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Simulation of the Impact of Social and Economic Institutions on the Size Distribution of Income and Wealth

By FREDERIC L. PRYOR*

In the vast literature on the distribution of income, relatively little systematic attention has been paid to the mutual influence of the distributions of income and wealth on each other, and to the impact of intergenerational grants on the entire process. The purpose of this essay is to utilize a simulation model in which the distributions of income and wealth are analyzed together. The model permits me to explore the influences of such socioeconomic variables as the pattern of intergenerational grants, the rules of inheritance, the patterns of mate selection, differential fertility of various income classes, and the patterns of governmental redistributions of income and wealth. So many simplifying assumptions must be made that the model, as it is presented below, cannot be directly used for determining the current size distribution of income and wealth; nevertheless, the results of the model suggest certain neglected factors, particularly the shape of the intergenerational savings function, that must be taken into account if we are to gain a clearer picture of the causal forces operating in the real world. The most important biases (mostly leading to greater income equality) arising from the simplifying assumptions of the model are also discussed below.

To take all the major long-run factors influencing the size distributions of income and wealth into account is an extremely complicated matter, and we face two alternative research strategies; either we model these factors mathematically, which requires some drastic simplifications in order to keep the equation system solvable,[1] or we take more factors into account by simulating their impact. While the latter procedure does not lead to a completely general solution it does permit us to investigate certain features of economic systems using parameters of particular interest.

To place this model in perspective, it is useful to note that variance of personal income can be derived from three sources; variances in the distribution of labor income, of property income, and of the interaction between these two variables.[2] In this model, variations of labor income are handled by assigning everyone a lifetime income equal to the average lifetime in-

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[1] Such an approach is followed by Joseph Stiglitz and, in a less formal fashion, by James Meade.

[2] The higher the correlation between individual labor and property incomes, the more unequal the distribution of income. (See formula in fn. 11.) The correlation can, of course, be negative, a situation that apparently arose in mandarin China where receivers of property income made a special point of avoiding manual work, even if it meant a lifetime of poverty. (See Hsiao-Tung Fei.)

come times a random variable with a mean of unity and a specified standard deviation. Variations in property income stem only from differential holdings of wealth; returns per unit of wealth are assumed equal; and, finally, no correlation is assumed between individual labor and property incomes. Thus the critical factor for a change in the distribution of income is a change in the distribution of wealth which, in turn, is greatly influenced by the different socioeconomic variables specified below.

I. The Basic Model[3]

The simulation model starts with 100 unmarried people with an arbitrary initial distribution of productive wealth. These people are "put to work" and both lifetime labor and property incomes are generated by means of a production function (a Cobb-Douglas function is used at first but a *CES* function is later tried) with assumptions of full employment and of mean factor payment equal to its marginal product. The total amount of property income is divided among all wealth holders in amounts proportionate to the quantity of wealth held by each; the total amount of labor income is distributed to the entire population by giving each a wage equal to the average wage times a normally distributed variable (which is supposed to represent a differential distribution of abilities such as intelligence or diligence).

The people in the model are then lined up according to income, and marriages are arranged according to one of three different rules: 1) A person can only marry another person next to him on the income scale (this is called the no-choice rule); 2) The chances for a person marrying anyone else are equal (the equal-choice rule); 3) A person can marry anyone but the chances are greater if the two are closer to each other on the income distribution (the limited-choice rule).[4]

The government can then step in and redistribute income (either progressively or regressively). This is followed by the accumulation or disposal of family wealth (positive or negative savings) so that a specified ratio of family wealth to family income is achieved; this intergenerational savings function is discussed in detail below. A simplifying assumption used in the model is that all wealth and net changes in wealth (net savings or dis-savings) are in the form of productive capital which yield property income.

At this point the various families have children according to their income in the following manner: First, the families are divided into three groups according to whether they are among those with the highest family incomes, lowest incomes, or in-between. (The percentage of families falling in each group can be varied.) Then the number of children are specified for each group, for example, the rich can be designated to have more or fewer children than the poor (or vice versa). Polygynous situations can be approximated by specifying many children for the rich and no children for the poor, since in such societies it is well known that only the wealthier can afford to support many wives and the low income men often do not marry.

The parents are then removed from the scene and family wealth is divided among

[3] The model rests on the pioneering work of Guy Orcutt. Other simulation models of income distribution have been made (for example, Hans-Juergen Krupp), but these are considerably different than mine and, moreover, have focused primarily on short-run problems.

[4] The probabilities of one person marrying another is, of course, changed when a couple is married and removed from the pool of eligibles. Therefore, the statements in the text must be considered as just approximately true. For the limited-choice model, the probability of marriage is inversely proportional to the difference in income rank between the two individuals. The calculations are simplified by not designating the sex of the individuals so that marriages between any two individuals may be possible.

the children according to one of three different rules: 1) One child can receive everything (the primogeniture rule);[5] 2) The property can be divided equally among all children (the equal-division rule); 3) The first child can receive half of the wealth and the remainder is divided equally among the rest (the compromise rule). At this point the government can also redistribute wealth (a type of inheritance tax). We now have a group of people with a given distribution of wealth of productive capital whom we put to work; the process is repeated many times to see if a stable distribution of income and wealth is achieved.

With only a few exceptions (discussed below) the processes converge toward an "equilibrium distribution" which, when attained, is maintained for all succeeding generations. In certain cases, however, the process is extremely slow and in order to avoid inordinate computer expenses, the following procedure was adopted. In every case the simulation was carried out twice, once starting from a highly unequal initial distribution of wealth (where the wealthiest 10 percent of individuals share the total societal wealth in equal portions) and once starting from a relatively equal wealth distribution (where the wealthiest 75 percent of individuals share in equal amounts the total wealth). Each simulation is then run for 30 generations (which represents 1000 years if a generation is calculated as $33\frac{1}{3}$ years, or 750 years if a generation is 25 years) and the end results of the simulations starting from different points are averaged; in almost all cases the two estimates were very similar to each other.

[5] I have used an extreme form of primogeniture in which the first child, regardless of sex, obtains the entire family's wealth. This variant of primogeniture leads to the most extreme wealth-holding inequalities and, in addition, is computationally simpler in that sexes do not need to be assigned to particular individuals for the model to work. A more usual case of primogeniture is, of course, when only the eldest son obtains the entire estate.

In the tables I use Gini coefficients as a measure of this inequality of the equilibrium size distribution of lifetime income. Gini coefficients are measured by calculating Lorenz curves and measuring the ratio between the "area of inequality" and the total income triangle (the coefficient ranges from 0.00, which represents total equality to 1.00, which represents total inequality). I have computed four other statistical measures of inequality but space does not permit their inclusion. In certain cases where it is useful to discuss relative speeds of convergence, the period is measured from the starting point to the point when an equilibrium Gini coefficient is achieved (plus or minus a small amount).[6]

Before the numerical results are presented, certain features of the model may perhaps be better understood if we examine in a qualitative fashion the effect of particular variables.

A. The Simplest Patterns

If we take a situation where every family, regardless of income, has two children; where there is no variation in labor income (the random factor is not yet introduced); and where families do not add to, or decrease, their inherited wealth (i.e., family wealth is passed on unchanged regardless of income, a situation occurring when land is the basic source of wealth and is not alienable), the observed patterns of convergence of the distribution of wealth are quite simple and are outlined in Table 1 below.

With primogeniture wealth accumulates eventually into a single hand.[7] The speed of convergence is inversely proportional to the degree to which people choose marriage partners in other income brackets. With

[6] The exact method employed is described in the Appendix.

[7] If the eldest son variant of primogeniture is used, then, of course, this extreme result will not obtain.

TABLE 1—BASIC EQUILIBRIUM PATTERNS OF THE DISTRIBUTION OF WEALTH

	Marriage rules		
Inheritance rules	No-choice rule	Limited-choice rule	Equal-choice rule
Primogeniture	Wealth concentrates to a single owner; fast convergence.	Same as no-choice rule, but convergence is slower.	Same as no-choice rule, but convergence is slower than other primogeniture situations.
Equal division	Wealth distribution remains the same as starting position.	Wealth becomes evenly distributed but convergence is slow.	Wealth becomes evenly distributed; convergence is faster than with limited-choice rule.

equal division of inherited property and with everyone marrying a person next to him on the income distribution, it should be readily apparent that no change will occur in the size distribution of income and wealth, as long as all families continue to have two children. In the other two cases with equal division of inherited property, wealth eventually becomes completely equally distributed.

The compromise rule of inheritance, where the first child received half of the property and the remainder is divided equally among the other children, is the same as the equal-division rule (under the assumption of two-child families).

The speed of conversion to equilibrium depends, of course, on the distribution of wealth at the starting point and at equilibrium. Starting from a highly unequal distribution of income, convergence is achieved in the primogeniture case in three to five generations; starting from the more equal distribution of wealth, convergence takes six to fifteen generations. For the equal-division cases where convergence occurs, the process generally takes somewhat longer. Although convergence speeds in the above examples appear to have little economic meaning, they become important when governmental policy measures are introduced (i.e., redistribution of income and/or wealth) and certain goals of income distribution are set.

B. *The Impact of a Random Variable Representing Abilities*

If we now introduce a random variable representing differential abilities so that labor incomes are the product of the average wage times a random variable (with a mean of unity), the basic patterns are modified in the following ways:

1) The greater the variation in the random element, the more unequal the equilibrium income distribution in those five cases in Table 1 where convergence is observed. This is because greater extremes in labor income are generated.

2) In situations where people are allowed choice in marriage partners, the convergence process is speeded up where there is an equal division inheritance rule and slowed down where there is primogeniture. This stems from the fact that there is a greater mixing of people of different wealth at the time of marriage (since the marriage rules are based on total income, not wealth alone). Where people marry those next to them on the income scale, a complication arises which is discussed below.

One important methodological point also arises. Introduction of a random element raises difficulties in determining the exact equilibrium distribution of income. In interpreting the various tables presented below, small differences in the coefficients should be overlooked and, although data are presented to three places, variations of several percent or less should usually be neglected.

Throughout the remaining simulations the random factor is set with a standard deviation of 15 percent, which is roughly similar to the variation of I.Q. test scores.

C. The Impact of Differential Fertility Rates

In all cases primogeniture leads to a highly unequal income distribution since, whatever the fertility pattern, only one child receives all the property. Similarly, in all cases where the rich have only one child (which leads automatically to primogeniture) a highly unequal equilibrium income distribution is also generated.

In nonprimogeniture cases, fertility affects the end results considerably. In cases where fertility increases with income, the system should converge relatively quickly to fairly even distributions. (Such a situation allegedly occurred in past eras in oriental despotic societies where the rulers encouraged polygyny so that the rich would have a much higher fertility rate than other classes and, at the same time, forbade primogeniture; the end result was supposed to be an economy with a relatively even distribution of income and no independent bases of wealth with which to challenge royal authority.)[8]

Simulation results of different patterns of fertility among the income classes are presented in Appendix Table 1. For simplicity, in the rest of the paper, I assume that fertility is the same in all income classes unless otherwise specified.

II. Quantitative Results: No Saving; Stationary Population; No Technological Change

We are now ready to begin the quantitative investigation. In Table 2 below, the results for the most simple situation with different marriage and inheritance rules are presented. There is no accumulation of wealth for the society as a whole and each family passes on only that wealth which it inherits.

TABLE 2—GINI COEFFICIENTS OF EQUILIBRIUM INCOME DISTRIBUTION ASSUMING DIFFERENT MARRIAGE AND INHERITANCE RULES

	Marriage Rules		
Inheritance Rules	No-Choice	Limited-Choice	Equal-Choice
Primogeniture	.307	.308	.297
Compromise	[a]	.064	.060
Equal Division	[a]	.064	.060

Assumptions: No net family capital formation; no capital or income redistribution; all families have two children; standard deviation of the random element is .15; labor share of national income is 75 percent.

[a] If the system starts from a highly unequal distribution of wealth, the equilibrium distribution of income is equal to its original value. If the system starts from a relatively equal wealth distribution where high ability people with no property might marry low ability people with property (since they would be next to each other on the income scale), then the equilibrium income distribution would be highly equal.

As expected, the inequality of income is higher with primogeniture than with the compromise or equal division inheritance rules (which give the same answers because each family has only two children). As we also expect, the equilibrium income distribution does not seem greatly affected by the marriage rules; in this simple model the major effect of the marriage rules appears on the speed at which equilibrium is achieved. Only in much more complicated models do the marriage rules appear to have much impact on the equilibrium size distribution of income. One puzzling phenomenon appears in the primogeniture case where the other measures of inequality give somewhat different results but this is due most likely to a change in the shape of the distribution of income, with slightly increasing inequality at the high income end and slightly de-

[8] Such systems are analyzed by K. Wittfogel.

creasing inequality at the middle and lower income levels. This particular result does not appear due to random factors since equilibrium was achieved from all starting points of the simulation and the various results given particular marriage and inheritance rules were very similar.

In the simulation model a wealth redistribution process can be set up so that all inherited wealth is taxed a given percentage and then the total amount of taxed wealth is distributed equally among all individuals. If the tax rate is positive, then such a process is progressive because the least wealthy end up with a net gain in wealth while the most wealthy end up with a net loss. This manner of specifying a capital tax on inherited wealth allows the total amount of privately held wealth to remain constant.

The results of imposing a capital redistribution tax are presented in Table 3 and can be summarized quite easily: The greater the redistribution of wealth, the more equal the equilibrium distribution of income and wealth. This, of course, is not surprising. The equilibrium income distributions are most unequal for the primogeniture cases and most equal when family property is evenly distributed among heirs, a result similar to the previous findings. The marriage rules again have a relatively small impact. It must be noted that with very high redistributions of wealth, the differences in the equilibrium distributions with the various marriage and inheritance rules are relatively small; it appears that after a certain point, the redistribution swamps the effects of other institutions.

Since the redistribution of wealth can be a deliberate tool of governmental policy, the rate of convergence to the equilibrium income distribution is of considerable interest. Several generalizations can be made: First, imposition of a redistribution of wealth greatly increases the speed of convergence and in almost all cases convergence is achieved within five generations (with the major portion of the changes occurring in the first two generations). Second, convergence appears to be faster when the model is started with relatively more unequal wealth distribution

TABLE 3—GINI COEFFICIENTS OF EQUILIBRIUM INCOME DISTRIBUTION WITH DIFFERENT REDISTRIBUTIONS OF WEALTH

Redistribution and inheritance rules	Marriage Rules: No-Choice	Limited-Choice	Equal-Choice
No redistribution of wealth			
Primogeniture	.307	.308	.297
Equal division	[a]	.064	.060
30 percent redistribution of wealth			
Primogeniture	.156	.158	.148
Equal division	.061	.063	.063
60 percent redistribution of wealth			
Primogeniture	.088	.091	.089
Equal division	.062	.062	.064
90 percent redistribution of wealth			
Primogeniture	.061	.066	.064
Equal division	.062	.062	.062

Assumptions: see Table 2
[a] See fn. a, Table 2

than with a relatively more equal wealth distribution. In other words, achieving a more equal distribution of income by means of a redistribution of wealth can be carried out more quickly in those cases where the differences between actual and desired distributions are greatest.

Introducing a redistributive income tax changes, of course, the equilibrium income distribution. In this case wealth is not affected; and since marriages are arranged according to relative income, this part of the system is not affected either. Under the assumptions in this section about no-net intergenerational accumulations of family capital, the speed of convergence toward the equilibrium income distribution should not be affected. However, once we allow net intergenerational accumulation of capital based on family income, then an income redistribution could have a number of effects on total savings, growth, and convergence and provides a much more interesting problem for analysis. Discussion of the effects of a redistributive income tax is therefore deferred until the rigid assumption about the integenerational transfer function is loosened.

III. Quantitative Results: Introduction of an Intergenerational Savings Function

Let us now allow the capital stock to change, while still keeping the total population constant, so that we can isolate some of the effects of different patterns of intergenerational savings and transfers. One of the most important results of the simulation exercise is to show that the shape of this function has crucial importance on the equilibrium income distribution. Before turning to the results, however, several theoretical questions deserve brief examination.

First we must inquire about the nature of the integenerational transfer function. Up to now this has been a *terra incognita* in the economic literature: empirical data with which to derive such a function are unavailable and theorists who insist on deducing propositions about consumer behavior from the neoclassical axioms of rational choice have not been able to say anything about the matter. One reasonable assumption, which is reflected in numerous *obiter dicta* on the subject, is that the amount of wealth passed on by a husband and wife to their children is primarily a function of lifetime family income and it is on this basis that the model is changed below. The shape of the function is still problematical and experiments are carried out with several different formulae.

Second, once we introduce changes in the net capital stock, a problem arises because we must also take into consideration the possibility of multiple stable growth paths;[9] thus the initial conditions of the system and the way in which the intergenerational transfer function are specified become quite important. In order to avoid such complications, care was exercised in designing the intergenerational transfer function so that only a single stable growth path, independent of the initial conditions, would be achieved. A number of tests were also made with different initial conditions (different initial capital stocks) in order to insure the correctness of the specification.

A third problem arises in the choice of the production function, i.e., the function showing the relationship between the capital and labor in the system and total production. All production functions used below exhibit diminishing returns to variations in the capital-labor ratio. This means, among other things, that the ratio of capital to labor asymptotically approaches a limit which represents a "steady-state equilibrium" where depreciation just equals gross savings and net

[9] This problem is analyzed in an abstract but lucid manner by Stiglitz.

capital formation is zero. Such diminishing returns can, however, be offset by technological change so that in "steady-state growth" production, the capital stock and the ratio of capital to labor rise at the same rate of growth. In this particular case (Harrod-neutral growth), labor productivity (the ratio of output to the constant labor force) also rises at the same rate while capital productivity asymptotically approaches a constant.

One last precautionary note must be added. Although the analysis below focuses almost exclusively on income distribution, it must be emphasized that the various types of intergenerational transfer functions, production functions, and redistributions of wealth and income lead to quite different equilibrium levels of production. Although there are a number of propositions in the economic literature linking the inequality of income to the growth rate of production (for example, the more unequal the distribution of income, the greater the aggregate production), the results below show exceptions to such generalizations. For those interested in pursuing the relationship between income distribution, the savings function and growth, some data on steady-state production equilibria are given in Appendix Tables 2, 4, and 6.

A. *The Impact of Linear and Non-Linear Intergenerational Transfer Functions*

Two types of integernerational transfer functions are used in the analysis below. The first is a simple linear function where intergenerational transfers of a particular family are a proportion of lifetime income: $S=Yz$ where S = intergenerational transfers; Y = family lifetime income; and z is the "savings constant" to be specified. If family income is low and inherited wealth is great, the intergenerational transfer (proportionate to income) might be less than the original inherited wealth, i.e., the family has "dipped into capital." A second type of intergenerational transfer function has a kink in it and is thus non-linear; such transfers are a function of income over and above some socially determined "subsistence level," below which no intergenerational transfers are made (i.e., the family doesn't pass on inherited wealth). Since negative intergenerational transfers (i.e., debts) are not permitted (although dipping into capital is allowed), the kink of the savings function occurs at $\overline{Y}$. The simple form selected for this non-linear function is $S=(Y-\overline{Y})z$, $S \geq 0$.

A brief digression is necessary to clear up certain ambiguities of this subsistence level. First, this is not necessarily the biological subsistence level nor does there need to be any explicit societal recognition that such a subsistance level actually exists. Rather, it is merely the income level below which families feel they must spend all of their funds for consumption in order to try to achieve a certain standard of living and as a result, they do not have any wealth left over to pass on to succeeding generations. If per capita income in the society rises and the socially determined subsistence level remains stationary, this subsistence level may eventually become such a small proportion of individual family income that the savings function is, for all intents and purposes, linear, i.e., the performance of the system asympototically approaches that of a system with an intergenerational transfer function of $S=Yz$. On the other hand, the socially determined subsistence level can also rise as society views on an adequate standard of living rise. Since this might be a reasonable approximation of reality, it is useful to tie the subsistence income to the rise in per capita income. For simplicity, I have set the subsistence income always equal to average per capita income which means that when a non-linear intergenerational transfer function is used, only those

TABLE 4—GINI COEFFICIENTS OF EQUILIBRIUM INCOME DISTRIBUTION ASSUMING DIFFERENT INTERGENERATIONAL TRANSFER FUNCTIONS

Transfer functions and inheritance rules	Marriage Rules No-Choice	Limited-Choice	Equal-Choice
	$S=Yz$		
$z=1.5$			
Primogeniture	.165	.165	.161
Equal division	.069	.067	.060
$z=2.0$			
Primogeniture	.167	.169	.162
Equal division	.066	.066	.063
$z=2.5$			
Primogeniture	.165	.170	.164
Equal division	.065	.063	.062
	$S=(Y-\bar{Y})z$		
$z=2.0$			
Primogeniture	.306	.301	.300
Equal division	.293	.266	.206
$z=2.5$			
Primogeniture	.308	.309	.306
Equal division	.296	.260	.199

Assumptions: All families have two children; standard deviation of random element is .15; labor share of national income is 75 percent; no income or capital redistributions; no technical change; no negative transfers.
Note: S=intergenerational transfers; Y=personal incomes; $\bar{Y}$=average income; z=a constant.

families with incomes above the average pass on wealth to the succeeding generation. Finally, it must be noted that the socially determined subsistence level can rise faster than average income and in this strange case with such a strong "demonstration effect," families might dip into capital (as long as they did not run into debt) until little capital would be left in the society as the subsistence level approaches the highest family incomes.

Experiments were also made with a third form of the intergenerational transfer functions: $S=W+Yz$, where W is the wealth inherited by the mother and father of the family from their parents. This function implies that the wealth inherited by the children is always greater than the wealth inherited by the parents, no matter how low the family income might be because of the low labor incomes received by the parents. Such a situation does not seem very realistic and these experiments were abandoned. The results of simulation experiments using both linear and non-linear intergenerational transfer functions for several different parameters of the savings coefficient (z) are presented in Table 4 above.[10]

The non-linear transfer function leads,

[10] If the capital-output ratio is greater than unity, the average z coefficient must be greater than unity or there must be a multiplicative constant in the production function if the capital stock in the economy is to be maintained. In the results reported in Table 4, z is placed larger than unity. If a multiplicative constant were used in the production function, z could be made less than unity (which, of course, makes more "real life" sense) but the results would be the same.

as one might expect, to a much greater inequality of income than the linear function; for in the former case, only the richer segments of the population pass on wealth to their children and this, in turn, concentrates wealth and property income. With a simple linear transfer function, the height of the savings coefficient (z) does not appear to affect the inequality of the equilibrium income distribution (the differences do not appear statistically significant) although, of course, a higher z leads to a higher steady-state production level. On the other hand, a higher savings coefficient does seem to affect the degree of income inequality when a non-linear function is used, although the direction of the effect depends upon the particular measure of inequality that is chosen and the inheritance rule that is followed.

With the introduction of intergenerational transfer functions based on income, we now have a situation where the marriage rules have a more important impact on the equilibrium income distribution than in the previous section where capital was passed on regardless of income. With primogeniture the effect of the marriage rules is small in almost all cases except with one nonreported inequality measure. On the other hand, with equal division of property, especially with a non-linear transfer function, the inequality of the equilibrium income distribution decreases as the marriage rules change toward equal choice.

Generalizing about the relationsips between equilibrium income distribution, inheritance rules, and transfer functions is more difficult. With a linear intergenerational transfer function, primogeniture leads to a more unequal distribution of personal income than other inheritance rules. With a non-linear transfer function, the results depend upon the measure of inequality chosen since the relationships between different parts of the income distribution are differentially affected. Thus no generalization is possible.

To summarize, with a linear transfer function where families at all income levels pass on wealth to the succeeding generation, the primary influence on the equilibrium distribution of income is the inheritance rules; and marriage rules or the height of the savings constant (z) have little effect. With a non-linear intergenerational transfer function, the equilibrium income distribution is affected by the marriage rules, the inheritance rules, and the height of the savings constant. Generalizations in these latter cases are difficult because the overall inequality and the shape of income distribution curve change simultaneously.

B. The Impact of Income and Capital Redistributions

A redistribution of capital in each generation not only affects the distribution of income but also, in the case of a non-linear intergenerational transfer function, the growth of the system. (This is because the share of income over the subsistence level is a smaller ratio of total income after a redistribution). The effect on the equilibrium distribution of income should increase with the severity of the redistribution and, one might suspect, would greatly affect the equilibrium distribution in the case of a non-linear, rather than a linear, transfer function. Relevant data are presented in Table 5 below which supports these conjectures.

An X percent redistribution of income in each generation should have a greater impact on the equilibrium distribution of income than an X percent redistribution of wealth because in the latter case only one source of income inequality is being changed. This differential impact of an income redistribution should also be greater in the case of a non-linear intergenerational savings function because the effect on the

TABLE 5—GINI COEFFICIENTS OF EQUILIBRIUM INCOME DISTRIBUTION WITH DIFFERENT CAPITAL REDISTRIBUTIONS

Redistribution taxes and inheritance rules	Marriage Rules		
	No-Choice	Limited-Choice	Equal-Choice
	$S=Yz,\quad (z=2.0)$		
No redistribution			
Primogeniture	.167	.169	.162
Equal division	.066	.066	.063
30 percent capital redistribution			
Primogeniture	.121	.120	.123
Equal division	.061	.060	.060
60 percent capital redistribution			
Primogeniture	.086	.081	.087
Equal division	.063	.064	.063
90 percent capital redistribution			
Primogeniture	.065	.064	.064
Equal division	.062	.062	.062
	$S=(Y-\overline{Y})z,\quad (z=2.0)$		
No redistribution			
Primogeniture	.306	.301	.300
Equal division	.293	.266	.206
30 percent capital redistribution			
Primogeniture	.233	.229	.226
Equal division	.220	.190	.126
60 percent capital redistribution			
Primogeniture	.144	.145	.133
Equal division	.134	.103	.089
90 percent capital redistribution			
Primogeniture	.069	.070	.068
Equal division	.063	.062	.064

Assumptions: See Table 4

share of income over the subsistence level vis-à-vis the total national income would be greater. And finally, the effect of an income distribution should increase with the extent of the redistribution and (using the same arguments employed in the case of the capital redistribution) should affect the equilibrium distribution more in the case of the non-linear than of the linear transfer function. These conjectures receive support in the data presented in Tables 5 and 6 although certain exceptions do arise, especially when other inequality measures are used. Although the underlying reasons for these exceptions are obscure, part of the difficulty may lie in the changing shape of the income distribution curve that accompanies the overall changes in equilibrium inequality.

Again, we observe the fact that convergence to the equilibrium income distribution is very much speeded up with the imposition of either a redistribution of income or wealth tax program. Again, convergence is achieved usually in a few generations although the parameters of the equilibrium are somewhat different than those discussed in Section II.

Since these income or capital taxes may serve as deliberate policy tools by the government to achieve particular income distribution goals, we must turn briefly to the convergence properties of the system. In the case of the linear intergenerational transfer functions, the situation is very similar to the case where there was no net capital formation in each family and convergence occurs very rapidly, usually in

TABLE 6—GINI COEFFICIENTS OF EQUILIBRIUM INCOME DISTRIBUTION WITH DIFFERENT INCOME REDISTRIBUTIONS

Redistribution taxes and inheritance rules	Marriage Rules No-Choice	Limited-Choice	Equal-Choice
	$S=Yz, \quad (z=2.0)$		
No redistribution			
Primogeniture	.167	.169	.162
Equal division	.066	.066	.063
30 percent income redistribution			
Primogeniture	.112	.115	.112
Equal division	.046	.044	.043
60 percent income redistribution			
Primogeniture	.062	.064	.062
Equal division	.024	.026	.024
90 percent income redistribution			
Primogeniture	.015	.016	.015
Equal division	.006	.006	.006
	$S=(Y-\overline{Y})z, \quad (z=2.0)$		
No redistribution			
Primogeniture	.306	.301	.300
Equal division	.293	.266	.206
30 percent income redistribution			
Primogeniture	.216	.215	.212
Equal division	.202	.176	.130
60 percent income redistribution			
Primogeniture	.124	.123	.121
Equal division	.118	.105	.064
90 percent income redistribution			
Primogeniture	.031	.031	.030
Equal division	.029	.026	.019

Assumptions: See Table 4

several generations. In the case of the non-linear intergenerational transfer function, the system approaches the equilibrium distribution equally rapidly; but in the wealth redistributions, the Gini coefficient of income inequality often cycles around the equilibrium value several times, rather than asymptotically approaching it.

Since the greatest changes in income equality brought about by the income or wealth redistribution occur in the first generation and since differences between the actual degree of income inequality and the equilibrium value are quite small after a few generations, one lesson seems clear: other things being equal, redistribution taxes appear an efficient method of changing the degree of income inequality in a nation. (One condition in the *ceteris paribus* clause is that such taxes do not have adverse effects on productivity.) Although radicals may be unwilling to wait a generation for the major effects to take place and prefer instead a revolution to accomplish redistribution aims, the destruction of capital and confusion following such events might lead to a situation where average income would be considerably lower and income not much more equally distributed than if mundane and undramatic redistributional taxes had been used instead.

C. The Impact of the Elasticity of Substitution

Up to now the results are based on a Cobb-Douglas production function which has an elasticity of substitution of unity. Since we have growth in the system, this assumption may have some impact on the results and a series of simulations were run using the *CES* function that has been explored by Kenneth Arrow et al.[11]

Summarizing the results in a capsule form is extremely difficult for a number of counteracting factors influenced the results: further, the results feature a number of small puzzles which are difficult to explain. For those wishing to explore these matters further, the results are presented in Appendix Tables 3 and 4.

IV. Quantitative Results: Introduction of Certain Dynamic Factors

A. The Impact of Harrod-Neutral Technological Change

If we introduce technological change such that the production arising from a given capital and labor stock is multiplied by an exponentially growing factor, per capita income grows, the capital stock grows (because savings increase), and eventually the system achieves a steady-state growth path. Using a Cobb-Douglas production function, factor income shares remain the same.

The results of the simulations can be easily summarized: the equilibrium income distributions do not seem greatly affected by the introduction of Harrod-neutral technical change. (Indeed, the only noticeable effects occur with the non-linear transfer function and these are relatively small.) Undoubtedly other types of technological change would complicate the results, but exploration of these matters must be left for future research. Results of the experiments with Harrod-neutral change are presented in Appendix Table 5.

B. The Impact of Population Change

If we now add to the analysis population growth which can arise from many different patterns of differential fertility, a large number of cases are open to explore. Simplification can be achieved once we realize that introduction of population change has two major effects: it raises the absolute value of total production; and it allows different rates of growth of the capital-labor ratio to occur through changes in the denominator of the fraction, rather than the numerator. Rather than multiply examples endlessly, it seems most useful to examine only several simple patterns of fertility in order to show how the system works. One financial constraint on this process of analysis must also be mentioned: the greater the number of people in the system, the more expensive the simulation becomes. To obtain the results reported in Table 7, I started the system with only fifty people; introduced a 10 percent population growth (per generation) and ran the system for only twenty-five generations. This led to an eightfold increase in population and, as a result, almost a quadrupling of computer cost. (It must also be noted that in order to limit population growth, "poor" and "rich" families are defined as the 20 percent of families on either end of the income distribution, while the "middle class" is the remaining 60 percent; this is slightly different than the income definitions used in calculating Appendix Table 1 below.) Equilibrium gross national products are presented in Appendix Table 6.

The most surprising result appears where the transfer function is linear: here differential fertility appears to have relatively little impact on the equilibrium distribution of income, a result which is somewhat different from the situation in Appendix Table 1 where no net capital formation takes place. In the case of the nonlinear transfer function, on the other hand, the expected impact of differential fertility can be observed in nonprimogeniture situations, i.e., the equilibrium

[11] See Arrow et al. For the derivation of factor shares I used the simple formulae derived by R. G. D. Allen.

TABLE 7—GINI COEFFICIENTS OF EQUILIBRIUM INCOME DISTRIBUTION WITH POPULATION CHANGE AND DIFFERENT FERTILITY PATTERNS

Number of children: Rich	M.C.	Poor	Marriage Rules: No-Choice	Limited-Choice	Equal-Choice
			$S=Yz,\ (z=2.0)$		
2	2	2			
Primogeniture			.167	.169	.162
Compromise			.066	.066	.063
Equal division			.066	.066	.063
3	2	2			
Promogeniture			.082	.178	.175
Compromise			.071	.068	.071
Equal division			.065	.069	.066
2	2	3			
Primogeniture			.182	.176	.175
Compromise			.075	.076	.073
Equal division			.074	.073	.072
			$S=(Y-\overline{Y})z,\ (z=2.0)$		
2	2	2			
Primogeniture			.306	.301	.300
Compromise			.293	.266	.206
Equal division			.293	.266	.206
3	2	2			
Primogeniture			.314	.313	.309
Compromise			.273	.253	.169
Equal division			.230	.185	.127
2	2	3			
Primogeniture			.312	.313	.310
Compromise			.309	.296	.194
Equal division			.306	.294	.219

Assumptions: Standard deviation of random element is .15; Cobb-Douglas production function with labor share of national income as 75 percent; no technological change; no income or capital redistribution; no negative transfers.

distribution appears more equal, the greater the number of children of the rich vis-à-vis other groups in the population.

It should also be noted that population growth gives rise to a somewhat more unequal equilibrium distribution of income than with no population growth and it seems likely that this effect would be greater if population growth were higher. This may be tied up with the results that with population growth, the equilibrium per capita income and the equilibrium capital-labor ratio are somewhat lower which, as a result, means that returns per unit of capital are higher and returns per unit of labor are lower. The exact interaction of these various factors is, however, complex.[12]

[12] Some insight can be gained into these matters by starting from the well-known formulas for the separation of the components of variance;

$$\mathrm{Var}(Y)=\mathrm{Var}(P)+\mathrm{Var}(W)+2\ \mathrm{Cov}(P,W)$$

and $\mathrm{Var}(P)\approx \bar{i}^2\ \mathrm{Var}\ (k)+\bar{k}^2\ \mathrm{Var}(i)+2\bar{i}\bar{k}\mathrm{Cov}(i,k)$

(over)

V. Application of the Model

Given the assumptions of the model, the numerical results obtained in the simulations cannot be directly applied to available data on the distribution of income. Certain biases resulting from the assumptions require special attention.

First, the model assumes that labor income of an individual is not positively correlated with his parents' income. Since it is generally believed that there is, indeed, a positive correlation, this means that the results presented above have a bias toward equality. In future simulations this fact could be built in the model either directly (a procedure which would require considerably more memory capacity of a computer than the program used in this study) or by using a different type of intergenerational savings function in which a fraction of the parents' income would be considered human capital transmitted to the children.

Second, the model assumes that labor income and property income are not positively correlated. Although data have not been published in a useable form empirically to investigate such matters, I strongly suspect that in the United States there is a positive correlation. Certainly those occupying important positions to which high labor incomes accrue are in a better position to invest their money in investments yielding high returns; further, the existence of great wealth often permits people to obtain positions yielding high labor incomes. If this correlation between labor and property income is positive, then the empirical results obtained with the simulation model show a greater income equality than actually exists. Repairing this fault in the model would not be difficult: labor income could be made a function of the random variable plus a given fraction of wealth.

Third, for technical reasons labor incomes in the model were bounded by limits of .5 and 1.5 of the average income, and since capital accumulation is a function of total income, certain limits are placed on the amount of capital accumulation that one individual can carry out. Since this does not permit the existence of a Henry Ford, a J. Paul Getty, or a John D. Rockefeller, who manage to accumulate enormous sums within a single lifetime, the results of the simulation model show a bias toward equality. This might be repaired in the model by designating one person in each generation who is destined to strike it rich at the expense of everyone else (who is "taxed" for this purpose).

Fourth, the distribution of income is calculated from labor and property income before family accumulation (or disaccumulation) takes place. This procedure omits a source making for greater income inequality, namely the income accruing to owners of recently accumulated wealth. In the model presented above, this should not make very much difference; but in more complicated models, this factor must be taken into consideration. On a more general level, the simulation model is based on the assumption that capital accumulation for the entire society occurs through the net addition to inherited productive capital by various families in the system. An alternative method for achiev-

where Y = total personal income; P = property income; W = work income, i = return on wealth; k = wealth; $\mathrm{Var}(Y)$ = variance of Y; $\mathrm{Cov}(i, k)$ = covariance between i and k; and a bar over a letter indicates an average value. In the case discussed in the text, $\bar{i}^2$ increases, $\mathrm{Var}(k)$ remains the same, the $\bar{k}^2\mathrm{Var}(i)$ factor still remains zero (since there is no variation in return per unit of capital) and the covariance term remains roughly the same. Thus the overall variance in property income rises and this, in turn, leads to an increase in the variance of overall personal income. Certain other puzzling phenomena remain, particularly in the data for the non-linear savings function; these are due in part to the fact that unlike most other simulations in this essay, equilibrium was achieved extremely slowly and often by the twenty-fifth generation (the cutoff point), this equilibrium point had not been reached.

ing economic growth that does not involve inheritance occurs when claims on productive capital are accumulated through part of a person's lifetime and then are converted into consumption by the end of the person's life. If generations overlap and if the maximum accumulated wealth is greater for each succeeding generation, then societal capital accumulation could occur without any inheritances.[13] Although such extreme a situation does not seem very likely, such a process may be occurring in part and the equilibrium income results of the simulation model would have a further bias toward equality. This factor could be incorporated into the model, but considerably more memory space in the computer would be required.

Fifth, in the simulation model factor incomes are distributed according to the marginal productivity theory. This is not, however, a crucial aspect of the model since by substituting different coefficients in the production function one can easily change the share of income received for the services of labor and capital. Although I have not considered any interactions between relative factor shares and some of the other parameters of the system (on the grounds that no convincing evidence of such a relationship has been found), such an effect could easily be incorporated into the simulation.[14]

Sixth, the model is allowed to run to equilibrium which, in many cases, takes twenty or so generations. Much more useful for policy purposes would be examination of situations where, starting with the current size distribution of income and wealth, the model were allowed to run only for several generations. The purpose of letting equilibrium be achieved is to give some general results about the direction of change that may be of use to analysts facing many different size distributions of income and wealth.

Seventh and finally, our knowledge about the actual parameters of the system that would influence the equilibrium size distribution of income and wealth is quite limited. Most importantly, we have no statistical idea about the shape of the intergenerational savings function which proves such a critical factor in determining the final equilibrium positions. We do not know the relative importance of more complicated inheritance arrangements where wealth is passed two generations away through particular types of trust arrangements. Our quantitative notions about marriage patterns and inheritance rules may be greater, but such matters still need considerable quantitative analysis before parameters can be derived for use in the model. Except for some imaginative work by Robert Summers, little work has been done on calculating lifetime size distributions of income.[15] Thus, even if the simulation model were more sophisticated, we would not have the requisite knowledge of the proper parametrs for running the model for predictive purposes.

VI. Some Speculations

Although the simple simulation model has some obvious shortcomings and must be viewed as a starting point for more sophisticated models, the results obtained point toward a number of factors neglected by economists interested in the size distribution of income. Moreover, the results permit several different answers to some puzzling questions regarding changes in the degree of inequality of the size distribution of income over time.

Among Western non-Marxist economists there seems to be some agreement that the distributions of income and wealth

[13] Such a situation is explored by James Tobin.

[14] I would like to thank Steven Resnick for his ideas on this theme.

[15] Lifetime income distributions in the United States are estimated by Summers. I would like to thank him for sending me a copy of this extremely useful study.

of nations become increasingly unequal during the early stages of the development process but that after the development process is well underway, this distributional tendency is reversed and the income distribution becomes increasingly more equal. Certain empirical evidence supports this proposition. For instance, in the United States estimates of the distribution of wealth show an increasing inequality throughout the ninteenth century, reaching a high point at the turn of the century. Since then, however, these data show that the inequality of wealth holding has declined so that the Gini coefficient of wealth inequality was roughly the same in 1962 as in 1860.[16] In this century the inequality of personal wealth holdings appears to have decreased in the United Kingdom as well.[17] Finally, income inequality has decreased in a great many developed nations in the last 30 to 60 years and there appears to be an inverse relationship in the West between the level of development and the Gini coefficient of income inequality on both a time-series and a cross-section basis.[18] A number of theoretical arguments concerning the causes of such shifts in the distribution of income have been offered[19] and in this digression I would like to add one additional explanation based on the simulation results above.

In a highly underdeveloped nation the major source of wealth is land; inheritances consist primarily of intergenerational transfers of a fixed amount of land; and the equilibrium distribution of income in such a case is described in Section II. As industrialization begins and accumulated industrial capital becomes an important source of wealth, it seems likely that the intergenerational transfer function approaches the non-linear form described in Section III. This is because the biological level of subsistence is still a substantial proportion of average income and it is unlikely that people with relatively low incomes could pass on a very significant proportion of their lifetime income to their heirs. As per capita income rises, this biological subsistence income becomes an increasingly smaller share of average incomes and it seems likely that the income level below which no intergenerational transfers take place does not rise as fast as average income. If so, then the non-linear intergenerational transfer function asymptotically approaches the linear case.

In such a situation, three stages in the distribution of wealth and income can be distinguished: a stagnant stage in which the distribution of wealth and income remain relatively constant; the initial stages of industrialization in which the distribution of wealth and income become increasingly more unequal (under the impact of a non-linear intergenerational savings function); and a later stage of industrialization in which the distribution of wealth and income become more equal when the intergenerational transfer function becomes more linear.

Since we know very little about intergenerational transfers at any stage of development, this scenario of development

[16] For relevant data see Lee Soltow, Robert Lampman, and James Smith.

[17] Data are presented by Lampman, p. 214.

[18] The four most extensive recent international comparisons of the size distribution of income are by Simon Kuznets, Irving Kravis, Harold Lydall (who only covers labor income), and Richard Weisskoff.

[19] Kuznets (1955) focuses on the shift from rural to urban areas as the most important casual factor. (This model is investigated more throughly on a theoretical level by Henri Theil, and on an empirical level by Weisskoff.) Stiglitz bases an explanation for the same phenomenon on the relationship in the process of economic growth of the starting point to the equilibrium production level. R. Albert Berry focuses on unemployment and changes in particular market imperfections. Others have focused on more politcal factors such as the increase of political mobilization accompanying economic development that leads to greater progressive redistribution of income and wealth by the government after a particular point of development.

must remain speculative; nevertheless, it does provide a focus for future empirical research.

VII. Two General Conclusions

A great many different cases have been discussed, and these are but a small fraction of the possible cases that can be generated by the model. Nevertheless, two general conclusions can be drawn:

First, for a general theory of the size distribution of income, we must take into account the influence of the size distribution of wealth; and this means we must bring into the analysis a number of important social and economic variables such as the pattern of intergenerational transfers of income and wealth, the rules of inheritance in the society, and differential patterns of fertility of income classes. It is impossible to generalize about long-run changes in the size distribution of income and wealth in capitalism without specifying many more variables than economists have usually done. Blanket predictions about increasing concentration of income and wealth, for example, the orthodox Marxist analysis of such problems, implicitly make too many vital assumptions to be of much use.

Second, the simulation model presented in this paper provides a starting point for such a broader type of analysis of the distribution of income and wealth. In order to bring the analysis closer to actual situations in particular countries, much more empirical and theoretical work needs to be done. On the theoretical side, we need to consider many more complications than those presented in this essay; on the empirical side we need to have a much clearer picture of the critical parameters. The model does, however, point to one extremely important factor—the shape of the intergenerational savings function—which has been neglected by previous analysts and which I hope to have demonstrated is critical in predicting changes in the size distribution of income and wealth.

Appendix

Determination of the Speed of Convergence to an Equilibrium Income Distribution

Due to the influence of random factors in the inheritance-marriage simulation program, the generated income distribution does not completely converge to a single income distribution, but rather to a band of income distributions around the equilibrium. A number of curve-fitting methods were attempted in order to derive the equilibrium distribution but these proved unsatisfactory and the following alternative method was adopted. This is a modification of a method suggested by Richard N. Cooper, to whom I would like to express my thanks.

First, an unweighted average and standard deviation of the Gini coefficient of income equality were calculated for the last five generations in the thirty-generation simulation. The sixth to last Gini coefficient was then tested to see if it fell within the .95 confidence limit of the calculated average. If this was the case, then the unweighted average and standard deviation were recalculated to include this datum and the next Gini coefficient was examined in a like manner. This process was stopped when the examined coefficient did not meet the test; the number of generations was then determined, and average coefficient for the other indicators of inequality (for example, the standard deviation of the logarithms of income, the share of income accounted for by the top 10 percent, etc.) were recorded.

Then, with the calculated standard deviation of the Gini coefficient from the above process, I started from the first generation to see at what generation the Gini coefficient had a significant chance of belonging to the calculated equilibrium. When this point was reached, the generation number was recorded. The conversion point was considered to lie between this point and the earliest generation to be included in the calculation outlined in the first step.

Such a procedure is based on the assumption that the income distribution converges by and large within thirty generations. Whether such a convergence occurred at all was determined by visual inspection of the entire series of calculated Gini coefficients.

Appendix Table 1—Gini Coefficients of Equilibrium Income Distributions Assuming Differential Fertility Rates[c]

Number of children[a] Rich / M.C. / Poor	Marriage Rules: No-Choice	Limited-Choice	Equal-Choice
2 2 2			
Primogeniture	.307	.308	.297
Compromise	[b]	.064	.060
Equal division	[b]	.064	.060
3 2 1			
Primogeniture	.309	.309	.306
Compromise	.075	.074	.076
Equal division	.074	.069	.074
1 2 3			
Primogeniture	.308	.308	.298
Compromise	.303	.304	.301
Equal division	.308	.300	.295
1 3 1			
Primogeniture	.310	.306	.300
Compromise	.306	.305	.295
Equal division	.308	.302	.295
3 1 3			
Primogeniture	.310	.306	.303
Compromise	.168	.160	.135
Equal division	.168	.158	.130

Assumptions: No net family capital formation; no capital or income redistributions; standard deviation of random element is .15; labor share of national income is 75 percent.

[a] The poor are those 25 percent of families with the lowest income; the rich are those 25 percent of families with the highest incomes.

[b] If the system starts from a highly unequal distribution of wealth, the equilibrium distribution of income is equal to its original value. If the system starts from a relatively equal wealth distribution where high ability people with no property might marry low ability with property (since they would be next to each other on the income scale), then the equilibrium income distribution would be highly equal.

[c] A Gini coefficient of zero represents total equality; a coefficient of unity represents total inequality.

APPENDIX TABLE 2—EQUILIBRIUM GROSS NATIONAL PRODUCTS WITH COBB-DOUGLAS PRODUCTION FUNCTIONS AND DIFFERENTIAL SAVINGS FUNCTIONS[a]

	Marriage rules					
	No-choice		Limited choice		Equal choice	
	Inheritance rules					
	Primogeniture	Equal division	Primogeniture	Equal division	Primogeniture	Equal division
	$S=Yz$: any income or capital redistribution					
$z=1.5$	114	114	114	114	114	114
$z=2.0$	126	126	126	126	126	126
$z=2.5$	136	136	136	136	136	136
	$S=(Y-\overline{Y})z$					
No redistributions						
$z=2.0$	79	78	79	75	79	71
$z=2.5$	85	84	85	79	85	73
$z=2.0$, income redistributions of R percent						
$R=0.0\%$	79	78	79	75	79	71
$R=30.0$	70	69	70	65	70	57
$R=60.0$	58	57	58	55	58	47
$R=90.0$	37	36	37	34	37	31
$z=2.0$, capital redistributions of R percent						
$R=0.0\%$	79	78	79	75	79	71
$R=30.0$	70	70	70	66	70	58
$R=60.0$	58	58	58	54	58	52
$R=90.0$	47	48	49	48	47	47

Assumptions: All familes have two children; standard deviation of random element is .15; labor share of national income is 75 percent; no negative saving; no technical change.

Notes: S=personal savings; Y=personal income; $\overline{Y}$=average income; z=a constant.

[a] For the equilibrium *GNP* using the non-linear saving function, production at the 30th generation was used as the equilibrium value.

APPENDIX TABLE 3—GINI COEFFICIENTS OF EQUILIBRIUM INCOME DISTRIBUTION WITH DIFFERENT PRODUCTION FUNCTIONS

Production functions, transfer and inheritance rules	Marriage Rules: No-Choice	Limited-Choice	Equal-Choice
	$S=Yz$, $(z=2.0)$		
CES, $s=1.5$			
Primogeniture	.208	.208	.200
Equal division	.061	.060	.059
Cobb-Douglas			
Primogeniture	.167	.169	.162
Equal division	.066	.066	.063
CES, $s=0.5$			
Primogeniture	.102	.101	.102
Equal division	.079	.076	.074
	$S=(Y-\overline{Y})z$, $(z=2.0)$		
CES, $s=1.5$			
Primogeniture	.234	.233	.237
Equal division	.220	.188	.117
Cobb-Douglas			
Primogeniture	.306	.301	.300
Equal division	.293	.266	.206
CES, $s=0.5$			
Primogeniture	.404	.402	.405
Equal division	.400	.377	.327

Assumptions: All familes have two children; standard deviation of random element is .15; no technological change; no income or capital redistribution; no negative transfers.

Notes: S = intergenerational transfers; Y = personal income; $\overline{Y}$ = average income; z = constant; L = labor force; K = capital stock; P = total production; b = a constant; s = elasticity of substitution.

Production functions: Cobb-Douglas: $P=L^{.75}K^{.25}$

CES: $P=[.75L^{-b}+.25K^{-b}]^{-1/b}$, where $b=(1/s)-1$

APPENDIX TABLE 4—EQUILIBRIUM GROSS NATIONAL PRODUCTS WITH DIFFERENT PRODUCTION FUNCTIONS[a]

Production functions	No-choice: Primogeniture	No-choice: Equal division	Limited-choice: Primogeniture	Limited-choice: Equal division	Equal-choice: Primogeniture	Equal-choice: Equal division
			$S=Yz$, $(z=2.0)$			
CES, $s=1.5$	131	131	131	131	131	131
Cobb-Douglas	126	126	126	126	126	126
CES, $s=0.5$	117	117	117	117	117	117
			$S=(Y-\overline{Y})z$, $(z=2.0)$			
CES, $s=1.5$	75	74	75	72	75	67
Cobb-Douglas	79	78	79	75	79	71
CES, $s=0.5$	86	85	86	82	86	73

Assumptions: All families have two children; standard deviation of random element is .15; no negative savings; no technical change; no capital or income redistributions.

Notes: S = personal savings; Y = personal income; $\overline{Y}$ = average income; z = a constant; s = elasticity of substitution.

[a] See fn. a, Appendix Table 2.

APPENDIX TABLE 5—GINI COEFFICIENTS OF EQUILIBRIUM INCOME DISTRIBUTION WITH HARROD-NEUTRAL TECHNICAL CHANGE

Transfer functions and inheritance rules	No-Choice	Limited-Choice (Marriage Rules)	Equal-Choice
$S=Yz$, $(z=2.0)$		No technical change	
Primogeniture	.167	.169	.162
Equal division	.066	.066	.063
		Technical change = 10% per generation	
Primogeniture	.165	.165	.161
Equal division	.069	.067	.060
		Technical change = 20% per generation	
Primogeniture	.165	.165	.161
Equal division	.069	.067	.060
$S=(Y-\overline{Y})z$, $(z=2.0)$		No technical change	
Primogeniture	.306	.301	.300
Equal division	.293	.266	.206
		Technical change = 10% per generation	
Primogeniture	.308	.305	.309
Equal division	.300	.253	.190
		Technical change = 20% per generation	
Primogeniture	.308	.305	.309
Equal division	.300	.253	.191

Assumptions: All families have two children; standard deviation of random element is .15; labor share of national income is 75 percent; no income or capital redistribution; no negative savings.

Notes: S = intergenerational transfers; Y = personal income; $\overline{Y}$ = average income; z = a constant.

APPENDIX TABLE 6—EQUILIBRIUM GROSS NATIONAL PRODUCTS WITH DIFFERENT FERTILITY RATES AND POPULATION GROWTH[a]

Fertility Patterns: Number of children — Rich	M.C.	Poor	No-choice: Primo-geniture	No-choice: Compro-mise	No-choice: Equal division	Limited-choice: Primo-geniture	Limited-choice: Compro-mise	Limited-choice: Equal division	Equal-choice: Primo-geniture	Equal-choice: Compro-mise	Equal-choice: Equal division
						$S=Yz$, $(z=2.0)$					
3	2	2	535	535	535	535	535	535	535	535	535
2	2	2	535	535	535	535	535	535	535	535	535
						$S=(Y-\overline{Y})z$, $(z=2.0)$					
3	2	2	337	314	298	336	309	281	336	271	252
2	2	2	337	335	335	336	331	328	336	282	288

Assumptions: Standard deviation of random element is .15; Cobb-Douglas production function with labor share of national income is 75 percent; no technological change; no income or capital redistribution; no negative savings; rich and poor are top and bottom 20 percent of income distribution, respectively.

Basic parameters: 50 familes, 25 generation

Abbreviations: S = personal savings; Y = personal income; $\overline{Y}$ = average income; z = a constant

[a] For the equilibrium GNP production at the 25th generation was used as the equilibrium value.

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