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SINGULAR VALUE INEQUALITIES FOR MATRICES WITH NUMERICAL RANGES IN A SECTOR

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Abstract. Let $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, where A_{22} is $q \times q$, be an $n \times n$ complex matrix such that the numerical range of A is contained in $S_\alpha = \{z \in \mathbb{C} : \Re z > 0, |\Im z| \leqslant (\Re z)\tan\alpha\}$ for some $\alpha \in [0, \pi/2)$. We obtain the following singular value inequality:

$$\sigma_j(A/A_{11}) \leqslant \sec^2(\alpha)\sigma_j(A_{22}), \qquad j = 1, \dots, q,$$

where $A/A_{11} := A_{22} - A_{21}A_{11}^{-1}A_{12}$ and $\sigma_j(\cdot)$ means the j-th largest singular value. This strengthens some recent results on determinantal inequalities. We also prove

$$\sigma_j(A) \leqslant \sec^2(\alpha)\lambda_j(\Re A), \qquad j = 1, \dots, n,$$

where $\lambda_j(\cdot)$ denotes the j-th largest eigenvalue, complementing a result of Fan and Hoffman.

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