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Full Length Research Paper

Simultaneous robot-world and hand-eye calibration using dual-quaternions and Kronecker product

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Two solutions of the homogeneous matrix equation AX=ZB, that allows a simultaneous computation of the transformations from robot world to robot base and from robot tool to robot wrist coordinate frames, are proposed. The presented methods introduce the Kronecker product and dual quaternions to solve the rotations and translations problem, simultaneously, with no propagation error. The experimental results in a simulated and a real environment are reported and analyzed.

Key words: Hand/eye calibration; robot/world calibration; dual quaternion; Kronecker product.

INTRODUCTION

A hand-eye configuration refers to the setup in which a camera is rigidly mounted on the hand of a robot manipulator. This configuration has been found many applications in industrial field such as electronic assembly line (Zhuang, 1998), welding guidance (Chen et al., 2006) and robot-based measurement (Li et al., 2009). In order to use a hand-mounted camera for guiding a robot to execute a task, the hand-eye calibration must be done in advance. Hand-eye calibration is the process of computing the position and orientation of the sensor coordinate system with respect to the robot hand coordinate system. The hand-eye calibration problem was usually formulated as solving homogeneous transformation equations of the forms $AX = XB$ (Shiu and Ahmad, 1989), where X is the 4×4 homogeneous transformation from the robot hand coordinate frame to the sensor coordinate frame, A is the measurable 4×4 homogeneous transformation of the robot hand from its first to second position, and B is the measurable 4×4 homogeneous transformation of the sensor and also, from its first to second position. A number of approaches have been devised for robot hand-eye calibration. These approaches can be divided into two categories. In the first category, the problem is solved by a two stage linear method (Shiu and Ahmad, 1989; Tsai and Lenz, 1989; Wang, 1992; Park and Martin, 1994). More specifically, the unknown rotation of the camera frame with respect to the hand frame is solved in the first stage, using a number of relative robot hand motions and the induced relative camera motions. The obtained rotation together with the measured quantities is used to solve for the unknown position of the camera frame with respect to the robot hand frame. The main drawback in these methods is that rotation estimation errors propagate to position estimation errors. In the second category, a nonlinear least squares procedure (Horaud and Dornaika, 1995) and dual quaternions (Daniilidis, 1999) are applied to solve simultaneously, for both the rotation and position of the camera frame with respect to the robot hand frame.

There has been another homogeneous transformation equation $AX=ZB$ derived by Zhuang et al. (1994), allowing the simultaneous estimation of both the transformations from the world frame to the robot-base frame and from the robot hand frame to sensor frame. Where A is the known homogeneous transformation from hand pose measurements, B is computed using the calibrated manipulator internal-link forward kinematics, X is the unknown transformation from the robot hand frame to sensor frame, and Z is the unknown transformation from the world frame to the robot-base frame. There are two categories for solving equation $AX=ZB$. One is to solve the problem using a two stage quaternion method

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(Zhuang et al., 1994). The unknown rotations associated with *X* and *Z* are firstly determined and then the translations associated with *X* and *Z* are found using the obtained rotations. Consequently, there has been propagation error in quaternion method. Additionally, the quaternion method requires the rotational angle associated with matrix *A* not to equal 180 degrees. The other method is to apply nonlinear minimization to compute both the rotations and positions associated X and Z, simultaneously (Dornaika, 1998).

This paper concentrates on solving *AX=ZB* and to describes two solutions to *X* and *Z*. Our robot is used for noncontact freedom surface measurement, which is a reaching movement. Although our methods do not belong to nonlinear minimization technology, they solve for two rotations and two translations, simultaneously and the propagation errors are eliminated maturely. The remainder of the paper is organized as follows: the two methods using dual-quaternion and Kronecker product to solve the equation *AX=ZB* are firstly stated. Next, simulation results of a MOTOMAN robot are presented to demonstrate the capabilities of the calibration technique. Finally, some conclusions are given.

SOLUTION TO *AX=ZB* USING DUAL QUATERNIONS

Dual quaternions and line transformations

This section outlines briefly the dual quaternions. First quaternions are explained, followed by a short description of dual numbers. Finally, the dual quaternions and their relevant properties are introduced.

Quaternions

It was Introduced by Hamilton, quaternions are an extension of the complex numbers to R^4. One definition of quaternions is as pairs $(q_0, \vec{q})$, where $q_0 \in R$ is called scalar part and $\vec{q} \in R^3$ is called vector part. The following operations exist:

$$\mathbf{a}+\mathbf{b}=(a_0+b_0)+(\vec{a}+\vec{b}) \quad (1)$$

$$\lambda\mathbf{a}=\lambda a_0+\lambda\vec{a} \quad (2)$$

$$\mathbf{a}\cdot\mathbf{b}=(a_0b_0-\vec{a}^{\mathrm{T}}\vec{b})+(a_0\vec{b}+b_0\vec{a}+\vec{a}\times\vec{b}) \quad (3)$$

The norm of a quaternion is defined as $\|\mathbf{a}\|=\mathbf{a}\overline{\mathbf{a}}$, where $\overline{\mathbf{a}}$ is the conjugate quaternion $(a_0, -\vec{a})$. A unit quaternion is a quaternion of norm one. For every rotation in R^3 about an axis $\vec{n}$ ($\|n\|=1$) with an angle *θ*, a corresponding unit quaternion $\mathbf{q}=(\cos\frac{\theta}{2}, \sin\frac{\theta}{2}\vec{n})$ exists that maps a vector $\vec{x}\in R^3$ to the vector $\mathbf{q}(0,\vec{x})\overline{\mathbf{q}}$.

Dual numbers

A dual number is defined as:

$\hat{a}=a+\varepsilon a'$ With $\varepsilon\neq 0$ and $\varepsilon^2=0$

where a is called real part and a' is called dual part. The following operations exist:

$$\hat{a}+\hat{b}=(a+b)+\varepsilon(a'+b') \quad (4)$$

$$\hat{a}\cdot\hat{b}=ab+\varepsilon(ab'+a'b) \quad (5)$$

Dual quaternions

Dual-quaternions are defined as $\hat{q}=(\hat{s},\hat{\vec{q}})$, where $\hat{s}$ is a dual number and $\hat{\vec{q}}$ is a dual vector.
The operations have the definitions:

$$\hat{a}+\hat{b}=(\hat{a}_0+\hat{b}_0)+(\hat{\vec{a}}+\hat{\vec{b}}) \quad (6)$$

$$\lambda\hat{a}=\lambda\hat{a}_0+\lambda\hat{\vec{a}} \quad (7)$$

$$\hat{a}\hat{b}=(\hat{a}_0\hat{b}_0-\hat{\vec{a}}^{\mathrm{T}}\hat{\vec{b}})+(\hat{a}_0\hat{\vec{b}}+b_0\hat{\vec{a}}+\hat{\vec{a}}\times\hat{\vec{b}}) \quad (8)$$

The norm of a dual quaternion is defined as $\|\hat{q}\|=\hat{q}\overline{\hat{q}}$, and is a dual number with a positive real part. If the norm is equal to one; the dual-quaternion is named as a unit dual-quaternion.

Line transformations with unit dual-quaternions

A line in space with direction $\vec{l}$ through a point $\vec{p}$ can be represented with the six-tuple $(\vec{l},\vec{m})$, where $\vec{m}$ is called the line moment and is equal to $\vec{p}\times\vec{l}$. The line moment is normal to the plane through the line and the origin, with magnitude equal to the distance from the line to the origin. The constraints $\|\vec{l}\|=1$ and $\vec{l}^{\mathrm{T}}\vec{m}=0$ guarantee that the degrees of freedom of an arbitrary line in space are four.

Problem: A line given by its dual quaternion $\hat{l}_a=\vec{l}_a+\varepsilon m_a$ is transformed with $(R,\vec{t})$ into a line $\hat{l}_b=\vec{l}_b+\varepsilon m_a$. Show that a unit dual quaternion exists such that $\hat{l}_b=\hat{q}\hat{l}_a\overline{\hat{q}}$, where $\hat{q}=q+\varepsilon q'$, q is a unit quaternion associated to R and

$q' = \frac{1}{2}tq$, $t = [0 \quad \vec{t}]^{\mathrm{T}}$. Applying a rotation R and a translation $\vec{t}$ to a line $(\vec{l}_b, \vec{m}_b)$, we obtain the transformed line $(\vec{l}_a, \vec{m}_a)$:

$$\begin{aligned}
\vec{l}_a &= \mathbf{R}\vec{l}_b \\
\vec{m}_a &= \vec{p}_a \times \vec{l}_a = (R\,\vec{p}_b + \vec{t}) \times (R\,\vec{l}_b) \\
&= R(\vec{p}_b \times \vec{l}_b) + \vec{t} \times R\,\vec{l}_b \\
&= R\,\vec{m}_b + \vec{t} \times R\,\vec{l}_b
\end{aligned}$$

The cross-product is tackled with the identity:

$$(0, \vec{t} \times \vec{q}) = \frac{1}{2}(\mathbf{q}\bar{\mathbf{t}} + \mathbf{q}\mathbf{t})$$

where t is the translation quaternion $(0, \vec{t})$, and q is the rotation quaternion. Using the identity, we obtain:

$$\begin{aligned}
\vec{l}_a &= q\vec{l}_b\bar{q} \\
\vec{m}_a &= q\,\vec{m}_b\bar{q} + \frac{1}{2}(ql_b\bar{q}\bar{t} + tql_b\bar{q})
\end{aligned}$$

We define a new quaternion $q' = \frac{1}{2}qt$ and a dual quaternion.

Derivation of *X* and *Z*

Let $\hat{q}_A, \hat{q}_X, \hat{q}_Z, \hat{q}_B$ be the unit dual-quaternions associated with A, X, Z, and B, respectively. Equation AX=ZB can be rewritten as:

$$\hat{q}_A\hat{q}_X = \hat{q}_Z\hat{q}_B \tag{9}$$

Using $\hat{q}_A = a + \varepsilon a'$, $\hat{q}_B = b + \varepsilon b'$, $\hat{q}_X = x + \varepsilon x'$ in (9):

$$\begin{aligned}
ax &= zb \\
a'x + ax' &= zb' + z'b
\end{aligned} \tag{10}$$

Describing Equation (10) in matrix equation form, we have:

$$\begin{aligned}
\tilde{a}x - \underset{\sim}{b}z &= 0 \\
\tilde{a}'x + \tilde{a}x' - \underset{\sim}{b}'z - \underset{\sim}{b}z' &= 0
\end{aligned} \tag{11}$$

Where

$$\tilde{a} = \begin{bmatrix} a_0 & -\vec{a}^{\mathrm{T}} \\ \vec{a} & a_0\mathrm{I} + \Omega(\vec{a}) \end{bmatrix} \quad \underset{\sim}{b} = \begin{bmatrix} b_0 & -\vec{b}^{\mathrm{T}} \\ \vec{b} & b_0\mathrm{I} - \Omega(\vec{b}) \end{bmatrix}$$

and for vector $v=[v_1,v_2,v_3]^{\mathrm{T}}$, its anti-symmetric matrix:

$$\Omega(\mathrm{v}) = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix}$$

Furthermore:

$$\begin{bmatrix} \tilde{a} & 0_{4\times4} & \underset{\sim}{b} & 0_{4\times4} \\ \tilde{a}' & \tilde{a} & \underset{\sim}{b}' & \underset{\sim}{b} \end{bmatrix} \begin{bmatrix} x \\ x' \\ z \\ z' \end{bmatrix} = 0 \tag{12}$$

Supposed there have been $\mathrm{n}(\mathrm{n} \geq 3)$ pose measurements $s_i = \begin{bmatrix} \tilde{a}_i & 0_{4\times4} & \underset{\sim}{b}_i & 0_{4\times4} \\ \tilde{a}_i' & \tilde{a}_i & \underset{\sim}{b}_i' & \underset{\sim}{b}_i \end{bmatrix}$, an 8n × 16 matrix $T = [s_1, s_2 \cdots, s_n]^T$ is constructed. Let $T = U\sum V^{\mathrm{T}}$ to be singular value decomposition. The diagonal entries of Σ are necessarily equal to the singular values of *T*. The columns of *U* and *V* are, respectively, left- and right-singular vectors for the corresponding singular values.

Suppose two column vectors responding to zero singular value are v_1 and v_2 in right-singular vectors matrix:

Let $v_1^{\mathrm{T}} = [\alpha_1^{\mathrm{T}}, \beta_1^{\mathrm{T}}, \omega_1^{\mathrm{T}}, \mu_1^{\mathrm{T}}]^{\mathrm{T}}$ and $v_2^{\mathrm{T}} = [\alpha_2^{\mathrm{T}}, \beta_2^{\mathrm{T}}, \omega_2^{\mathrm{T}}, \mu_2^{\mathrm{T}}]^{\mathrm{T}}$, the solutions of equation $[s_1, s_{2,} \cdots, s_n]^{\mathrm{T}}[x \quad x' \quad z \quad z']^{\mathrm{T}} = 0$ are:

$$x = \lambda_1\alpha_1 + \lambda_2\alpha_2 \tag{13a}$$

$$x' = \lambda_1\beta_1 + \lambda_2\beta_2 \tag{13b}$$

$$z = \lambda_1\omega_1 + \lambda_2\omega_2 \tag{13c}$$

$$z' = \lambda_1\mu_1 + \lambda_2\mu_2 \tag{13d}$$

Since $\hat{q}_X$ and $\hat{q}_Z$ are unit dual-quaternions, vectors x , x' z and z' also satisfies the following constraints:

$$\lambda_1^2\alpha_1^{\mathrm{T}}\alpha_1 + \lambda_2^2\alpha_2^{\mathrm{T}}\alpha_2 + +2\lambda_1\lambda_2\alpha_1^{\mathrm{T}}\alpha_2 = 1 \tag{14a}$$

$$\lambda_1^{\ 2}\alpha_1^{\ \mathrm{T}}\beta_1 + \lambda_2^{\ 2}\alpha_2^{\ \mathrm{T}}\beta_2 + \lambda_1\lambda_2(\alpha_1^{\ \mathrm{T}}\beta_2 + \alpha_2^{\ \mathrm{T}}\beta_1) = 1 \tag{14b}$$

$$\lambda_1^2\omega_1^{\mathrm{T}}\omega_1 + \lambda_2^2\omega_2^{\mathrm{T}}\omega_2 + 2\lambda_1\lambda_2\omega_1^{\mathrm{T}}\omega_2 = 1 \tag{14c}$$

$$\lambda_1^2\omega_1^{\mathrm{T}}\mu_1 + \lambda_2^2\omega_2^{\mathrm{T}}\mu_2 + \lambda_1\lambda_2(\omega_1^{\mathrm{T}}\mu_2 + \omega_2^{\mathrm{T}}\mu_1) = 0 \tag{14d}$$

The procedure for solving *X* and *Z* is thus summarized.

Step 1

Construct matrix *T* using (a_i, a_i') and (b_i, b_i'), which are found from A_i and B_i, i =1…n.

Step 2

Employ singular value decomposition on matrix *T* and determine two column vectors responding to zero singular value to be v_1 and v_2 in right-singular vectors matrix.

Step 3

Compute λ_1 and λ_2 using Equation (14a) - (14d) and calculate x, x', z and z' using Equation (13a) - (13d).

Step 4

Recover rotational matrixes R_x and R_z and translational vectors t_x and t_z.

SOLUTION TO *AX=ZB* USING KRONECKER PRODUCT

Definition 1

The stack operator maps an *n* by *m* matrix into an *nm* by 1 vector. The stack of the *n* by *m* matrix A, denoted as *vec*(A), is the vector formed by stacking the columns of A into an *nm* by 1 vector:

$$vec(A) = [a_{11}, a_{12}, \cdots, a_{1n}, \cdots a_{mn}]^{\mathrm{T}}$$

Definition 2

The Kronecker product is a binary matrix operator that maps two arbitrarily dimensioned matrices into a larger matrix with special block structure. Given the *n* by *m* matrix A and the *p* by *q* matrix B, their Kronecker product, denoted as $A \otimes B$, is the *np* by *mq* matrix with the block structure:

$$A \otimes B = \left[a_{ij} B\right] = \begin{pmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{pmatrix}$$

Extend the equation *AX=ZB*, we have:

$$R_A R_X = R_B R_Z \tag{15}$$

$$R_A t_X + t_A = R_Z t_B + t_Z \tag{16}$$

Using stack operator in Equation (15) and (16), it gives:

$$\begin{bmatrix} R_A \otimes I_3 & -I_3 \otimes R_B^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} vec(R_X) \\ vec(R_Z) \end{bmatrix} = 0 \tag{17}$$

$$\begin{bmatrix} I_3 \otimes t_B^{\mathrm{T}} & -R_A & I_3 \end{bmatrix} \begin{bmatrix} vec(R_X) \\ t_X \\ t_Z \end{bmatrix} = t_A \tag{18}$$

where I_3 is a 3 by 3 identity matrix, let $vec(R_X) = x$ and $vec(R_Z) = z$, Equation (15) and (16) are simplified as:

$$\begin{bmatrix} R_A \otimes I_3 & -I_3 \otimes R_B^{\mathrm{T}} & 0 & 0 \\ 0 & I_3 \otimes t_B^{\mathrm{T}} & -R_A & I_3 \end{bmatrix} \begin{bmatrix} x \\ z \\ t_X \\ t_Z \end{bmatrix} = \begin{bmatrix} 0 \\ t_A \end{bmatrix} \tag{19}$$

The solutions to *x*, *z*, t_x and t_z can be determined by least square technique. The rotation part *R*x and *R*z constructed by *x* and *z* may not be an orthogonal matrix due to noise, and a Rodrigues' rotation formula can be utilized to make *R***x** and *R*z orthogonal.

SIMULATION AND EXPERIMENTAL RESULTS

Simulation results

We study the estimation errors of the hand-eye transformation and the robot-to-world transformation against number of pose measurements for "X" number of poses.

The proposed algorithms are tested by numerical simulation. Various samples of *X*, *Z* and A_i are generated from, in which B_i can be computed. The following procedures demonstrate the numerical simulation:

1) Nominal values for the parameters of both the hand-eye transformation *X* and the robot-to-world transformation *Z* are provided.
2) Also, *n* matrices $\boldsymbol{A}_1$; …, $\boldsymbol{A}_n$ are provided, from which *n* hand positions that can be computed with $B_i = Z^1 A_i X$ are also provided.
3) Uniform distribution noise is added to both camera and robot positions, the homogeneous transformations, *X* and *Z*, are estimated in the presence of this noise using the two methods described in this paper and the quaternion method proposed by Zhuang.
4) We study the estimation errors of the hand-eye transformation and the robot-to-world transformation against number of pose measurements.

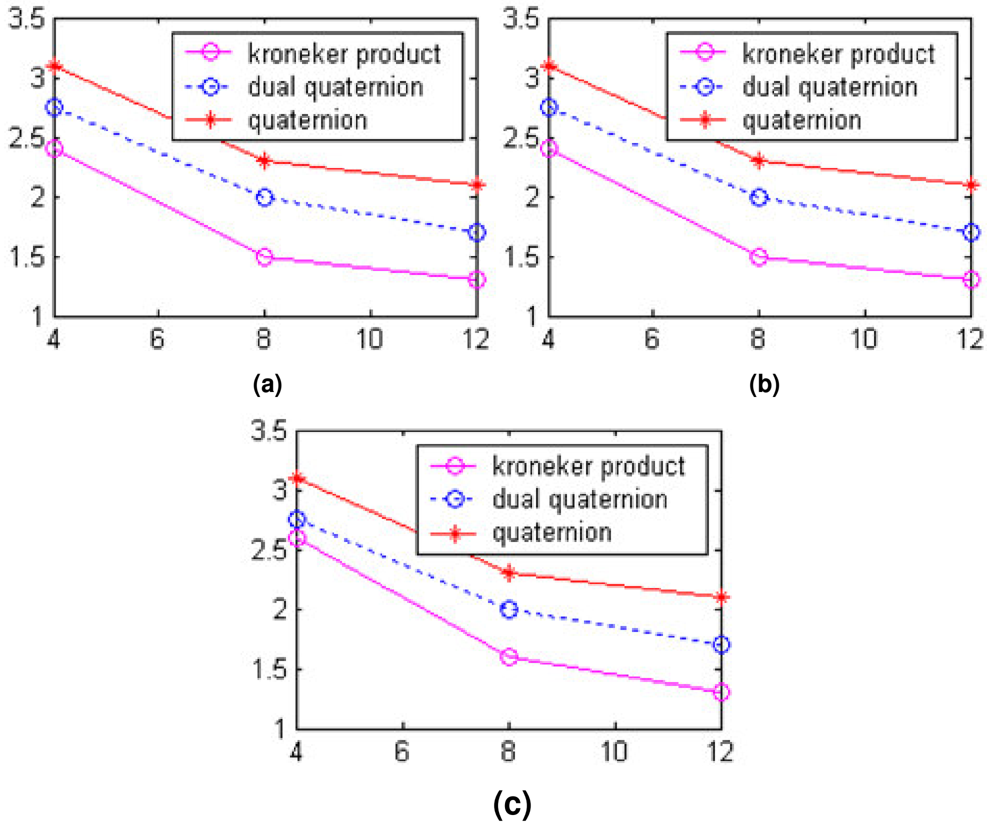

Figure 1. Errors against number of pose measurements. (a) Only matrix *A* has noise; (b) Only matrix *B* has noise; (c) Both *A* and *B* have noise.

The following conditions are simulated:

1) Only matrix *A* has noise.
2) Only matrix *B* has noise.
3) Both *A* and *B* have noise.

Since both rotations and translations may be represented as vectors, adding noise to a transformation consists of adding random numbers to each one of the vectors' components. Uniformly, distributed random noise processes for translation, U(-1, 1) mm, and for rotation, U(-0.5, 0.5) degrees, were added to the *Z-Y-Z* Euler angles and positions of *A* and *B*. Let σ_1 denote the norm of the difference between the computed *X* and theoretical *X*, σ_2 denote the norm of the difference between the computed Z and theoretical *Z*. The total error is defined as the sum of $\sigma_1$and σ_2.

Figure 1 shows the rotation and translation errors as a function of number of pose measurements with uniform noise added to the rotational part of the robot and camera positions. For each pose, the simulations are repeated 10 times and the average errors are obtained with the three aforementioned methods. Data were analyzed applying a Monte Carlo simulation.

The results show that the use of the Kronecker product approach delivers more accurate solutions than dual-quaternion method (as shown in Figure 1). Therefore, we can conclude that the methods proposed in this paper are more accurate than the linear quaternion method.

Experimental results

In this Section, we report some experimental results obtained with MOTOMAN HP3 robot (Figure 2). The data set was obtained with 7 different positions of the hand-eye device with respect to a planar circular-marked calibrating object with 15 mm radius. The camera calibration was implemented by using Tsai method (Tsai, 1987) and the extrinsic parameter matrix B is obtained by Using some conclusion described in Zhang's research (Zhang, 2000).

Our tests compare the linear quaternion method with the two methods developed in this paper. Table 1 summarizes the results, where the second column shows the sum of squares of the absolute error in rotation and

.

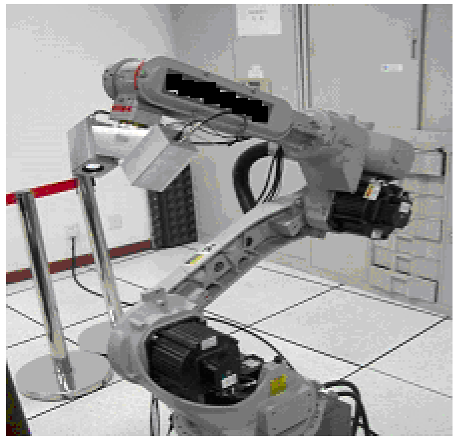

Figure 2. Hardware setup.

Table 1. The formulation AX = ZB USED with 7 different positions. These data were obtained using a HP3 robot.

	E
Quaternion method	3.131
Dual-quaternion method	3.147
Kronecker product method	2.933

translation:

$$E = \sum \left\| A_i X - ZB_i \right\|^2$$

The total error in the quaternion method is only slightly smaller than the error in the dual quaternion method. Importantly, the total error in the Kronecker product method is significantly smaller than the error in the quaternion and the dual quaternion method. These errors do not completely obey the simulation results, because the robot's kinematic chain is not perfectly calibrated and therefore, there are errors associated with the robot's translation parameters.

Conclusions

In this paper, we addressed the problem of robot-to-world and hand-eye calibration by solving homogeneous transformation equations of the form *AX* = *ZB*. We developed two methods. One uses dual-quaternions to solve *AX* = *ZB*. This method can determine the rotational and translational parts of matrix *X* and *Z*, since dual-quaternions can describe the rotation and translation in unified form. The other uses stack operator on sizes of *AX=ZB*, firstly and then Kronecker product is utilized for solving the linear equations. These two methods solve simultaneously for two rotations and two translations and the propagation errors are eliminated significantly compared with to the linear quaternion methods.

In order to verify the performance of the proposed methods, we performed both simulations and real experiments with the MOTOMAN HP3 robot.

The simulation shows that the results obtained with the Kronecker product method are significantly more accurate than the dual quaternion method. This is due to the fact that the residual error is inevitable when solving the four nonlinear equations in the dual-quaternions. Remarkably, the results obtained with both the Kronecker product method and the dual quaternion methods are more accurate than those obtained with the quaternion method. The data from the real experiment on MOTOMAN HP3 robot mostly agree with the simulation. However, the differences between the simulation and the real experiment results do not obey the simulation results completely because there are errors associated with the robot's translation parameters.

The two methods proposed in this paper may be useful in solving other problems that can be formulated into homogeneous transformation equations of the form *AX* = *ZB*. The future work should focus on using the transformation chain *AX=ZB* in order to calibrate robot kinematic parameters, which could cause errors associated with the robot hand translation parameters described by matrix *A*.

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