

# Simultaneous All-Optical Half-Adder and Half-Subtractor Based on Two Semiconductor Optical Amplifiers

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**Abstract**—We propose and demonstrate all-optical simultaneous half-addition and half-subtraction of two return-to-zero on-off keying data streams using only two semiconductor optical amplifiers and without any assist light. In the experimental demonstration, we realize the arithmetic functions with an extinction ratio larger than 14 dB and achieve error-free signal processing at the repetition rate of 10 Gbit/s.

**Index Terms**—Logic-based optical processing, nonlinear optical signal processing, optical signal processing, semiconductor optical amplifiers.

## I. INTRODUCTION

IN THE current telecommunications, many essential functions, such as packet forwarding, time-to-live (TTL) decrementing and address recognition, are relying on the electrical operations. However, in the future all-optical networks, all-optical signal processing is required to overcome speed limitation of electronics, enhance energy efficiency and improve transparency by omitting redundant optical-to-electrical (O/E) and electrical-to-optical (E/O) conversions. A wide range of researches have been conducted on the optical digital logic gates (e.g. AND, OR, NOT and XOR) and optical logic operation modules (e.g. counters, adders, subtractors and shift registers) to achieve the all-optical signal processing [1]–[3].

In particular, several all-optical half-adder and half-subtractor schemes have been proposed. Both half-adder and half-subtractor can be used in the arithmetic-logic unit, encryption and decryption of secure networks and optical packet checksum calculation [4]. Besides, half-subtractor can be applied to TTL decrementing, routing loop control and dual-direction binary counters [5]. These proposed schemes are mainly realized by means of semiconductor optical amplifier (SOA) and periodically poled lithium niobate waveguide (PPLN) based circuits, thanks to the advantages of SOA and

PPLN in the aspects of high nonlinearity, strong compactness and wide wavelength window. Two cascaded SOA based interferometers are applied for the constitution of a 10 Gbit/s half-adder without any assist light [6]; a half-adder using an SOA and a PPLN is demonstrated at the repetition rate of 5 Gbit/s for the simplified structure [7]; a 10 Gbit/s half-adder is realized by employing cross-gain modulation (XGM) in the four SOAs [8]; a 40 Gbit/s simultaneous half-adder and half-subtractor scheme is simulated based on the cascaded sum and difference frequency generation (SFG + DFG) using only one PPLN waveguide [9]; a 5 bit/s simultaneous half-addition and half-subtraction is demonstrated by using two parallel SOAs and a PPLN waveguide [4].

The optimization of an optical logic operation circuit design needs to take into consideration the number of active devices used, the power consumed, the number of assist light sources needed, the number of functions realized, the operation speed achieved as well as the signal quality obtained. In this letter, we propose and demonstrate simultaneous half-addition and half-subtraction at the data rate of 10 Gbit/s by using only two SOAs without any assist light. Compared to the current electrical operations, this all-optical scheme can provide transparency in the optical networks.

## II. OPERATION PRINCIPLE

A half-adder adds two one-bit binary numbers ( $A$  and  $B$ ) and outputs a sum of these two bits and a carry. In the half-addition module, an AND gate and an exclusive-OR (XOR) gate are implemented, whose outputs are corresponding to the carry and the sum, as listed in Table I. The XOR gate results in the value of logic '1' if exactly one of the inputs equals to logic '1', which can be expressed as  $\overline{A}B + A\overline{B}$ . A half-subtractor performs the function of subtraction for two bits. It has two inputs, which are minuend and subtrahend, and two outputs, difference and borrow. The output of difference functions as the XOR gate. Borrow outputs logic '1' only when minuend is logic '0' and subtrahend is logic '1', i.e.  $(0-1=1)$ . The logic function of the borrow can be represented as  $\overline{A}B$  or  $\overline{A}\overline{B}$ , depending on whether  $A$  is minuend or subtrahend. Therefore, there are four logic functions, AND ( $AB$ ), XOR ( $A \oplus B$ ),  $\overline{A}B$  and  $\overline{A}\overline{B}$ , in the half-adder and half-subtractor modules, among which XOR function is a combination of  $\overline{A}B$  and  $\overline{A}\overline{B}$  ( $A \oplus B = \overline{A}B + \overline{A}\overline{B}$ ).

The operation principle of the proposed scheme is illustrated in Fig. 1. Two input signals ( $A$  and  $B$ ) in OOK format are

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TABLE I  
TRUTH TABLE OF HALF-ADDER AND HALF-SUBTRACTER

Input Signal		Half-Adder		Half-Subtractor		
A	B	Carry	Sum	Borrow A-B	Borrow B-A	Difference
0	0	0	0	0	0	0
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0
Logic Function		$AB$ AND	$A \oplus B$ XOR	$\bar{A}B$	$A\bar{B}$	$A \oplus B$ XOR

centered at two different wavelengths ( $\lambda_A$  and  $\lambda_B$ ). The two signals have different powers. Signal A has lower power, acting as probe, and signal B has higher power, acting as pump. The optical pulses of the two signals temporally overlapped to each other and are injected into the SOA synchronously. The operation is based on XGM and four-wave mixing (FWM) in the SOA. When both data bits of signal A and signal B are logic '1's, the SOA is saturated by the high-intensity pulse of signal B, due to the carrier depletion, and XGM effect occurs. The low-intensity pulse of signal A is suppressed and there is no output at the wavelength of  $\lambda_A$ . In addition, FWM effect occurs with the coexistence of both pulses of signal A and signal B and a new signal is generated at  $\lambda_{FWM}$  ( $\lambda_{FWM} = 2\lambda_A - \lambda_B$ ), which realizes the function of  $AB$ . If the data bit of signal A is logic '1' and the data bit of signal B is logic '0', the low-intensity pulse of signal A is amplified and output at the wavelength of  $\lambda_A$ , where the function of  $\bar{A}B$  is achieved. With the same setup but reversed roles of the two signals, the function of  $A\bar{B}$  can be obtained at the wavelength of  $\lambda_B$ . Finally, the function of  $A \oplus B$  can be easily attained through the combination of both  $\bar{A}B$  and  $A\bar{B}$  by a power coupler.

### III. EXPERIMENTAL DEMONSTRATION AND DISCUSSION

Fig. 2 shows the experiment setup of the proposed half-adder and half-subtractor. Two mode-locked laser diodes (MLLD) are used to produce two 2.6 ps (FWHM) Gaussian shaped optical pulse trains at 1547 nm (signal A) and 1553 nm (signal B). The pulse trains are modulated by two intensity modulators with 10 Gbit/s  $2^{15} - 1$  PRBS data and data, separately. In one branch, a 20 m SMF is added after the modulation to mismatch the data pattern between the two branches. Then, the two modulated pulse trains are combined for the amplification.

After that, a  $1 \times 4$  coupler is used to split the signal into four branches. In these four branches, four 2 nm bandpass filters (BPF), two tunable optical delay lines and four attenuators are used to extract the target data, align the time difference and control the powers. The signals in the first two branches are coupled into SOA 1 after optimized polarization control. The average power of signal A is attenuated to  $-7.52$  dBm (peak power of 8.33 dBm), while signal B has the average power of 1.67 dBm (peak power of 17.52 dBm), which is used as a pump signal. XGM, which is insensitive to polarization, occurs with the existence of signal B and the function of  $\bar{A}B$  is achieved, which is extracted by a 0.6 nm BPF centered

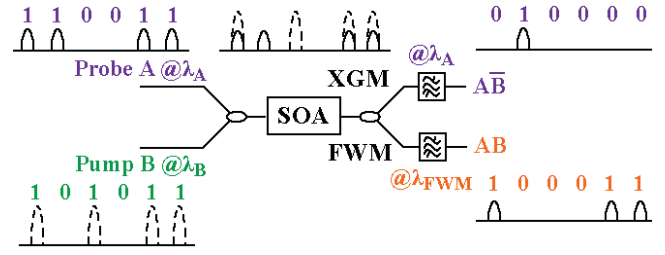


Fig. 1. Operation principle of the proposed scheme.

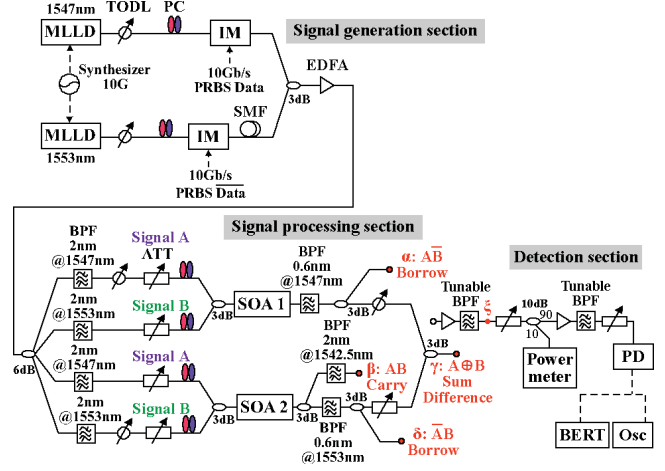


Fig. 2. Experiment setup of 10 Gbit/s half-adder and half-subtractor.

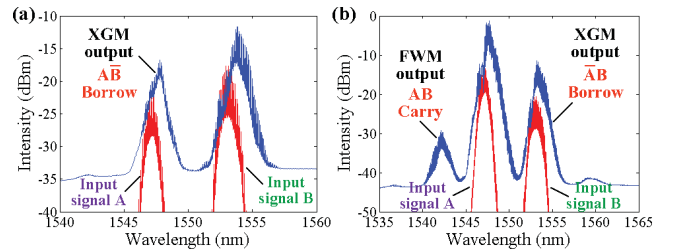


Fig. 3. (a) and (b) Input and output spectra, measured before and after SOA 1 and 2. (Resolution: 0.01 nm.)

at 1547 nm. The output spectrum before filtering is shown in Fig. 3(a).

The signals in the other two branches are coupled into SOA 2 after the individual polarization control, since the generation of FWM in the SOA is polarization dependent. The average powers of signal A and signal B are adjusted to 4.28 dBm and  $-5.31$  dBm (peak powers of 20.13 dBm and 10.53 dBm), respectively. In this case, signal A is used as a pump signal and the function of  $\bar{A}B$  is filtered out by a 0.6 nm BPF at 1553 nm. Besides, a new signal is generated when both input signals are '1', i.e.  $AB$ , resulted from the FWM effect in SOA 2. Since the wavelengths of the input signals are red-shifted, due to self-phase modulation (SPM) and XGM effects in the SOA, the signal of  $AB$  is extracted at 1542.5 nm by a 2 nm BPF. The output spectra of  $\bar{A}B$  and  $AB$  before filtering are shown in Fig. 3(b). After balancing the power by an optical attenuator and aligning the time by an optical delay line, the signals of  $\bar{A}B$  and  $AB$  are combined by a 3 dB coupler to produce the function of  $A \oplus B$ . The average

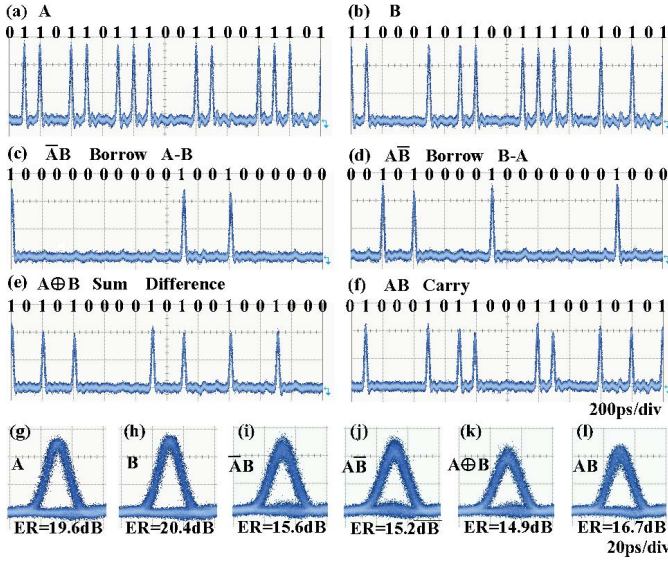


Fig. 4. Waveforms of (a) and (b) input signals and (c)–(f) output signals. (g)–(l) Corresponding eye diagrams. (Output signals are measured at point  $\zeta$ .)

powers of  $\overline{AB}$ ,  $\overline{AB}$ ,  $AB$  and  $A \oplus B$  at points  $\alpha$ ,  $\delta$ ,  $\beta$  and  $\gamma$  are  $-15.93$ ,  $-14.02$ ,  $-17.56$  and  $-15.31$  dBm, respectively. In the experiment, SOA 1 and SOA 2 are biased at 389 mA and 423 mA and their 10–90% gain recovery time are about 25 ps and 30 ps, respectively. The data rate is limited by SOA 2. To achieve a higher data rate and guarantee a good performance, we appeal to use SOAs with fast gain recovery.

It is worthy noting that there is a tradeoff in choosing the wavelength separation of the two input signals. A large separation can avoid beat noise in the output of sum and difference, where the signals at two different wavelengths exist. However, a large wavelength separation leads to a low conversion efficiency of FWM. Therefore, a compromise should be taken under the consideration of above facts. In the experiment, the wavelengths (1547 nm and 1553 nm) of the input signals are selected for the optimized performance. To meet the requirements of applications and realize wavelength transparency, wavelength conversions may be needed for the input and output signals.

Fig. 4 shows waveforms and eye diagrams of the input signals A and B and the output signals of borrows, difference, sum and carry. The output signals are measured at point  $\zeta$ . The binary data are labeled above the waveforms. In the borrow, there is an output pulse only when the data bit of signal A is logic ‘1’ and the data bit of signal B is logic ‘0’ for the case of  $A-B$ , and vice versa. The outputs of sum and difference are corresponding to the logic function of  $A \oplus B$ , where either the data bit of signal A or the data bit of signal B is logic ‘1’. Additionally, the carry is achieved when both input signals A and B are logic ‘1’. Open and clear eye diagrams are obtained for all outputs. Based on the experimental observation, the qualities of the input signals have small influence on the XGM outputs but affect the FWM output significantly. In the experiment, the extinction ratios (ERs) of the output signals are all larger than 14 dB.

BER performance is depicted in Fig. 5. Error-free operations are all achieved. Compared to the original signal A and B,

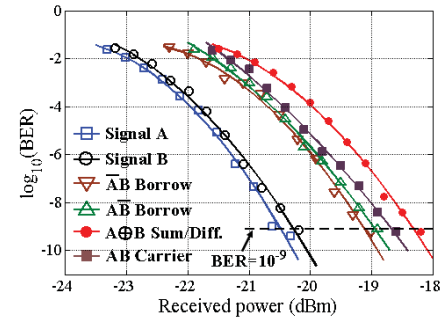


Fig. 5. BER performances for the input and output signals.

which are measured directly after the intensity modulations, the outputs of borrows ( $\overline{AB}$  and  $\overline{AB}$ ) show about 1.5 dB power penalty at BER of  $10^{-9}$ . The function of  $A \oplus B$  for sum and difference exhibits a 2 dB power penalty. In addition, the output of carry has a power penalty of about 1.8 dB.

#### IV. CONCLUSION

In conclusion, we have proposed and demonstrated an all-optical half-addition and half-subtraction of two 10 Gbit/s RZ-OOK signals with the pattern length of  $2^{15}-1$  simultaneously in the same setup. Only two SOAs are employed and no assist light source is needed. Error-free operations are all achieved. Due to the multi-functions and the use of integrable components, the proposed scheme has the potential to be applied for the future optical systems, such as arithmetic logic unit and encryption/decryption device.

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