Simulation of rail voltage and earth current in a PC-based traction power simulator

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ABSTRACT

Electrical circuit simulation by setting up nodal voltage equations and inverting a conductance matrix is a powerful CAE tool. Running large models on PCs, however, requires economic computing and storage requirements. This paper describes an algorithm for compacting intermediate storage necessary for solving circuit models with a large, sparse conductance matrix. The algorithm is based on targeted directed elimination followed by Gaussian elimination with back substitution. An example is given of an electrical calculation in a timestepping railway traction simulator. The circuit represents a double-track light rail system with one branch. Outputs are given for train voltage, substation busbar feeding conditions and rail voltage, the latter also giving information about earth current.

INTRODUCTION

Simulation of electric railway traction networks can be used to calculate the effects of substation rating and spacing, for a given traffic pattern, on catenary-rail voltage, substation feeder current and busbar voltage, rail voltage and earth current. Control of rail voltage and earth current is important because the voltage must be limited to safe values (typically 60 V continuous and 430 V on system faults), and the current must be prevented from corroding nearby metallic pipes and structures [1-3]. Moreover, since DC traction currents contain harmonics produced by traction converters and substation rectifiers, earth current also causes inductive interference to nearby telecommunications circuits. The factors affecting rail voltage and earth current are the electrification voltage, feeding system, load current distribution, rail resistance and rail-ground conductance. This paper considers their efficient computation in a DC traction network.

Traction simulators treat the track and equipment as an electric network, setting up and solving a nodal circuit model at discrete timesteps taking account of the power demand of the traction vehicles. Separate program modules are provided for calculating train movement and for solving the power network. The technical development of traction simulators is mature, reference [4], for example, describing a comprehensive facility developed over many years. With recent technological improvements in computation speed and storage capability,

it is now feasible to run models on low-cost PCs. The resulting problem in minimising the necessary computation and storage resources is to devise an economical way to store and manipulate the circuit conductance matrix.

The technique described in this paper is based on manipulation of the circuit conductance matrix exploiting its sparsity. The algorithm assigns optimal node numbering to the circuit model and reduces the size of the matrix by identifying and eliminating corresponding elements. Solution is then obtained by LU decomposition and Gaussian elimination. An example is presented of the calculation of train voltage, rail voltage and substation feeding conditions in a light rail transit (LRT) network.

RAIL VOLTAGE AND EARTH CURRENT IN DC RAILWAYS

Factors affecting voltage and current generation

In most electric railways, power is fed to the traction vehicles by an overhead catenary or third rail, and returned to the substations using the running rails. Figure 1 shows an equivalent circuit of a traction feeding system with one rectifier substation and train. Although the catenary can be treated as a lumped resistance, the rails and earth form a distributed transmission line with series resistance and shunt conductance, the latter arising because of the poor insulation between the rails, sleepers, ballast and ground. Traction return current thus flows back to the substation not only along the rails, but also through the ground.

The rail voltage and current may be calculated analytically for simple cases such as that of Figure 1 where the rail resistance and conductance to earth are uniform [5]. A useful technique is to derive the rail current and voltage in terms of the transmission line equations using the track characteristic resistance and propagation constant [6]. Boundary conditions may be found in terms of the sending and receiving end reflection coefficients using the effective termination resistances of the relevant track sections. Figure 1 shows the form of rail voltage and current obtained for the simple feeding model illustrated.

The earth leakage current flows through the rail conductance to ground and is, per unit track length

$$I_g(x) = G\, V(x) \qquad\qquad\qquad (1)$$

Evaluating the absolute rail voltage thus gives the leakage current per unit track length directly. The corrosive effect of leakage currents over a period of time is found from the gross leakage charge over a particular section, and a useful measure of the effectiveness of the track earth may be obtained by evaluating the accumulative leakage [7]. Because traction vehicles are moving continuously and change the pattern of rail voltage and current, the gross leakage charge has a time variation which can be found by simulating a particular traffic service over a fixed route with repeated runs covering perturbations in the schedule.

Estimate of rail voltage using single conductor model

Although it is possible to calculate rail voltage and current with many current sources and sinks using a transmission line model [8], most methods are based on solution of an equivalent electrical circuit with the track represented as a series of finite-length cells. Thus Figure 1 may be modelled as a one-ladder network
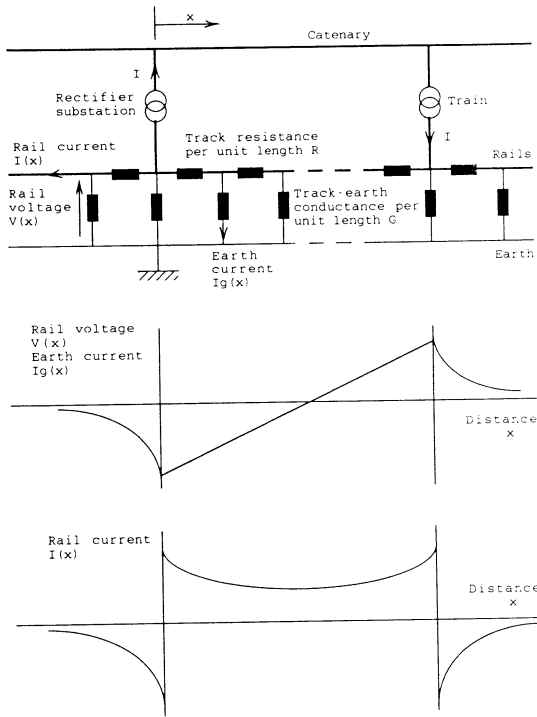
Figure 1: Traction feeding system for DC railway, with rail voltage,
earth current and rail current distribution.

and solved to find the rail voltage. Figure 2 gives typical results from a model of a 20 km section divided into 100 m track increments (cells) with four trains (current sources) and two substations (current sinks). The lowest values of rail potential are at the substations and the highest are at some of the train positions. The graph is in effect a freeze-frame picture derived from a master program supplying the train and substation positions and currents calculated ignoring track conductance to ground [9]. The circuit is modelled by writing nodal voltage equations assembling the matrix set

$$[G] [V] = [I] \tag{2}$$

where [G] is a square circuit conductance matrix, and [V] and [I] are column vectors of node voltages and branch currents. The vector [V] is obtained by inverting [G]. Even for the relatively simple system of Figure 1, where each rail node is connected to only two adjacent nodes, the computation and storage requirements for PC solution can be significant. It can be shown that appropriate node numbering results in a tridiagonal conductance matrix [G] of bandwidth equal to three [4]. LU decomposition may then be used to express the matrix [G] as the product of lower and upper triangular matrices for solution by Gaussian elimination and backward substitution.
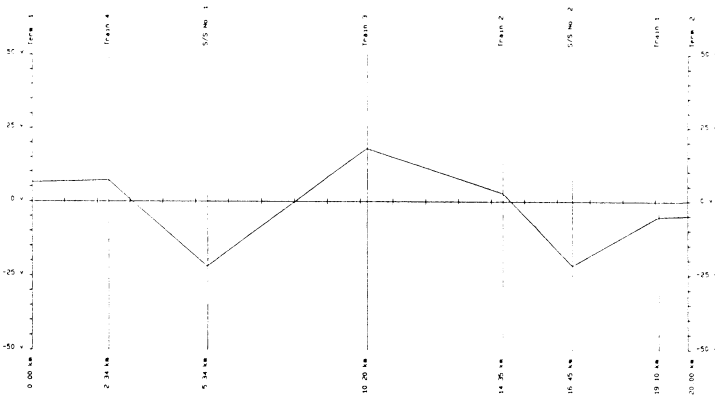
Figure 2: Estimate of rail voltage using one-ladder network representation.

RAIL VOLTAGE SIMULATION BASED ON 2-LADDER NETWORK

In order to calculate simultaneously the train voltage and rail voltage, a two-ladder circuit representation of the track is required. Figure 3 shows the circuit model of a 25.2 km double-track 750 V DC LRT system with a 12.5 km branch and three substations. The two-ladder model and branch increase the order of and remove the tridiagonality from the conductance matrix. The latter, however, is still highly sparse so its inversion algorithm can exploit the sparsity to ensure economy of computation time and storage.

System modelling
Overview The two-ladder network module is the electrical calculation routine in a simulation facility comprising a control program, train movement calculation routine and electrical circuit solution. The control program arranges the train data according to the run sequence giving position and power demand. This is then converted into a track and branch sequence. The solution starts by reading for each train the position, power and current demand; it then models and solves the circuit as before for train voltages, other node voltages (e.g. branch points, substation busbar and paralleling switch) and continuous rail voltage along each track. If the current demand is not satisfied due to substation power limitations, it is adjusted and the circuit resolved until actual current is equal to demand current. At the end of the calculation, conversion is remade to the run sequence to preserve the train order for movement recalculation.

Node identification A coherent node numbering scheme is necessary for a regular conductance matrix since the nodes are arranged sequentially by location in software. Dynamic train nodes are identified by positive, and static nodes by negative numbers. The latter are divided into first-ladder nodes connected with the train power collection equipment (substations, paralleling stations and branch catenary joints), and second-ladder nodes connected with the power return (rail, crossbond and branch rail), both being connected through train nodes. Nodes may connect with either two, three or four others. Detailed node numbering follows that in reference [9] except that idle trains are zero, cross bonds -6 and normal rail nodes -10. 3-dimensional arrays represent node locations and codes.
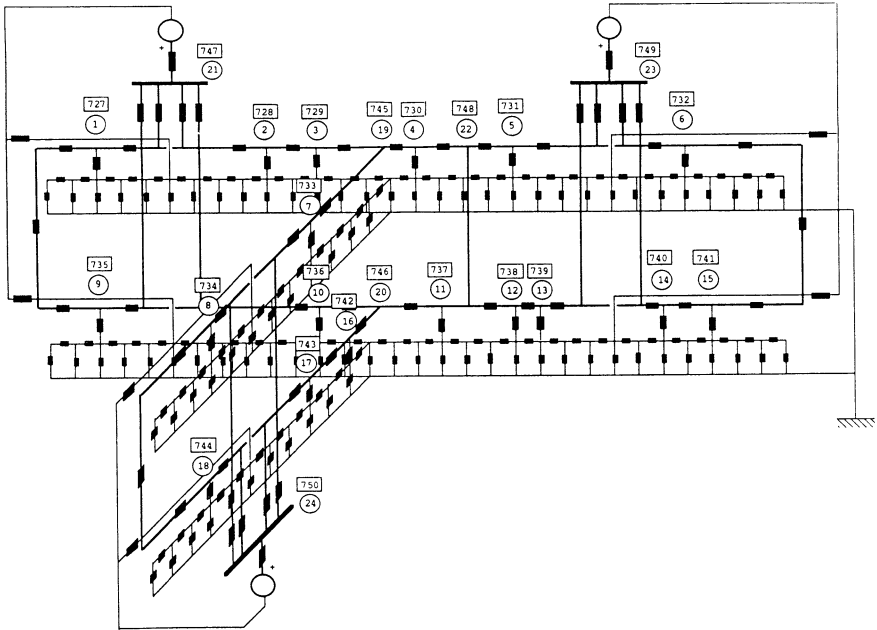
Figure 3: Two-ladder DC railway traction power system model (circles: train sequence numbers; squares: corresponding rail node numbers).

Circuit components Data are required for the system, substations, trains, paralleling stations, branch joints, overhead catenary, cross bonds and track, and are similar in format to those given in reference [9]. In the first ladder, the trains are represented by Norton sources with parallel conductance. The effective train loads change dynamically and are calculated at each timestep and read from a data file. Train catenary nodes connect with three other nodes. Substations are also represented by Norton sources with parallel conductance. Branch joints and paralleling stations are represented by three or four switches. In the second ladder, the normal rail nodes connect with two other nodes, whilst the crossbond nodes, branch rail joint nodes and train rail nodes connect with three others. The substation return current is extracted from the rails near the substations and the nodes are represented by normal rail nodes with modified self conductance.

Data manipulation and processing
Conductance matrix structure The conductance matrix is sparse symmetrical. Each nonzero off-diagonal element g_{ki} is the negative of the branch conductance between nodes k and i and each diagonal element g_{kk} is the sum of the branch conductances which terminate on node k, including those to ground. The conductance matrix sparsity requires an algorithm which stores and manipulates only nonzero elements. Optimal arrangement of the matrix is achieved by considering node numbering and exploiting regularity. The nodes are numbered in a specific geographical order in the sequence in reference [9]. The matrix is then split into five submatrices [A] - [E] storing different types of nodes. Matrix [A] stores normal rail nodes, has three columns with one self and two mutual

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conductances. Matrices [B], [C] and [D] store other rail, train and branch joint nodes. They are combined in [B] and assigned four columns containing mutual conductances only. Matrix [E], storing substation paralleling switch nodes, is assigned five columns and stores one self and four mutual conductances. Index matrices [IA], [IB] and [IC] determine the column location of each nonzero nondiagonal element in the corresponding matrix [A], [B] or [C].

Solution algorithm   The first stage uses targeted directed elimination [10] to find and exploit symmetry in the conductance matrix by eliminating redundant elements. As an example, consider mutual conductance elements a_{i0}, a_{i1} and a_{i2} in the ith row of matrix [A] and the corresponding elements in matrix [IA] which enable symmetric elements to be found in the jth row of either matrix [A] or [B]. If from the column locations of ia_{i1} of a_{i1} a symmetric element a_{j1} can be found in the jth row of the lower triangular portion of [G], this can be set equal to zero. For asymmetric positioning, the elements a_{i2} in row i and a_{j2} in the target row j do not have the same column index ($ia_{i2} \neq ia_{j2}$), so a newly created element is stored to replace a_{j1} and given a new column location equal to that of a_{i2}. When a_{i2} and a_{j2} have the same column locations ($ia_{i2} = ia_{j2}$), a_{j1} and ia_{j1} are set equal to zero. When the target row appears in matrix [B], the removed element b_{j1} receives the element created by a_{i2}. Completion of the procedure for matrix [A] results in a conductance matrix which is still symmetric but with more nonzero elements. Further gains from applying the procedure to matrices [B] and [E] are marginal, so standard Gaussian elimination with backward substitution is applied to solve the remaining simultaneous equations with a full element storage scheme for the complete conductance matrix.

Sample results
The network of Figure 3 with a cell length of 100 m has been simulated with all branch joint and paralleling station switches closed. Power is distributed according to the total system load. There are 18 trains, each equivalent to a load of 3.75 Ω. 750 nodes are formed in which 646 are in matrix [A], 100 in matrix [B] and 4 in matrix [E]. The simulation predicts branch joint voltage levels of 621.7 V and 618.6 V, which are low since they are at the centre of the trunk line, far from the substations. The busbar voltages and feeding currents are given in Table 1. The higher loading on the second trunk substation reflects the positions of the trains near that substation.The train voltages are obtained by unscrambling the node sequence into the original run sequence (Table 2). The lowest are 598.4 V and 599.5 V which correspond to trains 11 and 4 at the midpoint of the trunk line.

Table 1: Busbar voltages and feeding currents

Line	Location	Voltage	Current
Trunk	2.6 km	738.5 V	573.4 A
Trunk	19.6 km	733.0 V	847.7 A
Branch	8.0 km	739.0 V	551.3 A

Table 2: Simulated train voltages using two-ladder network representation

Up track (T: trunk; B: branch)			Down track (T: trunk; B: branch)		
Train	Voltage	Location	Train	Voltage	Location
1	725.5 V	0.90 km (T)	9	725.0 V	0.70 km (T)
2	644.8 V	5.40 km (T)	10	627.8 V	7.40 km (T)
3	611.3 V	7.80 km (T)	11	598.4 V	12.5 km (T)
4	599.5 V	12.4 km (T)	12	609.3 V	15.6 km (T)
5	629.4 V	16.3 km (T)	13	641.3 V	17.3 km (T)
6	695.0 V	23.0 km (T)	14	679.0 V	21.9 km (T)
7	633.7 V	4.00 km (B)	15	680.4 V	23.7 km (T)
8	721.8 V	10.8 km (B)	16	607.3 V	2.50 km (B)
			17	639.2 V	5.20 km (B)
			18	721.2 V	11.1 km (B)

The rail voltages for both trunk and branch lines are shown in Figure 4. The positive voltages reflect the choice of substation negative as the datum. The rail voltage at the substations or branch joints is low due to the substation return current and the larger effective leakage conductance at the branch joint which causes an outflow of current from the node.
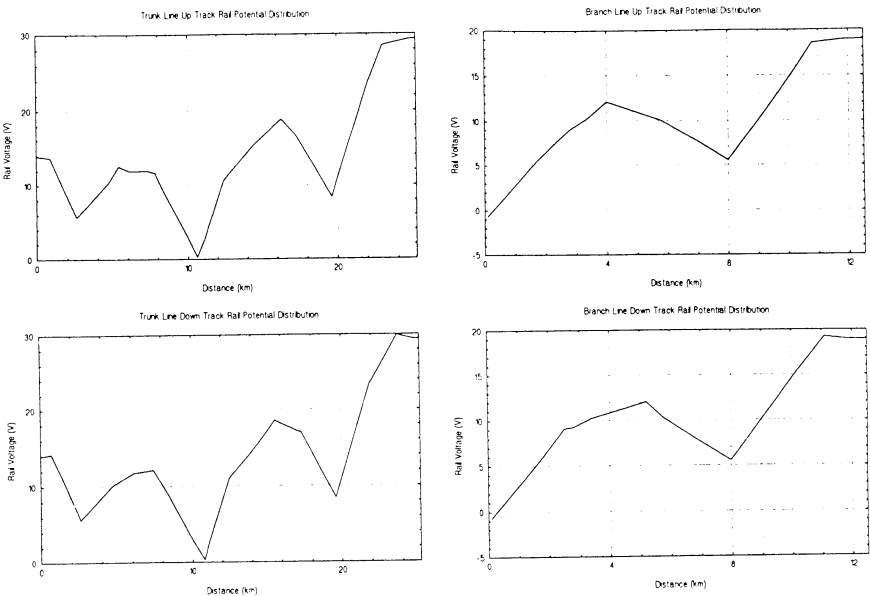
Figure 4: Simulated rail voltages for up and down tracks
on trunk and branch line.

CONCLUDING REMARKS

Railway traction networks are long, thin circuits which when modelled by nodal voltage equations give rise to large sparse matrices. To solve such a set of equations on a PC requires efficient methods for storing the circuit conductance matrix and for manipulating and inverting the matrix elements. This paper has described a method for efficient processing of a large two-ladder electrical circuit capable of evaluating rail voltage in addition to train and substation voltage and current for a multi-train operation on a large LRT system. The technique uses targeted directed elimination followed by Gaussian elimination with backward substitution. The work reported represents an improvement to an existing simulation facility. Future enhancements are planned to deal with regenerative operation with voltage-controlled and inverting substations and optimisation of the iteration process whereby the train power demand is more closely matched to that of the available traction network.

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