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Single and Double β Decay in $N \approx Z$ Nuclei

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We study single and double β decay for $N \approx Z$ nuclei, using an extended $P + QQ$ interaction with four force strengths. The Gamov-Teller (GT) transition is examined using the proton-neutron QRPA (pn-QRPA) in many j shells and using a single j shell model calculation. First, we calculate the single β-decay strength. The integrated strengths of $N \approx Z$ nuclei in the fp shell region are reproduced well by the pn-QRPA, although some quenching factor is necessary for several cases. The model accounts for the experimental observation that the GT strength increases with decreasing $|N - Z|$. The single j shell model calculation indicates that the GT strength becomes very large at $N = Z$. Second, we perform calculations for two-neutrino double β-decay transition strengths for ^{76}Ge and ^{82}Se. The results suggest fairly good applicability of the extended $P + QQ$ interaction to single and double β decay.

§1. Introduction

There is presently interest in the theoretical study of unstable nuclei far from the stability line, because of increasing experimental information owing to the development of radioactive ion beam facilities. Single beta (β)- and two-neutrino double-beta ($2\nu\beta\beta$)-decay strengths are fundamental properties to understand the microscopic structure of such exotic nuclei. The Gamov-Teller (GT) transition of proton-rich nuclei is important in studying nuclear astrophysics [1] as well as nuclear structure. [2-6] We believe that p-n interactions play significant roles in the β and $2\nu\beta\beta$ decay of nuclei with $N \approx Z$, because protons and neutrons involved in this decay are in the same orbit and wave functions have large overlap. [7] The β-decay transition of nuclei with $N \approx Z$ is thought to be enhanced by p-n correlations. It has been shown that the calculated ratio of β decay [8-11] to $2\nu\beta\beta$ decay [12-16] is extremely sensitive to the strength of p-n pairing correlations. The resulting halflives are shorter than those obtained from calculations that ignore the isoscalar p-n pairing.

There are mainly two nuclear models which are extensively used in the calculation of β- and $2\nu\beta\beta$-decay processes: the proton-neutron quasiparticle random phase approximation (pn-QRPA) method and the shell model. The pn-QRPA [8,17-19] has been the most popular theoretical tool and has been employed to calculate β- and $2\nu\beta\beta$-decay strengths. The shell model method [20-22] is capable of handling much larger configuration spaces now than a few years ago. Both methods reproduce quite successfully the experimental β- and $2\nu\beta\beta$-decay strengths, though quenching fac-

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tors are necessary.[9),10),23)-25)] The pn-QRPA gives results comparable to the large scale shell model.[27)] Double β decay occurs whenever single β decay is forbidden due to energy conservation or suppressed due to angular momentum mismatch. $2\nu\beta\beta$ decay, which is a process consistent with the Standard Model of the electroweak interaction, has been observed in some cases with lifetimes of 10^{19}–10^{21} years. Various approaches to study $2\nu\beta\beta$ decay have been reviewed recently by Suhonen and Civitarese.[26)] It is pointed out[27)-29)] that the pn-QRPA solutions for realistic force strengths are often close to the collapse point. To avoid this problem of collapse and instability, the renormalized pn-QRPA has recently been investigated.[29)] A large scale shell model calculation free from the difficulty of the pn-QRPA was recently performed for nuclei in the fp shell region.[27)]

We have recently proposed[30)] an interaction extended by introducing isovector and isoscalar p-n forces to the conventional $P+QQ$ force. The extended $P+QQ$ interaction, which is considered to be a simple but good approximation of realistic effective interactions,[31)] accurately reproduces various nuclear properties such as binding energies,[32)] energy levels, and $E2$ transitions.[30),33)] The extended $P+QQ$ model succeeds in describing the significant roles of p-n interactions in determining various nuclear properties of $N \approx Z$ nuclei.[32)] That study showed that our interaction is very useful to determine what types of interactions contribute to nuclear properties under consideration.

In this paper, we apply the extended $P+QQ$ model to the analysis of β and $2\nu\beta\beta$ decay in $N \approx Z$ nuclei in the fpg shell region. One purpose is to test the applicability of our model to β and $2\nu\beta\beta$ decay. Another purpose is to analyze the effects of p-n interactions on β and $2\nu\beta\beta$ decay, since these decay transitions are expected to be good indicators to probe the p-n interactions, as mentioned above. We also investigate β- and $2\nu\beta\beta$-decay strengths in respective isotopes. We use ordinary pn-QRPA, not the renormalized pn-QRPA, because our model does not give rise to the collapse problem, as shown below.

Our paper is organized as follows. In §2, we briefly review the extended $P+QQ$ model. Section 3 presents the pn-QRPA formalism. Section 4 discusses single β decay using the pn-QRPA in many j shells. The GT strength is analyzed using the single j shell model. In §5, $2\nu\beta\beta$ decay is investigated. Section 6 presents conclusions.

§2. Model Hamiltonian

The extended $P+QQ$ interaction we proposed in Ref. 30) is composed of an isospin-invariant pairing plus QQ plus quadrupole pairing (P_2) force and the J-independent isoscalar p-n force $H^{\tau=0}_{\pi\nu}$. The Hamiltonian can be rewritten in the following form by the rearrangement of four nucleon operators in the QQ force $(c^\dagger c c^\dagger c) \to (c^\dagger c^\dagger c c)$:

$$H = H_{sp} + H_{\rm int}, \tag{2·1a}$$
$$H_{sp} = \sum_{\alpha\rho} \epsilon_a c^\dagger_{\alpha\rho} c_{\alpha\rho}, \tag{2·1b}$$

$$H_{\rm int} = \sum_{a\le b}\sum_{c\le d}\sum_{J\tau} V(ab,cd;J\tau)\sum_{M\kappa} A^\dagger_{JM\tau\kappa}(ab)A_{JM\tau\kappa}(cd), \qquad (2\cdot1c)$$

with

$$A^\dagger_{JM\tau\kappa}(ab) = [c^\dagger_a c^\dagger_b]^{\tau\kappa}_{JM}/\sqrt{1+\delta_{ab}}. \qquad (2\cdot1d)$$

The single particle state α in the spherical shell-model basis denotes a set of quantum numbers $\alpha = \{n_a, l_a, j_a, m_\alpha\}$. The symbols JM and $\tau\kappa$ denote the angular momentum and isospin of a nucleon pair, respectively. The interaction matrix element $V(ab,cd;J\tau)$ is given by

$$\begin{aligned} V(ab,cd;J\tau) = &-\frac{1}{2}g_0\delta_{J0}\delta_{\tau1}\delta_{ab}\delta_{cd}\sqrt{(2j_a+1)(2j_c+1)} - \frac{1}{2}g_2\delta_{J2}\delta_{\tau1}q(ab)q(cd) \\ &-5\chi\frac{1}{\sqrt{1+\delta_{ab}}}\frac{1}{\sqrt{1+\delta_{cd}}}\{q(ac)q(db)(-1)^{j_a+j_b-J}W(j_aj_cj_dj_b:2J) \\ &+(c\leftrightarrow d)\} - k^0\delta_{\tau0}\delta_{ac}\delta_{bd}. \end{aligned} \qquad (2\cdot2)$$

Here $p_0(ab)=\sqrt{(2j_a+1)}\delta_{ab}$ and $p_2(ac)=q(ac)/b_0^2=(a\|r^2Y_2/b_0^2\|c)/\sqrt{5}$, where b_0 is the harmonic-oscillator range parameter. Our interaction has only four force parameters, g_0, χ, g_2 and k^0, which are, respectively, the strengths of the pairing, QQ, quadrupole pairing, and J-independent isoscalar p-n forces.

As shown in Ref. 30), the J-independent isoscalar p-n force $H^{\tau=0}_{\pi\nu}$ is reduced to

$$\begin{aligned} H^{\tau=0}_{\pi\nu} &\equiv -k^0\sum_{a\le b}\sum_{JM} A^\dagger_{JM00}(ab)A_{JM00}(ab) \\ &= -\frac{1}{2}k^0\left\{\frac{\hat{n}}{2}\left(\frac{\hat{n}}{2}+1\right) - \hat{T}^2\right\}, \end{aligned} \qquad (2\cdot3)$$

where the operator $\hat{n}$ stands for total number of valence nucleons and $\hat{T}$ for the total isospin (i.e., $\hat{n} = \hat{n}_p + \hat{n}_n = \sum_a(\hat{n}_{ap}+\hat{n}_{an})$ and $\hat{T} = \sum_a \hat{T}_a$). The isoscalar p-n force $H^{\tau=0}_{\pi\nu}$ is very important in determining the binding energy.[30),32)] As seen from the form of Eq. (2·3), $H^{\tau=0}_{\pi\nu}$ is considered to correspond to the volume, surface and symmetry energies in the liquid drop model. It is very important to note that $H^{\tau=0}_{\pi\nu}$ does not change the wave functions determined by the P_0+P_2+QQ force. We see this below in β- and $2\nu\beta\beta$-decay transitions.

For numerical calculations, we adopt the model space ($1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, $1g_{9/2}$) for the $N\approx Z = 20-50$ region. The lowest single-particle energy is determined as $\epsilon_{f_{7/2}} = B(^{41}\mathrm{Ca}) - B(^{40}\mathrm{Ca}) = -8.3633$ MeV from experimental binding energies. The other single particle energies are chosen so as to give the same level spacings as those used by Kisslinger and Sorensen.[34)] The pairing force strength g_0 and the quadrupole force strength χ are adjusted to qualitatively fit the experimental odd-even mass differences and lowest 2^+ energies of nuclei in the *fpg* shell region, respectively:

$$g_0 = 24A^{-1} \quad (\mathrm{MeV}), \qquad \chi = 350A^{-5/3} \quad (\mathrm{MeV}/b_0^4). \qquad (2\cdot4)$$

The strength of the QQ force is somewhat larger than the value $\chi = 240A^{-5/3}$ used in the conventional $P+QQ$ model.[35)] The quadrupole pairing force strength

g_2 is determined by the relation used by Hara et al., $g_2 = 0.2g_0$. [36] We fix [37] the strength k^0 of the isoscalar p-n force $H_{\pi\nu}^{\tau=0}$ so as to reproduce the average value of the symmetry energy [38), 39] as follows:

$$k^0 = \frac{224}{A}(1.0 - 2.2/A^{1/3}) \quad \text{(MeV)}. \tag{2·5}$$

The isoscalar p-n force with this force strength also reproduces the double differences of binding energies, as discussed in a previous paper. [37]

2.1. *BCS plus QRPA*

In this section, we present the QRPA based on the quasiparticle formalism. The generalization of the usual proton-proton (p-p) or neutron-neutron (n-n) QRPA to the p-n QRPA was carried out by Halbleib and Sorensen. [17] In the spherical shell-model basis, the use of the quasiparticle representation implies that the main part of the $\tau = 1$, $J = 0$ pairing correlations is taken into account as the self-consistent quasiparticle field. The quasiparticle creation and annihilation operators are defined by the Bogoliubov transformation

$$c_{\alpha\rho} = u_{a\rho} a_{\alpha\rho} + s_\alpha v_{a\rho} a^\dagger_{-\alpha\rho}, \tag{2·6a}$$
$$c^\dagger_{\alpha\rho} = u_{a\rho} a^\dagger_{\alpha\rho} + s_\alpha v_{a\rho} a_{-\alpha\rho}, \tag{2·6b}$$

where ρ denotes a proton or a neutron, $s_\alpha = (-1)^{j_a - m_\alpha}$, and the coefficients $u_{a\rho}$ and $v_{a\rho}$ satisfy $u^2_{a\rho} + v^2_{a\rho} = 1$. The Hamiltonian (2·1a) is transformed into

$$H = U_0 + \sum_{\alpha\rho} E_a a^\dagger_{\alpha\rho} a_{\alpha\rho} + : H_{\text{int}} :, \tag{2·7}$$

where U_0 is the BCS ground state energy and E_a the quasiparticle energy. The last term, $: H_{\text{int}} :$, represents the residual interaction between quasiparticles. Here, the symbol ": :" denotes the normal order product with respect to the quasiparticle operators $a^\dagger_{\alpha\rho}$ and $a_{\alpha\rho}$.

As discussed in the next section, the GT transition connects the 0^+ ground state of an even-even nucleus with any of the $J^\pi = 1^+$ states of neighboring odd-odd nuclei. Let us now assume that the spin-J states are described by the one-phonon states in the QRPA,

$$|\lambda; JM\rangle = O^{(\lambda)\dagger}_{JM(\rho\rho')}|\tilde{0}\rangle, \tag{2·8}$$

where $|\tilde{0}\rangle$ is the QRPA ground state defined by the relation $O^{(\lambda)}_{JM(\rho\rho')}|\tilde{0}\rangle = 0$. The QRPA phonon $O^{(\lambda)\dagger}_{JM(\rho\rho')}$ and its energy ω_λ are determined by the QRPA equations

$$[H, O^{(\lambda)\dagger}_{JM(\rho\rho')}] = \hbar\omega_\lambda O^{(\lambda)\dagger}_{JM(\rho\rho')}, \tag{2·9a}$$

$$O^{(\lambda)\dagger}_{JM(\rho\rho')} = \sum_{ab}\left(\psi^\lambda_{ab(\rho\rho')}[a^\dagger_{\alpha\rho} a^\dagger_{\beta\rho'}]_{JM} - \phi^\lambda_{ab(\rho\rho')}[a_{\alpha\rho'} a_{\beta\rho}]_{J\tilde{M}}\right), \tag{2·9b}$$

where $[\quad]_{J\tilde{M}} = (-1)^{J-M}[\quad]_{J-M}$, and $\psi^{\lambda}_{ab(\rho\rho')}$ and $\phi^{\lambda}_{ab(\rho\rho')}$ are the forward and backward QRPA amplitudes, which satisfy the orthonormalization conditions

$$\sum_{ab}\left\{\psi^{\lambda*}_{ab(\rho\rho')}\psi^{\lambda'}_{ab(\rho\rho')} - \phi^{\lambda*}_{ab(\rho\rho')}\phi^{\lambda'}_{ab(\rho\rho')}\right\} = \delta_{\lambda\lambda'}(1+\delta_{\rho\rho'}), \tag{2·9c}$$

$$\sum_{ab}\left\{\psi^{\lambda}_{ab(\rho\rho')}\phi^{\lambda'}_{ab(\rho\rho')} - \psi^{\lambda}_{ab(\rho\rho')}\phi^{\lambda'}_{ab(\rho\rho')}\right\} = 0. \qquad (\hbar\omega_\lambda > 0) \tag{2·9d}$$

When ρ and ρ' are both protons or both neutrons, $O^{(\lambda)\dagger}_{J=2M(\rho\rho')}$ is the conventional 2^+ phonon. This type of QRPA phonon describes the first 2^+ states and $B(E2)$ values in a wide range of nuclei,[34] from which the QQ force strength χ can be determined. As known from the interaction matrix element in Eq. (2·2), $O^{(\lambda)\dagger}_{J=2M(\rho\rho')}$ is created by the p-p or n-n correlations of the QQ force.

On the other hand, when ρ is a proton and ρ' is a neutron, $O^{(\lambda)\dagger}_{J=1M(\rho\rho')}$ is the pn-QRPA phonon, and is often used in the study of β and $2\nu\beta\beta$ decay. The advantage of the pn-QRPA is that it resolves the problem that the GT strength of single β decay obtained by a simple shell model calculation is too large. The interaction matrix element taken into account in the pn-QRPA equation is that of the $\tau = 0$, $J = 1$ p-n part of our interaction. The $\tau = 0$, $J = 1$ p-n pairing in the QQ force may affect both the β- and $2\nu\beta\beta$-decay transitions. The other $\tau = 0$, $J = 1$ p-n pairing is the $J = 1$ component of the isoscalar p-n interaction $H^{\tau=0}_{\pi\nu}$. As discussed in a previous paper,[30),32)] however, $H^{\tau=0}_{\pi\nu}$ does not change the wave function in isospin invariant systems. In the exact shell model calculation, the β- and $2\nu\beta\beta$-decay strengths do not depend on this interaction. We examine the dependence of the β- and $2\nu\beta\beta$-decay strengths on the force strengths in §§4 and 5.

§3. Single β decay

The GT_+ strength in single β decay is defined as

$$|M^{(\nu)}_{\mathrm{GT}_+}|^2 = \frac{1}{2J_i+1}\sum_{KM_iM_f}|\langle f|T^{(+)}_{J=1,K}|i\rangle|^2. \tag{3·1}$$

Here J_i denotes the total angular momentum of the initial state $|i\rangle$, and $T^{(+)}_{J=1,K}$ is the GT_+ transition operator,

$$T^{(+)}_{J=1,K} = \sum_{ab}\beta^+(ab)B^\dagger_{J=1,Kpn}(ab), \tag{3·2a}$$

with

$$\beta^+(ab) = \frac{1}{\sqrt{3}}\langle a||\sigma\tau_+||b\rangle, \tag{3·2b}$$

$$B^\dagger_{JM\rho\rho'}(ab) = [c^\dagger_{a\rho}c_{b\rho'}]_{JM}, \tag{3·2c}$$

where $\langle a||\sigma\tau_+||b\rangle$ is the reduced matrix element of the GT_+ transition operator. As in the case of ordinary spin, τ_+ is the isospin raising operator changing a proton into a neutron ($\tau_+|p\rangle = |n\rangle$), and σ is the usual Pauli spin operator.

We now use the pn-QRPA for evaluation of the GT_+ strength. Since the operator $B^\dagger_{JK}(ab)$ is represented by the phonon operators $O^{(\lambda)}_{JM}$ and $O^{(\lambda)\dagger}_{JM}$ as

$$B^\dagger_{J,Kpn}(ab) = u_{an}v_{bp}\sum_\lambda \left(\phi^\lambda_{ab}O^{(\lambda)}_{J\tilde{K}} + \psi^{\lambda*}_{ab}O^{(\lambda)\dagger}_{JK}\right) + v_{an}u_{bp}(-1)^J\sum_\lambda \left(\phi^{\lambda*}_{ab}O^{(\lambda)\dagger}_{JK} + \psi^\lambda_{ab}O^{(\lambda)}_{J\tilde{K}}\right), \tag{3·3a}$$

the GT_+ strength for $|0^+\rangle \to |\lambda; J=1, K\rangle$ decay is

$$|M^{(\nu)\lambda}_{\mathrm{GT}_+}|^2 = \sum_K |\sum_{ab}\beta^+(ab)\left(u_{an}v_{bp}\psi^{\lambda*}_{ab} - v_{an}u_{bp}\phi^{\lambda*}_{ab}\right)|^2. \tag{3·3b}$$

The integrated strength is given as a sum of strengths over excited states:

$$B(\mathrm{GT}_+) = \sum_\lambda |M^{(\nu)\lambda}_{\mathrm{GT}_+}|^2. \tag{3·4}$$

It is easily shown that the Ikeda sum rule,

$$S_- - S_+ = 3(N-Z), \tag{3·5a}$$

$$S_\mp = \sum_{k=1}^3\sum_f |\langle f|\sigma_k\tau_\mp|i\rangle|^2, \tag{3·5b}$$

which has received much attention in the past years, holds in the pn-QRPA.

We calculated the single β-decay strengths for many j shells using the pn-QRPA method. The integrated GT strengths $B(\mathrm{GT}_+)$ for the Ti, V, Fe, Co and Ni isotopes are shown in Fig. 1. The integration was done for the states below 10 MeV. Although the calculated values of $B(\mathrm{GT}_+)$ are larger than the experimental ones for all isotopes, our calculations accurately reproduce the behavior of the Fe and Ni isostopes that the GT strength decreases as the neutron number N increases. It should be noted that if one uses a quenching factor of 1/1.26, the large-scale shell model and pn-QRPA calculations reproduce the experimental results quite accurately. Our result is comparable to those of previous calculations. Thus, we can say that the extended $P+QQ$ model is useful in the analysis of single β decay. Quenching is also observed in the Ikeda sum rule. One way to explain this missing strength is to take account of the Δ-hole admixture. The QRPA calculations would agree with the data if the axial-vector coupling constants were renormalized in nuclei to give $g_A/g_V = 1.0$. However, recent experiments [40), 41)] show that the GT strength summed up to 50 MeV excitation energy is (93±5)% of the Ikeda sum rule value. This suggests that the effects of the Δ-hole are as small as 10% and that most of the quenching can be attributed to the admixture of the 2p-2h configurations.

It has recently been shown that the pn-QRPA may lead to an eventual collapse of the ground state, and the realistic p-n force strength may be close to the critical strength where the pn-QRPA solution collapses. [11)-16)] We here examine the dependence of the pn-QRPA solutions on the strengths of the QQ and isoscalar p-n forces.

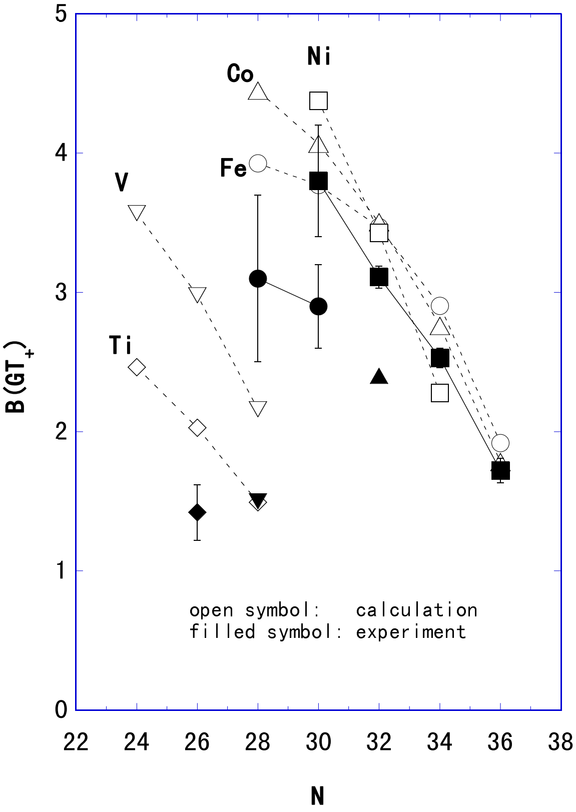

Fig. 1. The integrated GT strength of fp shell nuclei as a function of the neutron number N. The open symbols denote the calculated GT strengths, and the solid symbols the experimental strengths.

Figure 2 displays the plots of the β-decay strength for the ^{56}Fe nucleus: (a) the three lowest energies of the final 1^+ states in the neighboring odd-odd nucleus ^{56}Mn, and (b) the integrated GT strength $B(\mathrm{GT}_+)$ as a function of the QQ force strength χ. The isoscalar p-n force strength k^0 was fixed as $k^0 = 1.70$ (MeV) from Eq. (2·5), so as to reproduce the symmetry energy of ^{56}Fe. Figure 2(a) indicates a collapse at χ_{cr}, as expected. The integrated GT strength $B(\mathrm{GT}_+)$ increases as χ increases, and becomes singular at the critical point χ_{cr}. The increase of the GT strength with increasing χ is notable. It has been discussed that the GT strength depends on the deformation of nuclei.[42)-44)] Since the QQ force is strongly related to the deformation, our result seems to give support to this argument. The three lowest energies of the final 1^+ states in neighboring odd-odd nuclei and the integrated GT strength $B(\mathrm{GT}_+)$ as a function of k^0 are shown in Fig. 3, where the QQ force strength is chosen as $\chi = 0.427$ (MeV/b_0^4) from Eq. (2·4). As discussed in the previous section, if one performs an exact shell model calculation in an isospin-invariant system, $B(\mathrm{GT}_+)$ should be independent of k^0. In fact, Fig. 3(b) exhibits little dependence on the parameter k^0 in the pn-QRPA treatment. At first sight,

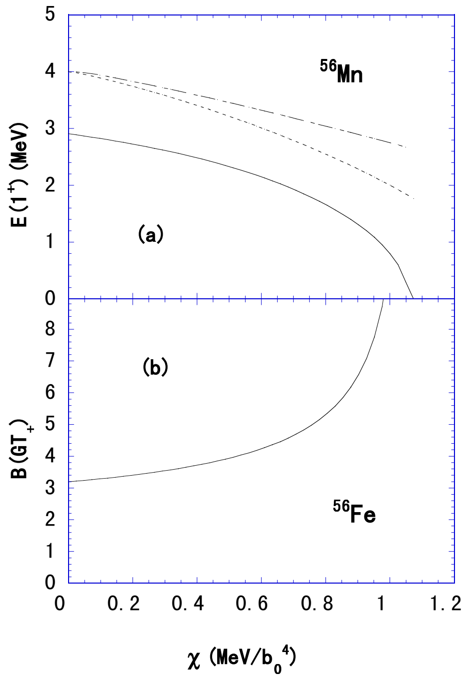

Fig. 2. The lowest three energies of the final 1^+ states for ^{56}Mn and the integrated GT strength of ^{56}Fe as a function of χ.

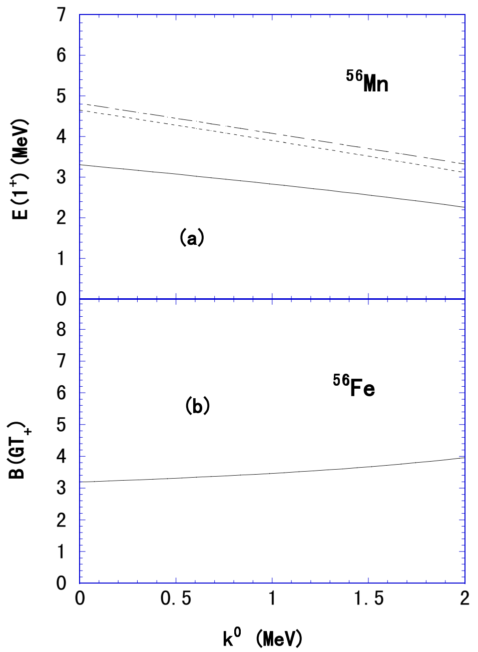

Fig. 3. The lowest three energies of the final 1^+ states for ^{56}Mn and the integrated GT strength of ^{56}Fe as a function of k^0.

this result appears to be opposite to the conclusion of Refs. 8)–11), that the β-decay transition is sensitive to isoscalar p-n correlations. We should, however, note that the QQ force includes an isoscalar p-n pairing component. Our result indicates that the β-decay strength depends strongly on this isoscalar pairing. In this sense, the result obtained here agrees with the the conclusions of Refs. 8)–11). The physically acceptable values of the QQ and isoscalar p-n forces are $\chi = 0.427$ (MeV/b_0^4) and $k^0 = 1.70$ (MeV) (where both force strengths are determined so as to reproduce the experimental first 2^+ energy and the symmetry energy for ^{56}Fe). Figures 2 and 3 show that the pn-QRPA solution is far from the critical point of collapse when reasonable force strengths are used in our model.

We here analyze the characteristic behavior of single β decay using a single j shell model. Though this shell-model space is tiny, the single j shell truncation makes it easy to understand the mechanism of the behavior. In previous papers,[30),32),33),37)] we performed the shell model calculation in a single j shell using the extended $P+QQ$ model to study the energy spectra and nuclear properties related to the binding energy in $f_{7/2}$ and $g_{9/2}$. The calculations accurately reproduce the experimental data for various properties, suggesting that single j truncation is sufficient for qualitative investigation in nuclei where a subshell ($f_{7/2}$ or $g_{9/2}$) dominates. Figure 4 shows the integrated GT strengths in the exact shell model calculation for even-even Ti isotopes. In this calculation, we used the pairing force strength $G = 0.59$ (MeV) and the QQ force strength $\chi = 1.29$ (MeV/b_0^4). The GT strength at $N = Z = 22$ is

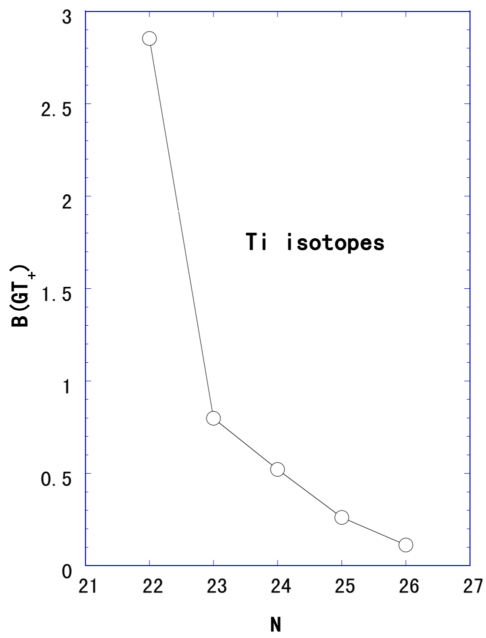

Fig. 4. The integrated GT strength of Ti isotopes as a function of the neutron number N in a single j shell model calculation.

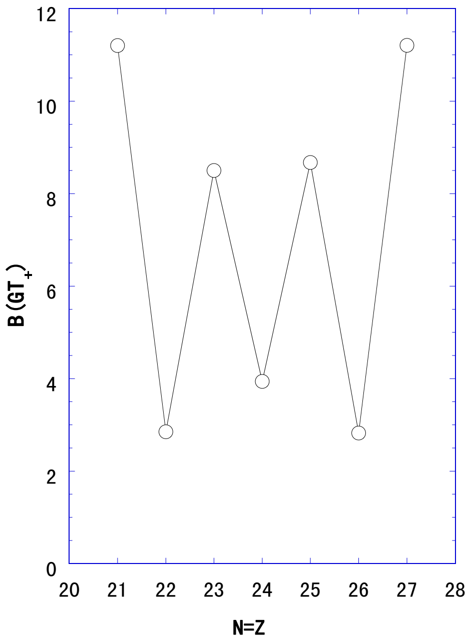

Fig. 5. The integrated GT strength of $N = Z$ nuclei in a single j shell model calculation.

four times larger than that at $N = 23$. This can be attributed to the large spatial overlaps between the single proton and neutron wave functions in $N = Z$ nuclei, since the Fermi energy for the proton is close to that for the neutron. The GT value shows a sudden decrease when one neutron is added to the $N = Z$ nucleus, and then it decreases gradually as the neutron number N is increased. The calculated and experimental GT strengths for Ni and Fe isotopes shown in Fig. 1 exhibit a tendency similar to this. This behavior can be easily understood with the pn-QRPA in a single j shell as follows: In a single j shell, the GT strength can be expressed as

$$|M^{(\nu)}_{\mathrm{GT}_+}|^2 \propto |\,(u_n v_p \psi - v_n u_p \phi)\,|^2. \qquad (3\cdot6)$$

Here $v_n = \sqrt{n_n/(4\Omega)}$ and $v_p = \sqrt{n_p/(4\Omega)}$, where n_n (n_p) denotes the valence neutron (proton) number, and $\Omega = j + 1/2$. Since the dominant component of the right-hand side (RHS) in Eq. (3·6) is the first term, $|u_n v_p \psi|^2$, $|M^{(\nu)\lambda}_{\mathrm{GT}_+}|^2$ is approximately proportional to $u_n^2 = 1 - n_n/4\Omega$. As a result, the GT strength decreases as N increases.

In addition, we examine the single GT strength in $N = Z$ even-even and odd-odd nuclei. Figure 5 displays the single GT strength obtained by the exact shell model calculation in a single j shell as a function of $N = Z$. We can see a staggered pattern with respect to odd and even numbers of $N = Z$. The GT strengths of odd-odd $N = Z$ nuclei are larger than those of even-even $N = Z$ nuclei. Though β decay of the $J = 0$, $T = 1$ ground states of odd-odd $N = Z$ nuclei is dominated by the Fermi-type transition to its analogue states, it is possible to observe this odd-even staggering of the GT strength. In fact, the odd-even staggering of the GT strength has been observed in $Z = 38 - 48$ nuclei.[45)]

The measurement of β and $2\nu\beta\beta$ decay may be used to obtain information on nuclear deformation. For instance, the study of GT β decay of highly deformed $N \approx Z$ nuclei may be very valuable.[42)] The role of deformation in β decay for nuclei in the fp region has been examined,[43),44)] and it was pointed out that the GT strength depends strongly on the shape of the parent nucleus. Since the QQ force is intimately related to the deformation, this fact is quite interesting.

§4. Two-neutrino double β decay

$2\nu\beta\beta$-decay transitions can be described as second-order processes in the perturbative treatment of the Standard Model of the electroweak interaction. The GT matrix elements describing $2\nu\beta\beta$-decay transitions involve a summation over the 1^+ states of the intermediate nucleus participant in the decay chain connecting the initial and final nuclei. The halflife of $2\nu\beta\beta$ decay connecting the 0^+ ground states of two even-even nuclei is given by

$$[T_{1/2}^{(2\nu)}(0^+ \to 0^+)]^{-1} = G^{2\nu}|M_{\rm GT}^{(2\nu)}|^2. \tag{4·1}$$

Here $G^{2\nu}$ is an integral kinematical factor,[46)] and $M_{\rm GT}^{2\nu}$ is the double GT matrix element defined by

$$M_{\rm GT}^{(2\nu)} = \sum_m \frac{\langle 0_f^+|T_{J=1,K}^{(-)}|m\rangle\langle m|T_{J=1,K}^{(-)}|0_i^+\rangle}{E_m - E_i + \Delta}, \tag{4·2}$$

where $T_{J=1,K}^{(-)} = (-1)^K T_{J=1,-K}^{(+)}$ and $\Delta = (E_i - E_f)/2$. The pn-QRPA has been the most powerful tool in the study of $2\nu\beta\beta$ decay of medium weight and heavy nuclei. Following the pn-QRPA approximation studied in the previous section, the initial state $|0_i^+\rangle$ and final state $|0_f^+\rangle$ of even-even nuclei can be approximated by the BCS ground states, and the intermediate state $|m\rangle$ in an odd-odd nucleus by the one phonon states $O^{(m)\dagger}|\tilde{0}\rangle$. Furthermore, Δ and $E_m - E_i$ can be respectively approximated as $\Delta \to -(\lambda_p - \lambda_n)$ and $E_m - E_i \to \omega_m + \lambda_p - \lambda_n$, where λ_p (λ_n) is the chemical potential for the proton (neutron). We calculated the $2\nu\beta\beta$ GT strength for Ge and Se isotopes using the pn-QRPA. The behavior of the $J^\pi = 1^+$ energies and $|M_{\rm GT}^{(2\nu)}|$ depending on χ and k^0 is similar to that of the GT strength shown in Figs. 2 and 3, though we omit the plots of this behavior. The pn-QRPA in our model does not give rise to the collapse problem. Figure 6 shows the calculated $2\nu\beta\beta$ GT strength $|M_{\rm GT}^{(2\nu)}|$ for Ge and Se isotopes as a function of the neutron number N. The experimental values for ^{76}Ge and ^{82}Se are also shown in Fig. 6. The two data sets are fit very well by the calculations. We can see the characteristic behavior of the calculated $2\nu\beta\beta$ GT strengths. Namely, the calculated results display the tendency that the strengths $|M_{\rm GT}^{(2\nu)}|$ decrease with increasing N, except for $N = 38$.

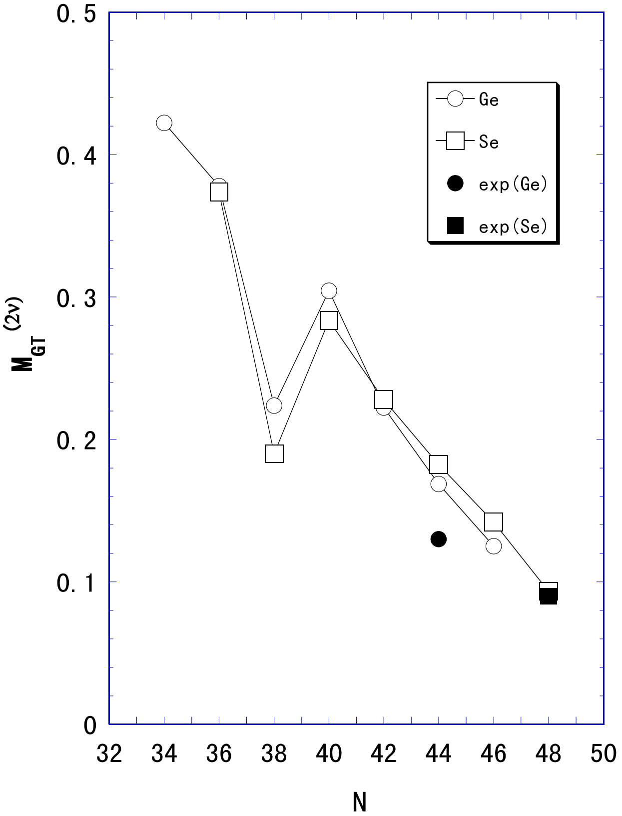

Fig. 6. The $2\nu\beta\beta$-decay strength of Ge and Se isotopes as a function of neutron number N. The open symbols are the calculated strengths and the solid symbols the experimental strengths.

The behavior at $N = 38$ is believed to be due to the shell structure, where the last neutron occupies the $p_{1/2}$ orbital. The calculated halflives of the $2\nu\beta\beta$ decay for ^{76}Ge and ^{82}Se are 1.26×10^{21} years and 1.01×10^{20} years, respectively. These values are compared with the experimental values, 1.80×10^{21} years for ^{76}Ge and 0.80×10^{20} years for ^{82}Se. This result is consistent with those of the Caltech pn-QRPA calculation.[27] The extended $P+QQ$ force seems to be applicable to the study of the double β decay. However, the calculated strength for ^{76}Ge is smaller than that found experimentally. It is known that the ground state correlations induced by the spin-isospin interaction reduce the value of $|M_{\rm GT}^{(2\nu)}|$ in comparison with the independent quasiparticle case.[8),28),29] The spin-isospin correlations causing such effects possibly affect the present results.

§5. Conclusion

We studied single and double β decay of $N \approx Z$ nuclei in the fpg region using an extended $P + QQ$ model proposed recently by the present authors. The GT transitions were examined using the pn-QRPA in many j shells and also using the single j shell model calculation. We calculated the single β-decay strength for the Ti, V, Fe, Co, and Ni isotopes. The integrated strengths are reproduced quite well with our model within the pn-QRPA, although quenching factors are necessary for several cases. Our calculation shows that the GT strength decreases, being proportional to the neutron number N. This observed behavior agrees well with the experimental results. The same behavior was also obtained with the exact single j shell model calculation. We analyzed this behavior using the pn-QRPA. In addition, we performed the calculations of halflives of $2\nu\beta\beta$ decay for the Ge and Se isotopes. The results obtained agree well with the experimental halflives of ^{76}Ge and ^{82}Se. It should be, however, stated that the spin-isospin interaction which suppresses the $2\nu\beta\beta$-decay transition may not been sufficiently included in the extended $P+QQ$ force. The calculated $2\nu\beta\beta$-decay strengths exhibit some systematic pattern with respect to neutron number N. This can be understood by the pn-QRPA in a single j shell. In this paper, however, we did not apply the pn-QRPA for many j shells to $N=Z$ nuclei, because the usual BCS approximation is not good. A generalized BCS treatment is necessary,[7] since the p-n interaction is quite strong for $N=Z$ nuclei. The single j shell model calculation showed that the GT strength becomes large at $N=Z$, and decreases with increasing N. Particularly, the GT strength for $N=Z$ nuclei is very strong for odd-odd nuclei and exhibits an odd-even staggering. We examined the pn-QRPA solution and the sensitivity of the β-decay transition with respect to the strengths of the QQ and isoscalar p-n forces. We found that the p-n part of the QQ force is very important for the β-decay transition. This is consistent with the results of Refs. 8)–16). By contrast, the β-decay transition has little dependence on the J-independent isoscalar p-n force $H_{\pi\nu}^{\tau=0}$. This is reasonable, because $H_{\pi\nu}^{\tau=0}$ does not change the wavefunctions.[30),32)] Thus we can conclude that the extended P+QQ interaction is a promising tool to study β and $2\nu\beta\beta$ decay as well as various nuclear properties, such as the binding energy[32)] and energy spectra[30),33)] investigated previously.

References

1) H. Bethe, Rev. Mod. Phys. **62** (1990), 801.
2) A. L. Williams et al., Phys. Rev. **C51** (1995), 1144.
3) W. P. Alford et al., Nucl. Phys. **A514** (1990), 49.
4) M. C. Vetterli et al., Phys. Rev. **C40** (1989), 559.
5) S. El-Katb et al., Phys. Rev. **C49** (1994), 3128.
6) T. Ronnquist et al., Nucl. Phys. **A563** (1993) 225.
7) A. L. Goodman, Adv. Nucl. Phys. **11** (1979), 263.
8) J. Engel, P. Vogel and M. R. Zirnbauer, Phys. Rev. **C37** (1988), 731.
9) K. Muto, E. Bender and H. V. Klapdor, Z. Phys. **A334** (1989), 177.
10) H. Homma, E. Bender, M. Hirsh, K. Muto, H. Klapdor-Keingrothaus and T. Oda, Phys. Rev. **C54** (1996), 2972.
11) J. Engel, M. Bender, J. Dobaczewski, W. Nazarewicz and R. Surman, Phys. Rev. **C60** (1999), 014302.
12) M. K. Cheoun, A. Bobyk, A. Faesslar, F. Simkovic and G. Teneva, Nucl. Phys. **A561** (1993), 74; **A564** (1993), 329.
13) G. Pantis, F. Simkovic, J. D. Vergados and A. Faessler, Phys. Rev. **C53** (1996), 695.
14) J. Schwieger, F. Simkov and A. Faessler, Nucl. Phys. **A600** (1996), 179.
15) F. Simkovic, J. Schwieger, V. Veselsky, G. Pantis and A. Faessler, Phys. Lett. **B393** (1997), 267.

16) A. Faessler and F. Simkovic, J. of Phys. **G24** (1998), 2139.
17) J. A. Halbleib and R. A. Sorensen, Nucl. Phys. **A98** (1967), 542.
18) P. Vogel and M. R. Zirnbauer, Phys. Rev. Lett. **57** (1986), 3148.
19) D. Cha, Phys. Rev. **C27** (1983), 2269.
20) E. Caurier, K. Langanke, G. Martínez-Pinedo and F. Nowacki, Nucl. Phys. **A653** (1999), 439.
21) K. Langanke, D. J. Dean, P. B. Radha, Y. Alhassid and S. E. Koonin, Phys. Rev. **C52** (1995), 718.
22) P. B. Radha, D. J. Dean, S. E. Koonin, T. T. S. Kuo, K. Langanke, A. Poves, J. Retamosa and P. Vogel, Phys. Rev. Lett. **76** (1996), 2642.
23) G. Martínez-Pinedo, A. Poves, E. Caurier and A. P. Zuker, Phys. Rev. **C53** (1996), R2602.
24) O. Civitarese, A. Faessler and T. Tomoda, Phys. Lett. **B194** (1987), 11.
25) K. Muto and H. V. Klapdor, Phys. Lett. **B201** (1988), 420.
26) J. Suhonen and O. Civitarse, Phys. Rep. **300** (1998), 123.
27) P. Vogel, nucl-th/9904065.
28) O. Civitarese, P. O. Hess, J. G. Hirsh and M. Reboiro, Phys. Rev. **C59** (1999), 194.
29) J. Toivanen and J. Suhonen, Phys. Rev. Lett. **75** (1999), 410.
30) M. Hasegawa and K. Kaneko, Phys. Rev. **C59** (1999), 1149.
31) A. Poves and A. P. Zuker, Phys. Rep. **70** (1981), 235.
32) K. Kaneko and M. Hasegawa, Phys. Rev. **C60** (1999), 024301.
33) M. Hasegawa, K. Kaneko and S. Tazaki, Nucl. Phys. **A674** (2000), 411.
34) L. S. Kisslinger and R. A. Sorensen, Rev. Mod. Phys. **35** (1963), 853.
35) D. R. Bes and R. A. Sorensen, Adv. Nucl. Phys. **2** (1969), 129.
36) K. Hara and S. Iwasaki, Nucl. Phys. **A348** (1980), 200.
K. Hara and Y. Sun, Nucl. Phys. **A529** (1991), 445.
Y. Sun, J.-ye Zhang, M. Guidry and C.-L Wu, Phys. Rev. Lett. **83** (1999), 686.
37) K. Kaneko and M. Hasegawa, nucl-th/9907022.
38) A. Poves and G. Martinez-Pinedo, Phys. Lett. **B430** (1998), 203.
39) J. Duflo and A. P. Zucker, Phys. Rev. **C52** (1995), R23.
40) H. Sakai et al., Nucl. Phys. **A649** (1999), 251c.
41) A. Arima, Nucl. Phys. **A649** (1999), 260c.
42) I. Hamamoto, Phys. Rev. **C60** (1999), 11305.
43) P. Sarriguren, E. Moya de Guerra, A. Escuderos and A. C. Carrizo, Nucl. Phys. **A635** (1998), 55.
44) P. Sarriguren, E. Moya de Guerra, A. Escuderos and A. C. Carrizo, Nucl. Phys. **A658** (1999), 13.
45) H. Schatz et al., Phys. Rep. **294** (1998), 167.
46) M. Doi, T. Kotani and E. Takasugi, Prog. Theor. Phys. Suppl. No. 83 (1985), 1.