# SIMULTANEOUS OBSERVATIONS OF PULSAR INTENSITY VARIATIONS AT PARKES AND OOTACAMUND* 

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## SUMMARY

Simultaneous observations at 326.5 MHz of the intensity variations of seven pulsars have been conducted over a $8000-\mathrm{km}$ baseline between Australia and India.

The theoretical interpretation of the drifting diffraction pattern is developed in some detail and a practical method is determined for measuring the velocity component along the baseline in the presence of intrinsic pulsar variability and random evolution of the pattern. It is shown that, because of the intrinsic variations, the usual method of determining velocity from the maximum cross-correlation leads to a large overestimate of the velocity.

For the majority of the pulsars observed, the intensity fluctuations at the two stations were significantly decorrelated for all time shifts. It is concluded that the most likely reason for the decorrelation is the presence of a scale size approximately equal to our receiver separation of 8000 km .

Three of the seven pulsars provided correlation functions which could be processed by the proposed method to give the velocity of pattern drift along the baseline. These velocities range from $40 \mathrm{~km} \mathrm{~s}^{-1}$ to $12 \mathrm{I} \mathrm{km} \mathrm{s}{ }^{-1}$ and, if pattern velocity is indicative of pulsar velocity, are not high enough to place these pulsars in a class of particularly high-velocity objects.

The pulsar PSR ${ }^{1642-03}$ provided clear diffraction patterns whose direction of drift reversed in an interval of about I hr . This behaviour is not consistent with a simple model of the interstellar electron irregularities but it may be possible to explain it with a multi-screen model.

## I. INTRODUCTION

Pulsars are known to vary in intensity on time scales ranging from seconds to months. The shorter variations, which are generally well correlated over a wide frequency band, are almost certainly intrinsic to the pulsar. Intensity variations on the intermediate time scale of a few minutes are believed to be caused primarily by interstellar scintillation, although significant intrinsic variability on a similar time scale is known to be present in a few pulsars. The indentification of the scintillation component rests largely on the existence of fine frequency structure of a random nature in the spectra of pulsars (see, for example, Rickett 1970) and on the broadening of the pulses by the scattering (see, for example, Ables, Komesaroff \& Hamilton 1970).

The first spaced-receiver observations of pulsar signals were made by Slee, Komesaroff \& McCulloch (1968) at 8 MHHz over a $325-\mathrm{km}$ baseline and by

[^0]Conklin et al. (1968) at 112.8 MHz over a $5000-\mathrm{km}$ baseline. The narrow-band properties of the scintillations at the longer metre wavelengths were not well recognized at the time so that the receiver bandwidths of $\sim \mathrm{r} \mathrm{MHz}$ used by these observers resulted in a high degree of smoothing of the scintillating component. It is therefore not surprising that neither experiment was able to detect the expected time lag between the amplitude changes at the spaced receivers. These observations did, in fact, verify that the wide-band variations on time scales of seconds to minutes were intrinsic to the pulsars.

Lang \& Rickett (1970) were the first investigators to conduct a spaced receiver experiment under conditions which allowed the detection of a drifting diffraction pattern. Their 408 MHz trans-Atlantic observations of PSR $1133+16$ over a baseline of 5500 km showed that an interstellar diffraction pattern was present and suggested that the drift velocity of the pattern was compatible with that expected from differential galactic rotation and the peculiar motion of the Earth with respect to nearby stars.

Galt \& Lyne (1972) conducted a $24-\mathrm{hr}$ trans-Atlantic experiment at 408 MHz on the circumpolar pulsar PSR $0329+54$ over a baseline of 6833 km . The resulting rotation of the projected baseline enabled them to derive a complete solution for the velocity of the diffraction pattern. The very high pattern velocity of $360 \mathrm{~km} \mathrm{~s}^{-1}$ supported previous suggestions (see, for example, Gunn \& Ostriker 1970) that pulsars belonged to a class of objects having very large proper motions, possibly due to their violent births in supernova explosions.

The present experiment was designed to extend measurements of the apparent drift velocities to a larger range of objects. The observations were made at 326.5 MHz in 1971 August on seven pulsars which could be observed with up to 3 hr overlap at stations in Australia and India. The instruments used were the CSIRO $64-\mathrm{m}$ reflector at Parkes and the TIFR $530 \times 30-\mathrm{m}$ radio telescope at Ootacamund. The baseline projected on to the plane normal to the line of sight was about 8000 km .

## 2. OBSERVATIONS

The receiver bandwidths of 340 kHz were accurately centred on $326 \cdot 5 \mathrm{MHz}$ and a linearly polarized component of the pulsar emission was accepted at each telescope. No attempt was made to maintain alignment of the polarization position angle at the two telescopes. The post-detection receiver outputs were smoothed with a time constant of about I per cent of the pulsar period and sampled 256 times per pulsar period. A gate of about two-thirds the pulse width was centred on the pulse by means of a continuously operating superposed epoch display with an exponential decay memory ( $\mathrm{e}^{-1}$ time $\sim 60 \mathrm{~s}$ ). Samples within the gate interval were averaged and recorded as a 7 -bit binary number ( 128 independent levels) for each pulse. Timing information accurate to $0 \cdot 1 \mathrm{~s}$ was also recorded. Similar measurements were made of the system noise in an identical gate placed half-way between the pulses; these were later used to estimate a base level, which was subtracted from the on-pulse measurements, resulting in a long sequence of pulse intensities suitable for correlation analysis.

Before the correlations were computed, however, the pulse-trains were smoothed by replacing groups of pulse intensities extending over 12 s by the average value located in time at the centre of the group. The autocorrelation functions and the
cross-correlation function of the time-series from the two stations were then computed using lags up to 20 per cent of the series length. The pulse-trains from the two stations could be aligned in time to $O \cdot 1 \mathrm{~s}$, which was sufficiently accurate to identify the common starting pulse. The timing of each observation was checked by cross-correlating a short series of 200 unsmoothed pulses; this invariably resulted in a sharp peak at zero lag due to the intrinsic pulse-to-pulse variability.

Some representative plots of the smoothed pulse intensities at Parkes and Ooty are shown in Fig. i. Fig. i(a) shows the very high degree of correlation of the intensity variations at the two sites for PSR 2045-16. The intensity changes are well developed on two times scales: (i) from one smoothing interval to the next, i.e. $\sim 12 \mathrm{~s}$; (ii) a longer time scale of several hundred seconds. The shorter fluctuations are, almost certainly, intrinsic to the pulsar; the longer changes probably consist of both intrinsic effects and interstellar scintillations. The fact that the amplitude is near zero for a significant proportion of the time suggests to us that the long-period fluctuations are probably dominated by the intrinsic component, which, we shall show in the next section, seriously complicates the measurement of pattern velocity and scale size.

Fig. 1 (b) shows the poorly correlated amplitude fluctuations for PSR $1749-28$. There can be little doubt in this case that the intensity variations are predominantly interstellar scintillations with a scale size less than our receiver separation of $\sim 8000 \mathrm{~km}$.

Fig. I(c) shows the variable degree of correlation of the intensity variations at the two sites for PSR 1642 -03. Again, because the instantaneous (zero-lag) crosscorrelation coefficient is significantly less than unity, we can be sure that much of the long-period variability is due to an interstellar diffraction pattern with a scale size of the order of our baseline length. Fig. i(c) also shows (from the lower correlation over the second half of the record) that the parameters of the diffraction pattern can change significantly in $\sim \mathrm{I} \mathrm{hr}$, a result which is not expected from a simple model of the interstellar scattering.

Table I summarizes all the observational data. The $\mathrm{e}^{-1}$ half-widths of the autocorrelation functions (column 5) suggest that, except for PSR $0031-07$, the intensity fluctuations have durations similar to those that have been ascribed to interstellar scintillations; in the case of PSR 0031 - 07 the intensity fluctuations are an order of magnitude shorter and are consistent with the strong intrinsic periodicity of 6.8 s noted in this pulsar by Sutton et al. (1970). Column 8 lists the maximum cross-correlation coefficients (including the effects of any intrinsic variability); the coefficients have been corrected for system noise using the signal-to-noise ratios and modulation indices listed in columns 6 and 7 . From the formula given in the next section it is clear that the corrections were less than 20 per cent. The extreme values of apparent lag $\tau_{0 x}$ given in column io were obtained by measuring the displacement from zero lag of the peak of the broad component of the cross-correlation function (see Fig. 2). In most cases $\tau_{0 \mathrm{x}}$ apparently indicates a very high pattern velocity of several hundred kilometres per second, but we will show in the next section that this parameter may grossly overestimate the pattern velocity in the presence of intrinsic variability and random changes in the pattern. We will show, however, that even in the presence of such variability the pattern speed can be estimated from the intersection of the auto- and cross-correlation functions.


Table I

```
    Summary of pulsar observations
        \(\begin{array}{cc}\text { (4) } & \begin{array}{c}(5) \\ \text { Mean } \mathrm{e}^{-1} \\ \text { half-width } \\ \text { of auto- }\end{array} \\ \text { Smoothing } \\ \text { interval } & \begin{array}{c}\text { correlation }\end{array} \\ \text { (pulses) } & (\mathrm{s}) \\ 13 & 133 \\ 29 & 112 \\ 17 & 202 \\ 21 & 217 \\ 9 & 204 \\ 49 & 388 \\ 7 & 155\end{array}\)
            (6)
Mean
sig./noise
ratio^
\(2 \cdot 2\)
22
\(2 \cdot 1\)
12
16
\(2 \cdot 1\)
\(3 \cdot 4\)
```


lags behind Ooty.
(9)
Dispersion
Dispersion
measure
$\left(\mathrm{pc} \mathrm{cm}^{-3}\right)$
$10 \cdot 6$
$35 \cdot 7$
10
$50 \cdot 4$
12.6
3.2
11.5
©
Mean max.
cross-
correlation $\ddagger$
0.86
0.54
0.78
0.54
0.97
0.86
0.91
(8)

* Geometric mean of the pulse amplitude/rms interpulse noise ratios at the two receivers after smoothing.
Average of the



Fig. 2 (a) and (b).


Fig. $2(e)$ and $(f)$.
Fig. 2. Correlation functions obtained from an analysis of all the data in each of six observations. Filled circles denote the autocorrelation (obtained by averaging the autocorrelations of the two stations); open circles outline the observed cross-correlation. Shortterm intrinsic variability can be seen as a narrow peak centred on zero lag. The vertical dashed lines dropped from the intersection of the auto- and cross-correlation curves in (a)-(d) give values for the parameter $\tau_{e x}$, which measures the drift velocity of the pattern. Fig. 2(e) and (f) are typical of the majority of our recordings, for which the crossing point $\tau_{e x}$ is poorly defined. Note that (a) corresponds to Fig. $\mathrm{I}(c)$ and (d) to Fig. $\mathrm{I}(a)$.


## 3. INTERPRETATION OF THE CORRELATION FUNCTION

It is useful to consider the general correlation function $\rho$ which gives the expected value of the correlation between $S(x, y, t)$, the signal intensity at time $t$ and at the point $(x, y)$, and the intensity $S(x+\xi, y+\eta, t+\tau)$ at time $(t+\tau)$ and at the point $(x+\xi, y+\eta)$. Assuming that $S$ is statistically stationary in both time and space, this correlation function is

$$
\begin{equation*}
\rho(\xi, \eta, \tau)=\frac{\left\langle\left[S(x, y, t)-\mu_{0}\right]\left[S(x+\xi, y+\eta, t+\tau)-\mu_{0}\right]\right\rangle}{\left\langle\left[S(x, y, t)-\mu_{0}\right]^{2}\right\rangle} \tag{I}
\end{equation*}
$$

where $\mu_{0}=\langle S(x, y, t)\rangle$ is the mean intensity. The autocorrelation function of $S$ is $\rho(0,0, \tau)$ and the cross-correlation of $S$ between any two stations separated by a distance $\xi_{0}$ may be written as $\rho\left(\xi_{0}, 0, \tau\right)$ by choosing the $x$-axis along the line between the stations.

Assuming that the intensity pattern in the $x y$-plane is statistically isotropic (i.e. scale size is not a function of direction) and that the form of the correlation function is the same for the temporal and spatial coordinates, Briggs (1972) has shown that

$$
\begin{equation*}
\rho(\xi, \eta, \tau)=f\left(A\left[\xi^{2}+\eta^{2}\right]+2 B \xi \tau+2 C \eta \tau+D \tau^{2}\right), \tag{2}
\end{equation*}
$$

where $A, B, C$ and $D$ are constants and the form of the function $f(\cdot)$ is determined by the form of the autocorrelation function. We will adopt this form for $\rho$ and discuss later the effect of the form of the correlation function being different for the temporal and spatial coordinates in the important case when this difference is caused by intrinsic variations in the source intensity. For the present we assume the source intensity is constant.

Here we are primarily interested in the translational velocity $V$ of the intensity pattern in the $x y$-plane. For a pattern which may evolve as it drifts, $V$ is defined as being that common velocity at which a set of arbitrarily spaced observers must move so that all cross-correlations have their maxima at $\tau=0$. In a coordinate system ( $x^{\prime}, y^{\prime}$ ) moving with velocity components $V_{x}$ and $V_{y}$ so that

$$
\begin{array}{ll}
x^{\prime}=x-V_{x} t & \xi^{\prime}=\xi-V_{x} \tau \\
y^{\prime}=y-V_{y} t & \eta^{\prime}=\eta-V_{y} \tau \tag{3}
\end{array}
$$

the observed correlation function is

$$
\begin{align*}
\rho_{\mathrm{v}}\left(\xi^{\prime}, \eta^{\prime} \tau\right)=f\left[A\left(\xi^{\prime}+V_{x} \tau\right)^{2}+A\left(\eta^{\prime}\right.\right. & \left.+V_{y} \tau\right)^{2} \\
& \left.+2 B \tau\left(\xi^{\prime}+V_{x} \tau\right)+2 C \tau\left(\eta^{\prime}+V_{y} \tau\right)+D \tau^{2}\right] . \tag{4}
\end{align*}
$$

For $V_{x}$ and $V_{y}$ to be the components of the pattern velocity, it is necessary that $\rho_{\mathrm{v}}\left(\xi^{\prime}, \eta^{\prime}, \tau\right)$ achieve its maximum at $\tau=0$ for arbitrary $\xi^{\prime}$ and $\eta^{\prime}$. At the maximum, $\partial \rho_{\mathrm{v}} / \partial \tau=0$, so that we must have $\partial \rho_{\mathrm{v}} /\left.\partial \tau\right|_{\tau=0}=0$ for all $\xi^{\prime}$ and $\eta^{\prime}$. Denoting the derivatives of $f$ by $f^{\prime}$, this condition becomes

$$
f^{\prime}\left[A\left(\xi^{\prime 2}+\eta^{\prime 2}\right] \cdot\left[\xi^{\prime}\left(A V_{x}+B\right)+\eta^{\prime}\left(A V_{y}+C\right)\right]=0\right.
$$

The only way this can be true for all $\xi^{\prime}$ and $\eta^{\prime}$ is that

$$
\begin{align*}
V_{x} & =-B / A \\
V_{y} & =-C / A, \tag{5}
\end{align*}
$$

thus expressing the drift velocity in terms of the constants of equation (2).

With these values for the velocity components of the moving system the correlation function becomes

$$
\begin{equation*}
\rho_{\mathrm{v}}\left(\xi^{\prime}, \eta^{\prime}, \tau\right)=f\left[A\left(\xi^{\prime 2}+\eta^{\prime 2}\right)+D^{\prime} \tau^{2}\right], \tag{6}
\end{equation*}
$$

where

$$
\begin{equation*}
D^{\prime}=D-\left(B^{2}+C^{2}\right) / A . \tag{7}
\end{equation*}
$$

If $D^{\prime}=0$ then $\rho_{\mathrm{v}}\left(\xi^{\prime}, \eta^{\prime}, \tau\right)$ has no temporal dependence; hence when

$$
D=\left(B^{2}+C^{2}\right) / A
$$

the pattern simply drifts without evolving in time.
Returning to the stationary frame, let us consider the time lag $\tau_{\mathrm{ex}}$ for which the cross-correlation between two stations on the $x$-axis, separated by $\xi_{0}$, is equal to the autocorrelation at the same lag, that is, $\rho\left(\xi_{0}, 0, \tau_{\mathrm{ex}}\right)=\rho\left(0, \mathrm{o}, \tau_{\mathrm{ex}}\right)$ or, in terms of $f, f\left(A \xi_{0}{ }^{2}+2 B \xi_{0} \tau_{\mathrm{ex}}+D \tau_{\mathrm{ex}}{ }^{2}\right)=f\left(D \tau_{\mathrm{ex}}{ }^{2}\right)$. Equating the arguments of $f$ gives $A \xi_{0}{ }^{2}+2 B \xi_{0} \tau_{\mathrm{ex}}+D \tau_{\mathrm{ex}}{ }^{2}=D \tau_{\mathrm{ex}}{ }^{2}$, which when solved for $\xi_{0} / 2 \tau_{\mathrm{ex}}$ yields (cf. Briggs, Phillips \& Shinn 1950)

$$
\begin{equation*}
\xi_{0} / 2 \tau_{\mathrm{ex}}=-B / A=V_{x}=V \cos \phi, \tag{8}
\end{equation*}
$$

where $\phi$ is the angle between the $x$-axis and the direction of drift.
If the function $f$ of equation (2), which must assume its maximum value of unity for zero argument, has additional (relative) maxima then there may be additional solutions to the equation $\rho\left(\xi_{0}, 0, \tau_{\mathrm{ex}}\right)=\rho\left(0, \mathrm{o}, \tau_{\mathrm{ex}}\right)$ for which the arguments are not equal. However, if there exists a crossing point at which the correlation value is greater than the correlation value at any secondary maximum of the autocorrelation function, then this is the crossing point corresponding to equality of the arguments.

Note that this crossing point of the autocorrelation and cross-correlation curves determines the component of the drift velocity projected on the baseline for a twostation experiment, regardless of the value of $D$ relative to the other parameters and hence regardless of whether the pattern evolves as it drifts. In contrast, consider the velocity defined by $V_{x}^{\prime}=\xi_{0} / \tau_{0 x}$, where $\tau_{0 \mathrm{x}}$ is the lag for maximum cross-correlation. $V_{x}^{\prime}$ has often been equated with $V / \cos \phi$ (e.g. Lang \& Rickett 1970; Galt \& Lyne 1972).

At the maximum, $\partial \rho\left(\xi_{0}, 0, \tau\right) / \partial \tau$ is equal to zero, so that $\tau_{0 \mathrm{x}}$ must be a root of $\left(B \xi_{0}+D \tau\right) f^{\prime}\left(A \xi_{0}{ }^{2}+2 B \xi_{0} \tau+D \tau^{2}\right)=0$. In addition, since $\rho(0,0, \tau)$ has its (global) maximum at $\tau=0$ we must have $\tau_{0 \mathrm{x}} \rightarrow 0$ as $\xi_{0} \rightarrow 0$. We first search for $\tau_{0 \mathrm{x}}$ among the zeros of $f^{\prime}\left(A \xi_{0}{ }^{2}+2 B \xi_{0} \tau+D \tau^{2}\right)$. Let $f^{\prime}(K)=0$, then $A \xi_{0}{ }^{2}+2 B \xi_{0} \tau+D \tau^{2}=K$ and $\tau^{2} \rightarrow K / D$ as $\xi_{0} \rightarrow 0$ and hence cannot correspond to a solution for $\tau_{0 \mathrm{x}}$ unless $K=0$. For $K=0, \tau=-\left(B \xi_{0} / D\right) \pm\left(\xi_{0} / D\right)\left(B^{2}-A D\right)^{1 / 2}$, so that we must have $B^{2} \geqslant A D$ for $\tau$ real. Now from the condition that $\rho(\xi, 0, \tau) \rightarrow 0$ as $\tau \rightarrow \infty$ for all $\xi$, it can be seen that the contours of equal correlation in the $\xi \tau$-plane defined by $A \xi^{2}+2 B \xi \tau+D \tau^{2}=$ const must be elliptical (rather than hyperbolic or parabolic), and this requires $B^{2}<A D$. Thus $\tau_{0 x}$ cannot coincide with any real zero of $f^{\prime}\left(A \xi_{0}{ }^{2}+2 B \xi_{0} \tau+D \tau^{2}\right)$ for $\xi_{0} \neq 0$.

The only remaining root of $\partial \rho\left(\xi_{0}, 0, \tau\right) / \partial \tau=0$ is given by $B \xi_{0}+D \tau=0$ so $\tau_{0 \mathrm{x}}=-B \xi_{0} / D$ and $V_{x^{\prime}}^{\prime}=-D / B$. As a result, $V_{x}^{\prime}=V / \cos \phi$ only if

$$
-D \mid B=\left(V_{x^{2}}+V_{y}^{2}\right) / V_{x}=-\left(B^{2}+C^{2}\right) / A B,
$$

i.e. $D=\left(B^{2}+C^{2}\right) / A$. But, from equation (7), this is simply the condition for $D^{\prime}=0$. Hence $\xi_{0} / \tau_{0 \mathrm{x}}=V / \cos \phi$ only if the pattern does not evolve as it drifts.

If it can be assumed that the pattern does not evolve but only drifts then it is possible to find both $V$ and $\cos \phi$ from a two-station experiment since, if $D^{\prime}=0$, $\cos \phi=\left(\tau_{0 \mathrm{x}} / 2 \tau_{\mathrm{ex}}\right)^{1 / 2}$ and $V=\xi_{0} /\left(2 \tau_{\mathrm{ex}} \tau_{0 \mathrm{x}}\right)^{1 / 2}$.

While it is quite reasonable to assume that the form of the correlation function is the same (say Gaussian) for both the temporal and spatial coordinates when the source intensity is steady, the strong intrinsic intensity variations typical of pulsars can cause the form of the temporal part of the correlation function to be determined in large part by this source variation rather than the statistics of the medium. Most pulsars exhibit intensity variations which are rapid enough to be significantly decorrelated at time lags less than one pulse period, and as a result the temporal part of the correlation function has a narrow 'spike' at $\tau=0$. In addition, some pulsars have strong periodic intensity fluctuations, with periods many times the pulse period, which will cause the temporal part of the correlation function to have oscillatory components not found in the spatial part.

The effect of the intrinsic source fluctuations is to replace the pattern function $S(x, y, t)$, which would have been measured for a constant source of the same mean intensity by $S_{1}(x, y, t)=[\mathrm{I}+\Delta I(t)] S(x, y, t)$, where $\langle\mathrm{I}+\Delta I(t)\rangle=\mathrm{I}$, i.e. $\langle\Delta I(t)\rangle=0$. Now if $\Delta I(t)$ and $S(x, y, t)$ are statistically independent (as is surely true in the present case) the observed composite correlation function is

$$
\begin{align*}
\rho_{\mathrm{c}}(\xi, \eta, \tau) & =\frac{\left\langle\left[S_{1}(x, y, t)-\mu_{0}\right]\left[S_{1}(x+\xi, y+\eta, t+\tau)-\mu_{0}\right]\right\rangle}{\left\langle\left[S_{1}(x, y, t)-\mu_{0}\right]^{2}\right\rangle} \\
& =\frac{\sigma_{0}^{2}[\mathrm{I}+R(\tau)] \rho(\xi, \eta, \tau)+\mu_{0}^{2} R(\tau)}{\sigma_{1}{ }^{2}} \tag{9}
\end{align*}
$$

where

$$
\begin{align*}
R(\tau) & =\langle\Delta I(t) \Delta I(t+\tau)\rangle \\
\mu_{0} & =\langle[\mathrm{I}+\Delta I(t)][S(x, y, t)]\rangle=\langle S(x, y, t)\rangle \\
\sigma_{0}^{2} & =\left\langle\left[S(x, y, t)-\mu_{0}\right]^{2}\right\rangle \\
\sigma_{1}^{2} & =\left\langle\left[S_{1}(x, y, t)-\mu_{0}\right]^{2}\right\rangle \tag{ıо}
\end{align*}
$$

and $\rho(\xi, \eta, \tau)$ is the correlation function, as defined in equation (1), that would have been observed with a steady source.

Denoting by $\tau_{0 \mathrm{x}}{ }^{\prime}$ the lag for maximum correlation between two stations separated by a distance $\xi_{0}$ on the $x$-axis and, as before, by $\tau_{0 \mathrm{x}}$ the lag for maximum correlation if the source were steady, we see from equation (9) that $\left|\tau_{0 \mathrm{x}}{ }^{\prime}\right| \leqslant\left|\tau_{0 \mathrm{x}}\right|$. This happens because the position of the maximum of the sum or product of two convex functions always lies between their respective maxima. In the region of their maxima, $R(\tau)$ and $\rho\left(\xi_{0}, 0, \tau\right)$ can be safely assumed to be convex, and since $R(\tau)$ has its maximum at $\tau=0$ the result above follows. Consequently the use of $\tau_{0 \mathrm{x}}{ }^{\prime}$ in place of $\tau_{0 x}$ (and the latter cannot be measured if source fluctuations are present) will lead to overestimation of the drift velocity.

Fortunately the determination of $\tau_{\text {ex }}$ is not affected by source intensity variations. Since $\tau_{\mathrm{ex}}{ }^{\prime}$, the lag for equal cross-correlation and autocorrelation with source fluctuations present, is defined by $\rho_{\mathrm{c}}\left(\xi_{0}, \mathrm{o}, \tau_{\mathrm{ex}}{ }^{\prime}\right)=\rho_{\mathrm{c}}\left(0,0, \tau_{\mathrm{ex}}{ }^{\prime}\right)$, we have, using equation (9),

$$
\begin{align*}
\sigma_{0} 2\left[\mathrm{I}+R\left(\tau_{\mathrm{ex}}{ }^{\prime}\right)\right] \rho\left(\xi_{0}, \mathrm{o}, \tau_{\mathrm{ex}}{ }^{\prime}\right)+ & \mu_{0}^{2} R\left(\tau_{\mathrm{ex}}{ }^{\prime}\right) \\
& =\sigma_{0}^{2}\left[\mathrm{I}+R\left(\tau_{\mathrm{ex}}{ }^{\prime}\right)\right] \rho\left(\mathrm{o}, \mathrm{o}, \tau_{\mathrm{ex}}{ }^{\prime}\right)+\mu_{0}^{2} R\left(\tau_{\mathrm{ex}}{ }^{\prime}\right) \tag{II}
\end{align*}
$$

Removing the term common to both sides of this equation and dividing by $\sigma_{0}{ }^{2}\left[\mathrm{I}+R\left(\tau_{\mathrm{ex}}{ }^{\prime}\right)\right]$ yields $\rho\left(\xi_{0}, \mathrm{o}, \tau_{\mathrm{ex}}{ }^{\prime}\right)=\rho\left(\mathrm{o}, \mathrm{o}, \tau_{\mathrm{ex}}{ }^{\prime}\right)$. This is identical with the defining equation for $\xi_{\text {ex }}$; hence $\tau_{\text {ex }}{ }^{\prime}=\tau_{\text {ex }}$.

We conclude, therefore, that $\tau_{\mathrm{ex}}$ is the most useful parameter of the correlation curves for the determination of the drift velocity, since $\xi_{0} / 2 \tau_{\mathrm{ex}}=V \cos \phi$ even when the pattern evolves as it drifts and the source intensity varies. Either of these conditions is sufficient to preclude the use of $\tau_{0 \mathrm{x}}$ as a means of determining the velocity.

In practice the measured signal intensity will be subject to error due to noise. The effect of uncorrelated noise in the records from two stations is to reduce the value of their cross-correlation. If $\sigma_{p}$ is the rms variation in pulse amplitude (including both scintillation and intrinsic components) and $\sigma_{\mathrm{n}}$ is the rms variation in system noise, then the observed peak cross-correlation coefficient should be increased by the factor

$$
\left\{\left[\mathrm{I}+\left(\frac{\sigma_{\mathrm{n}}}{\sigma_{\mathrm{p}}}\right)_{\mathrm{A}}^{2}\right]\left[\mathrm{I}+\left(\frac{\sigma_{\mathrm{n}}}{\sigma_{\mathrm{p}}}\right)_{\mathrm{B}}^{2}\right]\right\}^{1 / 2}
$$

where the subscripts refer to the two stations. For $\sigma_{\mathrm{p}} / \sigma_{\mathrm{n}}=10$ this factor is $\mathrm{I} \cdot 0 \mathrm{I}$ and even for $\sigma_{\mathrm{p}} / \sigma_{\mathrm{n}}$ low as 2 it is still only $\mathrm{I} \cdot 25$.

## 4. DISCUSSION OF RESULTS

Trans-Atlantic spaced receiver experiments by Lang \& Rickett (i970) and Galt \& Lyne (1972) at 408 MHz measured cross-correlation coefficients which did not differ significantly from unity. However, it is clear from Fig. $\mathbf{I}(\mathrm{b})$ and (c) and Table I that some of the cross-correlation coefficients measured in the present experiment are significantly less than unity.

Fig. I shows no evidence for any local ionospheric effects. Ionospheric scintillation would produce strong uncorrelated variations with a time scale $\lesssim 60 \mathrm{~s}$. Ionospheric Faraday rotation of linearly polarized pulses is not expected to cause the observed decorrelation because the total expected Faraday rotation of $1-3$ rad should not change significantly in a few minutes. This conclusion is supported by the observation that PSR $1929+10$, with a high average polarized component of $\sim 70$ per cent, suffers much less decorrelation than PSR 1642-03, which possesses an average polarized component of only $\sim 10$ per cent.

Random evolution of a drifting diffraction pattern on a time scale similar to or less than that of the interstellar scintillations would reduce the peak cross-correlation when the peak occurs at a non-zero value of lag but cannot reduce the instantaneous (zero lag) correlation. Intrinsic variations tend to increase the zero lag value. Hence records such as those in Fig. $\mathrm{I}(\mathrm{b})$ and (c), for which the instantaneous correlation is only a fraction of unity, demonstrate positively that the pattern scale is less than the receiver separation of 8000 km . This conclusion is consistent with two pieces of information contained in Table I.
(i) The widths of the autocorrelation functions (column 5) are consistent with the movement of the receiver at a moderate speed of $\sim 30 \mathrm{~km} \mathrm{~s}^{-1}$ through a diffraction pattern having a scale size of a few thousand kilometres.
(ii) The most highly decorrelated pulsars are those with the lowest modulation indices (column 7) and the highest dispersion measures (column 9). Rickett (1970) and Lang (1971) have shown that the dispersion measure is a good indirect measure
of the strength of scattering along the line of sight to the pulsar. When the scattering is strong enough to reduce the scintillation bandwidth below the receiver bandwidth, the scintillations are smoothed with a resulting reduction in the modulation index. Stronger scattering also results in a reduced scale size for the diffraction pattern, which, if the scale size is less than the receiver spacing, results in a lower correlation between the scintillations at the spaced stations.

In summary, it appears from the lack of high instantaneous correlation between the stations, especially for the more highly dispersed pulsars, that the pattern scale size at $326 \cdot 5 \mathrm{MHz}$ is less than the $8000-\mathrm{km}$ spacing.

The effect of intrinsic variability on a time scale similar to that of interstellar scintillation is to shift the cross-correlation peak to smaller lags and increase the peak cross-correlation coefficient. Many pulsars may possess this type of variability (PSR 1919+21 is a well-known example) with the result that measurements such as those of Lang \& Rickett (1970) and Galt \& Lyne (1972), based on the position of the cross-correlation peak, may have seriously over-estimated the drift velocity and consequently the scale size of the pattern.

The likelihood of both intrinsic variability and temporal evolution of the diffraction pattern being present leaves only $\xi_{0} / 2 \tau_{\mathrm{ex}}$ as a reliable estimator of the drift velocity component along the base line. Fig. 2(a), (b), (c) and (d) show the correlation functions for the four observations for which $\tau_{\mathrm{ex}}$, the lag corresponding to the intersection of the auto- and cross-correlation functions, could be unambiguously determined. Fig. 2(e) and (f) show two examples of correlation functions for which the intersection point cannot be reliably determined. Indeterminate results of this type, which are consistent with strong intrinsic variability and/or temporal evolution of the diffraction pattern, were obtained for PSR $0031-07$, 1706-15, 1749-28, 1919+21 and 2045-16. Of these five pulsars, PSR 0031-07 and $1919+21$ are known from earlier observations of a different kind to possess strong intrinsic variations.

Thus two observations of PSR 1642-03, a single observation of PSR 1929+10 and one of the six observations of PSR 2045-16 provided good determinations of the parameter $\xi_{0} / 2 \tau_{\text {ex }}$. The results from these four unequivocal records are listed in Table II. The surprising reversal in the velocity component for PSR 1642-03 between the observations of August II and I4 prompted us to sub-divide

Table II
Details of clear interstellar diffraction patterns

| Pulsar | Date of observation | Record length (s) | Averaging interval (pulses) | Sig./noise ratio ${ }^{\star}$ | $\mathrm{e}^{-1}$ halfwidth of autocorrelation (s) | Maximum <br> cross-correlation $\dagger$ | Velocity along baseline $\ddagger$ ( $\mathrm{km} \mathrm{s}^{-1}$ ) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1642-03 | 1 1 .8.71 | 7609 | 29 | 33 | 115 | $0 \cdot 61$ | 40 |
| 1642-03 | 14.8.71 | 8025 | 29 | 10 | 109 | 0.47 | -52 |
| $1929+10$ | 14.8 .71 | 3319 | 49 | $2 \cdot 1$ | 388 | $0 \cdot 86$ | 35 |
| 2045-16 | 9.8.71 | 8637 | 7 | $5 \cdot 9$ | 158 | $0 \cdot 98$ | - 12 I |

[^1]each day's data into halves, for which the correlation functions were computed separately. The maximum cross-correlation coefficients for the first and second half-records of PSR $1642-03$ on August II were 0.68 and 0.54 respectively; the lags for maximum cross-correlation ( $\tau_{0 \mathrm{x}}$ ) were +27 s and -25 s , respectively, indicating that the velocity component along the baseline had reversed between the two sections of the recording. On August 14 the two half-records of PSR $1642-03$ possessed maximum cross-correlation coefficients of 0.61 and 0.33 respectively and $\tau_{0 x}$ equal to -45 s and -59 s respectively.

## 5. CONCLUSION

The determinations of pattern velocity listed in Table II are in fact lower limits, since we have measured only the component along the baseline. Except for the one measurement from PSR 2045-16 of $121 \mathrm{~km} \mathrm{~s}^{-1}$, the velocities are consistent with those expected from compounding the transverse component of the galactic differential rotation (a maximum of $26 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{kpc}^{-1}$ ) with the peculiar motion of the Earth with respect to the local standard of rest ( $27 \mathrm{~km} \mathrm{~s}^{-1}$ during these observations). Even the value of $121 \mathrm{~km} \mathrm{~s}^{-1}$ measured for PSR 2045-16 may not be so unusual if it is realized that a diffracting screen placed half-way between the pulsar and observer would result in a geometrical magnification factor of two for the velocity of the pattern at the Earth. There is thus no evidence from the three pulsars which gave positive determinations of the pattern velocity that these objects possess extremely high proper motions.

It is clear, however, from the reversal of the velocity component for PSR 1642-03 on August II, which occurred in the interval of $\sim$ i hr, that we cannot interpret our observations in terms of a simple model of the interstellar medium. It seems probable that the line of sight to the pulsar is occupied by many diffraction ' screens', each moving with its own peculiar transverse velocity, which may not be related to the general galactic rotation. At any one time, the velocity of the diffraction pattern formed by the ensemble of screens may be dominated by a particular region along the line of sight.

The present experiment has demonstrated that observations of pulsar intensity fluctuations at spaced receivers will result in incorrect deductions about the velocity and scale size of an interstellar diffraction pattern unless it is recognized that intrinsic variability and evolving patterns are the rule rather than the exception. Both these factors, unless eliminated by the correct method of analysis, will result in the serious overestimation of the velocity and scale size of the pattern.

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[^0]:    * Radiophysics Publication RPP 1706, CSIRO, Sydney, Australia.

[^1]:    * Geometric mean of the average pulse amplitude/rms noise ratios at the two receivers after smoothing.
    $\dagger$ Corrected for the effects of noise; includes the effects of intrinsic variability.
    $\ddagger$ Positive sign means that the velocity is directed from Ooty towards Parkes.

