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Single valued neutrosophic multisets

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Abstract. In this paper, we have investigated single valued neutrosophic multisets in detail. Several operations have been defined on them and their important algebraic properties are studied.We have further introduced the notion of distance and similarity measures between two single valued neutrosophic multisets. An application of single valued neutrosophic multisets in medical diagnosis has been discussed.

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1. Introduction

The concept of multisets stemmed from the violation of one of the basic properties of classical set theory, which states that an element can occur in a set only once. The term "multiset" was first introduced by N. G. Bruijn [8]. Multisets, often referred to as "bags", are collections of objects that may contain a finite number of duplicates. Multisets are mathematical structures that come handy in areas like database enquiries related to computer science. Also, in dealing with the problems of constructing mathematical models for real-life situations, the data at hand are mainly imprecise and indeterministic. In 1965, Zadeh [31] came up with his remarkable theory of fuzzy sets where he introduced the notion of partial belongingness of an element in a set. Later these two concepts have been combined to generate Fuzzy Multisets [18] and were applied in many areas of computer science. After the introduction of Intuitionistic fuzzy sets by Atanassov [4, 5] in 1986, the theory of Intuitionistic Fuzzy Multisets [21] have been developed. On the other hand, in 1995, Florentin Smarandache incorporated the concept of Neutrosophic Logic [22, 23] which sprouted from a branch of philosophy, known as "Neutrosophy", meaning "the study of neutralities" and gave birth to the theory of Neutrosophic

Sets. Unlike intuitionistic fuzzy sets which associate to each member of the set a degree of membership μ and a degree of non-membership ν; $\mu, \nu \epsilon [0, 1]$, neutrosophic sets characterize each member x of the set with a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$, each of which belongs to the non-standard unit interval $]0^-, 1^+[$. Thus, although in some cases intuitionistic sets consider a particular indeterminacy or hesitation margin, $\pi = 1 - \mu - \nu$, neutrosophic sets are capable of handling uncertainty in a better way since in case of neutrosophic sets indeterminacy is taken care of separately. Further, in 2005, Wang et al, introduced the notion of Single-Valued Neutrosophic Sets (SVNS) [25], which differ from neutrosophic sets only in the fact that in the former's case, $T_A(x), I_A(x), F_A(x) \epsilon [0, 1]$ and can be applied to solve many practical problems. Research involving single valued neutrosophic sets and interval neutrosophic sets together with their applications are now on full swing. Notions of similarity, entropy, subsethood measure etc. of neutrosophic sets have been introduced and their applications in several areas [7, 17, 20, 28, 29] are being executed by many authors.

Again the theory of soft sets was initiated by D. Molodstov [19] in 1999 for modelling uncertainty present in real life. Roughly speaking, a soft set is a parameterized classification of the objects of the universe. Molodstov had shown several applications of soft sets in different areas like integration, game theory, decision making etc. Later Maji et al. [11] defined several operations on soft sets. Perhaps this is the only theory available with a parameterization tool for modelling uncertainty. H. Aktas and N. Cagman [1] have shown that fuzzy sets are special cases of soft sets. Later many authors [12, 13, 16] have combined soft sets with other sets to generate hybrid structures like fuzzy soft sets, intuitionistic fuzzy soft sets, generalized fuzzy soft sets, vague soft sets etc. and applied them in many areas like decision making, medical diagnosis, similarity measure etc. Few authors [2, 3, 6, 14, 15] have also defined the notions of soft multisets, fuzzy soft multisets, neutrosophic soft sets etc. Very recently S. Ye and J. Ye [30] combined the concepts of single valued neutrosophic sets along with the theory of multisets and proposed a new theory of Single Valued Neutrosophic Multisets (SVNMS in short).

In this paper we have slightly modified the definition of SVNMS and studied its properties. The initial contributions of this paper involve the introduction of various new set-theoretic operators on SVNMS and their properties. Later, the notion of single valued neutrosophic sets has been applied in solving a decision making problem regarding medical diagnosis.

2. Preliminaries

In this section we give the definition and some important results regarding single valued neutrosophic sets [25] and multisets.

Definition 2.1 ([25])**.** Let X be a space of points, with a generic element in X denoted by x. A single valued neutrosophic set A in X is characterized by a truth-membership function T_A, an indeterminacy-membership function I_A and a falsity-membership function F_A. For each point x in X, $T_A(x), I_A(x), F_A(x) \epsilon [0, 1]$.

When X is continuous, a SVNS A can be written as

$$A = \int_X \langle T(x), I(x), F(x) \rangle \diagup x, x \in X$$

When X is discrete, a SVNS A can be written as

$$A = \sum_{i=1}^{n} \langle T(x_i), I(x_i), F(x_i) \rangle \diagup x_i, x_i \in X$$

Definition 2.2 ([25])**.** The complement of a SVNS A is denoted by $c(A)$ and is defined by $T_{c(A)}(x) = F_A(x)$, $I_{c(A)}(x) = 1 - I_A(x)$ and $F_{c(A)}(x) = T_A(x)$,$\forall x \in X$.

Definition 2.3 ([25])**.** A SVNS A is contained in another SVNS B i.e.$A \subset B$ iff $T_A(x) \leq T_B(x)$, $I_A(x) \leq I_B(x)$ and $F_A(x) \geq F_B(x)$, $\forall x \in X$.

Definition 2.4 ([25])**.** Two SVNS A and B are said to be equal iff $A \subseteq B$ and $B \subseteq A$.

Definition 2.5 ([25])**.** The union of two SVNMS A and B is a SVNMS C, written as, $C = A \cup B$, whose truth-membership, indeterminacy membership and falsity membership functions are related to those of A and B as $T_C(x) = max\,(T_A(x), T_B(x))$, $I_C(x) = max\,(I_A(x), I_B(x))$ and $F_C(x) = min\,(F_A(x), F_B(x))$, $\forall x \in X$.

Definition 2.6 ([25])**.** The intersection of two SVNMS A and B is a SVNS C, written as, $C = A \cap B$, whose truth-membership, indeterminacy membership and falsity membership functions are related to those of A and B as $T_C(x) = min\,(T_A(x), T_B(x))$, $I_C(x) = min\,(I_A(x), I_B(x))$ and $F_C(x) = max\,(F_A(x), F_B(x))$, $\forall x \in X$.

Definition 2.7 ([25])**.** The difference between two SVNMS A and B is a SVNS C, written as, $C = A \setminus B$, whose truth-membership, indeterminacy membership and falsity membership functions are related to those of A and B as $T_C(x) = min\,(T_A(x), F_B(x))$, $I_C(x) = min\,(I_A(x), 1 - I_B(x))$ and $F_C(x) = max\,(F_A(x), T_B(x))$, $\forall x \in X$.

Definition 2.8 ([25])**.** The truth-favorite of a SVNS A, denoted by $\triangle A$, whose truth-membership, indeterminacy-membership and falsity-membership functions are defined as $T_{\triangle A}(x) = min\,(T_A(x) + I_A(x), 1)$, $I_{\triangle A}(x) = 0$ and $F_{\triangle A}(x) = F_A(x)$, $\forall x \in X$.

Definition 2.9 ([25])**.** The falsity-favorite of a SVNS A, is a SVNS denoted by ∇A, whose truth-membership, indeterminacy-membership and falsity-membership functions are defined as $T_{\nabla A}(x) = T_A(x)$, $I_{\nabla A}(x) = 0$ and
$F_{\nabla A}(x) = min\,(F_A(x) + I_A(x), 1)$, $\forall x \in X$.

Definition 2.10 ([24])**.** Let $\mathscr{U} = \{x_1, x_2, ..., x_n\}$ be the universal set. A crisp bag or multiset M of $\mathscr{U}$ is characterized by a function $C_M\,(.)$, $(C_M : U \to \mathbb{N})$ corresponding to each $x \in \mathscr{U}$, known as the count function. A multiset M is expressed as $M = \left\{\frac{k_1}{x_1}, \frac{k_2}{x_2}, ..., \frac{k_n}{x_n}\right\}$ such that x_i appears k_i times in M.

3. Single Valued Neutrosophic Multisets

The notion of single valued neutrosophic multisets were first defined in [30]. Here the three count functions are real valued but in this paper we have redefined the notion of SVNMS with positive integer valued count functions for the sake of practical use.

Definition 3.1. A Single Valued Neutrosophic Multiset (SVNMS) A, defined on a universe X is such that corresponding to each element of the set there exists a function, namely, the count function $C_f : X \to \mathbb{N}$, which denotes the number of times that particular element occurs in the set, such that each $x \epsilon X$ is characterized by three sequences of lengths $C_f(x)$, namely, a truth-membership sequence $\left(T_A^1(x), T_A^2(x), ..., T_A^k(x)\right)$, an indeterminacy-membership sequence $\left(I_A^1(x), I_A^2(x), ..., I_A^k(x)\right)$ and a falsity-membership sequence
$\left(F_A^1(x), F_A^2(x), ..., F_A^k(x)\right)$.
When the universe under consideration $X = \{x_1, x_2, ..., x_n\}$, is discrete, a Single Valued Neutrosophic Multiset A over X is represented as

$$A = \sum_{i=1}^{n} \left\langle \left(T_A^1(x_i), T_A^2(x_i), ..., T_A^{k_i}(x_i)\right), \left(I_A^1(x_i), I_A^2(x_i), ..., I_A^{k_i}(x_i)\right), \left(F_A^1(x_i), F_A^2(x_i), ..., F_A^{k_i}(x_i)\right) \right\rangle$$

where the element $x_i \epsilon X$ is repeated $k_i = C_f(x_i)$ times in X.

Remark 3.2. The truth-membership sequence is always a decreasing sequence of membership values whereas the indeterminacy-membership and falsity-membership sequences are such that they can assume the membership values in any order and $0 \leq T_A^r(x_i) + I_A^r(x_i) + F_A^r(x_i) \leq 3$, $\forall x_i \epsilon X$, $i = 1, 2, ..., n$ and $r = 1, 2, ..., k_i$. This has been done to keep parity with the definitions of Fuzzy and Intuitionistic fuzzy multisets.

Definition 3.3. The length of an element $x_i \epsilon X$ of a single-valued neutrosophic multiset A, defined on the set X, is defined as $l(x_i : A) = C_f(x_i), i = 1, 2, ..., n$.

Definition 3.4. The cardinality of a single-valued neutrosophic multiset A is defined as $Card(A) = \sum_{i=1}^{n} \sum_{j=1}^{l(x_i:A)} \left(T_A^j(x_i) + I_A^j(x_i)\right)$, $x_i \epsilon X$, $i = 1, 2, ..., n$.

Example 3.5. Suppose $X = \{x_1, x_2, x_3\}$ denotes three shirts displayed for sale in a particular shop. We now set forth to register the opinion of a domain of customers about the quality of shirts based on whether the shirts are made up of "good fabric", a level of indeterminacy on the part of the customers and whether they feel that the shirt is made up of "a not so good fabric". Based on the opinion of the domain of customers concerned, a single-valued neutrosophic multiset can be defined on X as follows:
$A = \langle (0.6, 0.4), (0.5, 0.3), (0.2, 0.3) \rangle /x_1 + \langle 0.2, 0.4, 0.7 \rangle /x_2$
$+ \langle (0.8, 0.6, 0.5), (0.2, 0.2, 0.3), (0.1, 0.3, 0.4) \rangle /x_3$
Also, $l(x_1 : A) = 2$, $l(x_2 : A) = 1$, $l(x_3 : A) = 3$ and $Card(A) = 5$.

Definition 3.6. An absolute single-valued neutrosophic multiset $\tilde{A}$ is a SVNMS where $T^i(x) = 1$, $I^i(x) = 1$ and $F^i(x) = 0$, $\forall x \epsilon X$, $i = 1, 2, ..., l(x : \tilde{A})$.

Definition 3.7. A null single-valued neutrosophic multiset $\tilde{\Phi}$ is a SVNMS where $T^i(x) = 0$, $I^i(x) = 0$ and $F^i(x) = 1$, $\forall x \epsilon X$, $i = 1, 2, ..., l(x : \tilde{\Phi})$.

4. Operations over Single Valued Neutrosophic Multisets

In this section we have defined several set theoretic and algebraic operations on SVNMS. These operations are different from the operations that have been defined in [30]. Moreover, in our case the behaviour of indeterminacy membership is similar to the behaviour of truth membership whereas in [30] the indeterminacy membership is similar to the behaviour of falsity membership.

4.1. **Set-theoretic operations over Single valued neutrosophic multisets.**

Definition 4.1. A SVNMS A is said to be contained in another SVNMS B iff $\forall x_i \,\epsilon\, X,\ i = 1, 2, ..., n,$
(i) $l(x_i : A) \leq l(x_i : B)$
(ii) $T^r_A(x_i) \leq T^r_B(x_i)$
(iii) $I^r_A(x_i) \leq I^r_B(x_i)$
(iv) $F^r_A(x_i) \geq F^r_B(x_i)$, $\forall x_i \,\epsilon\, X, i = 1, 2, ..., n$ and $r = 1, 2, ..., l(x_i : A)$.

Definition 4.2. Two SVNMS A and B are said to be equal iff they are subsets of one another.

Remark 4.3. Let A and B be two SVNMS over the universe X. In order to carry out any operation (set-theoretic or algebraic) between A and B it is verified at first whether $l(x_i : A) = l(x_i : B)$, $\forall x_i \,\epsilon\, X$. If $l(x_m : A) \neq l(x_m : B)$ for any $x_m \,\epsilon\, X$ then without any loss of generality, a sufficient number of 0's are appended with the truth-membership and the indeterminacy membership values and a sufficient number of 1's are appended with the falsity-membership values respectively to the sequences of smaller length thereby making the lengths equal and faciltating the execution of operations.

Example 4.4. Suppose two SVNMS A and B are given by,
$A = \langle 0.5, 0.4, 0.4\rangle /x_1 + \langle (0.3, 0.2), (0.5, 0.4), (0.6, 0.7)\rangle /x_2 + \langle 0.8, 0.3, 0.2\rangle /x_3$
$B = \langle (0.8, 0.6), (0.4, 0.5), (0.1, 0.3)\rangle /x_1 + \langle (0.5, 0.4), (0.7, 0.7), (0.5, 0.3)\rangle /x_2 + \langle 0.9, 0.5, 0.2\rangle /x_3$
Here $A \subseteq B$.

Definition 4.5. The union of two SVNMS A and B over X, denoted by $A\cup B$, is a SVNMS over X whose truth-membership, indeterminacy-membership and falsity-membership values are given by
$T^r_{A\cup B}(x_i) = max\,(T^r_A(x_i), T^r_B(x_i))$
$I^r_{A\cup B}(x_i) = max\,(I^r_A(x_i), I^r_B(x_i))$
$F^r_{A\cup B}(x_i) = min\,(F^r_A(x_i), F^r_B(x_i))$
$\forall x_i \,\epsilon\, X,\ i = 1, 2, ..., n;\ r = 1, 2, ..., l$ where $l = max\,\{l(x_i : A), l(x_i : B)\}$.

Definition 4.6. The intersection of two SVNMS A and B over X, denoted by $A \cap B$, is a SVNMS over X whose truth-membership, indeterminacy-membership and falsity-membership values are given by
$T^r_{A\cap B}(x_i) = min\,(T^r_A(x_i), T^r_B(x_i))$
$I^r_{A\cap B}(x_i) = min\,(I^r_A(x_i), I^r_B(x_i))$
$F^r_{A\cap B}(x_i) = max\,(F^r_A(x_i), F^r_B(x_i))$
$\forall x_i \,\epsilon\, X,\ i = 1, 2, ..., n;\ r = 1, 2, ..., l$ where $l = max\,\{l(x_i : A), l(x_i : B)\}$.

Example 4.7. Let two SVNMS A and B, defined over the universe $X = \{x_1, x_2, x_3\}$ be given by,

$$A = \langle (0.5, 0.3), (0.1, 0.1), (0.7, 0.8) \rangle /x_1 + \langle (0.7, 0.68, 0.62), (0.3, 0.45, 0.5), (0.34, 0.28, 0.49) \rangle /x_2$$
$$+ \langle (0.67, 0.5, 0.3), (0.2, 0.3, 0.4), (0.4, 0.5, 0.7) \rangle /x_3$$

$$B = \langle 0.75, 0.2, 0.15 \rangle /x_1 + \langle (0.43, 0.37, 0.28, (0.5, 0.2, 0.3), (0.7, 0.8, 0.9)) \rangle /x_2$$
$$+ \langle (1.0, 0.86, 0.79), (0.01, 0.1, 0.2), (0.0, 0.3, 0.2) \rangle /x_3$$

$$\therefore A \cup B = \langle (0.75, 0.3), (0.2, 0.1), (0.15, 0.8) \rangle /x_1 +$$
$$\langle (0.7, 0.68, 0.62), (0.5, 0.45, 0.5), (0.34, 0.28, 0.49) \rangle /x_3$$
$$+ \langle (1.0, 0.86, 0.79), (0.2, 0.3, 0.4), (0.0, 0.3, 0.2) \rangle /x_3$$

Definition 4.8. The truth-favorite of a SVNMS A, denoted by $\triangle A$, is a SVNMS and is characterized by the truth-membership, indeterminacy-membership and falsity-membership values which are respectively defined as
$T^r_{\triangle A}(x_i) = min\,(T^r_A(x_i) + I^r_A(x_i), 1)$
$I^r_{\triangle A}(x_i) = 0$
$F^r_{\triangle A}(x_i) = F^r_A(x_i)$
$\forall x_i \,\epsilon\, X,\ i = 1, 2, ..., n;\ r = 1, 2, ..., l$ where $l = max\,\{l(x_i : A), l(x_i : B)\}$.

Definition 4.9. The falsity-favorite of a SVNMS A, denoted by ∇A, is a SVNMS and is characterized by the truth-membership, indeterminacy-membership and falsity-membership values which are respectively defined as
$T^r_{\nabla A}(x_i) = T^r_A(x_i)$
$I^r_{\nabla A}(x_i) = 0$
$F^r_{\nabla A}(x_i) = min\,(F^r_A(x_i) + I^r_A(x_i), 1)$
$\forall x_i \,\epsilon\, X,\ i = 1, 2, ..., n;\ r = 1, 2, ..., l$ where $l = max\,\{l(x_i : A), l(x_i : B)\}$.

Example 4.10. Considering the SVNMS A of example 4.7 we have,

$$\triangle A = \langle (0.6, 0.4), (0.0, 0.0), (0.7, 0.8) \rangle /x_1$$
$$+ \langle (1.0, 1.0, 1.0), (0.0, 0.0, 0.0), (0.34, 0.28, 0.49) \rangle /x_2$$
$$+ \langle (0.87, 0.8, 0.7),\ (0.0, 0.0, 0.0),\ (0.4, 0.5, 0.7) \rangle /x_3$$

$$\nabla A = \langle (0.5, 0.3), (0.0, 0.0), (0.8, 0.9) \rangle /x_1$$
$$+ \langle (0.7, 0.68, 0.62), (0.0, 0.0, 0.0), (0.64, 0.73, 0.99) \rangle /x_2$$
$$+ \langle (0.67, 0.5, 0.3), (0.0, 0.0, 0.0), (0.6, 0.8, 1.0) \rangle /x_3$$

Proposition 4.11. *It has been observed that single-valued neutrosophic multisets satisfy the following properties under set-theoretic operations:*

1. Commutative Property
(i) $A \cup B = B \cup A$
(ii) $A \cap B = B \cap A$
2. Associative Property
(i) $A \cup (B \cup C) = (A \cup B) \cup C$
(ii) $A \cap (B \cap C) = (A \cap B) \cap C$

3. Idempotent Property
(**i**) $A \cup A = A$
(**ii**) $A \cap A = A$
(**iii**) $\triangle\triangle A = \triangle A$
(**iv**) $\nabla\nabla A = \nabla A$
4. Absorptive Property
(**i**) $A \cup (A \cap B) = A$
(**ii**) $A \cap (A \cup B) = A$
5. (**i**) $A \cup \tilde{A} = \tilde{A}$
(**ii**) $A \cap \tilde{A} = A$
(**iii**) $A \cup \tilde{\Phi} = A$
(**iv**) $A \cap \tilde{\Phi} = \tilde{\Phi}$

Proof. The proofs are straight-forward. □

4.2. **Algebraic Operations over SVNMS.**

Definition 4.12. The addition between two SVNMS A and B over the universe X, denoted by $A \oplus B$, is a SVNMS over X, whose truth-membership, indeterminacy-membership and falsity-membership values are defined as
$T^r_{A\oplus B}(x_i) = T^r_A(x_i) + T^r_B(x_i) - T^r_A(x_i).T^r_B(x_i)$
$I^r_{A\oplus B}(x_i) = I^r_A(x_i) + I^r_B(x_i) - I^r_A(x_i).I^r_B(x_i)$
$F^r_{A\oplus B}(x_i) = F^r_A(x_i).F^r_B(x_i)$
$\forall x_i \,\epsilon\, X,\ i = 1, 2, ..., n;\ r = 1, 2, ..., l.$

Definition 4.13. The multiplication between two SVNMS A and B over the universe X, denoted by $A \otimes B$, is a SVNMS over X, whose truth-membership, indeterminacy-membership and falsity-membership values are defined as
$T^r_{A\otimes B}(x_i) = T^r_A(x_i).T^r_B(x_i)$
$I^r_{A\otimes B}(x_i) = I^r_A(x_i).I^r_B(x_i)$
$F^r_{A\otimes B}(x_i) = F^r_A(x_i) + F^r_B(x_i) - F^r_A(x_i).F^r_B(x_i)$
$\forall x_i \,\epsilon\, X,\ i = 1, 2, ..., n;\ r = 1, 2, ..., l.$

Example 4.14. Let the SVNMS under consideration be those stated in example 4.7 Then we have

$A \oplus B = \langle (0.875, 0.3), (0.28, 0.1), (0.105, 0.0) \rangle /x_1$
$+ \langle (0.829, 0.7894, 0.7264), (0.65, 0.56, 0.65), (0.238, 0.224, 0.441) \rangle /x_2$
$+ \langle (1.0, 0.93, 0.853), (0.208, 0.37, 0.52), (0.0, 0.15, 0.14) \rangle /x_3$

$A \otimes B = \langle (0.375, 0.0), (0.02, 0.0), (0.745, 0.8) \rangle /x_1$
$+ \langle (0.301, 0.252, 0.174), (0.15, 0.09, 0.15), (0.802, 0.856, 0.949) \rangle /x_2$
$+ \langle (0.67, 0.43, 0.237), (0.002, 0.03, 0.08), (0.4, 0.65, 0.76) \rangle /x_3$

5. Proposed distance measure between two SVNMS

In this section, the notion of distance, denoted by $d(A, B)$, in general, between two SVNMS A and B defined over the universe $X = \{x_1, x_2, ..., x_n\}$ has been proposed in the sense of Hamming, Normalized Hamming, Euclidean and Normalized Euclidean.

Suppose that, $l_i = max_i \{l(x_i : A), l(x_i : B)\}$ and $L = max_i \{l_i\}$, $i = 1, 2, ..., n$.

Definition 5.1. The Hamming distance between A and B is given by,

$$d_N(A, B) = \sum_{i=1}^{n} \sum_{r=1}^{l_i} \left(|T_A^r(x_i) - T_B^r(x_i)| + |I_A^r(x_i) - I_B^r(x_i)| + |F_A^r(x_i) - F_B^r(x_i)| \right)$$

Definition 5.2. The Normalized Hamming distance between A and B is given by,

$$l_N(A, B) = \frac{1}{3nL} \sum_{i=1}^{n} \sum_{r=1}^{l_i} \left(|T_A^r(x_i) - T_B^r(x_i)| + |I_A^r(x_i) - I_B^r(x_i)| + |F_A^r(x_i) - F_B^r(x_i)| \right)$$

Definition 5.3. The Euclidean distance between A and B is given by,

$$e_N(A, B) = \sqrt{\sum_{i=1}^{n} \sum_{r=1}^{l_i} \left(|T_A^r(x_i) - T_B^r(x_i)|^2 + |I_A^r(x_i) - I_B^r(x_i)|^2 + |F_A^r(x_i) - F_B^r(x_i)|^2 \right)}$$

Definition 5.4. The Normalized Euclidean distance between A and B is given by,

$$q_N(A, B) = \sqrt{\frac{1}{3nL} \sum_{i=1}^{n} \sum_{r=1}^{l_i} \left(|T_A^r(x_i) - T_B^r(x_i)|^2 + |I_A^r(x_i) - I_B^r(x_i)|^2 + |F_A^r(x_i) - F_B^r(x_i)|^2 \right)}$$

Remark 5.5. It has been observed that the proposed distance measures as stated above satisfies the following properties:
(i) $d_N(A, B) \epsilon [0, 3nL]$
(ii) $l_N(A, B) \epsilon [0, 1]$
(iii) $e_N(A, B) \epsilon \left[0, \sqrt{3nL}\right]$
(iv) $q_N(A, B) \epsilon [0, 1]$

Proof. The proofs are straight-forward. □

Proposition 5.6. *It has been observed that in general, whatever might be the notion in which the distance between any two SVNMS be defined, the distance measure* $d(A, B)$ *between any two SVNMS A and B satisfies the following properties:*
(i) $d(A, B) \geq 0$ *and the equality holds iff* $A = B$.
(ii) $d(A, B) = d(B, A)$
(iii)$d(A, B) \leq d(A, C) + d(B, C)$, *where C is a SVNMS over X.*

Proof. The proofs of (i) and (ii) are straightforward. We give the outline of the proofs of (iii) only.

Consider three arbitrary SVNMS A, B and C. Then for $r = 1, 2, ..., l_i$ we have

$$|T_A^r(x_i) - T_B^r(x_i)| = |T_A^r(x_i) - T_C^r(x_i) + T_C^r(x_i) - T_B^r(x_i)|$$
$$\leq |T_A^r(x_i) - T_C^r(x_i)| + |T_B^r(x_i) - T_C^r(x_i)|$$

Similarly it can be shown that,

$$|I_A^r(x_i) - I_B^r(x_i)| \leq |I_A^r(x_i) - I_C^r(x_i)| + |I_B^r(x_i) - I_C^r(x_i)|$$
$$|F_A^r(x_i) - F_B^r(x_i)| \leq |F_A^r(x_i) - F_C^r(x_i)| + |F_B^r(x_i) - F_C^r(x_i)|$$

Hence it follows that,

$\sum_{i=1}^{n}\sum_{r=1}^{l_i}\left(\left|T_A^r(x_i)-T_B^r(x_i)\right|+\left|I_A^r(x_i)-I_B^r(x_i)\right|+\left|F_A^r(x_i)-F_B^r(x_i)\right|\right)$
$\leq \sum_{i=1}^{n}\sum_{r=1}^{l_i}\left(\left|T_A^r(x_i)-T_C^r(x_i)\right|+\left|I_A^r(x_i)-I_C^r(x_i)\right|+\left|F_A^r(x_i)-F_C^r(x_i)\right|\right)$
$+\sum_{i=1}^{n}\sum_{r=1}^{l_i}\left(\left|T_B^r(x_i)-T_C^r(x_i)\right|+\left|I_B^r(x_i)-I_C^r(x_i)\right|+\left|F_B^r(x_i)-F_C^r(x_i)\right|\right)$

Again, we see

$\left|T_A^r(x_i)-T_B^r(x_i)\right|^2=\left|T_A^r(x_i)-T_C^r(x_i)+T_C^r(x_i)-T_B^r(x_i)\right|^2$
$\leq \left|T_A^r(x_i)-T_C^r(x_i)\right|^2+\left|T_B^r(x_i)-T_C^r(x_i)\right|^2$

Similarly we have
$\left|I_A^r(x_i)-I_B^r(x_i)\right|\leq\left|I_A^r(x_i)-I_C^r(x_i)\right|+\left|I_B^r(x_i)-I_C^r(x_i)\right|$
$\left|F_A^r(x_i)-F_B^r(x_i)\right|\leq\left|F_A^r(x_i)-F_C^r(x_i)\right|+\left|F_B^r(x_i)-F_C^r(x_i)\right|$

and hence,

$\sum_{i=1}^{n}\sum_{r=1}^{l_i}\left(\left|T_A^r(x_i)-T_B^r(x_i)\right|^2+\left|I_A^r(x_i)-I_B^r(x_i)\right|^2+\left|F_A^r(x_i)-F_B^r(x_i)\right|^2\right)$
$\leq \sum_{i=1}^{n}\sum_{r=1}^{l_i}\left(\left|T_A^r(x_i)-T_C^r(x_i)\right|^2+\left|I_A^r(x_i)-I_C^r(x_i)\right|^2+\left|F_A^r(x_i)-F_C^r(x_i)\right|^2\right)$
$+\sum_{i=1}^{n}\sum_{r=1}^{l_i}\left(\left|T_B^r(x_i)-T_C^r(x_i)\right|^2+\left|I_B^r(x_i)-I_C^r(x_i)\right|^2+\left|F_B^r(x_i)-F_C^r(x_i)\right|^2\right)$

Thus the proofs follow automatically from the above results. □

Remark 5.7. From the aforementioned observations it can be concluded that the proposed notion of distance measures actually define metrics over the set of all single-valued neutrosophic multisets and hence if $\mathcal{N}_{\mathrm{m}}$denotes the collection of all SVNMS over a universe X then $(\mathcal{N}_{\mathrm{m}}, d)$ defines a metric space over X.

Example 5.8. Let us consider the SVNMS A and B as stated in example 4.7. Then we have
$d_N(A,B)=7.83$, $l_N(A,B)=0.29$, $e_N(A,B)=1.772$ and $q_N(A,B)=0.3418$.

6. Similarity Measure between two SVNMS

In this section the notion of similarity measure between two SVNMS has been stated. The various types of similarity measures between two SVNMS are proposed as follows:

Definition 6.1. The Distance Based Similarity Measure is defined as,

$$S_d(A,B)=\frac{1}{1+d(A,B)}$$

where the distance measure can be taken in any of the methods as mentioned in Section 5.

Definition 6.2. The Similarity Measure based on membership degrees is defined as,

$$S_m(A,B)=\frac{\sum_{i=1}^{n}\sum_{r=1}^{l_i}\left\{min\left(T_A^r(x_i),T_B^r(x_i)\right)+min\left(I_A^r(x_i),I_B^r(x_i)\right)+min\left(F_A^r(x_i),F_B^r(x_i)\right)\right\}}{\sum_{i=1}^{n}\sum_{r=1}^{l_i}\left\{max\left(T_A^r(x_i),T_B^r(x_i)\right)+max\left(I_A^r(x_i),I_B^r(x_i)\right)+max\left(F_A^r(x_i),F_B^r(x_i)\right)\right\}}$$

Remark 6.3. In some cases, especially while dealing with real life problems, the elements of the universe under consideration are associated with weights in order to specify the varying degrees of importance of the elements at hand. In this case we have considered this possibility and accordingly the similarity measure between two SVNMS have been defined where the universe over which the sets are defined have weights associated to its constituent elements.

Definition 6.4. The weighted similarity measure in the sense of Majumdar and Samanta [17] is defined as,

$$S_w\left(A,B\right)$$
$$= \frac{\sum_{i=1}^{n} w_i \left[\sum_{r=1}^{l_i} \{T_A^r(x_i).T_B^r(x_i)+I_A^r(x_i).I_B^r(x_i)+F_A^r(x_i).F_B^r(x_i)\}\right]}{\sum_{i=1}^{n} w_i \left[\left(T_A^r(x_i)^2+I_A^r(x_i)^2+F_A^r(x_i)^2\right).\left(T_B^r(x_i)^2+I_B^r(x_i)^2+F_B^r(x_i)^2\right)\right]}.\left\{\frac{1}{\sum_{i=1}^{n} w_i}\right\}$$

where $0 \le w_i \le 1$, $i = 1, 2, ..., n$ and w_i are the weights associated with the x_i's respectively where $x_i \,\epsilon\, X$.

Definition 6.5. The proposed weighted similarity measure in the sense of Dengfeng and Chuntian [9] is defined as follows:

Let A be a SVNMS over a universe X. We define a function $\psi_A : X \to [0,1]$ as,

$$\psi_A(x) = \frac{\sum_{r=1}^{l} \{T_A^r(x) + I_A^r(x) + (1 - F_A^r(x))\}}{3l}$$

where $l = l(x : A)$ is the length of an element $x \,\epsilon\, X$.

Define the weighted similarity measure between two SVNMS A and B over the same universe X as,

$$S_w^{'}\left(A,B\right) = 1 - \sqrt[p]{\frac{\sum_{i=1}^{n} w_i \left|\psi_B(x_i) - \psi_A(x_i)\right|^p}{\sum_{i=1}^{n} w_i}}$$

where $0 \le w_i \le 1$, $i = 1, 2, ..., n$ and w_i are the weights associated with the x_i's respectively where $x_i \,\epsilon\, X$.

Here p is a positive integer called the "similarity degree".

Example 6.6. We consider the SVNMS A and B as stated in example 4.7. Then taking into consideration the result $l_N\left(A,B\right) = 0.29$ from example 5.8 we have,
$S_d\left(A,B\right) = 0.775$ and $S_m\left(A,B\right) = 0.485$.

Proposition 6.7. *1. The distance based similarity measure satisfies the following properties:*
(i) $0 \le S_d\left(A,B\right) \le 1$ *and equality occurs iff* $A = B$
(ii) $S_d\left(A,B\right) = S_d\left(B,A\right)$
(iii) *For* $A \subset B \subset C$, $S_d\left(A,C\right) \le S_d\left(A,B\right) \wedge S_d\left(B,C\right)$
2. The similarity measure based on membership degrees satisfies the following properties:
(i) $0 \le S_m\left(A,B\right) \le 1$ *and equality occurs iff* $A = B$
(ii) $S_m\left(A,B\right) = S_m\left(B,A\right)$

(**iii**) *For* $A \subset B \subset C$*,* $S_d(A,C) \leq S_d(A,B) \wedge S_d(B,C)$
3. The weighted similarity measures satisfy the following properties:
(**a**).(**i**) $0 \leq S_w(A,B) \leq 1$ *and equality occurs iff* $A = B$
(**ii**) $S_w(A,B) = S_w(B,A)$
(**b**).(**i**) $0 \leq S_{w'}(A,B) \leq 1$ *and equality occurs iff* $A = B$
(**ii**) $S_{w'}(A,B) = S_{w'}(B,A)$
(**iii**) *For* $A \subset B \subset C$*,* $S_{w'}(A,C) \leq S_{w'}(A,B) \wedge S_{w'}(B,C)$
(**iv**) *Corresponding to any two SVNMS, for an ascending sequence of integral values of* p *a decreasing sequence of similarity measures is obtained.*

Proof. We only prove 3(b)(iii) and 3(b)(iv) since the remaining proofs are straightforward.

Let $A \subset B \subset C$ then it is easy to prove that

$$\{\psi_C(x_i) - \psi_A(x_i)\} \geq \{\psi_B(x_i) - \psi_A(x_i)\}$$

since $T^r_C(x_i) \geq T^r_B(x_i)$, $I^r_C(x_i) \geq I^r_B(x_i)$ and $F^r_C(x_i) \leq F^r_B(x_i)$, $\forall x_i \,\epsilon\, X$, $i = 1, 2, ..., n$ and $r = 1, 2, ..., max\,\{l\,(x_i : B)\,, l\,(x_i : C)\}$.

Thus,

$$\sum_{r=1}^{l(x_i:C)} \left\{T^r_C(x_i) + I^r_C(x_i) + \left(1 - F^r_C(x_i)\right)\right\} \geq \sum_{r=1}^{l(x_i:B)} \left\{T^r_B(x_i) + I^r_B(x_i) + \left(1 - F^r_B(x_i)\right)\right\}$$

and hence,

$$\{\psi_C(x_i) - \psi_A(x_i)\} - \{\psi_B(x_i) - \psi_A(x_i)\} = \{\psi_C(x_i) - \psi_B(x_i)\} \geq 0.$$

So,

$$|\{\psi_C(x_i) - \psi_A(x_i)\}|^p \geq |\{\psi_B(x_i) - \psi_A(x_i)\}^p|$$
$$\Rightarrow \sum_{i=1}^n w_i\,|\{\psi_C(x_i) - \psi_A(x_i)\}|^p \geq \sum_{i=1}^n w_i\,|\{\psi_B(x_i) - \psi_A(x_i)\}|^p$$
$$\Rightarrow S_w(A,C) \leq S_w(A,B)$$

Similarly, it can be shown that $S_w(A,C) \leq S_w(B,C)$.

Next, suppose that for a particular $p \,\epsilon\, \mathbb{N}$, $a_i = |\psi_B(x_i) - \psi_A(x_i)|$ and $q_i = w_i$, $i = 1, 2, ..., n$.
Thus,

$$\sqrt[p]{\frac{\sum_{i=1}^n w_i\,|\psi_B(x_i) - \psi_A(x_i)|^p}{\sum_{i=1}^n w_i}} = \left(\frac{\sum_{i=1}^n q_i\,(a_i)^p}{\sum_{i=1}^n q_i}\right)^{\frac{1}{p}} = f(p), say.$$

Hence, $f(p)$ is a power mean function [10] thereby being strictly monotonically increasing in nature i.e. for $p_1, p_2 \,\epsilon\, \mathbb{N}$ with $p_1 < p_2$, $f(p_1) < f(p_2)$.

$$\Rightarrow \left(\frac{\sum_{i=1}^n q_i\,(a_i)^{p_1}}{\sum_{i=1}^n q_i}\right)^{\frac{1}{p_1}} < \left(\frac{\sum_{i=1}^n q_i\,(a_i)^{p_2}}{\sum_{i=1}^n q_i}\right)^{\frac{1}{p_2}}$$

$$\Rightarrow 1 - \left(\frac{\sum_{i=1}^n q_i\,(a_i)^{p_1}}{\sum_{i=1}^n q_i}\right)^{\frac{1}{p_1}} > 1 - \left(\frac{\sum_{i=1}^n q_i\,(a_i)^{p_2}}{\sum_{i=1}^n q_i}\right)^{\frac{1}{p_2}}$$

This proves the fact that as p increases, the value of the similarity measure $S_{w'}(A, B)$ decreases and hence it may be concluded that for a sequence of ascending integral values for p we obtain a decreasing sequence of similarity measures between two SVNMS A and B, which completes the proof. □

Remark 6.8. From the aforementioned properties it is clear that the weighted similarity measure $S_w(A, B)$ in the sense of [17] is not an actual similarity measure in the actual sense of the term since it does not satisfy all the properties of the axiomatic definition of a similarity measure [26]. We thus term this particular measure of similarity a "weighted quasi-similarity measure".

7. An application of Single valued neutrosophic multisets in medical diagnosis

Since real-life situations involve uncertainties, while constructing a mathematical model of practical importance we need to incorporate variables that can deal with uncertainties. In this section we state an example with a view to show how the theory of single-valued neutrosophic multisets can be used in diagnosing a medical condition.

A fever is one of the most common medical signs and it is characterized by an elevation of body temperature above the normal range of $97.7°F - 98.5^{\circ}F$. It can be caused by many medical conditions ranging from benign to potentially serious. Besides an elevated body temperature there are additional symptoms such as shivering, sweating, loss of appetite etc. which are associated with fever. Moreover, there are specific patterns of temperature changes during a fever according to which a fever may be classified as Continuous fever, Pel-Ebstein fever, Remittent fever and Intermittent fever, which in turn may be classified into Quotidian fever, Tertian fever and Quartan fever. These fever patterns along with the associated symptoms aid in the diagnosis of a particular disease.

Consider the case of a person primarily diagnosed with fever associated with shivering, headache, muscle and joint ache, cough, running nose accompanied with sneezing, loss of appetite, chest pain and fatigue. The associated symptoms that are prominent in the person hint at the fact that the person might be suffering from Tuberculosis, Influenza or Common Cold.

Tuberculosis is an infectious disease, typically of the lungs and is characterized by remittent fever accompanied by bad cough , at times accompanied by blood, pain in the chest, fatigue, loss of appetite, chills and night sweating. Influenza or Flu is characterized by remittent fever, extreme chills, shivering, cough, nasal congestion and runny nose accompanied by sneezing, body ache, particularly in the joints, fatigue and headache. On the other hand, Common Cold is characterized by remittent fever accompanied by runny nose, shivering, cough, body ache, headache and sneezing.

For the sake of diagnosis, the patient is kept under supervision for a day and his fever pattern along with the other symptoms are monitored thrice, at intervals of 8 hrs, starting from 6:00 hrs in the morning, then at 14:00 hrs and finally at 22:00 hrs.

The medical findings of the patient are represented in a tabular form as follows:

Symptoms

Timings	T	Sh	Sw	H	MJ	C	LA	CP	F	RNS
6:00hrs	$97.5°F$	-	-	-	m	m	m	-	m	m
14:00hrs	$100°F$	m	-	m	m	m	m	-	m	m
22:00hrs	$101.2°F$	h	-	m	m	m	m	-	m	m

Table 7.1. Table representing the medical findings of the patient.

Here T, Sh, Sw, H, MJ, C, LA, CP, F and RNS denote body temprature, shivering, sweating, headache, muscle and joint pain, cough, loss of appetite, chest pain, fatigue and running nose with sneezing respectively and these symptoms altogether constitute the univrsal set. On the other hand, the symbols 'm' and 'h' are abbreviations for “moderate” and “high” respectively which denote the qualitative intensity of the elements of the universal set.
The above findings of the patient can be summarized and represented with the help of a SVNMS, denoted by P_f (symbolic representation for the patient's findings) over the above mentioned universe as follows:

$$A = \langle (0, 0.5, 0.7), (0.2, 0.1, 0), (0.9, 0.1, 0)\rangle /T + \langle (0, 0.5, 0.9), (0.1, 0.1, 0), (1, 0.3, 0)\rangle /Sh + \langle (0, 0, 0), (0.1, 0, 0), (0.9, 0.9, 1)\rangle /Sw + \langle (0, 0.5, 0.4), (0.1, 0.2, 0.2), (0.8, 0.3, 0.3)\rangle /H + \langle (0.5, 0.5, 0.5), (0, 0.1, 0), (0.2, 0.3, 0.4)\rangle /MJ + \langle (0.55, 0.5, 0.4), (0.2, 0, 0), (0.5, 0.4, 0.5)\rangle /C + \langle (0.5, 0.5, 0.45), (0.2, 0.2, 0.1), (0.4, 0.3, 0.4)\rangle /LA + \langle (0, 0, 0), (0.1, 0.1, 0), (0.8, 0.9, 0.9)\rangle /CP + \langle (0.58, 0.5, 0.5), (0.3, 0.2, 0.1), (0.3, 0.3, 0.4)\rangle /F + \langle (0.5, 0.4, 0.4), (0.1, 0.1, 0), (0.2, 0.3, 0.4)\rangle /RNS$$

Suppose the standard symptomatic characteristics of the diseases are represented by the following SVNMS as,

$$TB = \langle (0, 0.5, 0.6), (0, 0.1, 0), (0.9, 0.4, 0.2)\rangle /T + \langle (0.6, 0.6, 0.5), (0.1, 0.1, 0), (0.2, 0.1, 0.1)\rangle /Sh + \langle (0.5, 0.5, 0.7), (0.1, 0.2, 0.1), (0.4, 0.3, 0.1)\rangle /Sw + \langle (0, 0, 0), (0, 0.1, 0), (0.8, 0.8, 0.9)\rangle /H + \langle (0, 0, 0), (0.1, 0.1, 0.1), (0.9, 1, 0.9)\rangle /MJ + \langle (1, 0.9, 0.9), (0.1, 0, 0.1), (0, 0, 0.1)\rangle /C + \langle (0.6, 0.6, 0.5), (0.3, 0.2, 0.2), (0.4, 0.5, 0.4)\rangle /LA + \langle (0.7, 0.6, 0.6), (0.2, 0.1, 0.1), (0.2, 0.3, 0.1)\rangle /CP + \langle (0.5, 0.4, 0.3), (0.4, 0.4, 0.3), (0.4, 0.5, 0.3)\rangle /F + \langle (0.4, 0.3, 0.3), (0.3, 0.3, 0.2), (0.5, 0.4, 0.5)\rangle /RNS$$

$$Inf = \langle (0.8, 0.7, 0.7), (0, 0.1, 0.1), (0.1, 0.1, 0.2)\rangle /T + \langle (0.9, 0.8, 0.8), (0, 0, 0.1), (0.1, 0.2, 0.1)\rangle /Sh + \langle (0, 0, 0), (0, 0, 0), (0.9, 0.9, 0.1)\rangle /Sw + \langle (0.6, 0.6, 0.5), (0.2, 0.1, 0.1), (0.3, 0.3, 0.2)\rangle /H + \langle (0.9, 0.8, 0.8), (0.1, 0.1, 0), (0.1, 0, 0)\rangle /MJ + \langle (0.6, 0.6, 0.5), (0.1, 0.2, 0.1), (0.3, 0.4, 0.2)\rangle /C + \langle (0.6, 0.5, 0.5), (0.2, 0.2, 0.1), (0.4, 0.3, 0.3)\rangle /LA + \langle (0, 0, 0), (0.1, 0.2, 0.1), (0.8, 0.9, 0.8)\rangle /CP + \langle (0.5, 0.4, 0.3), (0.4, 0.4, 0.3), (0.4, 0.5, 0.3)\rangle /F + \langle (0.4, 0.3, 0.3), (0.3, 0.3, 0.2), (0.5, 0.4, 0.5)\rangle /RNS$$

$$CC = \langle (0.1, 0.5, 0.7), (0.1, 0.1, 0.2), (0.9, 0.4, 0.3)\rangle /T + \langle (0.9, 0.8, 0.8), (0.1, 0, 0.1), (0, 0.2, 0.1)\rangle /Sh + \langle (0, 0, 0), (0.2, 0.1, 0.1), (0.9, 0.4, 0.3)\rangle /Sw + \langle (0.6, 0.6, 0.5), (0.2, 0.1, 0.1), (0.3, 0.2, 0.2)\rangle /H + \langle (0.7, 0.6, 0.6), (0.3, 0.1, 0.1), (0.2, 0.3, 0.3)\rangle /MJ + \langle (0.6, 0.6, 0.5), (0.1, 0.2, 0.1), (0.3, 0.4, 0.2)\rangle /C + \langle (0.6, 0.5, 0.5), (0.2, 0.2, 0.1), (0.4, 0.3, 0.3)\rangle /LA + \langle (0.5, 0.5, 0.4), (0.1, 0.1, 0), (0.4, 0.3, 0.3)\rangle /CP + \langle (0.5, 0.4, 0.3), (0.4, 0.4, 0.3), (0.4, 0.5, 0.3)\rangle /F + \langle (0.7, 0.7, 0.6), (0.2, 0.2, 0.1), (0.3, 0.3, 0.4)\rangle /RNS$$

. The disease of the patient is diagnosed using the weighted similarity measure (Definition 6.5.). Suppose, for the sake of diagnosis, the highest priority is assigned to headache and muscle and joint ache and consequently symptoms such as, shivering, sweating, coughing, running nose and shivering, chest pain, fatigue, loss of appetite and body temperature have been prioritized. Let the respective weights assigned to T, Sh, Sw, H, MJ, C, LA, CP, LA, CP, F, RNS be 0.2, 0.7, 0.7, 0.9, 0.9, 0.6, 0.3, 0.4, 0.3 and 0.6.

The decision making process involves calculating the weighted similarity measures between the SVNMS P_f and the respective SVNMS representing the diseases. The set bearing the highest measure of similarity with respect to P_f is the disease that has affected the person. In order to confirm the obtained result, the process is repeated for more than one integral values of p i.e. the similarity degree.
The calculations have been represented in a tabular form as follows:

		Disease Sets		
Similarity Degree (p)	Patient Data	TB	Inf	CC
$p = 1$	P_f	0.69	0.89	0.87
$p = 2$	P_f	0.68	0.86	0.85
$p = 3$	P_f	0.66	0.85	0.83

Table 7.2. Table showing the similarity measures between the disease sets and P_f.

Thus, from the above findings it is clear that the patient has been suffering from Influenza.

8. Conclusion

In this paper a new hybridized concept, namely Single Valued Neutrosophic Multiset has been studied. Various set theoretic and algebraic operators have been defined and their properties have been discussed. The notions of distance and similarity measures have also been incorporated. Finally an example citing the applicability of single valued neutrosophic multisets in problems pertaining to medical diagnosis has been stated. Being characterized by an indeterminacy membership value, single valued neutrosophic sets provide a far more generalized tool in handling uncertainty as compared to fuzzy sets or intuitionistic fuzzy sets. Since single valued neutrosophic multisets have resulted by merging together the concepts of multisets and single valued neutrosophic sets, the former are a further generalization of the latter in the sense that in this case multiple occurrances of an element with varying degrees of membership values are taken into consideration and thus have more degrees of freedom compared to the latter. Moreover, the notions of distance and similarity measures are stated with a view to aid in the widespread applicability of SVNMS in fields like medical diagnosis, data retrieval on the web or in multicriteria decision making problems.

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