

*SECOND EDITION*

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# Simulation of Dynamic Systems

with MATLAB<sup>®</sup> and Simulink<sup>®</sup>

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