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Single-Spin Polarization Effects and the Determination of Timelike Proton Form Factors*

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Abstract

We show that measurements of the proton's polarization in $e^+e^- \rightarrow p\bar{p}$ strongly discriminate between analytic forms of models which fit the proton form factors in the spacelike region. In particular, the single-spin asymmetry normal to the scattering plane measures the relative phase difference between the timelike G_E and G_M form factors. The expected proton polarization in the timelike region is large, of order of several tens of percent.

1 Introduction

The form factors of hadrons as measured in both the spacelike and timelike domains provide fundamental information on the structure and internal dynamics of hadrons. Recent measurements [1] of the electron-to-proton polarization transfer in $\vec{e}^- p \rightarrow e^- \vec{p}$ scattering at Jefferson Laboratory show that the ratio of Sachs form factors [2] $G_E^p(q^2)/G_M^p(q^2)$ is monotonically decreasing with increasing $Q^2 = -q^2$, in strong contradiction with the G_E/G_M scaling determined by the traditional Rosenbluth separation method. The Rosenbluth method may in fact not be reliable, perhaps because of its sensitivity to uncertain radiative corrections, including two-photon exchange amplitudes [3]. The polarization transfer method [1, 4] is relatively insensitive to such corrections.

The same data which indicate that G_E for protons falls faster than G_M at large spacelike Q^2 require in turn that F_2/F_1 falls more slowly than $1/Q^2$. The conventional expectation from dimensional counting rules [5] and perturbative QCD [6] is that the Dirac form factor F_1 should fall with a nominal power $1/Q^4$, and the ratio of the Pauli and Dirac form factors, F_2/F_1 , should fall like $1/Q^2$, at high momentum transfers. The Dirac form factor agrees with this expectation in the range Q^2 from a few GeV^2 to the data limit of 31 GeV^2 . However, the Pauli/Dirac ratio is not observed to fall with the nominal expected power, and the experimenters themselves have noted that the data is well fit by $F_2/F_1 \propto 1/Q$ in the momentum transfer range 2 to 5.6 GeV^2 .

The new Jefferson Laboratory results make it critical to carefully identify and separate the timelike G_E and G_M form factors by measuring the center-of-mass angular distribution and by measuring the polarization of the proton in $e^+e^- \rightarrow p\bar{p}$ or $p\bar{p} \rightarrow \ell^+\ell^-$ reactions. The advent of high luminosity e^+e^- colliders at Beijing, Cornell, and Frascati provide the opportunity to make such measurements, both directly and via radiative return.

Although the spacelike form factors of a stable hadron are real, the timelike form factors have a phase structure reflecting the final-state interactions of the outgoing hadrons. In general, form factors are analytic functions $F_i(q^2)$ with a discontinuity for timelike momentum above the physical threshold $q^2 > 4M^2$. The analytic structure

and phases of the form factors in the timelike regime are thus connected by dispersion relations to the spacelike regime [7, 8, 9]. The analytic form and phases of the timelike amplitudes also reflects resonances in the unphysical region $0 < q^2 < 4M^2$ below the physical threshold [7] in the $J^{PC} = 1^{--}$ channel, including gluonium states and di-baryon structures.

At very large center-of-mass energies, perturbative QCD factorization predicts diminished final interactions in $e^+e^- \rightarrow H\bar{H}$, since the hadrons are initially produced with small color dipole moments. This principle of QCD color transparency [10] is also an essential feature [11] of hard exclusive B decays [12, 13], and thus needs to be tested experimentally.

There have been a number of explanations and theoretically motivated fits of the F_2/F_1 data. Belitsky, Ji, and Yuan [14] have shown that factors of $\log(Q^2)$ arise from a careful QCD analysis of the form factors. The perturbative QCD form $Q^2 F_2/F_1 \sim \log^2 Q^2$, which has logarithmic factors multiplying the nominal power-law behavior, fits the large- Q^2 spacelike data well. Others [17, 18] claim to find mechanisms that modify the traditionally expected power-law behavior with fractional powers of Q^2 , and they also give fits which are in accord with the data. Asymptotic behaviors of the ratio F_2/F_1 for general light-front wave functions are investigated in [15]. Each of the model forms predicts a specific fall-off and phase structure of the form factors from $s \leftrightarrow t$ crossing to the timelike domain. A fit with the dipole polynomial or nominal dimensional counting rule behavior would predict no phases in the timelike regime.

As noted by Dubnickova, Dubnicka, and Rekalov, and by Rock [16], the existence of the T -odd single-spin asymmetry normal to the scattering plane in baryon pair production $e^-e^+ \rightarrow B\bar{B}$ requires a nonzero phase difference between the G_E and G_M form factors. The phase of the ratio of form factors G_E/G_M of spin-1/2 baryons in the timelike region can thus be determined from measurements of the polarization of one of the produced baryons. We shall show that measurements of the proton polarization in $e^+e^- \rightarrow p\bar{p}$ strongly discriminate between the analytic forms of models which have been suggested to fit the proton G_E/G_M data in the spacelike region.

2 Timelike Measures

The center-of-mass angular distribution provides the analog of the Rosenbluth method for measuring the magnitudes of various helicity amplitudes. The differential cross section for $e^-e^+ \rightarrow B\bar{B}$ when B is a spin-1/2 baryon is given in the center-of-mass frame by

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\beta}{4q^2} D, \quad (1)$$

where $\beta = \sqrt{1 - 4m_B^2/q^2}$ and D is given by

$$D = |G_M|^2 (1 + \cos^2 \theta) + \frac{1}{\tau} |G_E|^2 \sin^2 \theta; \quad (2)$$

we have used the Sachs form factors [2]

$$\begin{aligned} G_M &= F_1 + F_2 , \\ G_E &= F_1 + \tau F_2 , \end{aligned} \tag{3}$$

with $\tau \equiv q^2/4m_B^2 > 1$.

As we shall show, polarization observables can be used to completely pin down the relative phases of the timelike form factors. The complex phases of the form factors in the timelike region make it possible for a single outgoing baryon to be polarized in $e^-e^+ \rightarrow B\bar{B}$, even without polarization in the initial state.

There are three polarization observables, corresponding to polarizations in three directions which are perhaps best called longitudinal, sideways, and normal but often denoted z , x , and y , respectively. Longitudinal (z) when discussing the final state means parallel to the direction of the outgoing baryon. Sideways (x) means perpendicular to the direction of the outgoing baryon but in the scattering plane. Normal (y) means normal to the scattering plane, in the direction of $\vec{k} \times \vec{p}$ where \vec{k} is the electron momentum and \vec{p} is the baryon momentum, with x , y , and z forming a right-handed coordinate system.

The polarization \mathcal{P}_y does not require polarization in the initial state and is [16]

$$\mathcal{P}_y = \frac{\sin 2\theta \operatorname{Im} G_E^* G_M}{D\sqrt{\tau}} = \frac{(\tau - 1) \sin 2\theta \operatorname{Im} F_2^* F_1}{D\sqrt{\tau}} . \tag{4}$$

The other two polarizations require initial state polarization. If the electron has polarization P_e then [16]

$$\mathcal{P}_x = -P_e \frac{2 \sin \theta \operatorname{Re} G_E^* G_M}{D\sqrt{\tau}} , \tag{5}$$

and

$$\mathcal{P}_z = P_e \frac{2 \cos \theta |G_M|^2}{D} . \tag{6}$$

The sign of \mathcal{P}_z can be determined from physical principles. Angular momentum conservation and helicity conservation for the electron and positron determine that \mathcal{P}_z/P_e in the forward direction must be +1, verifying the sign of the above formula.

The polarization measurement in $e^+e^- \rightarrow p\bar{p}$ will require a polarimeter for the outgoing protons, perhaps based on a shell of a material such as carbon which has a good analyzing power. However, timelike baryon-antibaryon production can occur for any pair that is energetically allowed. Baryons such as the Σ and Λ which decay weakly are easier to study, since their polarization is self-analyzing.

Polarization \mathcal{P}_y is a manifestation of the T-odd observable $\vec{k} \times \vec{p} \cdot \vec{S}_p$, with \vec{S}_p the proton polarization. This observable is zero in the spacelike case, but need not be zero in the timelike case because final state interactions can give the form factors a relative phase.

Notice the factor $\sin 2\theta$. Without polarization in the initial state, and with single photon exchange, the only information transferred to the final state is the total energy plus information for the photon's polarization about the line—undirected—of the electron and positron momentum in the initial state. Similarly, without using polarization in the final state, we can use the undirected line of the baryon momenta. We can define a directed normal by taking a cross product, providing a direction by rotating from the lepton to the hadron direction through the smaller angle. The observable is the dot product of this directed normal with the baryon polarization. At 0° or 90° , one cannot define a directed normal, hence one cannot obtain nonzero polarization at these two angles, as reflected in the $\sin 2\theta$ factor.

Any model which fits the spacelike form factor data with an analytic function can be continued to the timelike region. Spacelike form factors are usually written in terms of $Q^2 = -q^2$. The correct relation for analytic continuation can be obtained by examining denominators in loop calculations in perturbation theory. The connection is $Q^2 \rightarrow q^2 e^{-i\pi}$, or

$$\ln Q^2 = \ln(-q^2) \rightarrow \ln q^2 - i\pi . \quad (7)$$

If the spacelike F_2/F_1 is fit by a rational function of Q^2 , then the form factors will be relatively real in the timelike region also. However, one in general gets a complex result from the continuation.

More sophisticated dispersion relation based continuations could give more reliable results, if there is data also in the timelike region to pin down the magnitudes there. So far, this is possible for the magnetic form factor alone [7] but not for both form factors.

3 Polarization in the timelike region

We begin by selecting some existing fits to the spacelike data. Since we are concentrating on the polarizations, which depend only on the ratios of the form factors, we concentrate in turn on fits to the ratio F_2/F_1 , rather than fits to the individual form factors. We attempt to present a representative selection of fits, and refer to others that are similar to the ones included.

Odd- Q fits. The JLab experimenters themselves note that the polarization transfer data is well fit for Q^2 in the 2 to 5.6 GeV² region by

$$\frac{F_2}{F_1} = \frac{1.25 \text{ GeV}}{Q} . \quad (8)$$

There is theoretical work which obtains similar forms [17, 18, 19]. Because of its simple analyticity, this form becomes purely imaginary in the timelike region. Simply to get the right ratio at $Q^2 = 0$, we choose to modify this form to

$$\frac{F_2}{F_1} = \left(\frac{1}{\kappa_p^2} + \frac{Q^2}{(1.25 \text{ GeV})^2} \right)^{-1/2} , \quad (9)$$

where $\kappa_p = 1.79$ gives the anomalous magnetic moment of the proton. The numerical effect of the $1/\kappa_p^2$ term is hardly noticeable for Q^2 above 2 GeV².

Fits involving logarithms. A number of authors [14, 15] have given fits to the F_2/F_1 data which have the power law fall-off expected from QCD, with logarithmic corrections that enable a good fit to the data. Belitsky, Ji, and Yuan [14] have motivated a form that has two powers of $\ln Q^2$, and one of their fits is, with $\Lambda = 300$ MeV,

$$\frac{F_2}{F_1} = 0.17 \text{ GeV}^2 \frac{\ln^2(Q^2/\Lambda^2)}{Q^2}. \quad (10)$$

We give here an improved fit which matches the above asymptotically and also matches low Q^2 data,

$$\frac{F_2}{F_1} = \kappa_p \frac{[1 + (Q^2/0.791 \text{ GeV}^2)^2 \ln^{7.1}(1 + Q^2/4m_\pi^2)]}{[1 + (Q^2/0.380 \text{ GeV}^2)^3 \ln^{5.1}(1 + Q^2/4m_\pi^2)]}. \quad (11)$$

This last form also contains the cut at the two-pion threshold in the timelike region.

Two-component fits. In 1973, Iachello, Jackson, and Lande [20] presented a model for the nucleon form factors based on a two-component, core and meson cloud, structure for the nucleon with parameters fit to the then existing data. The fit was updated by Gari and Krumpelmann [21] and later by Lomon [22]. Iachello [23] has recently noted that one of the original fits accords well with the newest JLab data. Continued for timelike q^2 , the fit is

$$\begin{aligned} F_1 &= \frac{1}{2}g \left[(1 - \beta_\omega - \beta_\phi) - \beta_\omega \frac{m_\omega^2}{q^2 - m_\omega^2} - \beta_\phi \frac{m_\phi^2}{q^2 - m_\phi^2} \right. \\ &\quad \left. + (1 - \beta_\rho) - \beta_\rho \frac{m_\rho^2 + 8\Gamma_\rho m_\pi/\pi}{q^2 - m_\rho^2 + (q^2 - 4m_\pi^2)\Gamma_\rho \alpha(q^2)/m_\pi} \right], \\ F_2 &= \frac{1}{2}g \left[(0.120 + \alpha_\phi) \frac{m_\omega^2}{q^2 - m_\omega^2} - \alpha_\phi \frac{m_\phi^2}{q^2 - m_\phi^2} \right. \\ &\quad \left. - 3.706 \frac{m_\rho^2 + 8\Gamma_\rho m_\pi/\pi}{q^2 - m_\rho^2 + (q^2 - 4m_\pi^2)\Gamma_\rho \alpha(q^2)/m_\pi} \right], \end{aligned} \quad (12)$$

where

$$\begin{aligned} \alpha(q^2) &= \left(\frac{q^2 - 4m_\pi^2}{q^2} \right)^{1/2} \\ &\quad \times \left\{ \frac{2}{\pi} \ln \left(\frac{\sqrt{q^2 - 4m_\pi^2} + \sqrt{q^2}}{2m_\pi} \right) - i \right\}. \end{aligned} \quad (13)$$

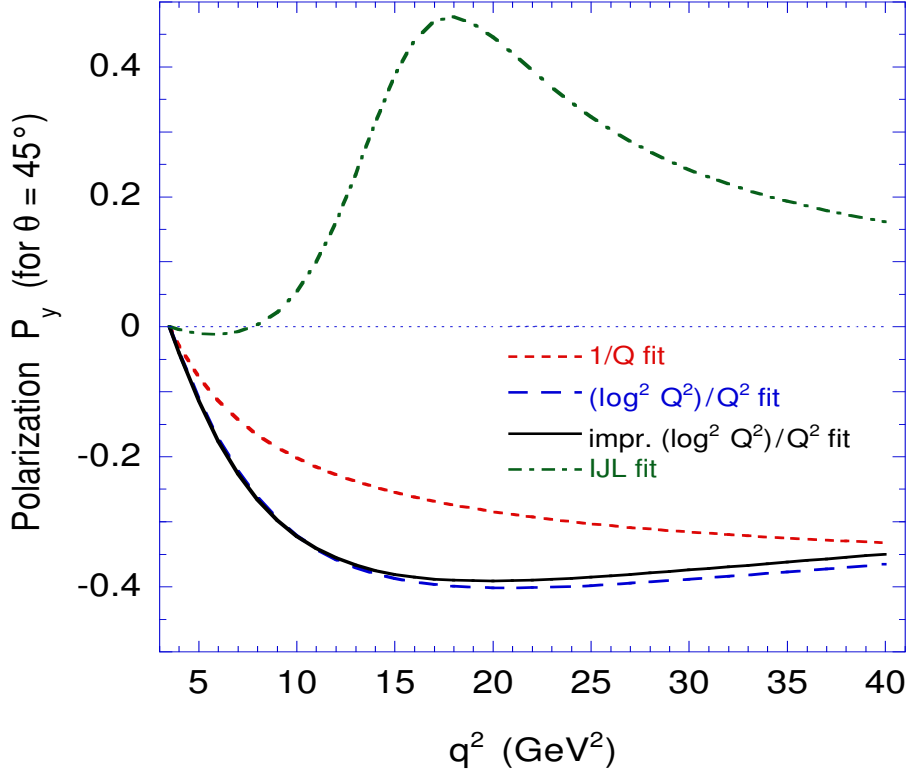


Figure 1: Predicted polarization \mathcal{P}_y in the timelike region for selected form factor fits described in the text. The plot is for $\theta = 45^\circ$. The four curves are for an $F_2/F_1 \propto 1/Q$ fit, using Eq. (9); the $(\log^2 Q^2)/Q^2$ fit of Belitsky *et al.*, Eq. (10); an improved $(\log^2 Q^2)/Q^2$ fit, Eq. (11); and a fit from Iachello *et al.*, Eq. (12).

The function $g = g(q^2)$ cancels in expressions for polarizations. The parameters are $\beta_\rho = 0.672$, $\beta_\omega = 1.102$, $\beta_\phi = 0.112$, $\alpha_\phi = -0.052$, $m_\rho = 0.765$ GeV, $m_\omega = 0.784$ GeV, $m_\phi = 1.019$ GeV, and $\Gamma_\rho = 0.112$ GeV.

Iachello [23] has also discussed extending the fits to the timelike region, and finds a complex phase from two sources. One source is a modification of the overall factor $g(q^2)$. The overall factor has no effect on G_E/G_M and no effect on quantities like polarizations that only depend on ratios. The other source of phase is the treatment of the rho widths. The phi and omega were approximated as zero width, but IJL [20] found that rho-width contributions were important for fitting data and incorporated a two-pion cut into an effective width term in the rho propagator. The extension to the timelike region seen above is a straightforward and expected analytic continuation of this term.

The expression for polarization \mathcal{P}_y , Eq. (4), leads to results shown in Fig. 1. The polarizations are shown for four fits listed above, and the polarizations are not small. They are very distinct from a purely polynomial fit to the spacelike data, which gives

zero \mathcal{P}_y .

The predictions for \mathcal{P}_x and \mathcal{P}_z are shown in Figs. 2 and 3. Both figures are for scattering angle 45° and $P_e = 1$. The phase difference $(\delta_E - \delta_M)$ between G_E and G_M is directly given by the $\mathcal{P}_y/\mathcal{P}_x$ ratio,

$$\frac{\mathcal{P}_y}{\mathcal{P}_x} = \frac{\cos \theta}{P_e} \frac{\text{Im } G_M^* G_E}{\text{Re } G_M^* G_E} = \frac{\cos \theta}{P_e} \tan(\delta_E - \delta_M) . \quad (14)$$

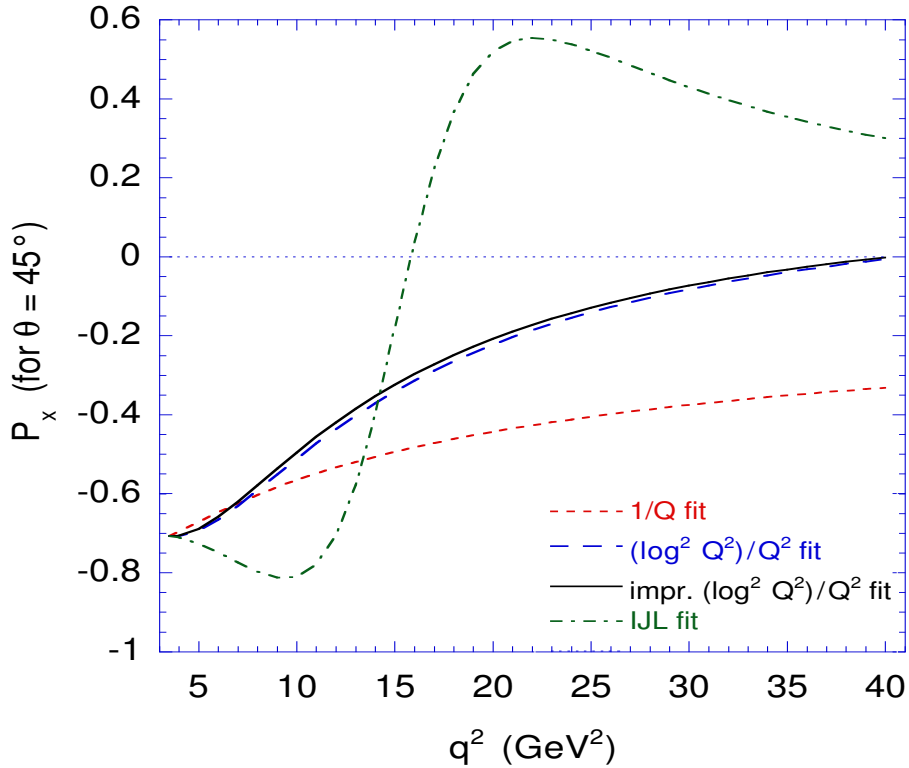


Figure 2: The predicted polarization \mathcal{P}_x in the timelike region for $\theta = 45^\circ$ and $P_e = 1$. The four curves correspond to those in Fig. 1.

The magnetic form factor in the IJL model is very small in the 10 to 20 GeV^2 region (taking the dipole form for comparison) and has a zero in the complex plane near $q^2 = 15 \text{ GeV}^2$. This accounts for much of the different behavior of the IJL model seen in the polarization plots. That the IJL ratio for G_E/G_M is strikingly large even by the standard set by the other three models also strongly affects the angular behavior of the differential cross section. This is witnessed by Fig. 4, which shows the angular behavior of $d\sigma/d\Omega$ for $q^2 = 10 \text{ GeV}^2$. The lower three models are also showing significant contributions from G_E ; at 90° , the difference between the curves shown and the value 0.5 is entirely due to $|G_E|^2$.

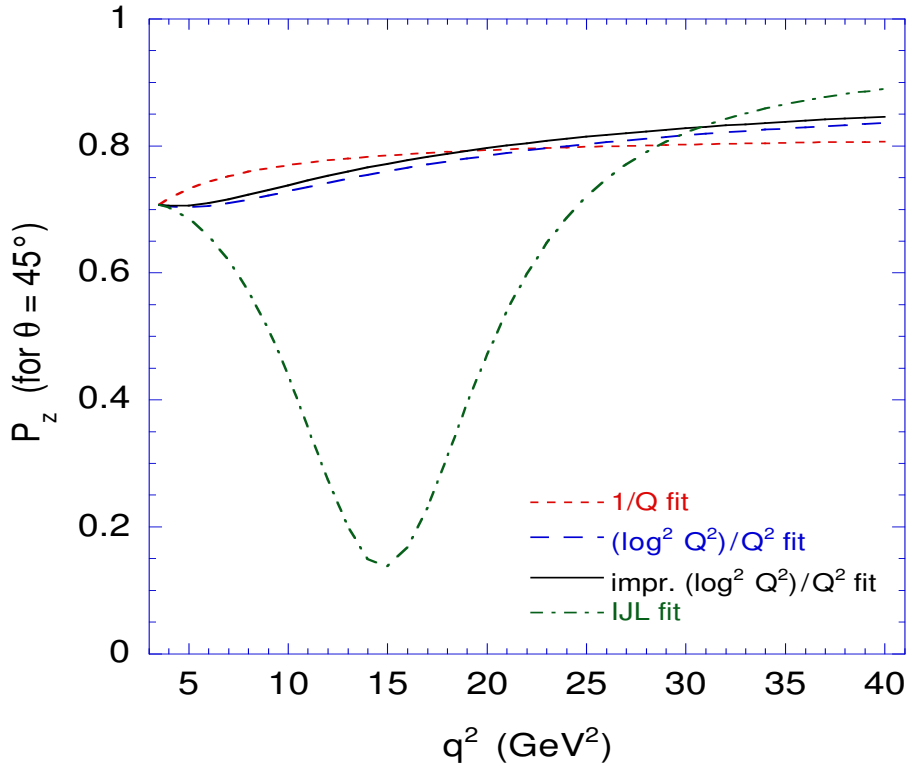


Figure 3: The predicted polarization \mathcal{P}_z in the timelike region for $\theta = 45^\circ$ and $P_e = 1$. The four curves correspond to those in Fig. 1.

4 Conclusions

We have discussed how to measure baryon form factors in the timelike region using polarization observables. Observing the baryon polarization in $e^-e^+ \rightarrow B\bar{B}$ for spin-1/2 baryons B may be the method of choice for determining the magnitude and the phase of the form factor ratio G_E/G_M . In the spacelike region, one recalls that at high Q^2 , the electric form factor makes a small contribution to the cross section, and the Rosenbluth method of separating it from the magnetic form factor, by its different angular dependence, is very sensitive to experimental uncertainties and radiative corrections [3]. The more direct method is to use polarization transfer [1, 4]. Similarly, in the timelike case, the angular distribution can be used to isolate $|G_E|$, but the numerical size of the G_E contribution is small in many models, whereas two of the three polarization observables are directly proportional to G_E . Additionally, the phase can only be measured using polarization.

The normal polarization \mathcal{P}_y is a single-spin asymmetry and requires a phase difference between G_E and G_M . It is an example of how time-reversal-odd observables can be nonzero if final state interactions give interfering amplitudes different phases.

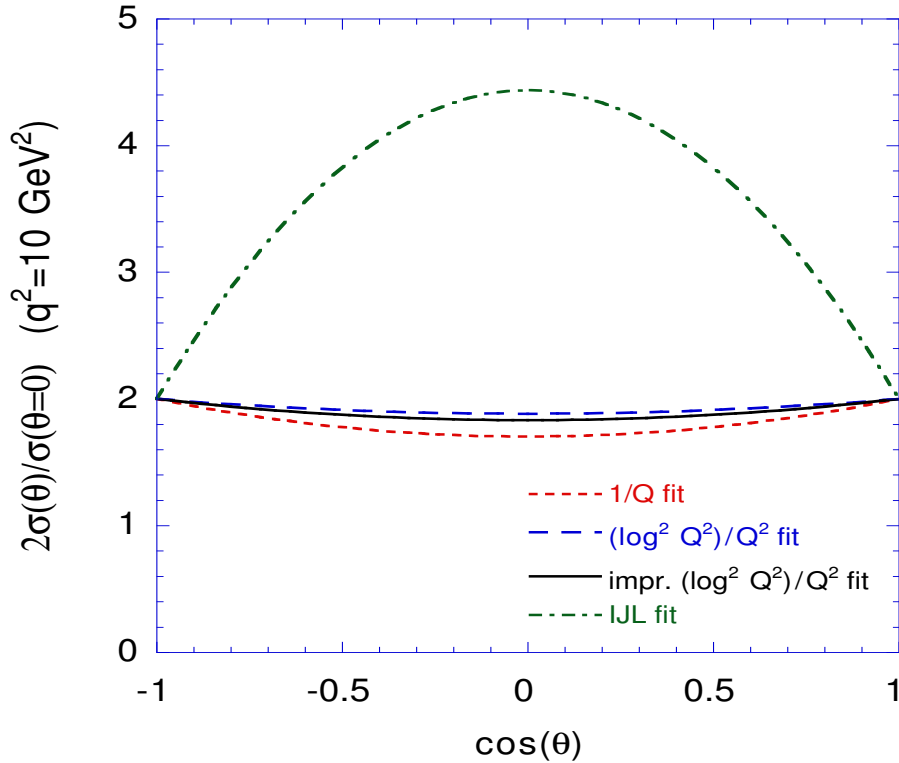


Figure 4: The predicted differential cross section $\sigma(\theta) \equiv d\sigma/d\Omega$. The four curves correspond to those in Fig. 1.

Its analog in the spacelike case is zero.

A strong current motivation for further baryon form factor study is the intriguing spacelike JLab data for F_2/F_1 or G_E/G_M on the proton [1]. We have selected a number of fits to this spacelike data, continued them to the timelike region, and predicted what size polarizations one may expect to see there. For the models we have examined the predicted polarizations are large and distinctive and should encourage experimental study.

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