Simultaneous state and unknown inputs estimation with PI and PMI observers for Takagi Sugeno model with unmeasurable premise variables

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17th Mediterranean Conference on Control and Automation MED'09
June 24-26, 2009, Makedonia Palace, Thessaloniki, Greece

Objective

Estimate simultanuously the state and unknown inputs of nonlinear systems described by a Takagi-Sugeno model using a Proportional Integral Observer (PIO) and Proportional Multiple Integral Observer (PMIO)

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Difficulties

Unmeasurable premise variables

Outline

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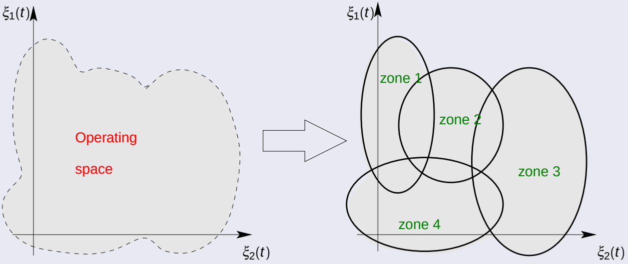
1. Takagi-Sugeno approach for modeling
   - Takagi-Sugeno principle
   - Takagi-Sugeno model
2. Proportional Integral Observer design
3. State and unknown input estimation
4. Numerical example
5. Conclusions

Outline

Takagi-Sugeno approach for modeling

Takagi-Sugeno principle

- Operating range decomposition in several local zones.
- A local model represent the behavior of the system in each zone.
- The overall behavior of the system is obtained by the aggregation of the sub-models with adequate weighting functions.



Nonlinear system　　Multiple model representation

Interests of Takagi-Sugeno approach

- Simple structure for modeling complex nonlinear systems.
- The specific study of the nonlinearities is not required.
- Possible extension of the theoretical LTI tools for nonlinear systems.

The considered system is given by the following equations

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(C_i x(t) + D_i u(t)) \end{cases}$$

- Interpolation mechanism : $\sum_{i=1}^{r} \mu_i(\xi(t)) = 1$ and $0 \leq \mu_i(\xi(t)) \leq 1, \forall t, \forall i \in \{1, ..., r\}$
- The premise variable $\xi(t)$ can be measurable (input $u(t)$,output $y(t)$,...) or unmeasurable (state $x(t)$,...).

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- Exact representation of the model $\dot{x}(t) = f(x(t), u(t))$.

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- Exact representation of the model $\dot{x}(t) = f(x(t), u(t))$.
- Only one T-S model for FDI with observer banks.
- Exemple of application: security improvement in cryptography systems.

Proportional Integral Observer design

Let us consider the T-S system represented by

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\left(A_i x(t) + B_i u(t) + E_i d(t) + W_i \omega(t)\right) \\ y(t) = Cx(t) + Gd(t) + W\omega(t) \end{cases}$$

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Assumption 1

Assume that the following assumptions hold:

- **A1.** The system is stable
- **A2.** The signals $u(t)$, $d(t)$ and $\omega(t)$ are bounded.

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Assume that the following assumptions hold:

- **A1.** The system is stable
- **A2.** The signals $u(t)$, $d(t)$ and $\omega(t)$ are bounded.

Assumption 2

The unknown inputs $d(t)$ are assumed to be constant:

- **A3.** $\dot{d} = 0$

(This assumption will be relaxed in the PMIO design).

Let us denote the estimated state by $\hat{x}$.

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The system can be re-written as follows:

$$\dot{x} = \sum_{i=1}^{r} \mu_i(\hat{x})(A_i x + B_i u + E_i d + W_i \omega + v)$$

where:

$$v = \sum_{i=1}^{r} (\mu_i(x) - \mu_i(\hat{x}))(A_i x + B_i u + E_i d + W_i \omega)$$

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The assumption **A3.** allows to write the system in the following augmented form:

$$\begin{cases} \dot{x}_a = \sum_{i=1}^{r} \mu_i(\hat{x}) \left(\tilde{A}_i x_a + \tilde{B}_i u + \tilde{\Gamma}_i \tilde{\omega}\right) \\ y = \tilde{C} x_a + \tilde{D} \tilde{\omega} \end{cases}$$

where:

$$\tilde{A}_i = \begin{bmatrix} A_i & E_i \\ 0 & 0 \end{bmatrix}, \quad \tilde{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \quad \tilde{\Gamma}_i = \begin{bmatrix} I & W_i \\ 0 & 0 \end{bmatrix}, \quad \tilde{\omega} = \begin{bmatrix} v \\ \omega \end{bmatrix}$$

$$\tilde{C} = \begin{bmatrix} C & G \end{bmatrix}, \quad \tilde{D} = \begin{bmatrix} 0 & W \end{bmatrix}, \quad x_a = \begin{bmatrix} x \\ d \end{bmatrix}$$

The proposed PI observer is given by the following equations:

$$\begin{cases} \dot{\hat{x}} = \sum_{i=1}^{r} \mu_i(\hat{x}) \left(A_i \hat{x} + B_i u + E_i \hat{d} + K_{Pi}(y - \hat{y}) \right) \\ \hat{y} = C\hat{x} + G\hat{d} \\ \dot{\hat{d}} = \sum_{i=1}^{r} \mu_i(\hat{x}) K_{Ii}(y - \hat{y}) \end{cases}$$

where $\hat{x}$ and $\hat{d}$ are the estimates of x and d.

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A similar reasoning makes it possible to transform the proposed PI observer in the following augmented form:

$$\begin{cases} \dot{\hat{x}}_a = \sum_{i=1}^{r} \mu_i(\hat{x}) \left(\tilde{A}_i\hat{x}_a + \tilde{B}_i u + \tilde{K}_i(y-\hat{y}) \right) \\ \hat{y} = \tilde{C}\hat{x}_a \end{cases}$$

where:

$$\tilde{K}_i = \begin{bmatrix} K_{Pi} \\ K_{Ii} \end{bmatrix}$$

Augmented state estimation error

$$e_a(t) = x_a(t) - \hat{x}_a(t)$$

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Objectives

The goal is to determine the gain matrices $\tilde{K}_i$ such that:

- The system which generates the state and unknown inputs errors is stable.
- $$\lim_{t \to \infty} e(t) = 0, \quad \text{pour} \quad \|\tilde{\omega}\|_2 = 0$$
  $$\|e(t)\|_2 < \gamma \|\tilde{\omega}\|_2 \quad \text{pour} \quad \|\tilde{\omega}\|_2 \neq 0$$

Lemma 1 [Tanaka and Wang 2001]

Consider the continuous-time TS-system defined by:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t))(A_i x(t) + B_i u(t)) \\ y(t) = Cx(t) \end{cases}$$

The system is stable and verifies the $\mathscr{L}_2$-gain condition:

$$||y(t)||_2 < \gamma ||u(t)||_2$$

if there exists a symmetric positive definite matrix P such that the following LMIs are satisfied for $i = 1, ..., r$:

$$\begin{bmatrix} A_i^T P + PA_i + C^T C & PB_i \\ B_i^T P & -\gamma^2 I \end{bmatrix} < 0$$

Theorem 1: PIO

The PI observer for the system is determined by minimizing $\bar{\gamma}$ under the following LMI constraints in the variables $P = P^T > 0$, M_i and $\bar{\gamma}$ for $i = 1, ..., r$:

$$\begin{bmatrix} \tilde{A}_i^T P + P\tilde{A}_i - M_i\tilde{C} - \tilde{C}^T M_i{}^T + I & P\tilde{\Gamma}_i - M_i\tilde{D} \\ \tilde{\Gamma}_i^T P - \tilde{D}^T M_i{}^T & -\bar{\gamma} I \end{bmatrix} < 0$$

The gains of the observer are derived from:

$$\tilde{K}_i = P^{-1} M_i$$

and the attenuation level is calculated by:

$$\gamma = \sqrt{\bar{\gamma}}$$

Proportional Multiple Integral Observer design

Let us consider the T-S system represented by

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\left(A_i x(t) + B_i u(t) + E_i d(t) + W_i \omega(t)\right) \\ y(t) = Cx(t) + Gd(t) + W\omega(t) \end{cases}$$

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$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\,(A_i x(t) + B_i u(t) + E_i d(t) + W_i \omega(t)) \\ y(t) = Cx(t) + Gd(t) + W\omega(t) \end{cases}$$

Assumption 3

The unknown input is assumed to be a bounded time varying signal with null q^{th} derivative:

- **A4.** $d^{(q)}(t) = 0$

Consider the generalization of the proportional multiple-integral observer to T-S systems of the PMI observer:

$$\begin{cases} \dot{\hat{x}} = \sum_{i=1}^{r} \mu_i(\hat{x})(A_i\hat{x} + B_i u + E_i\hat{d}_0 + K_{Pi}(y-\hat{y})) \\ \hat{y} = C\hat{x} + G\hat{d} \\ \dot{\hat{d}}_0 = \sum_{i=1}^{r} \mu_i(\hat{x})K_{Ii}^{0}(y-\hat{y}) + \hat{d}_1 \\ \dot{\hat{d}}_1 = \sum_{i=1}^{r} \mu_i(\hat{x})K_{Ii}^{1}(y-\hat{y}) + \hat{d}_2 \\ \vdots \\ \dot{\hat{d}}_{q-2} = \sum_{i=1}^{r} \mu_i(\hat{x})K_{Ii}^{q-2}(y-\hat{y}) + \hat{d}_{q-1} \\ \dot{\hat{d}}_{q-1} = \sum_{i=1}^{r} \mu_i(\hat{x})K_{Ii}^{q-1}(y-\hat{y}) \end{cases}$$

where $\hat{d}_i,\ i = 1,2,...,(q-1)$ are the estimation of the $(q-1)$ first derivatives of the unknown input $d(t)$.

The state and unknown inputs estimation errors are:

$$e = x - \hat{x}, \quad e_0 = \dot{d} - \dot{\hat{d}}_0, \ \ldots, \quad e_{q-1} = \dot{d}_{q-1} - \dot{\hat{d}}_{q-1}$$

Their dynamics are given in the following form:

$$\begin{cases} \dot{e} = \sum_{i=1}^{r} \mu_i(\hat{x})((A_i - K_{Pi}C)e + (\Gamma_i - K_{Pi}\bar{W})\tilde{\omega} + (E_i - K_{Pi}G)e_0) \\ \dot{e}_0 = \sum_{i=1}^{r} \mu_i(\hat{x})(-K_{Ii}^0 Ce + e_1 - K_{Ii}^0 \bar{W}\tilde{\omega} - K_{Ii}^0 Ge_0) \\ \dot{e}_1 = \sum_{i=1}^{r} \mu_i(\hat{x})(-K_{Ii}^1 Ce + e_2 - K_{Ii}^1 \bar{W}\tilde{\omega} - K_{Ii}^1 Ge_0) \\ \vdots \\ \dot{e}_{q-2} = \sum_{i=1}^{r} \mu_i(\hat{x})(-K_{Ii}^0 Ce + e_{q-1} - K_{Ii}^{q-2} \bar{W}\tilde{\omega} - K_{Ii}^{q-2} Ge_0) \\ \dot{e}_{q-1} = \sum_{i=1}^{r} \mu_i(\hat{x})(-K_{Ii}^{q-1} Ce - K_{Ii}^0 \bar{W}\tilde{\omega} - K_{Ii}^{q-1} Ge_0) \end{cases}$$

where:

$$\Gamma_i = \begin{bmatrix} I_n & W_i \end{bmatrix}, \quad \bar{W} = \begin{bmatrix} 0 & W \end{bmatrix}$$

In the augmented form, we have:

$$\begin{aligned}\dot{\tilde{e}} &= \sum_{i=1}^{r} \mu_i(\hat{x})((\tilde{A}_i - \tilde{K}_i\tilde{C})\tilde{e} + (\tilde{\Gamma}_i - \tilde{K}_i\bar{W})\tilde{\omega}) \\ \begin{bmatrix} e \\ e_0 \end{bmatrix} &= \bar{C}\tilde{e}\end{aligned}$$

where:

$$\tilde{e} = \begin{bmatrix} e \\ e_0 \\ e_1 \\ \vdots \\ e_{q-2} \\ e_{q-1} \end{bmatrix}, \quad \tilde{A}_i = \begin{bmatrix} A_i & E_i & 0 & \cdots & 0 & 0 \\ 0 & 0 & I_s & \cdots & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & I_s \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\tilde{K}_i = \left[K_{Pi}^T \;\; K_{Ii}^{0\,T} \;\; K_{Ii}^{1\,T} \;\; \ldots \;\; K_{Ii}^{q-2\,T} \;\; K_{Ii}^{q-1\,T}\right]^T$$

$$\tilde{C} = \begin{bmatrix} C & G & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$\tilde{\Gamma}_i = \begin{bmatrix} \Gamma_i^T & 0 & \ldots & 0 \end{bmatrix}^T$$

In the following, we are only interested with particular components e and e_0 of $\tilde{e}$:

$$\begin{bmatrix} e \\ e_0 \end{bmatrix} = \bar{C}\tilde{e}$$

where:

$$\bar{C} = \left[\begin{array}{cc|ccc} I_n & 0 & 0 & \cdots & 0 \\ 0 & I_s & & & \end{array}\right]$$

0 represents null matrix with appropriate dimensions.

Theorem 2

The PMI observer for the system that minimizes the transfer from $\tilde{\omega}(t)$ to $[e(t)^T \;\; e_0(t)^T]$ is obtained by finding the matrices $P = P^T > 0$, M_i and $\bar{\gamma}$ that minimize $\bar{\gamma}$ under the following LMI constraints for $i = 1, \ldots, r$:

$$\begin{bmatrix} \tilde{A}_i^T P + P\tilde{A}_i - M_i\tilde{C} - \tilde{C}^T M_i^T + \bar{C}^T\bar{C} & P\tilde{\Gamma}_i - M_i\bar{W} \\ \tilde{\Gamma}_i^T P - \bar{W}^T M_i^T & -\bar{\gamma} I \end{bmatrix} < 0$$

The gains of the observer are derived from:

$$\tilde{K}_i = P^{-1} M_i$$

and the attenuation level is calculated by:

$$\gamma = \sqrt{\bar{\gamma}}$$

Numerical example

Consider a continuous-time T-S system defined by:

$$A_1 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 1 & -8 \end{bmatrix}, \; A_2 = \begin{bmatrix} -3 & 2 & -2 \\ 5 & -3 & 0 \\ 1 & 2 & -4 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 5 \\ 0.5 \end{bmatrix}, \; B_2 = \begin{bmatrix} 3 \\ 1 \\ -7 \end{bmatrix}, \; E_1 = \begin{bmatrix} 0 & 7 \\ 0 & 5 \\ 0 & 2 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 0 & 6 \\ 0 & 3 \\ 0 & 1 \end{bmatrix}, \; W_1 = W_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \; W = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix},$$

and

$$C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \; G = \begin{bmatrix} 5 & 0 \\ 1 & 0 \end{bmatrix}$$

$d(t) = [d_1(t)^T \;\; d_2(t)^T]^T$.

The weighting functions depend on the first component x_1 of the state vector x and are defined as follows:

$$\begin{cases} \mu_1(x) = \frac{1-\tanh(x_1)}{2} \\ \mu_2(x) = 1-\mu_1(x) \end{cases} \quad (1)$$

The weighting functions obtained without perturbations and unknown inputs are shown in figure 1. This figure shows that the system is clearly nonlinear since μ_1 and μ_2 are not constant functions.

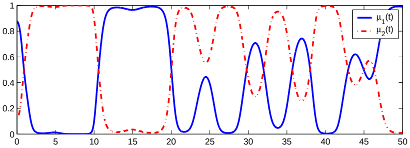

Figure: Weighting functions μ_1 and μ_2

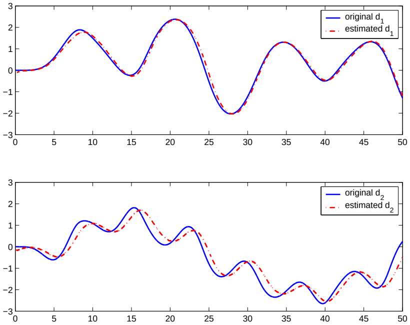

Figure: Unknown input estimation with a PI observer

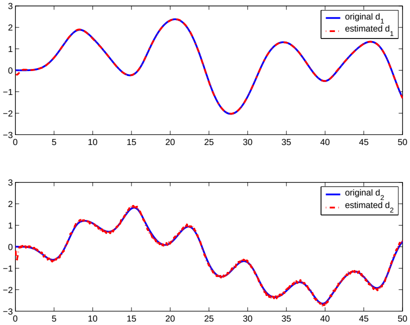

Figure: Unknown input estimation with a PMI observer

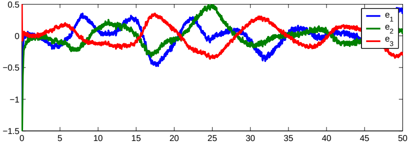

Figure: State estimation error with a PI observer

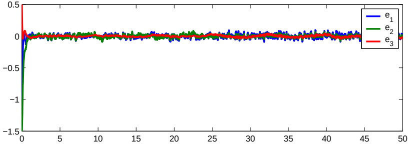

Figure: State estimation error with a PMI observer

Conclusions

- PI and PMI observers design for nonlinear systems represented by a Takagi-Sugeno structure.

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- Study and reduction of the conservatism when searching a common Lyapunov matrix P, which satisfies r LMIs.

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Perspectives

- Study and reduction of the conservatism when searching a common Lyapunov matrix P, which satisfies r LMIs.
- Extension to fault tolerant control of nonlinear systems.

Thank you for your attention!