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Simulation of Thermal Stabilization of Soil around Various Technical Systems Operating in Permafrost

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Abstract

This paper presents results of a numerical study using finite-difference methods of non-stationary heat transfer processes in permafrost from heat sources generated by various technical systems (production wells) as well as from cold sources used for thermal stabilization of the soil, for example, seasonal cooling devices. In the simulation a number of climatic factors (monthly average temperature variation and power of solar radiation which are characteristic of a considered geographic place), thermal parameters and lithology of the soil, as well as the technical features of considered engineering facilities are taken into account.

Mathematics Subject Classification: 35Q79, 35K61, 65N06

Keywords: Heat Transfer, Permafrost, Stefan Problem

1 Introduction

Permafrost (a soil which conserves a negative temperature during, at least, two years) takes place about 25% of the total land area of the globe [6]. For example, in Alaska, these areas cover 80% of the total area, in Canada — 50%.

Highlands may also be a permafrost zone (in China these areas takes place 11% of the territory, in Austria – 2%). In Russia permafrost covers 65% of the total area, and reserves of underground ice permafrost are about 19000 km^3, which allows to call this permafrost as underground glaciation. It had been thought that the average thickness of permafrost in these areas ranges from 10 to 800 meters. Ice-saturated rocks thawing due to global warming, or various technogenic influences will be followed by the earth surface subsidence and development of cryogenic hazardous geological processes, called thermokarst. Extraction and transportation of oil and gas also has a significant effect on permafrost, as heat flux from heated oil in wells and pipelines leads to permafrost thawing. Note that more than 75% of all Russian buildings and structures in the permafrost zone are constructed and operated on the base of principle of conservation of frozen soil foundation. Therefore the problem of reducing the intensity of thermal interaction in the "heat source — permafrost" zones is of particular importance for solving problems of energy saving, environmental protection, safety, cost savings and improve the reliability of various engineering structures. In order to reduce permafrost thawing near engineering structures there are used different methods of thermal stabilization of soils such as insulation materials and various devices to cool the soil, for example, seasonal cooling devices (SCDs), which operate without any external source of energy only by the laws of physics. In Russia there are produced various kinds of SCDs that are used for thermal stabilization of soils. In this paper, in simulation of thermal fields in the soil a seasonal vapor-fluid cooling device is considered. It is consisted of a hermetically sealed and filled by a cooling media metal pipe with diameter of 57 mm and length of 10 meters or more. It includes an aerial parts (condenser fins) up to 2.5 meters and an underground part. To simulate the propagation of heat from wells in permafrost the mathematical model is suggested, which takes into account not only climatic and physical factors [2], but also thermophysical parameters of applied thermal insulations, as well as any devices (such as SCDs) used for thermal stabilization (cooling) of the soil. This leads to solution of three-dimensional quasilinear thermal diffusivity equation for a Stephan problem in an area with engineering objects having different specifications and sizes, as well as multi-scale thermal insulation shell (thickness from a few millimeters to the sizes of domains up to 100 meters).

2 Mathematical model. Basic equations.

To describe heat (cold) propagation from various engineering devices in permafrost a quasi-linear heat equation will be used. Applicability of this approach to solution of Stefan problem is presented in [3]. Choice of this approach without explicit separation of the boundary of phase transitions, when the volumetric or specific heat of melting is introduced using Dirac delta function as a point of phase transition heat in heat capacity [4], is justified by the fact that in our case there are a number of moving fronts.

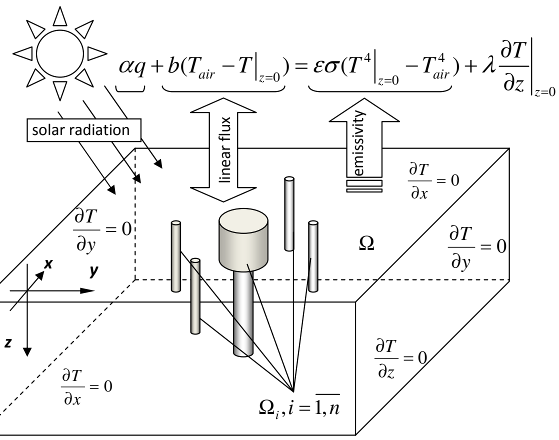


Figure 1: The main heat flows and boundary conditions

In this problem cyclic movement of fronts of phase transformation (in this case, we consider a zero temperature isotherm) arises due to seasonal climatic changes in air temperature (upper layers of the soil cyclically freezes and thaws). Other non-monotonic movement of the fronts of phase transformation arises in using SCDs for thermal stabilization (cooling) of the soil, when the laws of physics in the cold time of the year turns it on to cool the soil, and in the warmer months these devices are turned off.

Solar radiation can also affect the thermal processes in the soil. For example, in spring this influence is very significant, when the air temperature is still negative, but due to the power of solar radiation the surface temperature is increased and became positive.

This paper presents results of numerical simulations of thermal fields from different constructions in permafrost. Simulations of a well makes it possible to choose an upper part of the well to another structure, in order to reduce thermal influence on permafrost and to provide optimization of insulating shells of the well. To minimize the influence of thermal effects that can occur when different engineering units are used, and some simulations illustrate a process of thermal stabilization of the soil, when a well operates.

The computational domain is a three-dimensional box Ω, where x and y axes are parallel to the ground surface and z axis is directed downward. It includes n engeneering constructions $\Omega_i = \Omega_i(x, y, z)$, $i = 1, \ldots, n$, (Fig. 1). Let $T = T(t, x, y, z)$ be soil temperature at point (x, y, z) at time moment t and for $t = 0$ an initial temperature $T_0(x, y, z)$ is given in Ω.

In this case, following [3] and [4], to describe thermal fields in Ω taking into account the boundary conditions as in [2], let consider heat equation

$$\rho(c_\nu(T) + k\delta(T - T^*))\frac{\partial T}{\partial t} = \mathrm{div}\,(\lambda(T)\mathrm{grad}\,T), \tag{1}$$

where ρ is density [kg/m^3], T^* is temperature of phase transition $[K]$,

$$c_\nu(T) = \begin{cases} c_1(x,y,z), & T < T^*, \\ c_2(x,y,z), & T > T^*, \end{cases} \text{ is specific heat [J/kg K]},$$

$$\lambda(T) = \begin{cases} \lambda_1(x,y,z), & T < T^*, \\ \lambda_2(x,y,z), & T > T^*, \end{cases} \text{ is thermal conductivity coefficient [W/m K]},$$

$k = k(x,y,z)$ is specific heat of phase transition, δ is the Dirac delta function.

For equation (1) we have an initial condition

$$T(0,x,y,z) = T_0(x,y,z), \tag{2}$$

and let describe in more detail a boundary condition on the surface of the soil.

Let $q = q(t,x,y)$ be a total solar radiation in time moment t at point $(x,\,y)$, which depends on angle of incidence of the sun rays and on season in considered region, defined by geographical coordinates. Let $\alpha = \alpha(t,x,y)$ be a part of energy that is formed to heat the soil. Then the energy for the soil heating is $q_1 = \alpha q$.

Let q_2 be a thermal exchange of the soil surface $z = 0$ and air. If $T_{air} = T_{air}(t,x,y)$ denotes the temperature in the surface layer of air, which varies from time to time in accordance with the annual cycle of temperature. Then $q_2 = b(T_{air} - T(t,x,y,0))$, where $b = b(t,x,y)$ is a heat transfer coefficient.

On the other hand, $z = 0$ eliminates as a "heated black body" with emission $q_3 = \varepsilon\sigma(T^4(t,x,y,0) - T_{air}^4)$. Here $\sigma = 5,67\cdot 10^{-8}$ W/(m^{2}K^4) is Stefan–Boltzmann constant, $\varepsilon = \varepsilon(t,x,y)$ is the coefficient of emissivity. Also on surface $z = 0$ it is necessary to know an internal thermal flux $q_4 = \lambda\frac{\partial T(t,x,y,0)}{\partial z}$, where $\lambda = \lambda(T)$ is thermal conductivity coefficient.

Thus, a boundary condition on the upper surface $z = 0$ is defined by an flows balance equation, having the form

$$q_1 + q_2 = q_3 + q_4,$$

from which the following nonlinear boundary condition is obtained

$$\alpha q + b(T_{air} - T(t,x,y,0)) = \varepsilon\sigma(T^4(t,x,y,0) - T_{air}^4) + \lambda\frac{\partial T(t,x,y,0)}{\partial z}. \tag{3}$$

Note that the data for each month based on amount of solar radiation q and air temperature T_{air} is defined by specifying latitude and longitude, and is possible to be obtained from NASA climate databases. Other parameters b, ε,

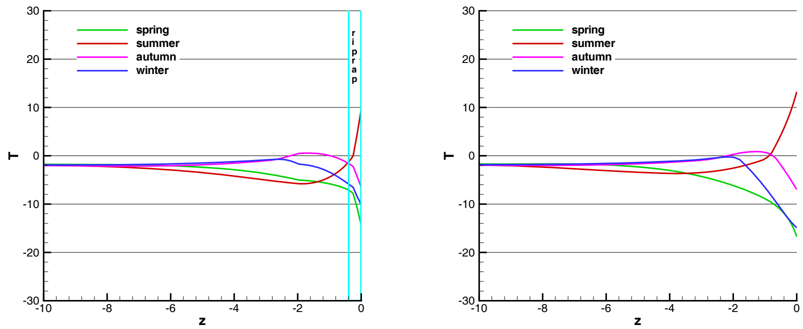
λ have to be given, and an iterative algorithm is developed to refine these values in dependence with the known thermal and thermo-physical characteristics of the soil in the considered region.

Usually, before starting construction in oil and gas fields a geophysical surveys of the area takes place, and shallow (up to 15 meters) exploration wells are drilled, in which temperature of the ground is measured for a number of time points and geophysical characteristics of the soil may be determined. These data is enough to specify parameters b, ε, λ, for a given geographic location. But even if we know only temperature of permafrost T_p at the depth below the zone of influence of cyclical changes of air temperature (according to our calculations and experimental data, the depth is approximately 10 meters), and in this case it is possible to determine these parameters too.

Let consider, for example, the results of calculations for one of the northern Russian oil and gas fields for which the parameters in boundary condition (3) have been defined. During numerical simulations of problem (1)–(3), with using a finite-difference methods for temperature $T_p = -1.5°\text{C}$ the following results are obtained. In Fig. 2b curves of temperature distribution are shown as a functions of depth in Ω, obtained for different seasons.

Let assume that in the comutational domain there is no artifical inner heat sources Ω_i. Furthermore, it is assumed that there is no thermal insulation of the ground surface (ripraps consisting for example of penopleks, sand and concrete) required to prevent premature thawing of permafrost base, as well as for technical capability to deploy technology to start drilling operations. Computations of temperature distribution in the simulated soil for several years are followed by an algorithm to specify parameters in boundary condition (3) so as to obtain the desired temperature distribution in the soil. Computations are performed up until the resulting temperatures in the soil with the chosen parameters do match (with some accuracy) with an experimental data, and is equal to T_p after 10 meters.

In Fig. 2a a comparison of the graphs for temperature changes in the soil in the presence of a two-meter riprap on the surface of the soil, consisting of sand (thickness 1.7 m) and concrete (thickness 0.30 m). Analysis of changes in temperature depending with the depth shows that riprap is indeed prevents heating in the summer, but also prevents penetration of cold in winter, which may allow to coservate permafrost. It is possible to include a number of devices appeared to be a cold or heat source in the ground. It is possible to be, for example, wells and SCDs. To simulate the thermal fields from these engineering devices on the specified boundaries of these objects Ω_i in accordance with specifications of these devices, we define temperatures T_i, which, in general, vary with time or dependent with z. Thus, the boundary conditions



(a) (b)

Figure 2: Profiles of temperatures for soil with $T_p = -1.5°C$: (a) — with riprap, (b) — no riprap.

are completed by

$$T\Big|_{\Omega_i} = T_i(t), \quad i = 1, \ldots, n. \tag{4}$$

In order to use numerical methods on the faces of parallelepiped Ω, located in the ground, we have to set, for example, boundary conditions

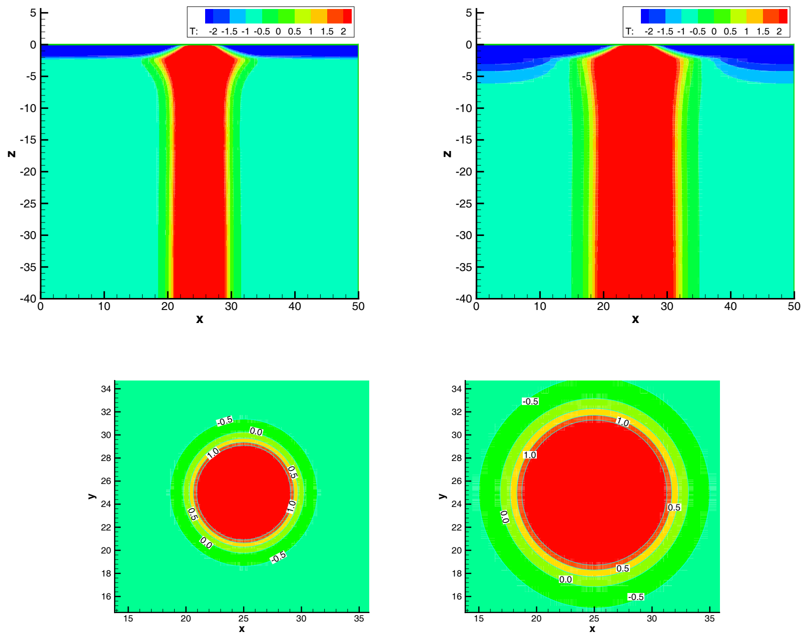
$$\frac{\partial T}{\partial x}\Big|_{x=\pm L_x} = \frac{\partial T}{\partial y}\Big|_{y=\pm L_y} = \frac{\partial T}{\partial z}\Big|_{z=-L_z} = 0, \tag{5}$$

which for larger sizes of the parallelepiped will have no essential influece on the solution. When we set (5) let assume $-L_x \le x \le L_x$, $-L_y \le y \le L_y$, $-L_z \le z \le 0$.

Thus, the simulation of heat transfer in three-dimensional domain with the phase transition is reduced to solving the initial-boundary value problem (1)–(5).

3 Numerical experiments

On the base of ideas in [4] a finite difference method is used with splitting by the spatial variables in three-dimensional domain to solve the problem (1)–(5). The base of this numerical method is an algorithm with good reliability in finding thermal fields of underground pipelines [1, 5], but in view of specificity, related to the possible phase transitions in the soil [2]. We construct an orthogonal grid, uniform, or condensing near the ground surface or to the surfaces of Ω_i. The original equation for each spatial direction is approximated by an implicit central-difference scheme and a three-point sweep method to solve a system of linear differential algebraic equations is used. On surface $z = 0$,



3 years 9 years

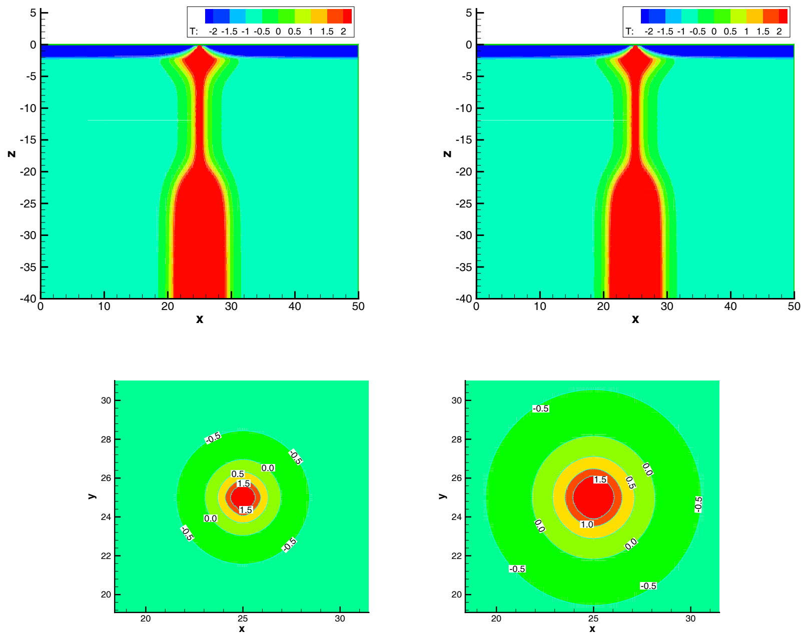
Figure 3: Termal fields for a non-insulated well

there is an algebraic equation of fourth degree, which is solved by Newton's method.

Consider a producing well operation with temperature +45°C. Soil temperature is $T_p = -0.7$°C, radius of the wells is 0.089 m. As a basic soil we will use a loam with the following parameters. Thermal conductivity: frozen — 1.82W/(m K), melted — 1.58W/(m K), volumetric heat: frozen — 2130kJ/(m^3 K), melted — 3140kJ/(m^3 K), volumetric heat of phase transition — $1.384 \cdot 10^5$kJ/(m^{3}K).

There is a layer of riprap of 2.5 m. The riprap consists of three layers: penoplex (0.2 m), sand (2.0 m) and the concrete slab on the top (0.3 m). Parameters: concrete slab with density 2500kg/m^3, thermal conductivity 1.69W/(m K), specific heat 0.84kJ/(kg K); sand with density 1600kg/m^3, thermal conductivity 0.47W/(m K), specific heat 0.84kJ/(kg K), penoplex with density 35kg/m^3, thermal conductivity 0.031W/(m K), specific heat 1.53kJ/(kg K).

In Fig. 3 thermal fields on February are shown for non-insulated well which is in exploitation during 3 and 9 years. The upper figures show the vertical section of the computational domain, the lower figures — the horisontal sections in the depth of 10 meters.

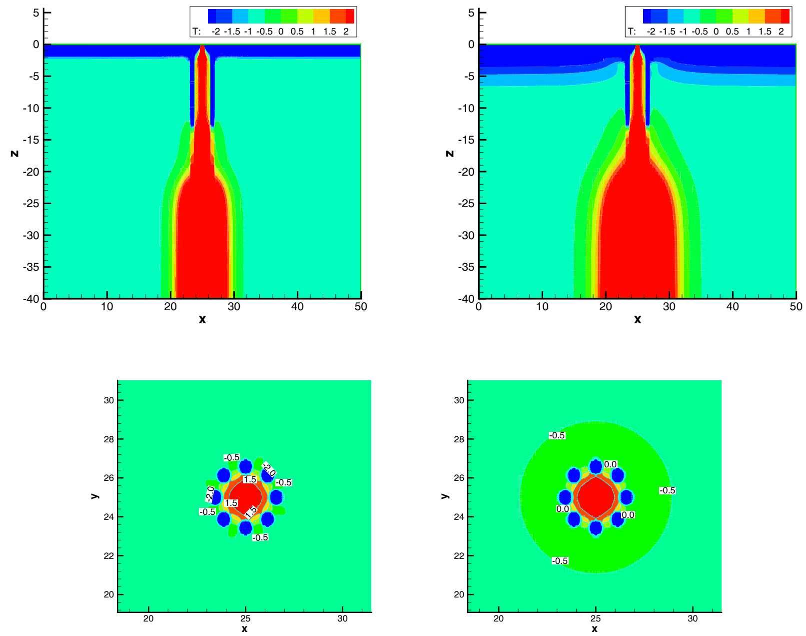


3 years 9 years

Figure 4: Thermal fields for an insulated well

As mentioned above it is required to minimize thermal effect on permafrost from various engineering facilities in order to prevent any changes in the soil, leading to accidents destruction wells and environment pollution. One such a way is to isolate the upper part of production well by various methods. The well has a cement shell with thickness of 0.176 m, and the the upper part up to 22 m has two additional insulating shells: a penoplex layer is inserted up to radius 0.410 m, cement — up to radius 0.5 m. In this case (see Fig. 4) the calculations show how the front of propagation of phase transformation (zero temperature isotherm, or the radius of thawing) is changed. The calculations show that the additional insulation allows 2 to reduce the radius of thawing from the well almost by 5 meters in 9 years of the well exploitation. To further reduce the radii of thawing from the well for stabilization (cooling) of soil, for example, SCDs may be used. These devices start to work, in our case, when $T_{air} < -5°$C, for warmer temperatures these devices are turned off. In accordance with the technical specifications we can approximately assume that the surface of SCDs during the operation is $T_{scd} = T_{air}/3$.

In Fig. 5 thermal fields for insulated well with 8 SCDs are shown. SCDs are at the distance of 1 m from the well. The calculations show that SCDs using allows to reduce radius of thawing at a depth of 10 meters for an insulated well for 9 years by 2 meters in addition.



3 years 9 years

Figure 5: Thermal fields for an insulated well with 8 SCDs

4 Conclusion

On the basis of a mathematical model taking into account a number of factors that influence on thermal field in permafrost a finite-difference method and a software are developed for numerical studies and simulations of heat transfer processes in upper layer of permafrost. In the proposed model the following actual physical factors are taken into account: thermal diffusion properties of soil, soil heterogeneity, heating the soil surface by air including heat emissivity, annual climatic cycle (monthly average air temperature and intensity of solar radiation). Initial and boundary parameters may be adapted to a specific geographic location. The developed software package makes it possible to carry out a series of numerical experiments to simulate thermal stabilization of permafrost soil by SCDs, to make long-term predictions about permafrost thawing in the presence of wells and other technical devices. The proposed approach and the results of numerical simulations will allow experts to review and make recommendations on optimal parameters of insulations and placement of wells and other technical systems in work sites of northern oil and gas fields.

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