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Single-Site Phase Velocity Measurement

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Summary

Mikumo & Aki attempted to determine phase velocities at a single site using seismometers and strainmeters oriented in the same horizontal direction. For the five earthquakes studied, they found a good agreement with the theoretically predicted velocities for body waves and, in some cases, for surface waves. Rodgers showed that for long periods (> 100 s), horizontal acceleration and tilt cannot easily be separated instrumentally, and King *et al.* have shown that near-station heterogeneities may result in azimuthal effects not considered by Mikumo & Aki. We rederive the method for single-site phase velocity determinations taking into account both of these effects. Our method uses strain and vertical acceleration measurements because the vertical acceleration record is neither affected by site effects nor contaminated by tilt.

Traditional methods for determining the velocities of seismic waves depend on the use of sets of widely-spaced seismometers. These instruments record simultaneously, and the velocity determination depends on measuring the travel times of different frequencies over known distances. For low frequency long wavelength waves, the separation of instruments must be large to permit satisfactory resolution, and at shorter wavelengths large spacing is an advantage as it prevents velocity estimates being contaminated by the small scale phase anomalies which result from refraction and diffraction scattering. The best results for phase velocity using these methods are only obtained when two or more stations lie close to a great circle including the seismic epicentre. This problem has been discussed by Brune & Dorman (1963).

We describe a method whereby it is, in principle, possible to determine the phase velocity of any seismic wave from the simultaneous measurement of a displacement (or its time derivative) and a spatial derivative (strain or tilt) at the same site. Mikumo & Aki (1964) have previously used this technique by the combined analysis of records from a *horizontal* seismometer and a horizontal strainmeter set up at a single station. For the five earthquakes studied they found a good agreement between measured and predicted phase velocities for body waves and in some cases for surface waves.

Since their work, Rodgers (1968) has shown that for long periods (> 100 s), horizontal acceleration and tilt cannot easily be separated instrumentally, and recent work by King *et al.* (1976) indicates that near-station crustal inhomogeneity may give rise to strain-strain and strain-tilt interactions. Thus, records from long-period horizontal seismometers are heavily contaminated by tilt and strain. Records written by vertical seismometers do not suffer this defect. In the present paper we rederive the principle of single-site velocity determination, taking into account the strain-coupling effects noted by King *et al.* and using vertical acceleration. Our discussion is presented for Rayleigh waves, and assumes that the ellipticity of particle motion at the surface is independent of frequency. The validity of this assumption may be tested if there is a site-calibrated horizontal seismometer at the same site.

The acceleration amplitude at the surface ($x_3 = 0$) associated with a Rayleigh wave travelling in a direction θ relative to a fixed co-ordinate frame (see Fig. 1) are:

$$\ddot{u}(\theta, \omega) = -\omega^2 \, a\mathbf{R}^T(\theta) \begin{bmatrix} \kappa \\ 0 \\ i \end{bmatrix} \tag{1}$$

and the strain components are:

$$\mathbf{e}(\theta, \omega) = ik\kappa a \, \mathbf{R}^T(\theta) \, \mathbf{W}\mathbf{R}(\theta), \tag{2}$$

where a is an amplitude constant, ω is the angular frequency, $k = \omega/c$ where c is the Rayleigh wave phase velocity, i is the imaginary unit and κ, α and β are respectively the ellipticity of particle motion and the P- and S-wave velocities at the surface. $\mathbf{R}(\theta)$ is the axis transformation:

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\mathbf{R}^T$ is the transpose of $\mathbf{R}$, and

$$\mathbf{W} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2(\beta^2/\alpha^2)-1 \end{bmatrix}$$

Let us define $\Omega_{ij}(\theta, \omega)$ as the ratio of the ijth component of the strain tensor $\mathbf{e}$ to the vertical acceleration $\ddot{u}_3(\theta, \omega)$:

$$\Omega_{ij}(\theta, \omega) = \frac{e_{ij}(\theta, \omega)}{\ddot{u}_3(\theta, \omega)}$$

$$= \frac{-\kappa}{\omega c} \, T_{ij}(\theta, \omega) \tag{3}$$

where $T_{ij}(\theta, \omega) = \big(\mathbf{R}^T(\theta) \, \mathbf{W}\mathbf{R}(\theta)\big)_{ij}$.

Hence, given any non-zero component of the strain, the vertical acceleration and the ellipticity, one can solve for the phase velocity as a function of frequency:

$$c(\omega) = \frac{-\kappa}{\omega \Omega_{ij}(\theta, \omega)} \, T_{ij}(\theta, \omega) \qquad \text{no sum on } i \text{ and } j. \tag{4}$$

The horizontal linear strain in direction λ (see Fig. 1) is $L(\lambda) = e_{11} \cos \lambda + e_{12}{}^2 \sin 2\lambda + e_{22} \sin^2 \lambda$, and the areal and volume strains, which are independnet of θ, are given by $A = e_{11} + e_{22}$ and $V = A + e_{33}$ respectively.

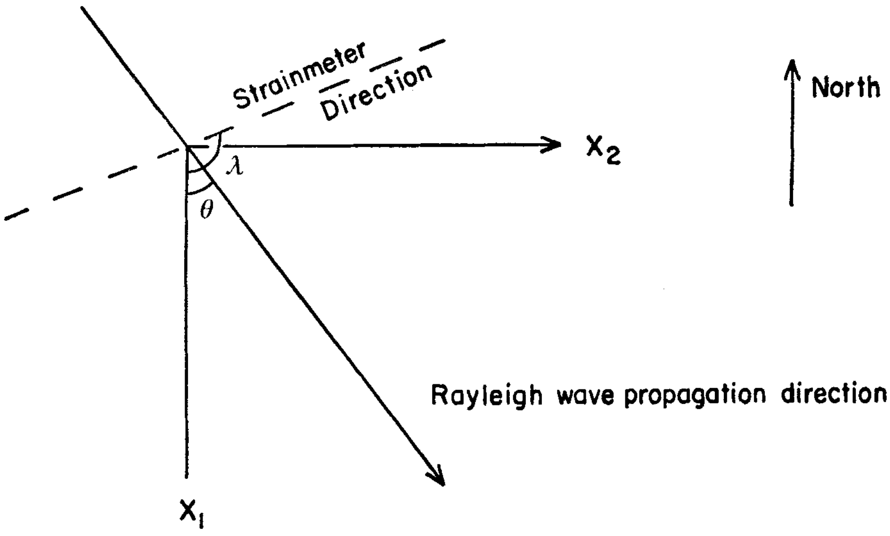

FIG. 1. Axes x_1, x_2 and x_3 are the normal spherical polar axes. e_θ, e_ϕ and e_r. θ and λ are angles describing Rayleigh wave propagation direction and linear strainmeter direction respectively.

Defining

$$\Omega^L(\lambda, \theta, \omega) = \frac{L(\lambda, \theta, \omega)}{\ddot{u}_3(\theta, \omega)} \tag{5}$$

$$\Omega^A(\omega) = \frac{A(\omega)}{\ddot{u}_3(\theta, \omega)} \tag{6}$$

and

$$\Omega^V(\omega) = \frac{V(\omega)}{\ddot{u}_3(\theta, \omega)} \tag{7}$$

one finds

$$c(\omega) = \frac{-\kappa \cos^2(\lambda-\theta)}{\omega\Omega^L(\lambda, \theta, \omega)} = \frac{-\kappa}{\omega\Omega^A(\omega)} = \frac{-2\beta^2 \kappa}{\alpha^2 \omega\Omega^V(\omega)}. \tag{8}$$

The simple formulation above can suffer from a major defect in the real earth. Although it is possible to achieve the necessary instrumental calibration accuracy for both strainmeters and vertical accelerometers, the former are very sensitive to lateral inhomogeneities in elastic properties close to the site. The general problem of strain tensor distortion due to departures from horizontally layered (radially stratified) earth models was discussed by King (1971) and the specific effects of particular types of site have been examined (with reference to tidal signals) by King & Bilham (1973), Beaumont & Berger (1974, 1976), Harrison (1975) and Itsueli *et al.* (1975). A recent paper by King *et al.* (1976) points out that $\ddot{u}_3$ is unaffected by these effects and also that the strain site effect may be described by a fourth-order tensor **B** with nine independent coefficients:

$$\boldsymbol{\varepsilon}(\omega) = \mathbf{e}(\omega) + \mathbf{Be}(\omega)$$

or

$$\varepsilon_{ij}(\omega) = (\delta_{ik}\delta_{jl} + B_{ijkl})\, e_{kl}(\omega) \tag{9}$$

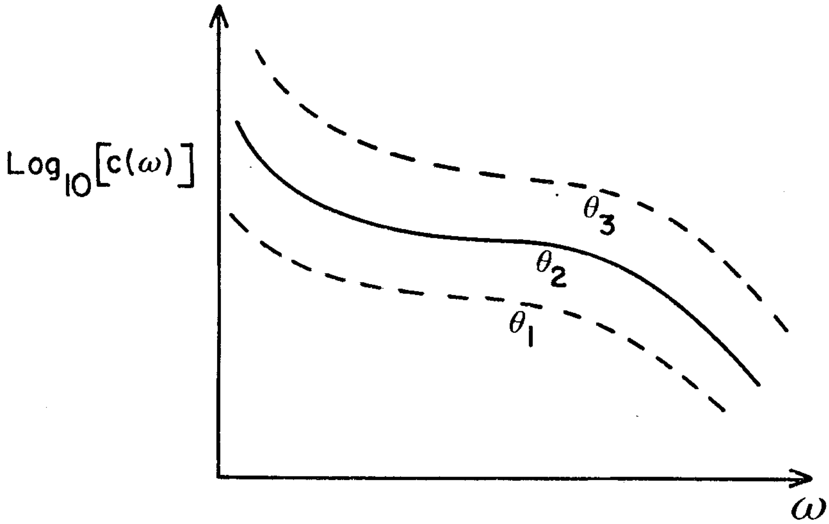

FIG. 2. Idealized plots of $\log_{10}\{c(\Omega)\}$ for Rayleigh waves arriving from three different directions θ_1, θ_2 and θ_3. The curves are shifted vertically relative to each other by the site effect, but their shape remains unchanged.

$\boldsymbol{\varepsilon}$ is the observed strain tensor and $\mathbf{e}$ is the tensor averaged over a sufficiently large area. Sufficiently large in this context means an area whose dimension in any direction is large compared to the dimension of the site inhomogeneity, yet small compared to the wavelength of the signal. This, and certain other important constraints on the validity of equation (9) are discussed by King *et al.* (1976).

For the Rayleigh wave example described above, the observed strain will be:

$$\varepsilon_{ij}(\theta, \omega) = ik\kappa a\, T'_{ij}(\theta) \tag{10}$$

where

$$T'_{ij} = T_{ij} + B_{ijkl}\, T_{kl} = T_{ij} + B_{ij11}\cos^2\theta + B_{ij12}\sin 2\theta + B_{ij22}\sin^2\theta \tag{11}$$

Using equation (10) in defining Ω_{ij} gives:

$$c(\omega) = \frac{-\kappa}{\omega\Omega_{ij}(\theta, \omega)}\, T'_{ij}(\theta) \qquad \text{(no sum on } i \text{ and } j\text{)} \tag{12}$$

instead of equation (4).

The horizontal linear strain is now $L'(\lambda) = (\varepsilon_{11}\cos^2\lambda + \varepsilon_{12}\sin 2\lambda + \varepsilon_{22}\sin^2\lambda) = ik\kappa a\big(D_{11}(\lambda)\cos^2\theta + D_{12}(\lambda)\sin 2\theta + D_{22}(\lambda)\sin^2\theta\big)$, where the D_{kl} are constants, assumed independent of frequency, for a particular linear strainmeter in direction λ, and are related to B by:

$$D_{kl}(\lambda) = (I+B)_{11kl}\cos^2\lambda + (I+B)_{12kl}\sin 2\lambda + (I+B)_{22kl}\sin^2\lambda. \tag{13}$$

The areal strain is $A' = \varepsilon_{11} + \varepsilon_{22} = ik\kappa a(E_{11}\cos^2\theta + E_{12}\sin 2\theta + E_{22}\sin^2\theta)$, and the volume strain is $V' = A' + \varepsilon_{33} = ik\kappa a\,(2\beta^2/\alpha^2)\,(F_{11}\cos^2\theta + F_{12}\sin 2\theta + F_{22}\sin^2\theta)$, where the E_{ij} or F_{ij} are constants for a particular areal or volume strainmeter, and are related to B by:

$$\begin{aligned} E_{kl} &= \delta_{kl} + B_{11kl} + B_{22kl} \\ F_{kl} &= \delta_{kl} + B_{11kl} + B_{22kl} + B_{33kl}. \end{aligned} \tag{14}$$

Then, instead of equations (8), we have:

$$c(\omega) = \frac{-\kappa}{\omega\Omega^L(\lambda, \theta, \omega)}(D_{11}(\lambda)\cos^2\theta + D_{12}(\lambda)\sin 2\theta + D_{22}(\lambda)\sin^2\theta)$$

$$= \frac{-\kappa}{\omega\Omega^A(\theta, \omega)}(E_{11}\cos^2\theta + E_{12}\sin 2\theta + E_{22}\sin^2\theta)$$

$$= \frac{-\kappa}{\omega\Omega^V(\theta, \omega)}(F_{11}\cos^2\theta + F_{12}\sin 2\theta + F_{22}\sin^2\theta). \quad (15)$$

In a heterogeneous half space, the ratio of the strain and the vertical acceleration is not only related to the ellipticity and the phase velocity, as in the homogeneous case, but also to θ, the azimuth of the Rayleigh waves and to the $D_{kl}(\lambda)$, E_{kl} or F_{kl} as appropriate. For any azimuth, therefore, the phase velocity–period curve (Fig. 2) will have some unknown multiplier, γ, acting on the velocity scale; in other words, we determine $c(\omega)/\gamma$ where γ is given by:

$$\gamma^L(\lambda, \theta) = (D_{11}\cos^2\theta + D_{12}\sin 2\theta + D_{22}\sin^2\theta)/\cos^2(\theta-\lambda) \quad \text{for linear strain,}$$

$$\gamma^A(\theta) = E_{11}\cos^2\theta + E_{12}\sin 2\theta + E_{22}\sin^2\theta \quad \text{for areal strain,}$$

$$\gamma^V(\theta) = F_{11}\cos^2\theta + F_{12}\sin 2\theta + F_{22}\sin^2\theta \quad \text{for volume strain.}$$

A method for evaluating the site coefficients B_{ijkl} (or the subsidiary coefficients D_{kl}, E_{kl} or F_{kl}) using tidal observations was described by King *et al.* (1976), and can be applied if the ocean load effect is known, or is known to be small. The function $\gamma(\theta)$ may then be calculated and $c(\omega)$ evaluated absolutely.

Equally, if $c(\omega)$ can be determined by other methods for some particular frequency, then the phase velocities at other frequencies may be determined and also the site coefficients.

The usefulness of this method lies in the fact that ‘ local ’ effects (those generated over distances less than the wavelength considered) are discounted. Thus the technique will be of particular value in determining the structure of the upper mantle, especially in those regions where the crustal structure is already known.

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