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### Authors

Miracky, J.F. Clarke, J.

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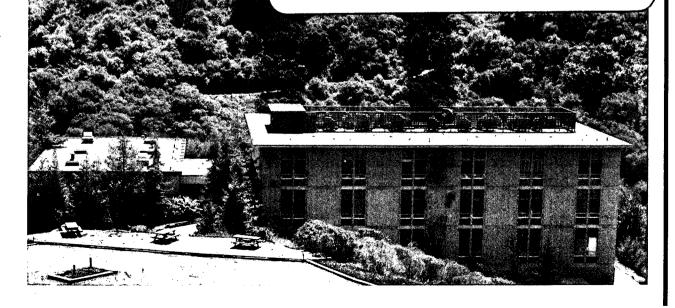
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J.F. Miracky and J. Clarke

May 1983

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#### SIMULATION OF THE NOISE RISE IN THREE-PHOTON JOSEPHSON PARAMETRIC AMPLIFIERS

Robert F. Miracky and John Clarke

#### Department of Physics University of California Berkeley, California 94720

and

#### Materials and Molecular Research Division Lawrence Berkeley Laboratory Berkeley, California 94720

#### ABSTRACT

Analog simulations of the three-photon Josephson parametric amplifier indicate that the noise rise observed in experimental devices arises from occasional hopping between a mode at the pump frequency,  $\omega_p$ , and a mode at the half-harmonic,  $\omega_p/2$ . The hopping is induced by thermal noise associated with the shunt resistance.

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The three-<sup>1-4</sup> and four<sup>5</sup>photon Josephson parametric amplifiers have been plagued by the problem of "noise rise", a noise temperature that increases roughly as the power gain.<sup>6-9</sup> Huberman <u>et al</u>.<sup>10</sup> first suggested that chaos might be the explanation of this phenomenon in the four-photon case, and there has subsequently been a considerable literature on the subect.<sup>7-9,11</sup> Pedersen and Davidson<sup>8</sup> concluded that chaos could indeed explain the noise rise in the four-photon amplifier, while Levinsen<sup>9</sup> concluded that chaos could not be the explanation in either the three- or four-photon case. Levinsen<sup>9</sup> emphasizes that an important aspect of the real amplifiers has usually been neglected in the other studies of chaotic behavior, namely the circuit that couples the signal to and from the junction. Since the gain of the amplifier is intimately related to the impedance presented by the coupling circuit, any model that attempts to explain the noise rise must include this element.

Recent<sup>12</sup> experimental studies of chaos in-a Josephson tunnel\_junc-\_\_\_ tion shunted by a resistor with significant self-inductance have brought to light an important effect due to the presence of thermal noise in a non-linear system exhibiting subharmonic modes and chaos. In this system, noise-induced "hopping" between nearby modes can occur, resulting in copious levels of noise at low frequencies. To test whether such hopping effects can occur in a Josephson parametric amplifier, we have simulated the three-photon mode with an analog simulator that includes both a calibrated thermal noise source and a model for the coupling circuit. We conclude, first, that chaos cannot explain the noise rise, and, second, that there is strong evidence that the hopping mechanism accounts for the observed noise.

The analog circuit, suggested by Levinsen, 'is illustrated in Fig. 1. We assume the resistively shunted junction model with  $\beta_{C} \equiv 2\pi I_{O}R_{J}^{2}C/$  $\Phi_{0}$  = 26.3 for all simulations discussed here; I is the critical current,  $1/R_{J}$  is the quasiparticle conductance, C is the self-capacitance, and  $\Phi_{o} = h/2e$  is the flux quantum. The pump is represented by a current source,  $I_p cos(\omega_p t)$ , and the details of its coupling circuit are assumed to be unimportant. We report simulations for  $\omega_p/\omega_o = 1.12$ , where  $\omega_o \equiv$  $(2\pi I_{O}/\Phi_{O})^{\frac{1}{2}}$  is the maximum plasma frequency. Although Kautz<sup>11</sup> states that chaotic solutions exist only for  $1/R_{\rm I}C \lesssim \omega \lesssim \omega_{\rm o}$ , we have discovered no significant difference between simulations for  $\omega_p/\omega_o = 1.12$ ,  $\omega_p/\omega_o =$ 0.93, and  $\omega_p/\omega_0 = 0.84$ , except in the pump amplitude required for maximum gain. In real parametric amplifiers, one generally uses a circulator to couple the signal into and out of the junction. We represent this element in our model by a resonant circuit tuned to the signal frequency,  $\omega_{c} = (L'C')^{-1_{2}}$ . We chose values of L', C' and of the series and load resistors, R' and R<sub>1</sub>, to achieve a Q for the coupling circuit,  $(L'/C')^{\frac{1}{2}}$  $(R' + R_{T})$ , of about 20. We also maintained the conditions  $R' = R_{T}$  and  $R' + R_{I} \approx R_{I}$  necessary to achieve near-optimum noise performance.<sup>4</sup> Finally, a white noise current,  $I_N(t)$ , was included to represent Nyquist noise in the shunt conductance of the junction.<sup>13</sup> The magnitude of this noise is characterized by  $\Gamma = 2\pi k_{\rm p} T / I_{\rm p} \Phi_{\rm c}$ .

We followed the standard procedure in setting up the correct pump and dc-bias levels. With low pump power and a small signal at  $\omega_S \approx \omega_p/2$ , we adjusted the dc-bias current,  $I_{dc}$ , until signal gain and an idler at frequency  $\omega_i = \omega_p - \omega_S \approx \omega_S$  were observed. We then increased the pump amplitude,  $I_p$ , adjusting  $I_{dc}$  to achieve maximum gain. Maximum gain oc-

curs when  ${}^4 \omega_{\rm S} = \omega_0 [J_0(n)]^{\frac{1}{2}} [1 - (I_{\rm dc}/I_0)^2]^{\frac{1}{4}}$ , where  $n \equiv 2eV_{\rm P}/\hbar\omega_{\rm P}$  and  $V_{\rm P}$  is the voltage across the junction at the pump frequency.

Figure 2 illustrates representative examples of our results. In Fig. 2(a), we plot the spectral density of the voltage across  $R_{_{\rm I}}$  for  $I_p/I_o = 0$  and  $I_p/I_o = 0.682$  (the point of maximum gain) in the absence of added thermal noise (the residual noise level corresponds to  $\Gamma$  = 6.1  $\times$  10<sup>-7</sup>). The idler is approximately equal to the signal in amplitude. The smaller peaks at frequencies below  $\omega_1$  and above  $\omega_2$  represent resonances associated with the coupling circuit.<sup>14</sup> The signal gain,  $G_{s}$ , is 19.9 dB, while the noise gain,  $\boldsymbol{G}_{_{\!\!N}},$  at frequency  $\boldsymbol{\omega}_{_{\!\!N}}$  increases by 31 dB as  $I_p/I_o$  is increased from 0 to its value for maximum gain. (The signal gain, G<sub>S</sub>, is the ratio of the power at  $\omega = \omega_S$  for  $I_P/I_Q = 0.682$  to that at  $I_p/I_o = 0$ ; the noise gain,  $G_N$ , is the ratio of the noise powers at  $\omega \approx \omega_{\rm S}$  for the same two pump levels.) For larger values of I<sub>p</sub> or I<sub>dc</sub>, a bifurcation occurs to an oscillation at  $\omega_p/2$ . This behavior is welldocumented experimentally,<sup>15</sup> and is a useful means of showing that the maximum gain condition has just been passed. At the bifurcation point, signal amplification drops sharply. Furthermore, there is no evidence for chaotic solutions at nearby bias points.

Figure 2(b) shows the results for the same range of pump amplitudes with  $\Gamma = 1.8 \times 10^{-4}$ , or T = 4.2K for I<sub>0</sub> = 1 mA. The maximum signal gain decreases to 14.0 dB from its noise-free value of 19.9 dB. More importantly, the noise level near  $\omega_{\rm S}$  has increased far more sharply than the signal, by 32 dB as the gain is increased from unity to its maximum value.

An important issue in the history of both the three- and four-photon amplifiers has been the exact dependence of the noise temperature,  $T_N$ , on

the signal gain,  $G_{s}$ . To address this question, in Fig. 3 we plot  $G_{N}/G_{s}$ (a quantity proportional to  $T_N$ ) versus  $G_S$ , where we varied  $G_S$  by varying I<sub>p</sub> with I<sub>dc</sub> kept fixed at is value for maximum gain. For  $\Gamma = 6.1 \times 10^{-7}$ (upper curve)  ${\rm T}^{}_{\rm N}$  rises somewhat as  ${\rm G}^{}_{\rm S}$  rises to about 7 dB, and is constant for higher values of  $G_{s}$ . Thus, for  $G_{s} > 7$  dB,  $T_{N}$  is independent of  $G_{\rm S}^{}$ , as one would expect for a useful amplifier. On the other hand, for  $\Gamma$  = 1.8  $\times$  10<sup>-4</sup> (lower curve), the noise temperature rises steadily as  $\rm G_S$ is increased, by 18 dB as the gain increases from 0 to 14 dB. We note that  $T_{N}$  rises more rapidly than  $G_{S}$  for values of  $G_{S}$  below about 10 dB, and more slowly than  $G_{S}^{}$  for higher values of  $G_{S}^{}$ . This deviation from a linear relationship between  ${\rm T}_{\rm N}$  and  ${\rm G}_{\rm S}$  is very compatible with the experimental results of Mygind <u>et al</u>.<sup>2</sup> for values of  $G_S$  up to about 8 dB. However, it is difficult to make meaningful comparisons of the magnitudes of  ${\rm T}_{\mathop{\rm N}}$  obtained from our simulations with those obtained from real devices without more accurate information on the parameters of the real junction and, in particular, of the coupling circuit.<sup>3,4</sup>

To explore whether there is a well-defined threshold for the onset of the noise rise, we measured  $G_S$  and  $G_N$  for several values of  $\Gamma$ ; the results are summarized in Table I. Even for values of  $\Gamma$  as low as 0.42 ×  $10^{-5}$  (corresponding to a temperature of 0.1K for  $I_o = 1$  mA), the noise rise is clearly present. The ratio  $G_N/G_S$  is almost independent of temperature over the range  $0.1K \leq T \leq 0.42$  (for  $I_o = 1$  mA). Thus, only a low level of thermal noise is necessary to trigger the noise rise.

From these results, we can draw three conclusions about the noise rise in three-photon parametric amplifiers. First, choas is excluded as a possible explanation. It is an inescapable fact that to achieve signif-

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icant levels of gain the amplifier must be operated at values of bias current and pump power below the threshold for bifurcation to period two, and well-removed from possible chaotic modes that might exist at higher bias levels. Second, the noise rise occurs only when a non-zero level of thermal noise is present. Third, the noise rise is most likely the result of occasional hopping, induced by thermal noise, between a bias point in the unbifurcated region to an unstable one in the bifurcated region. Qualitatively, one would expect this picture to give rise to a noise temperature that increases with gain: One increases the gain by biasing the amplifier nearer to the threshold for a period-doubling bifurcation, so that the likelihood of noise-induced hopping is inevitably increased.

Finally, we emphasize that it is most unlikely that the mechanism we have presented here for the noise-rise in the three-photon amplifier accounts for the noise-rise in the four-photon amplifier, for which the amplification process involves significantly different dynamics.

We are grateful to R. Y. Chiao for helpful discussions. This work was supported by the Director, Office of Energy Research, Office of Basic Energy Sciences, Materials Sciences Division of the U. S. Department of Energy under Contract Number DE-ACO3-76SF00098.

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Γ (× 10 <sup>-5</sup> )	G <sub>S</sub> (dB)	G <sub>N</sub> (dB)	$G_N/G_S$ (dB)
0.061	$19.9 \pm 0.1$	31.1 ± 1.1	11.2 ± 1.1
0.42	$18.8 \pm 0.1$	36.6 ± 0.7	17.8 ± 0.7
4.2	16.7 ± 0.1	36.3 ± 0.7	19.6 ± 0.7
8.4	15.6 ± 0.1	34.7 ± 0.5	19.1 ± 0.5
18.0	$14.0 \pm 0.1$	32.0 ± 0.7	18.0 ± 0.7

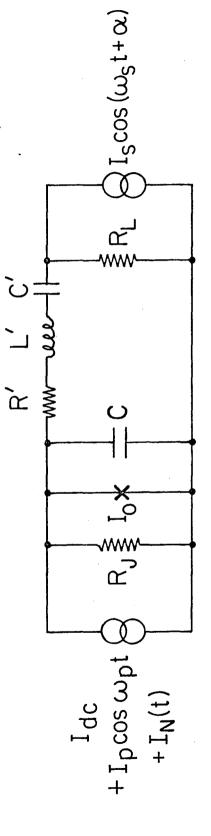
TABLE I. Dependence of  $\rm G_{s},~G_{N}$  and  $\rm G_{N}/G_{S}$  on level of thermal noise.

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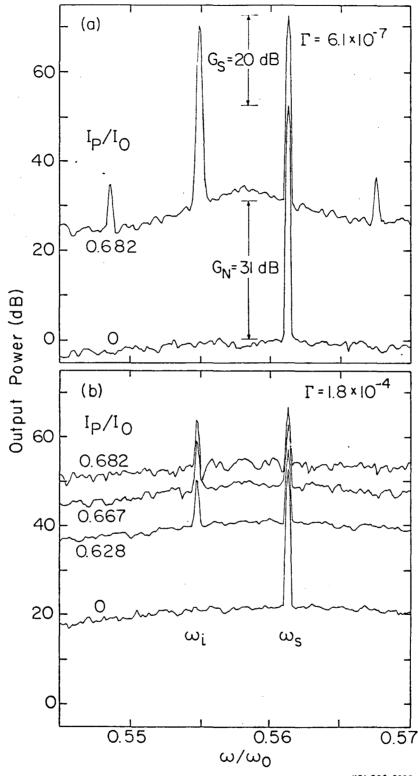
#### Figure Captions

- Fig. 1 Circuit for analog simulation of three-photon Josephson parametric amplifier.
- Fig. 2 Spectral density of voltage across  $R_L$  for analog parametric amplifier with  $\beta_C = 26.3$  and  $\omega_P/\omega_o = 1.12$  for various pump powers. In (a)  $\Gamma = 6.1 \times 10^{-7}$  while in (b)  $\Gamma = 1.8 \times 10^{-4}$ . In both cases, the OdB reference level is the noise level at  $\omega_S$  in (a).
- Fig. 3  $G_N/G_S$  (proportional to  $T_N$ ) vs.  $G_S$  for (a)  $\Gamma = 6.1 \times 10^{-7}$  and (b)  $\Gamma = 1.8 \times 10^{-4}$ . Solid line represent  $T_N \propto G_S$ .

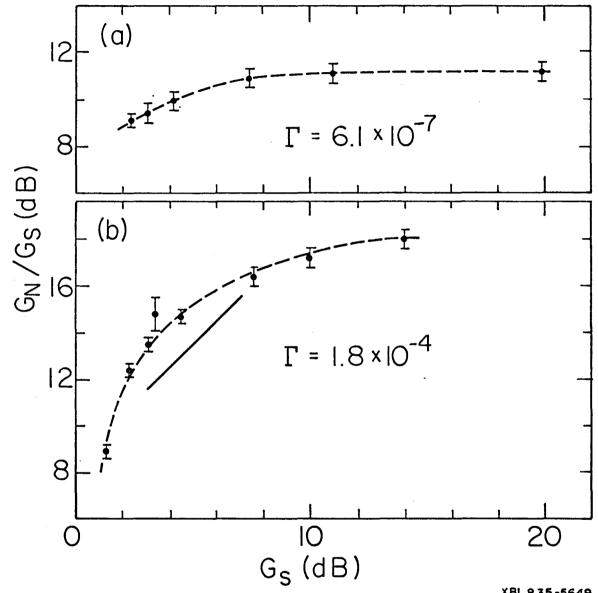


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