Letter

Single-photon nonlocality in quantum networks

Paolo Abiuso [1,*] Tamás Kriváchy [1,2,3,4,*] Emanuel-Cristian Boghiu [1] Marc-Olivier Renou [1]
Alejandro Pozas-Kerstjens [5,6] and Antonio Acín [1,7]

[1] *ICFO - Institut de Ciencies Fotoniques, The Barcelona Institute of Science and Technology, 08860 Castelldefels (Barcelona), Spain*
[2] *Department of Applied Physics, University of Geneva, CH-1211 Geneva, Switzerland*
[3] *Institute for Quantum Optics and Quantum Information - IQOQI Vienna, Austrian Academy of Sciences, Boltzmanngasse 3, 1090 Vienna, Austria*
[4] *Atominstitut, Technische Universität Wien, 1020 Vienna, Austria*
[5] *Departamento de Análisis Matemático, Universidad Complutense de Madrid, 28040 Madrid, Spain*
[6] *Instituto de Ciencias Matemáticas (CSIC-UAM-UC3M-UCM), 28049 Madrid, Spain*
[7] *ICREA - Institució Catalana de Recerca i Estudis Avançats, Lluis Companys 23, 08010 Barcelona, Spain*

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A single-photon maximally entangled state is obtained when a photon impinges on a balanced beamsplitter. Its nonlocal properties have been intensively debated in the quantum optics and foundations communities. It is however clear that a standard Bell test made only of passive optical elements cannot reveal the nonlocality of this state. We show that the nonlocality of single-photon entangled states can nevertheless be revealed in a quantum network made only of beamsplitters and photodetectors. In our protocol, three single-photon entangled states are distributed in a triangle network, introducing indeterminacy in the photons' paths and creating nonlocal correlations without the need for measurements choices. We discuss a concrete experimental realization and provide numerical evidence of the tolerance of our protocol to standard noise sources. Our results show that single-photon entanglement may constitute a promising solution to generate genuine network-nonlocal correlations useful for Bell-based quantum information protocols.

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I. BACKGROUND

Local hidden variables models cannot account for all the predictions of quantum theory. This was formalized in 1964 by J. S. Bell [1], and is now commonly termed nonlocality [2]. Nonlocality is a quantum property with no classical analog displayed in the so-called Bell tests, defined by the statistics obtained when performing appropriate local measurements on a well-chosen entangled state. Bell tests have been performed in many different systems, from massive particles [3] to photons [4,5], and using many different degrees of freedom, such as electronic levels, polarization, orbital angular momentum, or time bins. In most of these realizations, the relevant degrees of freedom used to encode the entanglement are transmitted to each distant observer by a physical carrier, such as, for instance, a photon.

In this work, we are interested in the question of whether single-particle quantum states can display nonlocal correlations with no classical analog. In particular, we consider the question in the context of single-photon entanglement, that is, the state

$$|\psi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|01\rangle_{AB} + |10\rangle_{AB}), \tag{1}$$

obtained when sending a single photon into a balanced beamsplitter. Here $|01\rangle_{AB}$ (resp. $|10\rangle_{AB}$) represents the situation in which the photon is sent to the right party B (resp. the left party A). The resulting state therefore consists of only one photon and entanglement is encoded in the two optical spatial modes.

Is the state (1) nonlocal? This question has been intensively debated in the quantum foundations and quantum optics community, e.g., Refs. [6–19]. In principle, a positive answer is provided by the following simple argument [8–10]: the two optical modes can be transferred to the population of two energy levels of two distant massive particles. Single-photon entanglement is therefore mapped into two-particle entanglement and a Bell test can now be implemented. The question is much subtler when considering only optical means. To obtain a nonlocal behavior, the two observers need to use local *active measurements* involving local oscillators creating extra local photons [6,7,13,16]: without these active measurements, measuring the information content of the state (1) would allow the observers to deduce if they received the photon sent by the source, destroying the indeterminacy in the photon path, i.e., the coherences in (1). Then, the statistics become classically simulable. One is therefore tempted to conclude that the observation of nonlocal effects in the single-photon entangled

*These authors contributed equally to this work.

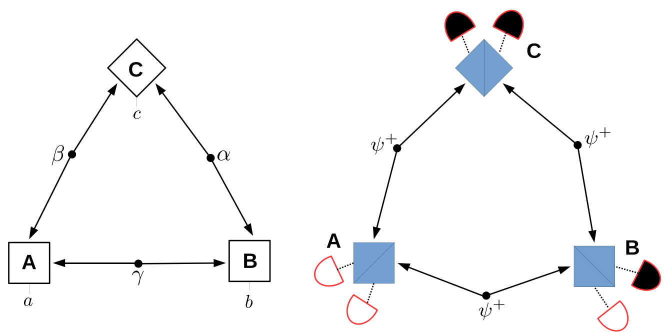

FIG. 1. (Left) Causal model for the triangle network: three independent sources $\{\alpha, \beta, \gamma\}$ prepare correlated states that are distributed among the three parties. Each of them produces an output through a local process acting on the received parts of the states. The form of the states and local processes depend on the theory, say classical or quantum, used to reproduce the correlations in the network. (Right) Schematics of the proposed quantum optical experiment. A, B, and C share single-photon entangled states $|\psi^+\rangle = (|01\rangle + |10\rangle)/\sqrt{2}$ prepared by the sources. Each party receives two optical modes that are mixed on a beamsplitter, the resulting output modes being measured by photodetectors. In the specific experimental instance depicted here, A does not detect any photon, B has one detector firing, and C has both detectors firing.

state by passive optical means, that is, phase shifters, beamsplitters, and photodetectors, is impossible.

The main result of this work is to show that this is not the case and one can indeed reveal the nonlocality of state (1) with only *passive measurements*. To do so, we go beyond standard Bell tests and consider setups defined by causal networks. These are causal structures involving several independent sources, each being distributed to a subset of the parties involved in the scenario, according to a structure defined by a network [20]. It is well understood that these networks offer new possibilities to design quantum experiments with no classical analog [21–26]. Here, we show that three copies of single-photon entangled states placed in a *triangle causal network* (cf. Fig. 1) can exhibit nonclassical correlations. Our main idea is to exploit the topology of the network to reintroduce indeterminacy in the photon path, necessary to exploit the coherences of these states. Remarkably, the obtained setup is not only passive in terms of the implemented measurements, but also because it does not require any active choice of measurements. That is, in our setup, there are no classical inputs and observers perform a single measurement on their received shares. These characteristics make the proposal, arguably, the simplest experimental demonstration of the nonlocality of the single-photon entangled state, as well as the first experimental proposal for genuine network nonlocality [26].

Beyond the fundamental motivation, our results are also relevant from an applied point of view. Correlations with no classical analog are the main resource for device-independent applications. For instance, the security of device-independent protocols for quantum random number generation [27,28] and quantum key distribution [29] is based on the observation of Bell inequality violations. For that, the simplest way of producing entangled states is through spontaneous parametric down conversion (SPDC). Entanglement can be encoded on different degrees of freedom of the resulting two photons. However, the state produced by SPDC is a mixture of the desired entangled state and vacuum [30]. In fact, a heralded preparation of a two-photon maximally entangled state is quite challenging [31]. In turn, single-photon entanglement can be easily prepared in a heralded way: an arbitrarily good approximation to it can be obtained when detecting photons in one of the two modes resulting from the SPDC process and sending the nonmeasured mode into a balanced beamsplitter (cf. Ref. [32]). Moreover, this form of entanglement does not require the control of any other light degrees of freedom, such as, e.g., polarization or orbital angular momentum. Therefore the design of simple setups to generate correlations with no classical analog from this state opens new avenues for the implementation of device-independent protocols.

II. THE TRIANGLE NETWORK

The considered Bell-type experiment consists of a triangle causal network where three observers, A, B, and C, receive states prepared by three sources, see Fig. 1. These states are measured producing outcomes a, b, and c with probability $p(abc)$. A classical description of the experiment compatible with the causal constraints defined by the network has the form (here $d\alpha$, $d\beta$, and $d\gamma$ are normalized measures)

$$p(abc) = \int d\alpha d\beta d\gamma \; p_A(a|\beta\gamma)p_B(b|\gamma\alpha)p_C(c|\alpha\beta). \quad (2)$$

The causal model therefore consists of classical variables α, β, and γ distributed by the sources and local response functions p_X, with $X = A,\ B,$ and C, producing the measurement outcomes. In analogy with standard Bell tests, we define probability distribution $p(abc)$ that can be written as Eq. (2) as causally classical or, simpler, *local*.

A quantum description of the experiment compatible with the causal network replaces the random variables by quantum states ρ_α, ρ_β, and ρ_γ and the local response functions by quantum measurements. Therefore quantum probabilities compatible with the triangle network have the form

$$p(abc) = \mathrm{Tr}\big[(\rho_\alpha \otimes \rho_\beta \otimes \rho_\gamma)\big(M_A^{(a)} \otimes M_B^{(b)} \otimes M_C^{(c)}\big)\big], \quad (3)$$

where $M_A^{(a)}$ denote the positive measurement operators defining the positive-operator valued measure (POVM) for A, $\sum_a M_A^{(a)} = \mathbb{1}_A$, and similarly for B and C. We slightly abuse the notation in Eq. (3) by not specifying the tensor products and different Hilbert spaces in which the different operators act, but this is clear from Fig. 1. We say that a quantum experiment, defined by states and measurements producing the outcome distribution $p(abc)$ according to Eq. (3), is *nonlocal* whenever this distribution cannot be described by a classical model (2). Our goal in what follows is to provide a nonlocal quantum experiment in the triangle network using only single-photon entangled states, beamsplitters and photodetectors.

The basic idea of the experimental proposal is depicted in Fig. 1: three parties A, B, and C share, for each pair AB, BC, CA, the single-photon entangled state $|\psi^+\rangle$, see Eq. (1). The initial state is thus

$$|\psi^+\rangle_{A_2B_1} \otimes |\psi^+\rangle_{B_2C_1} \otimes |\psi^+\rangle_{C_2A_1} \equiv |\Psi^+\rangle_{A_1A_2B_1B_2C_1C_2}. \quad (4)$$

Each party then receives its two optical inputs on modes X_1X_2 ($X = A, B, C$) and mixes them with a beamsplitter, which induces a unitary transformation $\mathcal{B}_{X_1X_2}(t, \phi)$ parametrized by its transmissivity t and phase ϕ. All parties use the same value for t, and the phases are all null for simplicity in the following (cf. Ref. [32]).

After passing through the beamsplitters, the photons end up in photodetectors. For each mode X_i, the operators describing a perfectly efficient photodetection correspond to the projectors onto the vacuum state $D^{\square}_{X_i} = |0\rangle\langle 0|_{X_i}$ (detector off) and the projector on its orthogonal complement $D^{\blacksquare}_{X_i} = \mathbb{1}_{X_i} - |0\rangle\langle 0|_{X_i}$ (detector firing). Indeed, we assume that the detectors do not resolve the number of photons but only their presence. The measurement obtained by mixing two modes with the beamsplitter and the ideal photodetectors can be accordingly expressed as a POVM for each party (here $\mathcal{B}_{X_1X_2} = \mathcal{B}_{X_1X_2}(t, 0)$)

$$\begin{aligned}
\Pi_t^{(0)}{}_{X_1X_2} &= \mathcal{B}^{\dagger}_{X_1X_2}\big(D^{\square}_{X_1} \otimes D^{\square}_{X_2}\big)\mathcal{B}_{X_1X_2},\\
\Pi_t^{(L)}{}_{X_1X_2} &= \mathcal{B}^{\dagger}_{X_1X_2}\big(D^{\blacksquare}_{X_1} \otimes D^{\square}_{X_2}\big)\mathcal{B}_{X_1X_2},\\
\Pi_t^{(R)}{}_{X_1X_2} &= \mathcal{B}^{\dagger}_{X_1X_2}\big(D^{\square}_{X_1} \otimes D^{\blacksquare}_{X_2}\big)\mathcal{B}_{X_1X_2},\\
\Pi_t^{(2)}{}_{X_1X_2} &= \mathcal{B}^{\dagger}_{X_1X_2}\big(D^{\blacksquare}_{X_1} \otimes D^{\blacksquare}_{X_2}\big)\mathcal{B}_{X_1X_2},
\end{aligned} \tag{5}$$

where the measurement labels stand respectively for no photon counts (0), a count in the left detector (L), a count in the right detector (R), or counts in both detectors (2). The crucial point is that when $t \neq 0$, the L and R measurements actually detect superpositions of photons in the incoming modes (see details in Ref. [32]).

The quantum experiment described here results in the output distribution

$$\begin{aligned}
&p_t(abc) = \mathrm{Tr}\big[\,|\Psi^+\rangle\langle\Psi^+|\,\big(\Pi_t^{(a)} \otimes \Pi_t^{(b)} \otimes \Pi_t^{(c)}\big)\big],\\
&a, b, c \in \{0, L, R, 2\},
\end{aligned} \tag{6}$$

which depends on the transmissivity t of the beamsplitters used by the parties and whose exact expression can be found in Ref. [32].

III. WITNESSING SINGLE-PHOTON NONLOCALITY

The first main result of this work is that the distribution p_t obtained from the experiment described in Fig. 1 (cf. previous section), is nonlocal (at least) for values of the beamsplitter transmissivity in the intervals $t \in (0, 0.215)$ and $t \in (0.785, 1)$.

We give in the following a sketch of the proof, which is analytical and detailed in Ref. [32]. First, we simplified the structure that classical strategies must follow in the triangle network (2). Specifically, all the local response functions p_A, p_B, and p_C in (2) can be assumed to be deterministic, and all the indeterminacy is therefore delegated to the classical sources $\{\alpha, \beta, \gamma\}$, which can all be assumed to be, w.l.o.g, real numbers uniformly distributed in the interval [0,1]. Therefore any local model is specified by deterministic triangle-local response functions $p_A p_B p_C$ that map all the points of the cube $[0, 1]^3$ to the observed outputs

$$\{\alpha, \beta, \gamma\} \to \{a(\beta, \gamma), b(\gamma, \alpha), c(\alpha, \beta)\}. \tag{7}$$

Secondly, we were able to identify strict constraints that need to be satisfied by all possible classical causal models simulating the considered experimental output $p_t(abc)$ in the triangle network. In particular, we exploited the cyclic symmetry and null components of the distribution. For example, all outputs of the form (here χ represents any of L or R) $\{(000), (00\chi), (2\chi\chi), (22\chi)\}$, or any of their permutations, have zero probability, due to the fact that there are initially three photons in the network, of which at most 2 can end up in the same photodetector. That is, in each run of the experiment the total number of clicks in the detectors must be 2 or 3. By taking all the relevant properties of p_t into account, one can identify constraints that need to be satisfied by any classical strategy, specified by the response functions (7), aiming at reproducing p_t. In fact, while the exact form of the response functions remains in general unknown, some of its marginals can be expressed in terms of the output p_t. These relevant marginals are nothing other than linear constraints on the response functions, parametrized by t. Together with standard normalization and positivity constraints, these define a linear program. The feasibility of such a linear program is, by definition, necessary for the existence of such local response functions. Therefore, when infeasible, no local model exists to simulate our experiment proposal. Results show that the linear program is infeasible for $t \in (0.785, 1)$ and $t \in (0, 0.215)$, proving the claims of this section. We refer to Ref. [32] for the technical details and the complete proof.

The techniques we used are similar to those introduced in Ref. [26] and generalized in Ref. [33]. However, their findings cannot be applied directly to our scenario. The reason behind this is that the works [26,33] are based on a token-counting approach to some physical "tokens" that are (i) generated from the sources, (ii) distributed to the parties in a coherent superposition of different ways, and (iii) counted at the output. In our experiment, the physical tokens are the photons, which however can be miscounted at the output, as more than one could enter in the same photodetector. For these reasons, in the proof [32], we had to extend these techniques so that they could be applied to our setup. As part of the proof, we showed that our distribution is nonlocal if and only if the distribution proposed in [26], which we dub p'_t, is nonlocal as well. While finishing this manuscript, we became aware of preliminary unpublished results [34], which prove nonlocality of p'_t for discrete points in the range $t \in (0.5, 0.785)$ as well. Nonlocality of p'_t in such interval has been conjectured already [35]. Given the above mentioned equivalence between the nonlocality of p_t and p'_t proven in this work, this would imply that the proposed ideal experiment is nonlocal for all transmissivities except $t \in \{0.0, 0.215, 0.5, 0.785, 1.0\}$, which are known to have local models (cf. Refs. [26,32]).

IV. NOISE TOLERANCE AND MACHINE LEARNING ANALYSIS

After proving the nonlocality of the outputs of the ideal noiseless experiment, we analyzed the robustness of our results against typical noise errors, by modeling imperfections which occur in experimental realizations of the optical network presented in Fig. 1. Therefore the resulting output distribution, $p_t^{Q,T,\nu}(abc)$ depends on additional noise parameters quantifying: the impurity of the generated single-photon entangled state (Q), the transmissivity of the optical channels (T) of the network, and the efficiency of the final photodetectors (ν). It follows that

$$p_t^{Q=0,T=1,\nu=1}(abc) \equiv p_t(abc), \tag{8}$$

that is, with no impurity, and perfect transmission and detection, we recover the idealized experiment. The details of the modeling employed are deferred to Ref. [32].

Inevitably, part of the key properties and symmetries of $p_t(abc)$ disappear as soon as noise is introduced in the network. This makes the analytic approach unworkable in this case. Consequently, in order to estimate the tolerance to the noises introduced above, we resorted to a technique recently introduced in Ref. [35]: there, a feed-forward neural network is shaped with the same topology of the causal network under study, and it is then asked to reproduce the target distribution $p_t^{(Q,T,\nu)}$. Each output of the neural network is thus *literally* an instance of a classical model [which can be therefore described by Eq. (2) in our case] trying to reproduce $p_t^{(Q,T,\nu)}$. For a fixed target distribution, the neural network is trained by minimizing the Euclidean distance from the neural network's local model to the target. When the target distribution is inside the local set, a sufficiently large neural network should be capable of learning it. Instead, a large distance between the machine's best guess and the target is taken as an indication of nonlocality. What it means to be "large" enough can be somewhat arbitrary, since some nonlocal behaviors are extremely close to the local set (as is the case here), and additionally the neural network's model is not guaranteed to converge to the optimal solution as it can get stuck in local minima during training. In order to gain deeper insight into the boundary between locality and nonlocality we examine transitions of the learning algorithm's behavior when adding noise to the target distribution, and retraining the machine independently for each target distribution. The very noisy case is guaranteed to be local and the machine learning results on those give a reference to which we can compare the nonlocal regime. By definition, this technique does not certify nonlocality in an absolute way, but has been shown to be reliable and efficient from the point of view of computational resources [35].

The results of the analysis are summarized in Figs. 2 and 3, where we consider only $t \geqslant 0.5$ because of the symmetry of the experiment when mirroring the beamsplitters $t' = 1 - t$. For the noiseless distribution (perfect visibility $r = 1$ in Fig. 2), the neural network's best guess is distant from the experimental output, corroborating the analytical proof of nonlocality for $t \in (0.785, 1)$. At the same time, the neural network hints at the locality of the output distribution for $t = 0.5$ and $t = 1$, which clearly have local strategies. A local model exists as well for $t \sim 0.785$ (cf. Refs. [26,32]) where the neural network struggles to get closer; however, note that the distance of 0.003 achieved there is already *very* close to the local set. Moreover, the same machine indicates (seemingly even stronger) nonlocality in the range $t \in (0.5, 0.785)$, in line with the conjecture of Ref. [35] and the results of Ref. [34].

The noise robustness is, however, small. In Fig. 2, an artificial noise is considered by adding a Werner state visibility to the source (1) of ideal experiment ($Q = 0, T = 1, \nu = 1$).

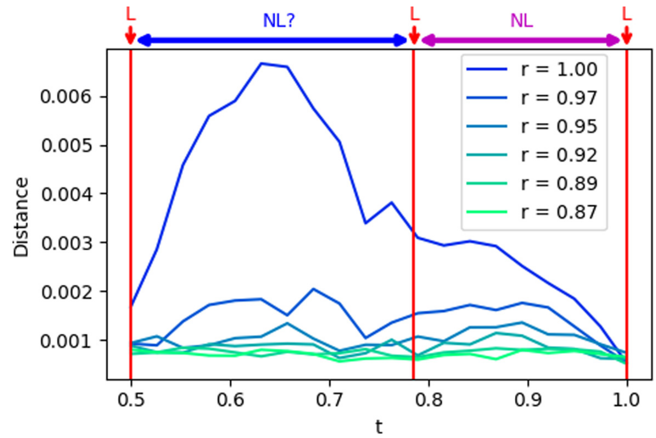

FIG. 2. Euclidean distance of machine learned local models to the target distributions $p_t(abc)$, for various levels of artificial noise on the singlets (1) (visibilities r of Werner states $r|\psi^+\rangle\langle\psi^+| + (1-r)\mathbb{1}/4$). With red vertical lines we depict the transmissivities t at which analytic local models exist ($t \in \{0.5, 0.785, 1\}$). At the top of the figure a purple line shows the regime where we have proven nonlocality, while the blue line shows the regime where we conjecture nonlocality, based on these numerics and the relation to the distribution in Ref. [26], which was studied numerically in Ref. [35].

The neural network seems to indicate that the points that are "most nonlocal" are $t \sim 0.85$ in the proven region (purple interval in Fig. 2), and $t \sim 0.65$ in the conjectured region (blue interval). For these two points we tested the tolerance to the physical noises introduced above, see Fig. 3: choosing $Q \simeq 0,7\%$ (cf. Ref. [32]), the neural network tries to learn $p_t^{(Q,T,\nu)}$ for different values of the transmissivity T and detector efficiency ν. Results show that nonlocality is more robust for $t = 0.65$, where it is lost when $T \lesssim 95\%$ or $\nu \lesssim 95\%$.

All data were obtained by representing each of the three response function ($p_A(a|\gamma\beta)$, $p_B(b|\gamma\alpha)$, $p_C(c|\alpha\beta)$) by a multilayer perceptron of depth 4 and width 20 with rectified linear activation functions. For each target distribution we retrained the neural network independently 30 times and kept the smallest distance among those.

V. DISCUSSION

We have proven how single-photon entangled states can be used to generate an outcome distribution with

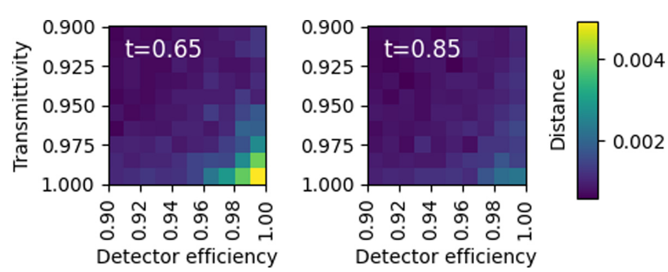

FIG. 3. Euclidean distance of machine learned local models from the noisy distribution $p_t^{(Q,T,\nu)}$ under an experimentally realistic noise model for $t = 0.65$ (left) and 0.85 (right), with $Q = 0.006875$ for both.

no classical analog in a triangle network. The considered setup only requires passive optical elements, namely beamsplitters, phase shifters and photodetectors, and involves a single measurement per observer. Our results not only challenge the current understanding of the nonlocal properties of single-photon entanglement, but also open new perspective for the use of this form of entanglement for quantum information applications, as they provide the first proposal of an experimental demonstration of genuine network nonlocality.

We have shown that the nonlocality of such proposal has (small) noise-tolerance to natural noises that can arise in its implementation, through a machine learning analysis. Such approach is however not exact, and it remains an open question to prove nonlocality in the noisy regime by other means, e.g., certifying it by inflation techniques [36–49], which would be crucial for an experimental implementation.

Finally, in Ref. [32], we show that our main result on the nonlocality of the ideal experimental proposal in the triangle network can be extended to any ring network with $N \geqslant 3$ parties, although increasing the number of parties does not improve the detectability of nonlocality in the proposed experiment with our current techniques.

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