Single atom as a macroscopic entanglement source

Ling Zhou,[1,2] Han Xiong,[1] and M. Suhail Zubairy[1]
[1]*Institute for Quantum Studies and Department of Physics, Texas A&M University, College Station, Texas 77843-4242, USA*
[2]*Department of Physics, Dalian University of Technology, Dalian 116024, China*
(Received 14 June 2006; published 24 August 2006)

We discuss the generation of a macroscopic entangled state in a single atom cavity-QED system. The three-level atom in a cascade configuration interacts dispersively with two classical coherent fields inside a doubly resonant cavity. We show that a macroscopic entangled state between these two cavity modes can be generated under large detuning conditions. The entanglement persists even under the presence of cavity losses.

DOI: 10.1103/PhysRevA.74.022321 PACS number(s): 03.67.Mn, 42.50.Dv

I. INTRODUCTION

Quantum entanglement lies at the heart of quantum computing and quantum information science. Cavity quantum electrodynamics (QED) provides an important testing ground for these ideas. For example, cavity QED can be used to not only store quantum information but also to act as a source of entanglement [1–8]. The generation of entanglement in cavity QED has been studied by many authors including the generation of entangled coherent state [1–3], single photon and vacuum entanglement [4], and two-atom entanglement [5].

More recently, generation of macroscopic entangled states via phase sensitive amplification has been discussed. Such continuous variables entanglement offers many advantages in quantum information processing [6]. For example, a quantum secure communication protocol using continuous variables Einstein-Podolsky-Rosen correlations was proposed in Ref. [7]. Conventionally, continuous variables entanglement is produced in a parametric down-conversion process [8]. Recently, based on the study concerning a two-mode correlated spontaneous emission laser (CEL) [9], it was shown that a CEL can lead to two-mode entanglement even when the average photon number can be very large [10,11]. The scheme using CEL is the result of many-atom dynamics. The scheme [12] with potential to produce macroscopic entangled states is still for atomic cloud. On the other hand, a one-atom laser, has been realized experimentally [13]. The entanglement between single atom and its emitted photon has been observed [14]. More recently, Morigi *et al.* [15,16] put forward a scheme where a single trapped atom allows for the generation of entangled light under certain conditions.

In this paper, we propose a scheme to produce a macroscopic entangled state using a single atom in a cavity QED system. We show that a two-mode coherent squeezed state can be generated from our system. In our scheme, a driven three-level atom in cascade configuration dispersively interacts with a two-mode field. We show that under appropriate conditions on the detunings and atom-field coupling, the classical driving fields can help to build up the field in the two modes of the cavity and at the same time an entanglement is generated between the two modes.

II. SYSTEM DESCRIPTION AND CALCULATIONS

We consider a three-level atom in a cascade configuration crossing or trapped in a two-mode field cavity. The atomic level configuration is depicted in Fig. 1. The two atomic transitions $|a\rangle \leftrightarrow |b\rangle$ and $|b\rangle \leftrightarrow |c\rangle$ interact with the two cavity modes with detunings $\mp\delta$ with $\delta = |\omega_1 - (E_a - E_b)| = |\omega_2 - (E_b - E_c)|$. The two atomic transitions (namely, $|a\rangle \leftrightarrow |b\rangle$ and $|b\rangle \leftrightarrow |c\rangle$) are also driven by two classical fields with the same detunings as their corresponding quantized field modes and Ω_1 and Ω_2 are the Rabi frequencies of the two classical fields. The dipole forbidden atomic transition between $|a\rangle$ and $|c\rangle$ are resonantly driven by another classical field of Rabi frequency Ω.

The Hamiltonian of our system under the dipole and rotating wave approximation and in the interaction picture is given by

$$\begin{aligned}\hat{H}_I = {} & g_1\left[\left(\hat{a}_1 + \frac{\Omega_1}{g_1}\right)\hat{\sigma}_{bc} + \left(\hat{a}_1^\dagger + \frac{\Omega_1}{g_1}\right)\hat{\sigma}_{cb}\right] \\ & + g_2\left[\left(\hat{a}_2 + \frac{\Omega_2}{g_2}\right)\hat{\sigma}_{ab} + \left(\hat{a}_2^\dagger + \frac{\Omega_2}{g_2}\right)\hat{\sigma}_{ba}\right] \\ & + \Omega(\hat{\sigma}_{ac} + \hat{\sigma}_{ca}) - \delta(\hat{\sigma}_{aa} + \hat{\sigma}_{cc}), \end{aligned} \tag{1}$$

where $\hat{\sigma}_{ij} = |i\rangle\langle j|$ $(i,j = a,b,c)$ are the atomic operators. $\hat{a}_1(\hat{a}_1^\dagger)$ and $\hat{a}_2(\hat{a}_2^\dagger)$ are the creation (annihilation) operators of the two cavity modes and g_1 and g_2 are the atom-field coupling constants and, in general, they are different.

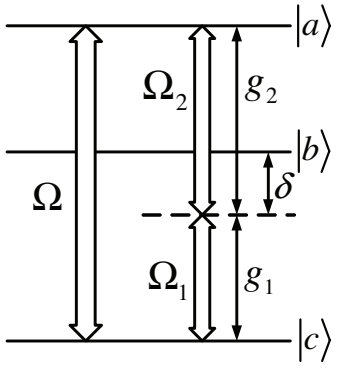

FIG. 1. The level configuration of the three-level atom. Two cavity modes and two classical fields interact with atomic transitions $|a\rangle \leftrightarrow |b\rangle$ and $|b\rangle \leftrightarrow |c\rangle$ with detunings $\mp\delta$, and another classical field with a Rabi frequency of Ω drives the dipole forbidden atomic transition between $|a\rangle$ and $|c\rangle$ resonantly.

The Heisenberg equations of motion for the atomic operators $\hat{\sigma}_{bc}$ and $\hat{\sigma}_{ba}$ are given by

$$i\frac{d\hat{\sigma}_{bc}}{dt} = -g_1\tilde{a}_1^\dagger(\hat{\sigma}_{cc}-\hat{\sigma}_{bb}) - g_2\tilde{a}_2\sigma_{ac} + \Omega\hat{\sigma}_{ba} - \delta\hat{\sigma}_{bc},$$

$$i\frac{d\hat{\sigma}_{ba}}{dt} = -g_1\tilde{a}_1^\dagger\hat{\sigma}_{ca} + g_2\tilde{a}_2(\hat{\sigma}_{bb}-\hat{\sigma}_{aa}) + \Omega\hat{\sigma}_{bc} - \delta\hat{\sigma}_{ba}, \quad (2)$$

where

$$\tilde{a}_j = \hat{a}_j + \frac{\Omega_j}{g_j},\ \tilde{a}_j^\dagger = \hat{a}_j^\dagger + \frac{\Omega_j}{g_j},\ j=1,2. \quad (3)$$

Under the large detuning condition when $\delta \gg \Omega, \Omega_j, g_1, g_2$, Eq. (2) can be solved adiabatically by taking $d\hat{\sigma}_{bc}/dt = d\hat{\sigma}_{ba}/dt = 0$. The adiabatic solutions for $\hat{\sigma}_{bc}$ and $\hat{\sigma}_{ab}$ can then be substituted into the Hamiltonian (1) and we obtain

$$\hat{H}_1 = \Omega(\hat{\sigma}_{ac}+\hat{\sigma}_{ca}) - \delta(\hat{\sigma}_{aa}+\hat{\sigma}_{cc}) + \frac{1}{\Omega^2-\delta^2}\{-\delta g_1^2(2\tilde{a}_1^\dagger\tilde{a}_1+1) \times(\hat{\sigma}_{bb}-\hat{\sigma}_{cc}) + \delta g_2^2(2\tilde{a}_2^\dagger\tilde{a}_2+1)(\hat{\sigma}_{aa}-\hat{\sigma}_{bb}) + \Omega[(g_1^2\tilde{a}_1^\dagger\tilde{a}_1 + g_2^2\tilde{a}_2^\dagger\tilde{a}_2)\hat{\sigma}_{ac} + (g_1^2\tilde{a}_1\tilde{a}_1^\dagger + g_2^2\tilde{a}_2\tilde{a}_2^\dagger)\hat{\sigma}_{ca}] + 2g_1g_2\delta(\tilde{a}_1\tilde{a}_2\hat{\sigma}_{ac} + \tilde{a}_1^\dagger\tilde{a}_2^\dagger\hat{\sigma}_{ca}) + g_1g_2\Omega(\tilde{a}_1\tilde{a}_2 + \tilde{a}_1^\dagger\tilde{a}_2^\dagger)(\hat{\sigma}_{aa}+\hat{\sigma}_{cc}-2\hat{\sigma}_{bb})\}. \quad (4)$$

If the atom is initially injected in level $|b\rangle$, it will remain confined to this level due to the large detuning approximation. The approximate effective Hamiltonian for this case reduces to

$$\hat{H}_b = \eta_1\tilde{a}_1^\dagger\tilde{a}_1 + \eta_2\tilde{a}_2^\dagger\tilde{a}_2 + \frac{1}{2}(\eta_1+\eta_2) + \xi(\tilde{a}_1\tilde{a}_2 + \tilde{a}_1^\dagger\tilde{a}_2^\dagger), \quad (5)$$

where

$$\xi = \frac{2g_1g_2\Omega}{\delta^2-\Omega^2},$$

$$\eta_1 = \frac{2g_1^2\delta}{\delta^2-\Omega^2},$$

$$\eta_2 = \frac{2g_2^2\delta}{\delta^2-\Omega^2}. \quad (6)$$

This Hamiltonian can be rewritten as

$$\hat{H}_b = (\eta_1+\eta_2)\hat{K}_0 + \xi(\hat{K}_- + \hat{K}_+) + \frac{1}{2}(\eta_1-\eta_2)\hat{N}_0, \quad (7)$$

where

$$\hat{K}_0 = \frac{1}{2}(\tilde{a}_1^\dagger\tilde{a}_1 + \tilde{a}_2^\dagger\tilde{a}_2 + 1),$$

$$\hat{K}_- = \tilde{a}_1\tilde{a}_2,$$

$$\hat{K}_+ = \tilde{a}_1^\dagger\tilde{a}_2^\dagger,$$

$$\hat{N}_0 = \tilde{a}_1^\dagger\tilde{a}_1 - \tilde{a}_2^\dagger\tilde{a}_2.$$

These operators can be verified to obey the SU (1, 1) commutation relations $[\hat{K}_-, \hat{K}_+] = 2\hat{K}_0$, $[\hat{K}_0, \hat{K}_\pm] = \pm\hat{K}_\pm$, and $[\hat{N}_0, \hat{K}_0] = [\hat{N}_0, \hat{K}_\pm] = 0$. We can therefore use the SU (1, 1) Lie algbra to expand the unitary evolution [17] operator $\hat{U} = e^{-i\hat{H}_b t}$ as

$$\hat{U} = e^{(A_+\hat{K}_+)}e^{(\ln A_0\hat{K}_0)}e^{-it/2(\eta_1-\eta_2)\hat{N}_0}e^{(A_-\hat{K}_-)}, \quad (8)$$

where

$$A_0 = a_0^2,$$

$$A_+ = A_- = \frac{-i\xi t}{\phi}a_0\sinh\phi \quad (9)$$

with

$$a_0 = \frac{1}{\cosh\phi + it\dfrac{(\eta_1+\eta_2)}{2\phi}\sinh\phi},$$

$$\phi^2 = \left[-\left(\frac{\eta_1+\eta_2}{2}\right)^2 + \xi^2\right]t^2. \quad (10)$$

We now consider the case when the two-mode field is initially prepared in a vacuum state $|0,0\rangle$. The time evolution of the field state can be obtained as

$$|\Psi_f(t)\rangle \Rightarrow \exp(A_+\hat{a}_1^\dagger\hat{a}_2^\dagger)\exp(\alpha_1\hat{a}_1^\dagger)\exp(\alpha_2\hat{a}_2^\dagger)|0,0\rangle \quad (11)$$

with

$$\alpha_1 = \frac{\Omega_2}{g_2}A_+ + \frac{\Omega_1}{g_1}[a_0e^{-it/2(\eta_1-\eta_2)}-1],$$

$$\alpha_2 = \frac{\Omega_1}{g_1}A_+ + \frac{\Omega_2}{g_2}[a_0e^{it/2(\eta_1-\eta_2)}-1]. \quad (12)$$

The SU (1, 1) Lie algebra yields

$$e^{A_+\hat{a}_1^\dagger\hat{a}_2^\dagger} = e^{(\vartheta^*\hat{a}_1\hat{a}_2 - \vartheta\hat{a}_1^\dagger\hat{a}_2^\dagger)}e^{A_+^*\hat{a}_1\hat{a}_2}e^{g(\hat{a}_1^\dagger\hat{a}_1+\hat{a}_2^\dagger\hat{a}_2+1)}.$$

Let $\vartheta = re^{i\varepsilon}$, $g = \ln\cosh r$, where r and ε are determined by the relation

$$A_+ = -e^{i\varepsilon}\tanh r. \quad (13)$$

The squeezed parameter r and ε are

$$r = \tanh^{-1}|A_+|,$$

$$\cos\varepsilon = -\frac{\mathrm{Re}(A_+)}{|A_+|},$$

$$\sin\varepsilon = -\frac{\mathrm{Im}(A_+)}{|A_+|}. \quad (14)$$

The state of the system can then be written as

$$|\Psi_f(t)\rangle = e^{(\vartheta^* \hat{a}_1 \hat{a}_2 - \vartheta \hat{a}_1^\dagger \hat{a}_2^\dagger)} |\alpha_1 \cosh r, \alpha_2 \cosh r\rangle$$
$$= S(\vartheta) D(\alpha_1 \cosh r) D(\alpha_2 \cosh r) |0,0\rangle. \quad (15)$$

It is obviously a two-mode coherent-squeezed state [18,19].

For the generation of macroscopic entangled state, we consider two quantities, namely, the mean photon number and the correlation functions involved in the entanglement criterion. The total average photon number of the two-mode field $N=\langle \hat{a}_1^\dagger \hat{a}_1 \rangle + \langle \hat{a}_2^\dagger \hat{a}_2 \rangle$ can be easily obtained

$$N = 2 \sinh^2 r + \cosh^2 r [(|\alpha_1|^2 + |\alpha_2|^2) \cosh 2r$$
$$- (\alpha_1 \alpha_2 e^{-i\varepsilon} + \alpha_1^* \alpha_2^* e^{i\varepsilon}) \sinh 2r]. \quad (16)$$

To determine the entanglement of state (15), we need the entanglement criterion for continuous variables system. Recently, different criteria have been proposed [20–23]. Here, we choose the summation of the quantum fluctuations proposed in Ref. [20]. According to this criterion, a state is entangled if the summation of the quantum fluctuations in the two EPR-like operators $\hat{u}$ and $\hat{v}$ satisfy the following inequality:

$$(\Delta \hat{u})^2 + (\Delta \hat{v})^2 < 2, \quad (17)$$

where

$$\hat{u} = \hat{x}_1 + \hat{x}_2, \hat{v} = \hat{p}_1 - \hat{p}_2,$$

and $\hat{x}_i = (\hat{a}_i e^{-i\psi} + \hat{a}_i^\dagger e^{i\psi})/\sqrt{2}$ and $\hat{p}_i = (\hat{a}_i e^{-i\psi} - \hat{a}_i^\dagger e^{i\psi})/\sqrt{2}i$ $(i=1,2)$ are the quadrature operators of the field. For the state (15) and by taking $\psi = \frac{1}{4}\pi$ we can derive that

$$(\Delta \hat{u})^2 + (\Delta \hat{v})^2 = 2(\cosh 2r - \sin \varepsilon \sinh 2r). \quad (18)$$

From Eqs. (12), (16), and (18), it is clear that the average photon number of the two-mode field depends on Ω_1 and Ω_2, however, the entanglement condition is independent of the strengths of the driving fields. We can thus change the average photon number of the field by manipulating Ω_1 and Ω_2 without affecting the entanglement of the two modes.

It is useful to consider the case when $\Omega_1 = \Omega_2 = 0$ which means $\alpha_1 = \alpha_2 = 0$ in Eq. (12). From Eq. (15), we have

$$|\Psi\rangle = S(\vartheta)|0,0\rangle = \frac{1}{\cosh r} \sum_n \tanh^n r |n,n\rangle. \quad (19)$$

This is a two-mode squeezed state which can also be generated by a parametric amplifier [24]. The total average photon number of the two-mode field for an initial vacuum state is

$$N = 2 \sinh^2 r. \quad (20)$$

The entanglement condition still has the form of Eq. (18).

Next we consider the effect of the cavity losses by including the cavity damping terms in the equation of motion for the density operators. The equation of motion for the density operator is given by

$$\dot{\hat{\rho}} = -i \Bigg[\xi(\hat{a}_1 \hat{a}_2 + \hat{a}_1^\dagger \hat{a}_2^\dagger) + \eta_1 \hat{a}_1^\dagger \hat{a}_1 + \eta_2 \hat{a}_2^\dagger \hat{a}_2 + \left(\eta_2 \frac{\Omega_2}{g_2} + \xi \frac{\Omega_1}{g_1} \right)$$
$$\times (\hat{a}_2 + \hat{a}_2^\dagger) + \left(\eta_1 \frac{\Omega_1}{g_1} + \xi \frac{\Omega_2}{g_2} \right) (\hat{a}_1 + \hat{a}_1^\dagger), \hat{\rho} \Bigg]$$
$$+ \kappa \sum_{i=1,2} (2\hat{a}_i \hat{\rho} \hat{a}_i^\dagger - \hat{a}_i^\dagger \hat{a}_i \hat{\rho} - \hat{\rho} \hat{a}_i^\dagger \hat{a}_i). \quad (21)$$

The resulting equations for the expectation values of the field operators are

$$\frac{d\langle \hat{a}_1^\dagger \hat{a}_1 \rangle}{dt} = -i \Bigg[\xi(\langle \hat{a}_1^\dagger \hat{a}_2^\dagger \rangle - \langle \hat{a}_1 \hat{a}_2 \rangle) + \left(\eta_1 \frac{\Omega_1}{g_1} + \xi \frac{\Omega_2}{g_2} \right)$$
$$\times (\langle \hat{a}_1^\dagger \rangle - \langle \hat{a}_1 \rangle) \Bigg] - 2\kappa \langle \hat{a}_1^\dagger \hat{a}_1 \rangle,$$

$$\frac{d\langle \hat{a}_1 \hat{a}_2 \rangle}{dt} = -i \Bigg[\xi(\langle \hat{a}_1^\dagger \hat{a}_1 \rangle + \langle \hat{a}_2^\dagger \hat{a}_2 \rangle + 1) + \left(\eta_1 \frac{\Omega_1}{g_1} + \xi \frac{\Omega_2}{g_2} \right) \langle \hat{a}_2 \rangle$$
$$+ \left(\eta_2 \frac{\Omega_2}{g2} + \xi \frac{\Omega_1}{g_1} \right) \langle \hat{a}_1 \rangle \Bigg] - [2\kappa + i(\eta_1 + \eta_2)] \langle \hat{a}_1 \hat{a}_2 \rangle,$$

$$\frac{d\langle \hat{a}_1 \rangle}{dt} = -i \Bigg[\xi \langle \hat{a}_2^\dagger \rangle + \left(\eta_1 \frac{\Omega_1}{g_1} + \xi \frac{\Omega_2}{g_2} \right) \Bigg] - (\kappa + i\eta_1) \langle \hat{a}_1 \rangle. \quad (22)$$

On interchanging the subscripts 1 and 2 and taking the Hermitian conjugate, we can obtain the remaining five differential equations of $\langle \hat{a}_2^\dagger \hat{a}_2 \rangle$, $\langle \hat{a}_1^\dagger \hat{a}_2^\dagger \rangle$, etc. These eight equations can be solved by using the standard techniques such as those based on Laplace transform method. We can then evaluate the average photon numbers and the quantity $(\Delta \hat{u})^2 + (\Delta \hat{v})^2$ for this system. These solutions are long and tedious and we do not reproduce them here. Instead, we present a numerical solutions for these equations in the next section.

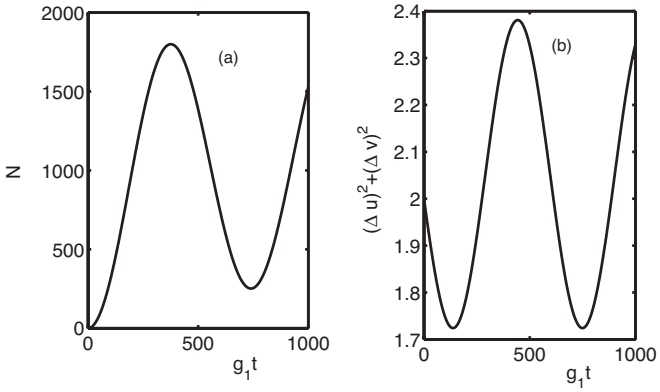

FIG. 2. The time evolution of the total average photon number N and $(\Delta \hat{u})^2 + (\Delta \hat{v})^2$. The two-mode field is entangled when $(\Delta \hat{u})^2 + (\Delta \hat{v})^2 < 2$ [Eq. (15)]. The parameters are $\Omega_1 = 10$, $\Omega_2 = 40$, $g_1 = 1$, $g_2 = 2$, $\delta = 1000$, and $\Omega = 200$.

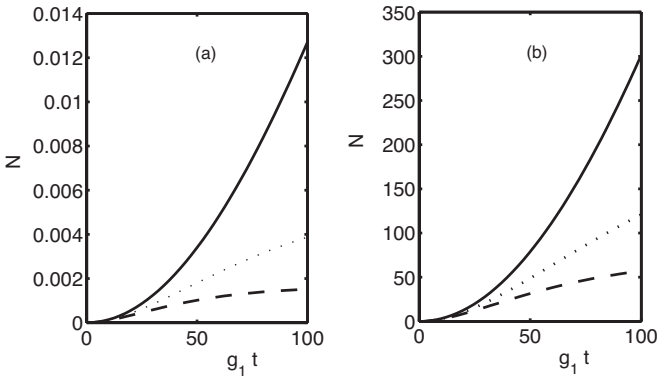

FIG. 3. The time evolution of total average photon number N. Solid lines in (a) and (b) correspond to two-mode squeezed vacuum state [Eq. (20)] and the two-mode coherent-squeezed state [Eq. (16)], respectively; dotted (κ=0.01) and dashed lines (κ=0.02) are plotted from Eq. (21). In (a) $\Omega_1=\Omega_2=0$ while for (b) $\Omega_1=10$, $\Omega_2=40$. For all plots, $g_1=1$, $g_2=2$ and $\delta=1000$, $\Omega=200$.

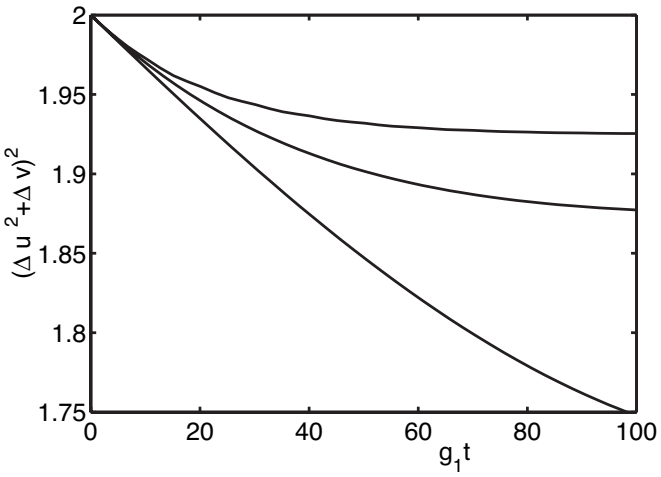

FIG. 4. The quantity $(\Delta\hat{u})^2+(\Delta\hat{v})^2$ changes with time for several values of the cavity decay rates κ. From top to bottom $\kappa=0.02, 0.01, 0$. Other parameters are $\Omega=200, g_1=1$, $g_2=2$, and $\delta=1000$.

III. DISCUSSION

We now discuss the entanglement properties of the amplified fields inside the doubly resonant cavity. In our plots, all of parameters are in expressed in units of g_1. In Fig. 2, we plot the average photon number and $(\Delta\hat{u})^2+(\Delta\hat{v})^2$ for the two-mode coherent-squeezed state from Eqs. (16) and (18), respectively. Both the quantities exhibit oscillations. This is a consequence of the terms proportional to η_1 and η_2 in Hamiltonian (5). The period of the oscillations can, however, be very large as η_1 and η_2 can be small. Thus we can have entanglement for sufficiently large interaction times.

In Figs. 3 and 4, we plot N and $(\Delta\hat{u})^2+(\Delta\hat{v})^2$ in the small time region where entanglement is present. In Fig. 3 we plot the total average photon number N as a function of time under two cases: $\Omega_1=\Omega_2=0$ [Fig. 3(a)] and $\Omega_1=10$, $\Omega_2=40$ [Fig. 3(b)]. Solid lines in Figs. 3(a) and 3(b) are plotted from Eqs. (20) and (16), respectively. Dotted lines and dashed lines are plotted from Eq. (21) with the inclusion of cavity losses. Comparing the two solid lines in Figs. 3(a) and 3(b), we note that the average photon number of two-mode coherent-squeezed state is extremely larger as compared to a two-mode squeezed vacuum state. Even with the inclusion of cavity losses (dotted lines and dashed lines), the average photon number of two-mode fields still increase dramatically for the driven system. Thus the two-mode fields still can be amplified even when cavity losses are present.

In Fig. 4, we show the time evolution of $(\Delta\hat{u})^2+(\Delta\hat{v})^2$ in the presence of cavity losses. Notice that the entanglement exists in a lossy cavity. It is worthwhile to point out that we plot $(\Delta\hat{u})^2+(\Delta\hat{v})^2$ and N for the same set of parameters. For this set of parameters, we obtain both amplification and entanglement at the same time.

Finally, we note that the classical field Ω not only affects entanglement between the two quantum fields but also the amplification of these fields. On the other hand, the two classical fields Ω_1 and Ω_2 mainly amplify the quantum field but plays no role on the entanglement criterion.

We note that Morigi *et al.* [15,16] have also considered the generation of two-mode squeezing in a single atom. Their situation is, however, different than ours. In their work, Morigi *et al.* consider both external and internal degrees of freedom. Under the large detuning limit, the atom's internal degrees of freedom are eliminated and a two-mode squeezed state at certain times is obtained. At those times the atom is decorrelated from the two cavity modes. At other times, the system is in a tripartite entangled state between the cavity modes and the center-of-mass degrees of freedom of the atom. In our scheme, the entangled states are generated over a wide range of interaction times. This is easily seen from Figs. 2 to 4. Moreover in our scheme, we can generate a two-mode coherent-squeezed state of large intensity.

IV. CONCLUSION

In summary, we discussed a scheme in which a single atom in the cascade configuration inside a doubly resonant cavity can lead to amplified fields that are entangled. The resulting field, under appropriate conditions, is a two-mode coherent-squeeze state. We show that the entanglement persists even in the presence of cavity losses [25].

ACKNOWLEDGMENTS

This work was supported by DARPA-QuIST, the Air Force Office of Scientific Research, and the Office of Naval Research.

[1] L. Davidovich, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. A **53**, 1295 (1996).
[2] X. B. Zou, K. Pahlke, and W. Mathis, Phys. Rev. A **65**, 064303 (2002); C. C. Gerry, *ibid.* **55**, 2478 (1997).
[3] E. Solano, G. S. Agarwal, and H. Walther, Phys. Rev. Lett. **90**, 027903 (2003).
[4] M. S. Zubairy, M. Kim, and M. O. Scully, Phys. Rev. A **68**, 033820 (2003).
[5] S. B. Zheng, Phys. Rev. A **69**, 055801 (2004); S. B. Zheng and G. C. Guo, Phys. Rev. Lett. **85**, 2392 (2000).
[6] S. L. Braunstein and P. van Loock, Rev. Mod. Phys. **77**, 513 (2005).
[7] G. He, J. Zhu, and G. Zeng, Phys. Rev. A **73**, 012314 (2006).
[8] D. Bouwmeester, *et al.*, Nature (London) **390**, 575 (1997).
[9] M. O. Scully, Phys. Rev. Lett. **55**, 2802 (1985); M. O. Scully and M. S. Zubairy, Phys. Rev. A **35**, 752 (1987).
[10] H. Xiong, M. O. Scully, and M. S. Zubairy, Phys. Rev. Lett. **94**, 023601 (2005).
[11] H.-T. Tan, S.-Y. Zhu, and M. S. Zubairy, Phys. Rev. A **72**, 022305 (2005).
[12] R. Guzman, J. C. Retamal, E. Solano, and N. Zagury, Phys. Rev. Lett. **96**, 010502 (2006).
[13] J. McKeever, A. Boca, A. D. Boozer, J. R. Buck, and H. J. Kimble, Nature (London) **425**, 286 (2003).
[14] B. Blinov, D. L. Moehring, L. M. Duan, and C. Monroe, Nature (London) **428**, 153 (2004).
[15] G. Morigi, J. Eschner, S. Mancini, and D. Vitali, Phys. Rev. Lett. **96**, 023601 (2006).
[16] G. Morigi, J. Eschner, S. Mancini, and D. Vitali, Phys. Rev. A **73**, 033822 (2006).
[17] M. Ban, J. Math. Phys. **33**, 3213 (1992).
[18] H. Zeng and F. Lin, Phys. Rev. A **52**, 809 (1995).
[19] C. T. Lee, Phys. Rev. A **42**, 1608 (1990).
[20] L. M. Duan, G. Giedke, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **84**, 2722 (2000).
[21] V. Giovannetti, S. Mancini, D. Vitali, and P. Tombesi, Phys. Rev. A **67**, 022320 (2003).
[22] E. Shchukin and W. Vogel, Phys. Rev. Lett. **95**, 230502 (2005).
[23] M. Hillery and M. S. Zubairy, Phys. Rev. Lett. **96**, 050503 (2006).
[24] D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer-Verlag, Berlin, 1994).
[25] C. J. Villas-Boas and M. H. Y. Moussa, Eur. Phys. J. D **32**, 147 (2005).