for all bands. Reduced neutron pairing would be expected in the $t$-bands. This would reduce the rotational $g$ factors and increase the proton contribution to the $M 1$ matrix element, but these effects have not yet been explored in detail.
The involvement of $t$-bands in the explanation of anomalous (weakly hindered) $K$-isomer decays has
already been demonstrated [3] in ${ }^{179} \mathrm{~W}$. The present data, together with their qualitative description by the TAC model, suggest a more widespread influence of $t$-bands, which provide a mechanism for the introduction of large-amplitude, high- $K$ components into $A \approx 180$ yrast bands. Hence, $t$-bands would have an important role to play in determining multiquasiparticle isomer decay rates. It is hoped that the new data for ${ }^{181}$ Re will stimulate more detailed theoretical evaluation of the phenomenon, including the effect of residual spin-spin interactions which have not been included in the TAC calculations. Generalized GallagherMoszkowski rules [11] favor parallel intrinsic-spin couplings for unlike nucleon pairs ( $n \uparrow p \uparrow$ and $n \downarrow p \downarrow$ ) and antiparallel couplings for like pairs ( $n \uparrow n \downarrow$ and $p \uparrow p \downarrow$ ). Residual interactions in even-even nuclei would therefore favor $s$-bands ( $n \uparrow n \downarrow$ ) over $t$-bands ( $n \uparrow n \uparrow$ ) with regard to bandhead energies (by $\sim 200 \mathrm{keV}$ ). However, there is no such favoring for the ${ }^{181} \mathrm{Re}, 9 / 2^{-}[514] \uparrow$ and $5 / 2^{+}[402] \uparrow s$-bands ( $p \uparrow n \uparrow n \downarrow$ ) compared to their respective $t$-bands ( $p \uparrow n \uparrow n \uparrow$ ) when all pair interactions are considered. Indeed, the higher spin of the bandhead for the high- $K$ coupling enables these $t$-bands to become yrast. Other couplings, such as to the $7 / 2^{+}[404] \downarrow$ proton, would form energetically unfavored $t$-bands ( $p \downarrow n \uparrow n \uparrow$ ) which would not become yrast.

In summary, clear signatures have been found for two $t$-bands involving $\left(i_{13 / 2}\right)^{2}, K \approx 8$ couplings in ${ }^{181} \mathrm{Re}$. In each case it is the $t$-band and not the $s$-band which gives rise to backbending in the associated yrast sequence. This is the first observation of $t$-bands in an odd- $Z$ nucleus, and the first observation of more than one $t$-band in a nucleus. TAC calculations provide a reasonable description of the experimental data. It is suggested that $t$-bands are of more general importance than has hitherto been recognized.
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[1] A. Johnson, H. Ryde, and J. Sztarkier, Phys. Lett. 34B, 605 (1971).
[2] F. S. Stephens, Rev. Mod. Phys. 47, 43 (1975).
[3] P. M. Walker et al., Phys. Rev. Lett. 67, 433 (1991); P. M. Walker et al., Nucl. Phys. A568, 397 (1994).
[4] S. Frauendorf, Nucl. Phys. A557, 259c (1993).
[5] S. Frauendorf, Phys. Scr. 24, 349 (1981).
[6] P. M. Walker et al., Phys. Lett. B 309, 17 (1993).
[7] T. Kutsarova et al., Nucl. Phys. A587, 111 (1995).
[8] A. Neskakis et al., Nucl. Phys. A261, 189 (1976).
[9] R. Bengtsson and S. Frauendorf, Nucl. Phys. A327, 139 (1979).
[10] S. Frauendorf and J. Meng, Z. Phys A 356, 263 (1996).
[11] K. Jain et al., Nucl. Phys. A591, 61 (1995).

