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Simulation of a Transformer Winding for Partial Discharge Propagation Studies

S. N. Hettiwatte, P. A. Crossley, *MIEEE*, Z. D. Wang, *MIEEE*, A. Darwin, *SMIEEE* and G. Edwards

Abstract—A simulation model of a continuous disc type 6.6kV transformer winding was used to study the propagation behaviour of partial discharge (PD) pulses. The model based on multi-conductor transmission line theory uses a single turn as a circuit element with the capacitance, inductance, and losses calculated as distributed parameters. Transfer functions that describe how the location of the PD source affects the current signals measured at the terminals of the winding were calculated. The paper shows how the position of the zeros in the frequency response of the measured current signals can be used to locate the source of the discharge. Sensitivity studies on the parameters of the model were used to investigate the effect of inaccuracies in the model on the position of the zeros and hence the location of the discharge.

Index Terms—Losses, Modelling, Multiconductor Transmission Lines, Partial Discharges, Power Transformers, Sensitivity, Simulation, Skin Effect and Transfer Functions.

I. INTRODUCTION

PARTIAL discharges (PD) are a major cause of insulation failure in EHV power transformers. If PD activity is not detected and ideally located before it develops into a full discharge, catastrophic failure can result and the resulting economic cost to the utility may be significant. The actual cost depends on the location and importance of the transformer, the availability and cost of alternative sources and/or routes of power and the capital cost of repair or replacement. On-line condition monitoring of EHV power transformers is advisable and ideally this should include immediate detection of increased PD activity and location of the discharge. Depending on the severity and location of a discharge the transformer will either be taken out of service immediately or at a convenient time or will be kept in service with increased monitoring. Early work [1] on PD location assumed the transformer behaved like a capacitive network,

but further studies [2] indicate this is only valid over a limited frequency range and is inadequate for studying PD propagation. A recent paper [5] modelled each section of the winding as a lumped circuit that takes into account capacitance, inductance and resistive and dielectric losses. This was valid when the dominant PD frequencies are up to a few hundred kHz, but is inadequate in the MHz region. This paper describes how a model based on multi-conductor transmission line theory can be used to simulate a transformer winding over a frequency range from a few hundred kHz to a few tens of MHz.

A wide frequency range is necessary because the spectra needed to describe the various types of partial discharges observed on a power transformer can extend from tens to hundreds of kHz for a surface or interface type PD and from hundreds of MHz to a few GHz for a small bubble void [6]. A model suitable for the latter was beyond the scope of this research because of the difficulties associated with the design and experimental validation of such a model and also because PD pulses of 1 - 100ns duration have minimal destructive power and are of limited practical significance. As a compromise, the model proposed in this paper is suitable for simulating the propagation of PD pulses of duration 100ns-10 μ s. This is appropriate for most practical cases.

Representing a transformer winding by a lumped circuit model requires a resolution appropriate for the chosen frequency. For example, previous research [5] has shown that representing each disc in the winding by a lumped PI circuit model is valid up to a few hundred kHz. To correctly model the winding at higher frequencies requires that each turn in the disc (and hence the entire winding) is modelled as a circuit unit. The electrical parameters of the winding are calculated on a per turn basis and the entire winding is represented by distributed multiconductor transmission line (MTL) model.

The transformer winding is considered as a single input multiple output (SIMO) system, where the input is the PD signal and the outputs are the current signals at the measuring terminals. Transfer functions are calculated from all possible PD locations to the line-end and neutral-end measuring terminals using a simulation program developed in Matlab[®]. Simulation results show that the zeros in the transfer functions contain information about the location of discharge.

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II. THE MULTI-CONDUCTOR WINDING MODEL

Fig.1 shows a transformer winding modelled by MTL. The voltage (V) and current (I) vectors at any point x along a multi-conductor transmission line can be expressed by wave equations (1) and (2), where $[Z]$ and $[Y]$ are impedance and admittance matrices of the line respectively, and $[P]^2 = [Z][Y]$, $[P_t]^2 = [Y][Z]$.

$$\frac{d^2 V}{dx^2} = [Z][Y]V = [P^2]V \quad (1)$$

$$\frac{d^2 I}{dx^2} = [Y][Z]I = [P_t^2]I \quad (2)$$

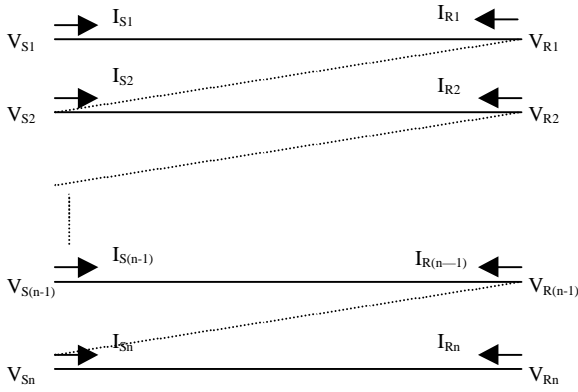


Fig. 1. Multi-conductor transmission line model

Equation (1) and (2) can be solved to find the voltage and current vectors at a distance x from the sending end.

$$V_x = V_1 e^{(-[P]x)} + V_2 e^{([P]x)} \quad (3)$$

$$I_x = Y_O (V_1 e^{(-[P]x)} - V_2 e^{([P]x)}) \quad (4)$$

In (3) and (4), V_1 and V_2 are voltage vectors to be determined by the terminal conditions and $[Y_O] = [Z]^{-1}[P] = [Y][P]^{-1}$ is the characteristic admittance matrix of the model.

With terminal conditions applied at the sending end 's' and the receiving end 'r' it is possible to express sending end and receiving end currents in terms of voltages.

$$\begin{pmatrix} I_S \\ I_R \end{pmatrix} = Y_O \begin{pmatrix} \coth([P]l) & \operatorname{cosech}([P]l) \\ \operatorname{cosech}([P]l) & \coth([P]l) \end{pmatrix} \begin{pmatrix} V_S \\ V_R \end{pmatrix} \quad (5)$$

Further, if matrix $[P]$ has eigenvectors $[Q]$ and eigenvalues $[\gamma]$, (5) can be written as

$$\begin{pmatrix} I_S \\ I_R \end{pmatrix} = \begin{pmatrix} [A] & -[B] \\ -[B] & [A] \end{pmatrix} \begin{pmatrix} V_S \\ V_R \end{pmatrix} \quad (6)$$

where:

$[A] = [Y][Q][\gamma]^{-1} \coth([\gamma]l)[Q]^{-1}$ and $[B] = [Y][Q][\gamma]^{-1} \operatorname{cosech}([\gamma]l)[Q]^{-1}$ are $n \times n$ matrices, and 'n' is the number of conductors in the model. I_S , I_R , V_S and V_R are vector quantities representing the values in Fig. 1.

A. PD injection

The transmission line model in Fig. 1 has terminal conditions:

$$I_R(i) = -I_S(i+1) \quad \text{for } i = 1 \text{ to } n-1 \quad (7)$$

$$V_R(i) = V_S(i+1) \quad \text{for } i = 1 \text{ to } n-1 \quad (8)$$

If a PD current pulse I_{PD} is injected into the k^{th} turn of the winding, (7) is modified when $i=k-1$:

$$I_R(k-1) + I_S(k) = I_{PD} \quad (9)$$

With this set of terminal conditions applied to (6), it is possible to simplify it to the following form:

$$\begin{pmatrix} I_S(1) \\ 0 \\ \vdots \\ 0 \\ I_{PD} \\ 0 \\ \vdots \\ 0 \\ I_R(n) \end{pmatrix} = [Y] \begin{pmatrix} V_S(1) \\ V_S(2) \\ \vdots \\ V_S(k) \\ \vdots \\ V_S(n) \\ V_R(n) \end{pmatrix} \quad (10)$$

where $[Y]$ is a $(n+1) \times (n+1)$ matrix.

If matrix $[Y]$ is inverted and re-arranged, it is possible to get

$$\begin{pmatrix} I_S(1) \\ V_S(2) \\ V_S(3) \\ \vdots \\ V_S(k) \\ \vdots \\ V_S(n) \\ V_R(n) \end{pmatrix} = [T] \begin{pmatrix} V_S(1) \\ 0 \\ \vdots \\ 0 \\ I_{PD} \\ 0 \\ \vdots \\ 0 \\ I_R(n) \end{pmatrix} \quad (11)$$

Hence, if the line-end voltage, the neutral-end current and the PD current are known all other voltages and currents can be calculated.

B. Transformer terminal conditions

The bushing of transformer can be simulated by a capacitance C_B connected at the line-end. Then,

$$I_S(1) = -j\omega C_B V_S(1) \quad (12)$$

If the neutral end is at earth potential,

$$V_R(n) = 0 \quad (13)$$

C. Transfer functions

With the above terminal conditions, the transfer function from the PD source current to the line-end current (TF_L) is

$$\frac{I_S(1)}{I_{PD}} = \frac{T(1,k)T(N,N) - T(N,k)T(1,N)}{T(N,N) + \frac{1}{j2\pi f C_B} (T(1,1)T(N,N) - T(N,1)T(1,N))} \quad (14)$$

where $N=n+1$. Similarly, the transfer function from the PD source current to the neutral-end current (TF_N) is

$$\frac{I_R(n)}{I_{PD}} = \frac{T(N,1)TF_L - j2pfC_B T(N,k)}{T(N,N)j2pfC_B} \quad (15)$$

III. CALCULATION OF ELECTRICAL PARAMETERS

The transformer winding used in this calculation has twenty-two (22) continuous disc type sections each having thirteen (13) turns. The impedance and admittance matrices in (1) and (2) can be expressed as

$$[Z] = R_S [I_n] + j2pf[L] \quad (16)$$

$$[Y] = [G] + j2pf[C] \quad (17)$$

where R_S = resistance, $[L]$ = inductance matrix, $[G]$ = conductance matrix, $[C]$ = capacitance matrix, $[I_n]$ = unit matrix and f is the frequency.

A. Capacitance

Capacitance calculation plays a major role in the accuracy of this model since inductance is also based on the capacitance value. Capacitance calculations are based on geometry of the winding and permittivity of the insulation. There are three components of capacitance: inter-turn (CIT), inter-section (CID) and capacitance to low voltage winding (CLV). CIT is calculated assuming two adjacent turns of the winding form a parallel plate capacitor. In calculating CID cross capacitance is also considered for more accuracy. Formula for capacitance between two coaxial cylinders is used in the calculation of CLV.

B. Inductance

In evaluating inductance, it is assumed that the magnetic flux penetration into the laminated iron core is negligible at frequencies above 1 MHz [7]. The inductance is calculated by assuming the winding consists of loss-less multi-conductor transmission lines surrounded by a homogeneous insulator. Hence [4]

$$[L][C] = [C][L] = \mu \epsilon [I_n] \quad (18)$$

where μ and ϵ are the permeability and permittivity of the insulation and I_n is the unit matrix. If no high frequency magnetic flux penetrates the iron core, the winding can be regarded as a conductor in free space surrounded by insulation. The inductance due to the flux external to the conductor (L_n) can be calculated using [3]

$$[L_n] = \frac{\epsilon_r}{c^2} [C_n]^{-1} \quad (19)$$

where $[C_n]$ = capacitance without insulation, ϵ_r = relative permittivity of insulation and c = velocity of light in free space. At high frequencies, the flux internal to the conductor also creates an inductance [4]

$$L_i = \frac{R_S}{2pf} \quad (20)$$

where R_S is the resistance due to the skin effect and f is the frequency. The total inductance is given by

$$[L] = [L_n] + L_i [I_n] \quad (21)$$

C. Resistance

In resistance calculation, the skin effect at high frequencies is taken into account. The resistance per unit length of conductor is given by

$$R_S = \frac{1}{2(d_1 + d_2)} \sqrt{\frac{pfm}{s}} \quad (22)$$

where d_1, d_2 = cross-sectional dimensions of rectangular conductor, μ = permeability of conductor, σ = conductivity and f = frequency.

D. Conductance

The conductance (G) is due to the capacitive loss in the insulation. It depends upon the frequency f , the capacitance C and the dissipation factor $\tan \delta$.

$$G = 2pf [C] \tan \delta \quad (23)$$

The $\tan \delta$ for the Nomex[®] paper insulation used in this transformer satisfies

$$\tan \delta = 0.07 \left(1 - \frac{6}{7} e^{-(0.308 f \times 10^{-6})} \right) \quad (24)$$

Hence

$$[G] = 0.44 f [C] \left(1 - \frac{6}{7} e^{-(0.308 f \times 10^{-6})} \right) \quad (25)$$

IV. SIMULATION RESULTS

The transfer functions between the PD source and the line-end (TF_L) and the neutral-end (TF_N) are calculated for different positions of the PD source along the winding. The position of the PD source is determined in terms of the number of turns from the line-end. The line-end terminal is connected to the 1st turn and the neutral-end terminal to the 286th turn (22 sections x 13 turns). Fig. 2 shows the transfer functions obtained for the line-end while Fig. 3 shows those obtained for the neutral-end.

Fig. 2 and Fig.3 show that the crests (or poles) in the transfer functions always occur at fixed frequencies and are not affected by the location of the PD source. The locations of poles in the transfer functions are listed in Table 1. It can be also seen that the troughs (or zeros) in the line-end transfer functions increase in frequency as the PD source moves away from the line-end, whereas the troughs in the neutral-end transfer functions decrease in frequency. The locations of zeros in the line-end transfer-functions are listed in Table 2 while those in the neutral-end transfer functions in Table 3.

TABLE 1 POSITION OF POLES (P) IN KHZ

P ₁	P ₂	P ₃	P ₄	P ₅	P ₆	P ₇	P ₈
70	271	508	753	1005	1265	1534	1813

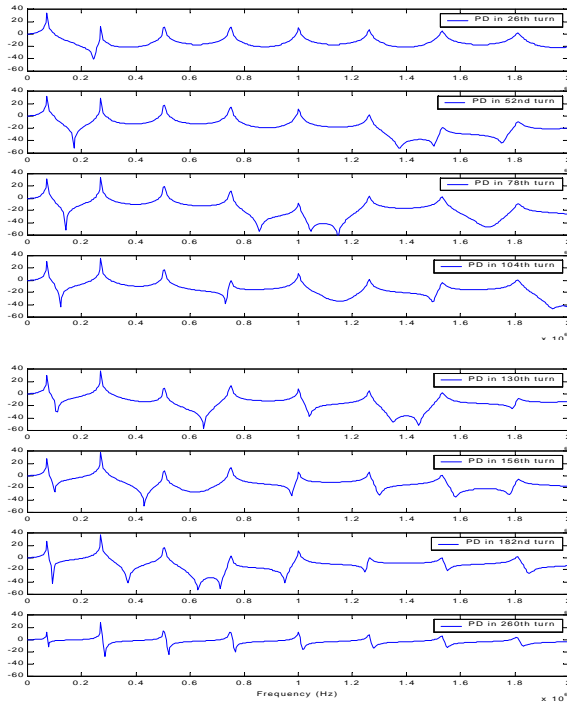


Fig. 2. Magnitudes of transfer functions (in dB) between PD source current and line-end current, TF_L (1 kHz ~ 2000 kHz)

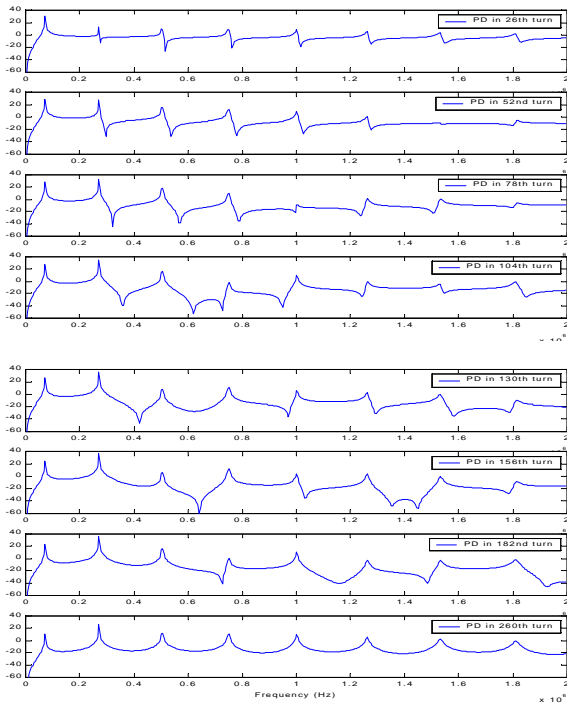


Fig. 3. Magnitudes of transfer functions (in dB) between PD source current and neutral-end current, TF_N (1 kHz ~ 2000 kHz)

TABLE 2 POSITION OF ZEROS (Z) OF TRANSFER FUNCTIONS TF_L IN KHZ

PD (turn no.)	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
26	275	520	765	1020	1280	1552
52	295	546	784	1030	1280	1542
78	321	572	792	996	1242	1510
104	358	624	730	954	1248	1550
130	419	975	1298	1585	1793	-
156	646	1036	1355	1454	1790	-
182	729	1488	1930	-	-	-
260	-	-	-	-	-	-

TABLE 3 POSITION OF ZEROS (Z) OF TRANSFER FUNCTIONS TF_N IN KHZ

PD (turn no.)	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
26	244	-	-	-	-	-
52	172	1375	1505	1755	-	-
78	141	860	1050	1150	-	-
104	122	736	1496	1945	-	-
130	109	654	1045	1354	1450	1795
156	99	432	981	1304	1585	1785
182	91	369	634	716	951	1250
260	74	287	524	768	1020	1282

V. SENSITIVITY ANALYSIS

Sensitivity analysis is used to investigate the effect on a system by varying one or more parameters in the system. The system is the transformer-winding model while the capacitance, inductance and losses are the parameters of the system. In simulating PD propagation through a transformer winding the transfer functions from the PD source to either measuring terminal will depend upon the location of the discharge. Hence, sensitivity analysis on the transfer functions is not the most appropriate method of analysis. It is preferable to consider the terminal impedance of the winding as the 'system' used for sensitivity analysis.

The terminal impedance (Z_S) can be derived using (11) with $I_{PD} = 0$ and $V_R(n) = 0$, and is given by

$$Z_S = \frac{V_S(1)}{I_S(1)} = \frac{T(N, N)}{T(1,1)T(N, N) - T(1, N)T(N, 1)} \quad (26)$$

A. Effect of capacitance

The effect of the capacitance on the terminal impedance of the winding when the capacitance is increased by 20% of its primary value is shown in Fig.4. The upper characteristic is the magnitude of Z_S while the lower characteristic is the phase angle. As seen from Fig. 4, an increase in capacitance will reduce all the significant frequencies and the characteristic shifts leftwards.

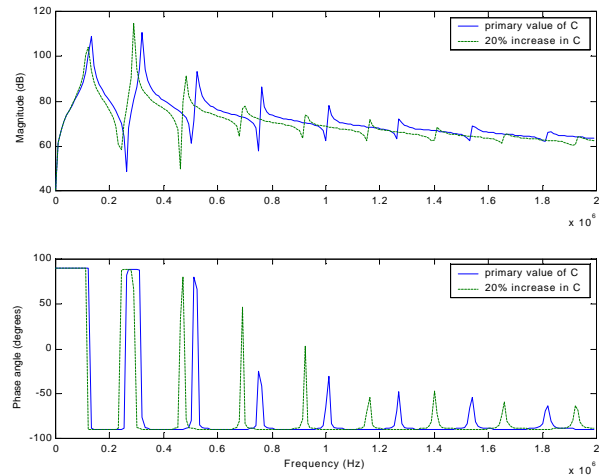


Fig. 4. Effect of capacitance on terminal impedance

B. Effect of inductance

The effect on Z_S when the inductance is increased by 20% of its primary value is shown in Fig.5. An increase in

inductance will have a similar effect to that of capacitance with reduction in all significant frequencies.

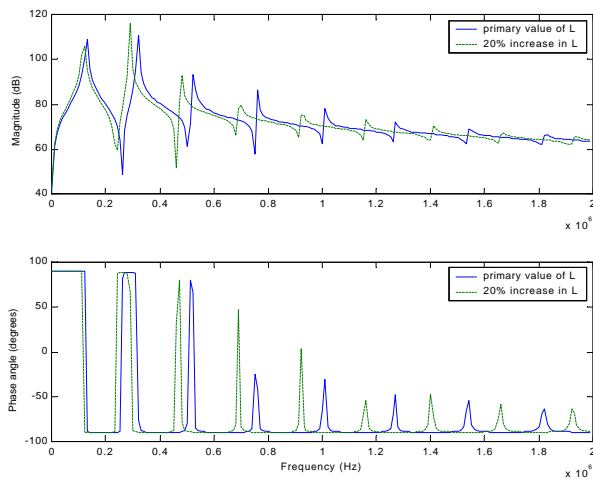


Fig. 5. Effect of inductance on terminal impedance

C. Effect of resistance

The effect of resistance on the terminal impedance of the winding is minimal in the frequency range $f = 1\text{kHz}$ to 2000kHz . Fig. 6 shows the terminal impedance calculated with the primary value of R_s and with $R_s = 0$. The effect is negligible within this frequency band.

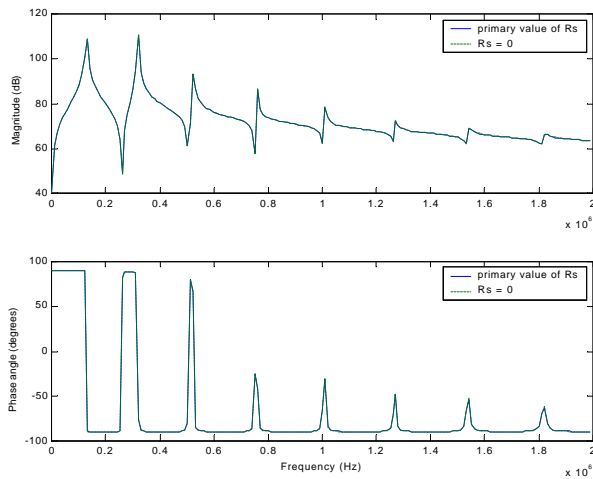


Fig. 6. Effect of resistance on terminal impedance

D. Effect of conductance

The effect of conductance on the terminal impedance is shown in Fig. 7. It is clear that in the frequency range $f = 1\text{kHz}$ to 2000kHz , conductance has negligible effect on the terminal impedance.

Further sensitivity studies conducted at higher frequencies ($1\text{kHz} \sim 13\text{MHz}$) showed that the behaviour of the terminal impedance of the winding is mainly governed by the value of the capacitance and the inductance. The effect of losses (resistance and conductance) on the terminal impedance is negligible. Since capacitance without insulation is used to derive the inductance (19), capacitance is the most sensitive parameter in the model.

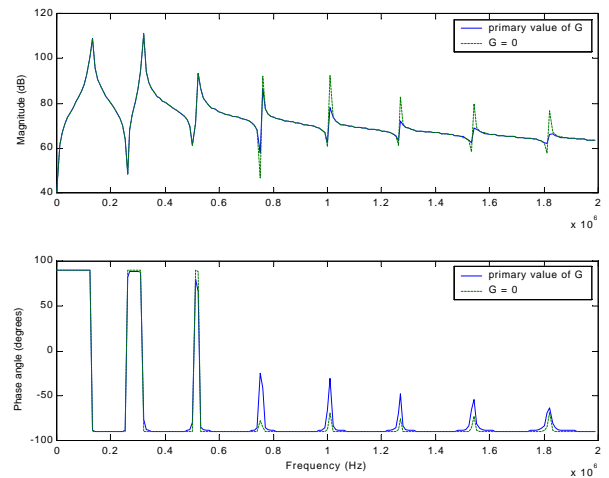


Fig. 7. Effect of conductance on terminal impedance

VI. CONCLUSION

In a continuous disc type transformer winding, the transfer function that describe the frequency characteristics between the source of discharge and the line-end or neutral-end measuring terminals can be used to detect and locate a discharge. The frequency of each zero in the simulated spectra increases as the discharge moves away from the measuring terminal. The frequency location of the first zero is a good indicator of where the PD is located. The poles only give local oscillation frequencies and are not affected by the position of PD source.

VII. ACKNOWLEDGEMENT

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IX. BIOGRAPHIES

Sujeewa Hettiwatte was born in Colombo, Sri Lanka, on 2 September 1968. He graduated from the University of Moratuwa, Sri Lanka, with first class honours in B. Sc. (Eng.) degree in 1994. He also completed his M. Eng. degree at the same university in 2000. Both degrees were in electronic and telecommunication engineering. His employment experience includes five years work experience at the Open University of Sri Lanka where he is employed as a Lecturer in Electrical and Computer Engineering. At present he is on Study Leave from the Open University pursuing research on partial discharge location in transformers with main emphasis on computer simulation and signal processing aspects of the project.

Zhongdong Wang was born in Hebei, China, on 19 January 1969. She studied electrical engineering at Tsinghua University from 1986 until 1993, receiving her B. Sc. and M. Sc. degrees in 1991 and 1993, respectively. She was employed as a lecturer at Tsinghua University before coming to UMIST for doctoral studies. She completed her PhD at UMIST in July 1999 and subsequently joined UMIST as a Lecturer in February 2000. Her research interests are in high voltage engineering, partial discharge monitoring in transformers and cables, fast transient distribution in windings, ageing of insulation, material and diagnosis detection of winding displacements.

Peter Crossley was born in Burnley, England, on 25 June 1956. He graduated with a BSc degree from UMIST and a PhD degree from the University of Cambridge in 1983. During the period 1977 – 1990, he was employed by GEC and later GEC Alstom on the design and application of digital protection relays. In 1991 he joined UMIST where he is a Reader in Power Systems. His research interests are in power system protection and condition monitoring. He has published more than 120 technical papers and is a joint author of a book on embedded generation.

Alan Darwin was born in Woodford, England, on 19 October 1945. He studied electrical engineering at Southampton University from 1964 until 1967, receiving a B. Sc. (Eng) degree in 1967. He joined English Electric (later GEC, GEC Alsthom and presently ALSTOM T&D Ltd Transformers) at Stafford in 1967 as a graduate trainee and joined the HV transformer design department in 1969. He has subsequently worked in the design department as Chief Designer, Chief Engineer and Chief Transformer Engineer. At present, he is Chief of Development. He is a Member of the Institution of Electrical Engineers and a Senior Member of the Institute of Electrical and Electronic Engineers.

Gwilym Edwards was born in Bangor, Wales, on 25 May 1961. He started his working life as an electrical craft apprentice with Babcock & Wilcox, working on a number of power stations throughout the UK. He joined the CEGB in 1986 at Dinorwig Power Station as an electrical craftsman. He subsequently received a HND in Engineering at Gwynedd College. Since 1990 he has worked as an electrical engineer in the Engineering Development Department for Edison Mission Energy First Hydro Company, responsible for generator motor rotors, transformers and 400kV plant.