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Simultaneous photometry and spectroscopy of the Be star 28 (ω) CMa – II. Line profile modelling

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ABSTRACT

We analyse a series of high-resolution line profiles of He I 6678 in the periodic Be star 28 CMa. A simple zeroth-order approximation to the eigenfunctions is used to model the line profile variations in terms of non-radial pulsation. In addition, we use the best available theory for the eigenfunction in a rotating star. In all cases the calculated fits to the observed profiles are poor. We conclude that non-radial pulsation is almost certainly not responsible for the line profile variations in this star. In an attempt to understand the very large line profile changes, but negligible light variation, we consider a 'patch' model. This consists of a circular area on the photosphere having the same temperature, but a different intrinsic line width. A simple model of this kind produces a good fit to the line profile variations and to the photometry. We conclude that modulation of the profile by a patch on, or close to, the photosphere may offer a plausible starting point for understanding the periodic variations in some Be stars.

Key words: line: profiles – stars: early-type – stars: emission-line, Be – stars: individual: 28(ω) CMa – stars: oscillations – stars: variables: other.

1 INTRODUCTION

A large fraction of Be stars exhibit strictly periodic light and line profile variations with periods of 0.5–2 d, consistent with the rotational period (the λ Eri stars). The shape and amplitude of the periodic light curve is strongly variable: a single-wave light curve may change into a double-wave and vice versa (Balona, Sterken & Manfroid 1991). The amplitude of the periodic variations may be unobservable in one season, yet may attain a value of 0.1 mag or larger at another time. On top of this, there may be irregular light fluctuations and occasional outbursts. The periodic variations have been interpreted as non-radial pulsation (NRP), but there is evidence to suggest that they may be better understood in terms of some form of rotational modulation (Balona 1995). The behaviour of the light and line profile variations is clearly different from the β Cep and 53 Per (SPB) stars. Therefore, periodic light and low-order line profile variations seem to be a unique characteristic of the λ Eri stars. The connection between the periodic variability and the enhanced mass loss is still an open question.

One of the stars studied by Štefl et al. (1998) is the bright Be star 28 CMa (B2–3 IV–Ve). Apart from the well-known spectroscopic period of $P_1 = 1.37$ d, they found a transient period of $P_2 = 1.48$ d which is weakly present in most He I lines, attains more power in Si II, Mg II and the higher Balmer lines, and is entirely dominant in Hα and Hβ. The transient period had already been suspected in the light curve by Balona et al. (1987), but with an amplitude too small to be considered significant. In Štefl, Aerts & Balona (1999, hereafter Paper I), we examined new spectroscopic data and new and existing photometry. We found that P_1 is still the dominant period in the He I 6678 line, but there are indications that P_2 may be present in the line wings. Evidence from the photometry is less conclusive, but indications of both periods were found.

One of the puzzles that remains unresolved is the contrast between the very large radial velocity variation and the negligible light variation at the same period. In the NRP model, a low light amplitude may be expected for modes of high degree, but the lack of more than one wave in the line profile variations and the large radial velocity amplitude implies a low degree. It cannot be a matter of the angle of inclination, since this has the same effect on the radial velocity as on the light amplitude. Clearly, the NRP model needs to be investigated more thoroughly. The starspot hypothesis has not been investigated.

In this paper, we investigate the NRP and rotational modulation hypotheses by attempting to model the variations of the He I 6678-Å line using the light amplitude and phase as a constraint. The detailed investigation of this bright star should assist in our understanding of the periodic variations in other Be stars.

2 THE NRP MODEL

In a star in which the pulsation period is much smaller than the

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period of rotation, the eigenfunction is well described by a simple spherical harmonic of degree ℓ and azimuthal order m. Since the period of rotation for Be stars is 1–2 d and the observed period in 28 CMa is 1.37 d, this condition does not hold. It is therefore necessary to include higher order terms in the expansion of the eigenfunction. Expressions for the pulsational velocity in the first-order theory are given by Carroll & Hansen (1982) and Aerts & Waelkens (1993). These theories express the rotationally modified pulsation variables as a truncated power series in Ω/ω, where Ω and ω are the rotation and pulsation frequencies respectively. It is evident that such expressions will be inaccurate for $\Omega/\omega > 1$, which limits their applicability. The results of first-order theory when $\Omega/\omega > 0.5$ cannot be trusted.

Another technique, which does not rely on series expansion, has been developed by Berthomieu et al. (1978) and extended by Lee & Saio (1987). This theory, like the first-order theory, neglects the effect of centrifugal force and the gravitational perturbation. As a result, the equations can be cast in a form in which they resemble those of a non-rotating star. This similarity transformation technique provides a complete treatment of the Coriolis force for all values of Ω/ω. Although it is still not a full treatment, it can be expected to provide more reliable results than the first-order theory, especially for $\Omega/\omega > 0.5$. Townsend (1997) discusses the eigenfunction of rapidly rotating early-type stars using this theory.

We estimate the radius of 28 CMa as $R \approx 6\ \mathrm{R}_\odot$ from its spectral type (Balona 1995). Using $v \sin i = 80\ \mathrm{km\,s^{-1}}$ gives a rotation period $P_{\mathrm{rot}} < 3.8$ d; therefore $\Omega/\omega > 0.36$ if $m = 0$. The frequency ratio, Ω/ω, will be higher for prograde modes ($m < 0$) and lower for retrograde modes ($m > 0$). For such a large ratio, departures of the eigenfunctions from pure spherical harmonics will be significant. The zeroth-order approximation, which has been used in the past, is very unlikely to give a good fit to the line profiles.

For pulsation with a relatively long period, as may occur in 28 CMa, most of the motion will be in the horizontal plane. This follows from the boundary condition that the pressure variation tends to zero at the surface of the star. The ratio of horizontal to radial displacement amplitudes in the approximation of a slowly rotating star is given by $K = g/(\omega^2 R)$ where g is the gravitational acceleration at the surface and ω is the angular frequency of pulsation in the corotating frame. If $P_1 = 1.37$ d is an axisymmetric mode ($m = 0$), then $K \approx 6$, assuming a mass of about 9 $\mathrm{M}_\odot$. For non-axisymmetric modes, the frequency in the corotating frame depends on the (unknown) period of rotation. The value of K will be larger for prograde modes, smaller for retrograde modes.

A rotating star with a non-uniform azimuthal temperature distribution will produce line profile variations because the component of rotational velocity will be weighted according to the non-uniform surface brightness distribution. Balona (1987) discusses this effect and introduces a pseudo-velocity, v_f, as a convenient measure of this effect. Clearly, $v_f < v \sin i$. Normally, the periodic compression of the atmosphere during NRP will produce a temperature variation. Rapid rotation will then introduce significant profile variations. In principle, it is possible to predict the relative temperature variation given the relative radial or horizontal amplitude. Alternatively, the relative temperature amplitude and phase may be left as free parameters to be determined from the line profile itself (Balona 1987).

It is a simple matter to calculate the line profile variations, given the pulsation parameters. The inverse problem is far more difficult. There are two approaches currently in use: the Doppler imaging method and the method of moments. Doppler imaging is very easy to apply and is the most common method, but while it can be used to estimate ℓ and (possibly) m, it is not capable of giving the velocity amplitudes and phases which are required to model the profiles. On the other hand, the method of moments does give these data. It uses a first-order approximation to the eigenfunction, which is probably still not a sufficiently good approximation in the case under study. The BRUCE code developed by Townsend (1997) uses the Lee & Saio (1987) expansion for the eigenfunction and is the best available approximation for modelling NRP in 28 CMa. With this code we can generate line profiles given the pulsational parameters. What we would like to do, however, is to solve the inverse problem.

Although two periods are present in 28 CMa, it is fortunate that the $P = 1.365$ d period dominates the He I 6678 line profile. The transient period may be important in the line wings, but the duration of the time series does not allow full resolution. Therefore it is probably safe to describe the core of the line by only one period and only one NRP mode. For a single mode, we need to solve for the following parameters: the projected rotational velocity, $v \sin i$; the intrinsic Gaussian line width, W_{i}; the velocity amplitude, V_{r}, and its phase, ϕ_{r}, and the angle of inclination, i. In modelling the line profiles of 28 CMa, we assume that there is no temperature variation. This is not physically realistic, but is necessary if the resulting light variation is to have a negligible amplitude, as observed in 28 CMa. Our aim is to show that, even under this unphysical assumption, the line profiles predicted by NRP still do not match the observed line profiles.

The intrinsic line profile is most often approximated by a Gaussian. To obtain a better estimate of the intrinsic profile, Dr J. Kubát (Astronomical Institute, Ondřejov) kindly provided theoretical profiles for the He I 6678 line at various angles of incidence. An effective temperature $T_{\mathrm{eff}} = 23\,000$ K, $\log g = 3.8$ was assumed in the atmospheric models. The theoretical intrinsic profile is actually very well described by the Gaussian $I = I_0 \exp(\Delta\lambda^2/2W_{\mathrm{i}}^2)$ with $W_{\mathrm{i}} = 10.2\ \mathrm{km\,s^{-1}}$. In the line profile modelling, we searched for the best fit using this fixed value of W_{i}. Since we found extremely poor agreement, we decided to relax the restriction on W_{i} and perform another set of solutions in which W_{i} was adjusted for best fit. In the theoretical intrinsic profile, the linear limb-darkening parameter, u, is not equal to the continuum value, as is sometimes assumed, but varies between $u = 0.07$ and 0.26.

Since neither the Doppler imaging nor the method of moments is applicable, we decided to attempt the most direct approach. For a given mode (ℓ, m), and given W_{i}, $v \sin i$, V_{r}, ϕ_{r} and i, we computed 10 line profiles at intervals of 0.1 period with $P = 1.365$ d. These calculated profiles were normalized to have the same equivalent width as the observed profile. As a measure of the goodness of fit, we used the standard deviation in the intensity, σ, over all 10 profiles as determined for that part of the profile within $100\ \mathrm{km\,s^{-1}}$ of line centre. The most likely pulsation parameters are those that minimize σ.

To ensure that the value of σ is a global minimum, not a local minimum, we calculated values of σ in a grid where $0 < V_{\mathrm{r}} < 100\ \mathrm{km\,s^{-1}}$ with an interval of $10\ \mathrm{km\,s^{-1}}$, $0 < \phi < 1$ period with an interval of 0.2 period and $10° < i < 90°$ with an interval of 10°. The procedure is computationally intensive and would not be possible to apply for more than one period. The parameters on the grid point with smallest σ were then used as starting values to a routine which minimized σ by varying $v \sin i$, V_{r}, ϕ_{r} and i. The Levenberg–Marquardt method of optimization by non-linear least-squares was used (Press et al. 1992).

2.1 Solutions using the zeroth-order approximation

Although the zeroth-order approximation ($\Omega/\omega \approx 0$) is not appropriate, it is still widely used in mode identification and line profile modelling. Therefore, we felt it would be important to discuss the results obtained using a simple spherical harmonic eigenfunction for comparison with previous work. One important point to bear in mind is the correct value of K, the ratio of horizontal to vertical displacement. The answer to this in the zeroth-order approximation is unequivocal: $K = g/(\omega^2 R)$ where g is the gravitational acceleration, R the stellar radius and $\omega = \omega_{\rm obs} + m\Omega$ the pulsation frequency in the corotating frame. However, the first-order approximation for K given by Schrijvers et al. (1997) was also used to determine the effect of a variation in K. We found solutions using both values of K, but the results are not significantly different.

Mode identification was accomplished by direct fitting, as described above. For a given value of (ℓ, m), the following quantities were determined: the projected rotational velocity, $v \sin i$, the vertical velocity amplitude, $V_{\rm r}$, the initial phase, ϕ, and the angle of inclination, i. No attempt was made to allow for polar flattening of the star, gravity darkening or variation of temperature with latitude. Solutions were made using the intrinsic line profile calculated from a suitable model atmosphere, as described above. Solutions were also made in which the width of the intrinsic line profile (assumed Gaussian) was allowed to vary for the best fit. For physically realistic cases, where K is fixed by the boundary conditions as described above, the fits were all extremely poor.

We therefore took the step of allowing K itself to be a free parameter, to be determined for the best fit. The reason for this is that we are not sure of the correct boundary conditions for rapidly rotating stars. Making this a free parameter at least gives an indication as to whether a reasonable fit can be achieved. Results were, once again, very disappointing. Finally, we allowed the intrinsic width, $W_{\rm i}$, also to be determined by the fit instead of being fixed at the most physically plausible value of $10.2\,{\rm km\,s^{-1}}$.

The solutions, with and without fixed $W_{\rm i}$, are shown in Table 1. The best fit is obtained, not surprisingly, when both K and $W_{\rm i}$ are allowed to vary. The best solution ($\ell = 2, m = +1$ and $W_{\rm i} = 29.3\,{\rm km\,s^{-1}}$) is shown in Fig. 1. The poor fit unequivocally demonstrates that it is impossible to obtain a satisfactory fit with a simple spherical harmonic approximation to the eigenfunctions, even when we drop the constraints on K and $W_{\rm i}$ imposed by physics.

Table 1. Best solutions using the zeroth-order approximation (the eigenfunction is a pure spherical harmonic). The columns give the mode (ℓ, m), projected rotational velocity, intrinsic line width, vertical component of velocity amplitude ($V_{\rm r}$), horizontal component of velocity amplitude ($V_{\rm h}$), initial phase (ϕ, in periods), angle of inclination (i degrees) and the standard deviation of the fit in intensity units (σ, unit continuum). The first line is the best solution with $W_{\rm i}$ fixed at the calculated value of $10.2\,{\rm km\,s^{-1}}$; the second line is the best solution when $W_{\rm i}$ is allowed to vary. In all cases, $V_{\rm h}$ was obtained by minimizing the error of the fit. Using standard boundary conditions to fix $V_{\rm h}$ from $V_{\rm r}$ produced worse fits.

(ℓ, m)	$v \sin i$	$W_{\rm i}$	$V_{\rm r}$	$V_{\rm h}$	$\phi_{\rm r}$	i	σ
(1,−1)	101.5	10.23	0.60	6.05	0.398	81.8	0.01974
	96.7	27.27	2.24	32.90	0.071	48.4	0.01897
(1, 1)	100.6	10.23	10.11	8.60	0.002	21.2	0.01973
	95.0	31.04	10.39	20.36	0.015	89.3	0.01892
(2,−2)	104.1	10.23	3.41	327.49	0.088	90.0	0.02913
	68.5	35.89	11.65	373.82	0.713	53.2	0.02293
(2,−1)	25.7	10.23	92.12	416.06	0.656	90.0	0.02160
	96.4	27.84	0.00	0.00	0.345	90.0	0.01916
(2, 1)	100.8	10.23	9.30	17.50	0.024	80.6	0.01960
	95.7	29.26	6.22	8.51	0.039	35.5	0.01886
(2, 2)	110.0	10.23	10.00	3.01	0.000	20.0	0.02108
	96.6	27.36	0.79	0.28	0.404	19.8	0.01916
(3,−2)	67.6	10.23	12.23	196.89	0.873	89.9	0.01980
	68.6	30.14	9.88	164.00	0.633	90.0	0.01929
(3,−1)	83.8	10.23	19.45	176.40	0.330	62.8	0.02154
	96.3	29.07	0.43	4.71	0.476	62.3	0.01907
(3, 1)	101.0	10.23	2.59	1.00	0.068	10.8	0.01972
	95.1	30.60	4.24	6.14	0.005	38.5	0.01895
(3, 2)	101.4	10.23	18.73	21.56	0.759	90.0	0.01956
	96.2	28.06	1.72	2.07	0.545	90.0	0.01916
(3, 3)	100.9	10.23	21.02	3.66	0.005	19.1	0.01964
	95.5	29.76	50.13	10.35	0.124	20.2	0.01901

2.2 Solutions using the similarity transformation

The similarity transformation developed by Lee & Saio (1987) is the best approximation we have at present for calculating the eigenfunctions of a rotating star. Townsend's BRUCE code, which implements the method, allows for variations in gravity and temperature with latitude. This, in turn, affects the equivalent width of the intrinsic line profile. The effect with temperature was taken into account using results of theoretical models.

Results using the BRUCE code are shown in Table 2. The code automatically determines the horizontal velocity amplitude, $V_{\rm h}$, from the given vertical velocity amplitude, $V_{\rm r}$, using the standard boundary condition. $V_{\rm h}$ is extremely large for prograde modes because the period in the corotating frame tends to infinity for these modes. Also shown in the table are results where the intrinsic line width is allowed to vary for the best fit, as was done for the zeroth-order approximation. Once again, a better fit is obtained for $W_{\rm i} > 20\,{\rm km\,s^{-1}}$. In fact, all fits with $W_{\rm i} = 10.2\,{\rm km\,s^{-1}}$ are rather poor. The effect of fixing the line width at this value is to introduce many more wiggles in the fitted curve which are not present in the observations. The larger intrinsic width smears out these wiggles and gives a better fit.

Contrary to expectations, the best fitting mode is a *retrograde* mode. The fits for (ℓ, m) = (1, +1), (2, +1) and (3, +1) are almost equally good. The solution for (3, −3) can be eliminated at once. Although the fit is reasonable near the line core (where the discriminant is calculated), it completely fails in the line wings. This is because the model assumes a horizontal pulsational velocity amplitude of over $100\,{\rm km\,s^{-1}}$, which is unphysical. The most plausible solution is (2, +1) for which the angle of inclination is 23°. The low value is expected for 28 CMa which is known as a 'pole-on' Be star. The fit for this mode with $W_{\rm i} = 37.2\,{\rm km\,s^{-1}}$ is shown in Fig. 2. The fit is not too bad, apart from the absorption 'blip' in the core. However, the pulsation parameters need to be physically plausible. Apart from the unphysically large intrinsic width, there is also a problem with the pulsation amplitudes. The vertical and horizontal displacement amplitudes vary with longitude and latitude. For the best-fitting $m = +1$ modes, the maximum vertical amplitude exceeds 0.15 stellar radii, while the maximum horizontal amplitude exceeds 0.25 stellar radii in all cases.

These large relative amplitudes pose a serious problem for the model. Not only are they larger than for almost any other pulsating star, but also they should give rise to considerable temperature variations and therefore a large light amplitude, which is not observed.

2.3 Biperiodic model

One might argue that, although He I 6678 is dominated by P_1, the

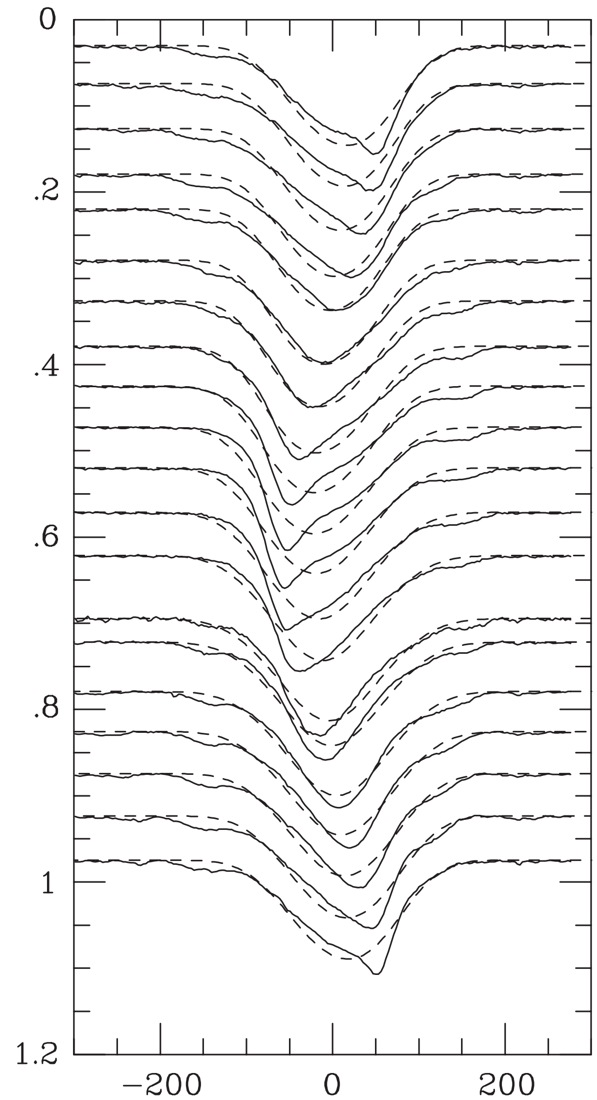

Figure 1. Observed line profiles (solid line) and profiles from a model with $(\ell, m) = (2, 1)$ pure spherical harmonic. The observed line profiles have been co-added in discrete phase bins. The profiles have been shifted so that the continuum measures the time in fractions of the period $P_1 = 1.37$ d. The abscissa is in km s^{-1} relative to the mean radial velocity of the star.

line profiles can be better fitted if one considers a biperiodic pulsational model for the periods P_1 and P_2. Direct fitting for such a model implies unrealistic computation times. For this reason we applied the biperiodic moment method by using a first-order approximation to the eigenfunction. We again find retrograde modes with $m = +1$ as best candidates. However, they once more do not lead to an acceptable fit for physically justified parameters.

Because of all the above-mentioned problems, we do not believe that the line profile variations in 28 CMa are the result of NRP.

3 THE STARSPOT MODEL

Balona (1990, 1995) has pointed out that the period/$v \sin i$ relationship for the periodic Be stars is consistent with the photometric period being identical to the rotational period of the star. Moreover, limits can be placed on how much the ratio of these periods may differ from unity. The data allow a deviation of about 5 per cent (one standard deviation). Since the period of Be stars is so close to the

Table 2. Best solutions using the similarity transformation method and *bruce*. The columns give the mode (ℓ, m), projected rotational velocity, intrinsic line width, vertical and horizontal components of velocity amplitude (V_r, V_h), initial phase (ϕ, in periods), angle of inclination (i degrees) and the standard deviation of the fit in intensity units (σ, unit continuum). The first line is the best solution with W_i fixed at the calculated value of 10.2 km s^{-1}; the second line is the best solution when W_i is allowed to vary. The value of V_h is obtained from V_r and the standard boundary condition.

(ℓ, m)	$v \sin i$	W_i	V_r	V_h	ϕ_r	i	σ
(1,−1)	99.6	10.2	61.3	7236.9	0.802	19.8	0.0406
	120.6	17.5	2.2	462187.0	0.809	29.4	0.0179
(1, 1)	99.8	10.2	23.2	56.7	0.797	88.8	0.0163
	54.3	51.1	39.0	126.1	0.789	82.2	0.0097
(2,−2)	100.0	10.2	20.9	1474.1	0.281	40.4	0.0170
	97.8	24.9	17.5	1225.3	0.275	39.3	0.0122
(2,−1)	102.9	10.2	44.5	978.4	0.214	52.5	0.0227
	99.9	20.2	21.9	53.2	0.800	19.8	0.0156
(2, 1)	95.9	10.2	23.6	57.2	0.787	71.3	0.0140
	79.7	37.2	35.5	51.1	0.785	23.0	0.0086
(2, 2)	104.6	10.2	70.8	22.1	0.780	16.9	0.0176
	99.7	20.7	83.5	24.9	0.808	15.6	0.0169
(3,−3)	107.1	10.2	1.2	44.6	0.282	74.9	0.0085
	102.3	21.4	4.5	120.4	0.239	61.7	0.0139
(3,−2)	100.7	10.2	20.6	1236.2	0.807	39.9	0.0174
	100.6	20.6	20.1	1129.0	0.806	39.5	0.0137
(3,−1)	82.9	10.2	19.5	268.0	0.283	56.6	0.0126
	100.0	20.6	12.8	231.5	0.223	58.0	0.0181
(3, 1)	98.5	10.2	11.1	21.0	0.779	42.0	0.0134
	84.4	27.3	19.9	41.7	0.783	41.5	0.0084
(3, 2)	101.6	10.2	27.1	38.5	0.280	78.7	0.0167
	93.8	26.3	27.2	41.1	0.284	76.0	0.0161
(3, 3)	103.1	10.2	108.0	31.3	0.764	24.2	0.0156
	100.5	23.5	61.6	34.1	0.769	39.0	0.0149

rotational period, the simplest hypothesis is to assume that it is the rotational period and try to model the profile and light variations by some kind of rotational modulation. This may take the form of a starspot, similar in nature to sunspots, or a corotating obscuration. In principle, it is possible to obtain a crude image of the stellar surface as is done for some Ap and RS CVn stars. However, we prefer to model the simplest possible spot – a single circular patch – in the hope that this may be sufficient to allow us to understand the line profile variations.

The effect of a circular spot has been discussed by Budding (1977), who derives mathematical expressions for the light curve. It may be possible to determine these parameters from the line profile by moments. The number of free parameters is about the same as for singly periodic NRP, so a search for the best fit between observed and calculated profiles is computationally feasible.

The line profile variations caused by a circular spot are determined by the following parameters: R_e, R_p – the equatorial and polar radii; F_e, F_p – the fiducial photospheric equatorial and polar fluxes; $v \sin i$ – the projected rotational velocity; i – the angle of inclination; u – the linear limb-darkening coefficient; W_i – the intrinsic line profile width in the photosphere; λ – the longitude (relative to some arbitrary epoch); β – the latitude; γ – the spot radius in degrees; F – the flux from the spot; W_s – the intrinsic line profile width in the spot. The same direct-fitting technique described above was used. First, a coarse solution was found by fitting to profiles spaced at 0.2 period, then the best solution was used as a starting value for the Levenberg–Marquardt optimization method.

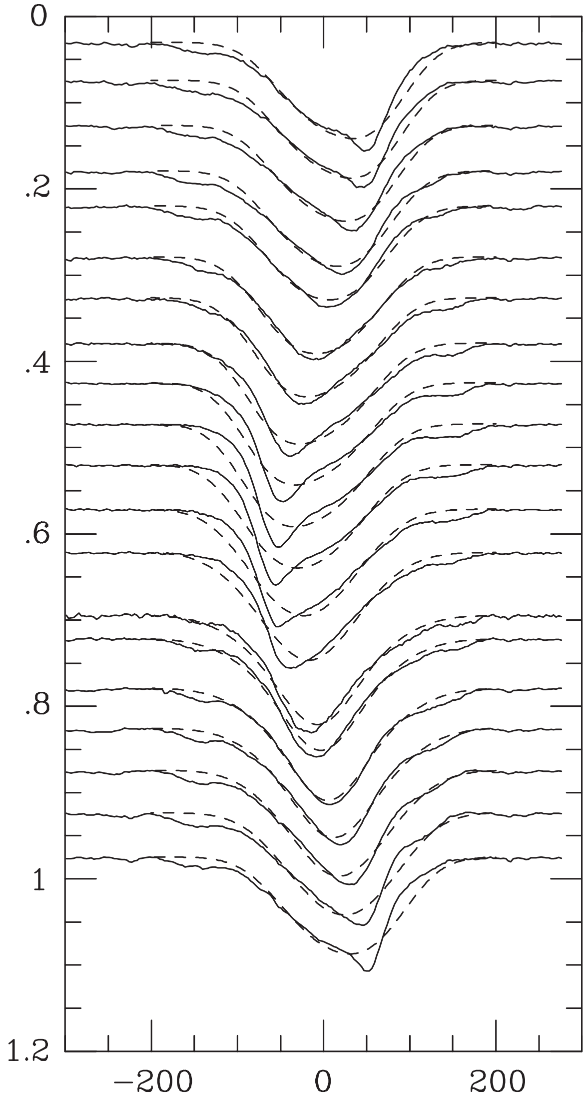

Figure 2. Observed line profiles (solid line) and profiles from a model with (ℓ, m) = (2, 1) using the rotationally-corrected eigenfunction generated by BRUCE.

One of the most important constraints is the very low light amplitude observed in 28 CMa. The very high ratio between the radial velocity and light amplitudes is one of the most puzzling aspects of Be stars. This led Balona (1995) to the conclusion that neither NRP nor conventional starspots could account for this ratio. In a conventional starspot, the spot has a different temperature from the surrounding photosphere. This leads to substantial light variations for even a moderately sized spot. In 28 CMa, the radial velocity amplitude is very large. To generate this amplitude requires a large spot. This automatically leads to a large light amplitude. Although it is possible to obtain a very good fit to the line profile variations in this way, the large light variations resulting from such a spot contradict observations. This, of course, is exactly the same problem as we encountered for NRP.

We have estimated, for example, that a spot model which matches the line profile variations and has the same intrinsic profile in the spot and in the photosphere requires that the spot brightness be one-quarter of the photospheric brightness. The resulting light amplitude is at least 0.2 mag. The parameters of some of these models are shown in Table 3. The following physical parameters were used for the star: $R_e = R_p = 6.0$ R$_\odot$; $F_e = F_p = 1.0$; $v \sin i = 80\,\mathrm{km\,s^{-1}}$; $u = 0.24$.

Table 3. Spot parameters for a selection of best-fitting spots using the following fixed parameters: $R_e = R_p = 6.0\,R_\odot$; $F_e = F_p = 1.0$; $v_e \sin i = 84$ km s^{-1}; $u = 0.24$ (see text). The spot parameters are as follows: λ – the longitude in degrees (relative to HJD 245 0000.000); β – the latitude in degrees; γ – the spot radius in degrees; F – the flux from the spot relative to the photosphere. The standard deviation of the fit to the observed line profiles is given by σ; Δm is the expected peak-to-peak light amplitude.

i	W_i	λ	β	γ	F	σ	Δm
25	20	80	30	40	2.0	0.00162	0.24
25	20	70	20	50	2.0	0.00149	0.29
35	20	70	10	60	2.0	0.00150	0.41
15	30	70	0	50	4.0	0.00094	0.36
25	30	70	0	60	4.0	0.00098	0.65
15	40	70	40	30	4.0	0.00086	0.24
20	40	65	30	40	4.0	0.00085	0.45
25	40	60	40	40	4.0	0.00087	0.50

Clearly, if the light amplitude is to be very small, the difference in temperature between the spot and the photosphere must be small. To obtain the observed profile variations, one must assume that the intrinsic line profile in the spot differs from the intrinsic line profile in the photosphere. One way in which this might be achieved is if the spot is the result of a difference in abundance of a particular element. This is indeed the case in the Bp and Ap stars. However, it cannot be true in 28 CMa because all elements show substantial line profile variations which are *in phase* (Baade 1982a,b). If helium is overabundant in the spot, then hydrogen is less abundant in the spot so that the radial velocities will be phase-shifted by 180° relative to each other. This is not observed.

If differences in equivalent width (i.e. abundance differences) are excluded by the above argument, then the only other way in which a radial velocity can be generated is by keeping the equivalent width of the intrinsic line profile in the spot the same as in the photosphere, but allowing the rms width of the intrinsic profile to differ: in other words, by assuming a difference in 'macroturbulence'. Instead of a spot, we will refer to a 'patch' when the temperature is the same as the surrounding photosphere.

A patch model can indeed reproduce the observed line profiles. A sequence of models is shown in Table 4. Models with Lorentzian intrinsic profiles agree a little better than those with Gaussian profiles, but the difference is not very significant. The deduced values of W_i and W_s are much larger than expected. They can be reduced to some extent by including the effects of varying gravity

Table 4. Spot parameters for a spot model in which the rms width of the intrinsic profile in the spot, W_s, is allowed to differ from the rms width in the photosphere, W_i. A Gaussian (top two lines) or Lorentzian (bottom two lines) intrinsic line profile is assumed. Refer to Table 3 for a description of the other symbols.

Model	$v \sin i$	i	W_i	W_s	λ	β	γ	F	σ	Δm
G1	94	13	33	18	88	42	40	1.0	0.00060	0.000
G2	81	15	48	25	88	25	45	1.05	0.00052	0.005
L1	77	10	31	14	88	23	43	1.0	0.00046	0.000
L2	78	9	30	14	89	25	42	1.05	0.00045	0.005

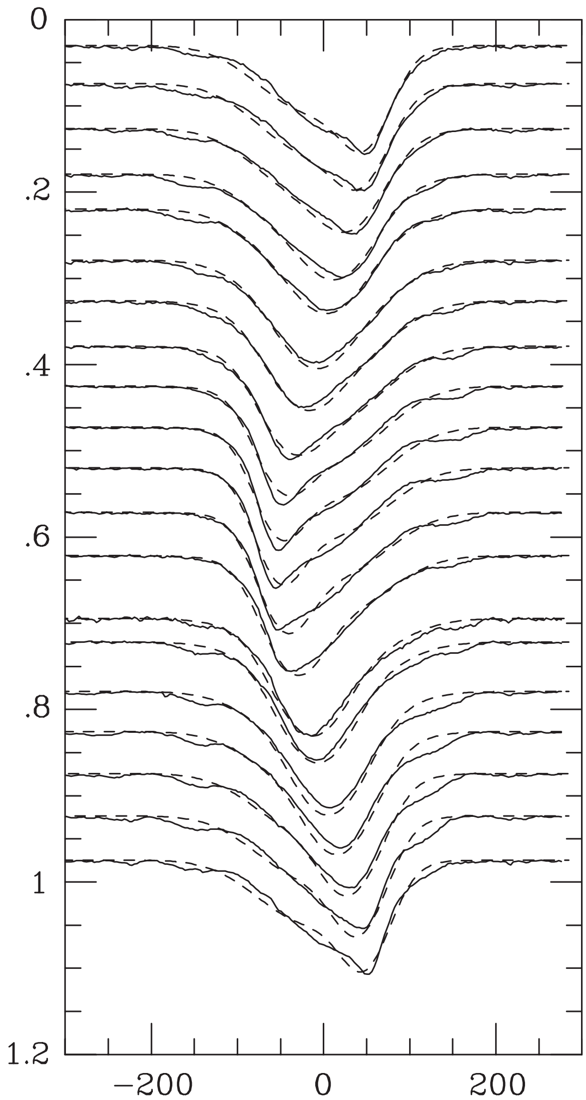

Figure 3. Observed line profiles (solid line) and profiles from spot model G2 of Table 4. Note the weak absorption features at about +150 km s^{-1} in the phase interval 0.3–0.7 and at -150 km s^{-1} at the other phases.

and temperature, but we do not show the calculations here. Our intention is not to propose that such a patch does indeed exist in 28 CMa, but to illustrate how easy it is to account for the line profile variations even with an extremely idealistic patch model. Fig. 3 shows the comparison with the observed profiles for one of these models. Even the characteristic absorption blip is rather well modelled.

4 WEAK ABSORPTION FEATURES IN THE FAR WINGS

It can be seen from Figs 1, 2 and 3 that, during some phase ranges, a weak absorption feature appears in the far wings of the line. It is difficult to determine whether these features are at fixed wavelengths but have varying intensity, or whether they have a variable velocity. If the velocity is variable, it is conceivable that the period may differ slightly from that of the main absorption line. We looked very carefully at the possibility that these weak features may be an observational artefact, but are convinced that this is not the case.

To investigate the nature of this weak absorption feature, the

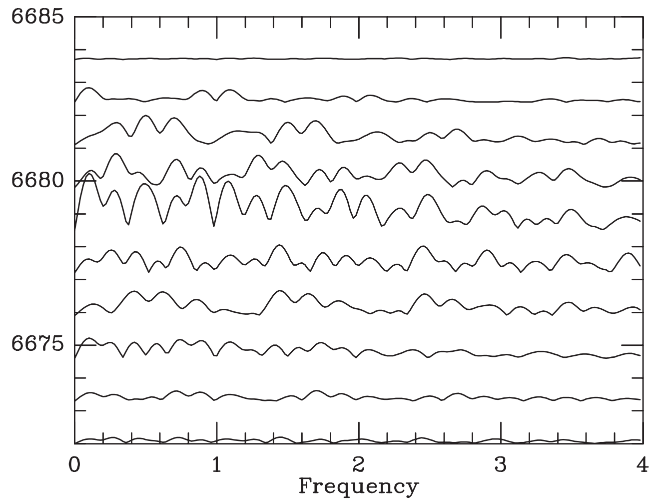

Figure 4. Periodograms of 10 wavebands across the line profile after starspot model G2 is removed. Refer to Table 4 for parameters of the model.

variations of the main He line need to be removed. To do this, we used model G2 of Table 4. We could have used any of the other starspot or NRP models, but we chose G2 because it is one of the most effective in removing the variations (because the fit is better than any of the NRP models). Fig. 4 shows the periodogram for 10 wavelength bins across the residual profiles. There appears to be nothing significant, suggesting that the weak absorption features are probably at constant radial velocity but variable intensity. However, the result is not conclusive.

What mechanism is responsible for this weak absorption feature? Clearly, it cannot be explained in terms of NRP, spot or patch. The feature is situated at ± 150 km s^{-1} from the main He I line. Since the projected equatorial velocity is no more than 90 km s^{-1}, it would have to be formed above the photosphere. If the material is in corotation, it must lie at a distance of nearly one stellar radius above the photosphere, assuming a Keplerian orbit. One possibility is an optically thick ring of varying thickness at this distance.

5 DISCUSSION AND CONCLUSIONS

We have attempted to fit the line profile variations of He I 6678 by an NRP model which uses a zeroth-order approximation to the eigenfunction. The best fit is a mode with $m = +1$, i.e. a *retrograde* mode, but the fit is too poor to be acceptable. We then used a much better approximation to the rotationally modified eigenfunction using the Lee & Saio (1987) theory. Using Richard Townsend's BRUCE code, we again found that the best fit is given by an $m = +1$ mode. The fit is still poor, but becomes acceptable if we are allowed to increase the intrinsic line width from the expected value of $W_i = 10$ km s^{-1} to at least 20 or 30 km s^{-1}. The most likely solution is $\ell = 2, m = +1$ with $W_i = 37.2$ km s^{-1} and $i = 23°$. The intrinsic line width seems to be unacceptably large. Even more serious, however, is the fact that the pulsation amplitude is much too large to be physically realistic and leads to a large light variation, in contradiction to observations.

In our analysis of NRP, we have gone far beyond what would be considered physically acceptable constraints in an attempt to reconcile the observed and predicted line profile variations. We have accepted parameters, such as a very large intrinsic line width, that would normally not be acceptable on physical grounds. Even then, the fit is poor. Thus, even if we assume that some other (unknown) factor is responsible for the large line broadening, NRP is still not an acceptable solution. We conclude that a physically plausible NRP model cannot be found. The principal conclusion of this paper is therefore that NRP is not responsible for the $P_1 = 1.37$-d variation.

We know from the HEROS results (Štefl et al. 1998) and from the analysis in Paper I that, apart from the period $P_1 = 1.37$ d which dominates most of the He I 6678 line, there is another period, $P_2 = 1.49$ d, which is present only in those lines (or parts of a line) formed high in the atmosphere. We have not considered this periodicity because it does not play a significant role in the He I 6678 line. We can say, however, that NRP cannot be responsible for this transient period either. Owing to the fact that it has negligible amplitude in the photosphere and an increasing amplitude above the photosphere, it must be a trapped mode. Therefore the period of such a mode must be of the same order as the thermal time-scale. In the upper atmosphere, however, the thermal time-scale is of the order of minutes, not days. Therefore no such mode can exist.

Because the observed periods of Be stars are consistent with the rotational period, it is natural to model the line-profile variations by means of a starspot. It is quite clear, however, that a normal starspot, in which there is a significant temperature difference between the spot and the photosphere, cannot produce the radial velocity variations and, at the same time, a very low light amplitude. The only way to avoid this is to assume that there is very little temperature difference. Instead, it is a difference in *intrinsic line profile shape* in the spot and in the photosphere which is largely responsible for the line profile variations. This would certainly explain the large velocity/light ratio in other Be stars as well (Balona 1995). The difference in intrinsic line profile is probably not a result of equivalent width differences, i.e. abundance differences, because the radial velocities measured for a variety of elements all appear to be in phase (Baade 1982a,b). We conclude that a difference in rms width, i.e. macroturbulence, is required. We will call this a 'patch' rather than a spot.

It turns out that a simple circular patch with a radius of about 45° situated at moderate latitudes gives a good fit to the line profiles while producing negligible light variations. Once again, however, we find intrinsic line widths which are much larger than expected. All this suggests that the atmospheres of Be stars are extremely turbulent. We do not understand why this should be the case, but we can at least state that a patch model, unlike NRP, does fit the line profiles quite well. Unfortunately, we know too little about Be stars to construct a detailed model that can be defended. It is clear, for example, that the role of magnetic fields is probably an important consideration (Smith 1994).

We may speculate that the patch in 28 CMa may be a cloud of gas suspended above the photosphere by a magnetic field. Attempts to detect magnetic fields on Be stars directly have not been successful (Barker 1986; Bohlender 1994). For example, Bohlender (1994) mentions the negative result of several seasons of polarimetric observations of *o* And. It should be remembered, however, that detection will succeed only if the magnetic field is organized over a large scale. Indirect evidence for the presence of magnetic fields in Be stars comes from the observations of X-ray flares in some of these stars, such as γ Cas. This star is well known to undergo rapid, chaotic X-ray fluctuations. There is evidence that the source of the X-ray flaring is on, or close to, the photosphere rather than from a compact companion (Smith et al. 1998). X-ray and UV flaring has also been found in λ Eri (Smith et al. 1997). The existence of flaring in the photosphere can only be explained by the presence of magnetic fields. One possibility is that energy released by the connection and disconnection of magnetic field lines triggers explosive ablations, causing the X-ray flaring.

One of the most puzzling aspects of 28 CMa, and one not reported in any other Be star, is the presence of weak absorption features flanking the He I 6678 line. These features do not seem to vary in velocity, but undergo changes in intensity with the $P_1 = 1.37$ d period. There are at present insufficient data to understand why they exist. It would certainly be interesting if the same phenomenon were to be found in other Be stars. There is a resemblance between these absorption features and the much stronger, stationary, absorption feature reported by Smith (1985) in the non-Be star α Vir. In this star a 'spike', or strong, narrow absorption feature, is first seen on the red wing of the Si III triplet lines. It grows and recedes, only to appear on the blue wing about four hours later. Smith attributes this to a quasi-toroidal mode of high degree. Short persisting spikes and ramps were observed on opposite wings of the He I and Si III lines in μ Cen (Rivinius et al. 1998). The authors argue that they are of photospheric origin. It is not clear how strong the correspondence is to 28 CMa; the impression we have is that the two weak absorption features in 28 CMa are always present, but are less visible as the wings of the He I 6678 strengthen. Confirmation of similar weak absorption features in other spectral lines in 28 CMa are required to remove entirely the possibility that they are an artefact.

The explanation for the transient period P_2 is at present unknown. Again, one can offer a speculation: that it may be the result of an obscuration just above the photosphere. Material flowing from the photosphere will tend to lag behind the rate of stellar rotation owing to loss of angular momentum with height. If there is a condensation of such material above the photosphere, it may give rise to the observed effects with a period somewhat smaller than the period of rotation.

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