Sketch Interpretation Using Multiscale Stochastic Models of Temporal Patterns

by

Tevfik Metin Sezgin

Submitted to the Department of Electrical Engineering and Computer Science
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Abstract

Sketching is a natural mode of interaction used in a variety of settings. For example, people sketch during early design and brainstorming sessions to guide the thought process; when we communicate certain ideas, we use sketching as an additional modality to convey ideas that can not be put in words. The emergence of hardware such as PDAs and Tablet PCs has enabled capturing freehand sketches, enabling the routine use of sketching as an additional human-computer interaction modality.

But despite the availability of pen based information capture hardware, relatively little effort has been put into developing software capable of understanding and reasoning about sketches. To date, most approaches to sketch recognition have treated sketches as images (i.e., static finished products) and have applied vision algorithms for recognition. However, unlike images, sketches are produced incrementally and interactively, one stroke at a time and their processing should take advantage of this.

This thesis explores ways of doing sketch recognition by extracting as much information as possible from temporal patterns that appear during sketching. We present a sketch recognition framework based on hierarchical statistical models of temporal patterns. We show that in certain domains, stroke orderings used in the course of drawing individual objects contain temporal patterns that can aid recognition. We build on this work to show how sketch recognition systems can use knowledge of both common stroke orderings and common object orderings. We describe a statistical framework based on Dynamic Bayesian Networks that can learn temporal models of object-level and stroke-level patterns for recognition. Our framework supports multi-object strokes, multi-stroke objects, and allows interspersed drawing of objects – relaxing the assumption that objects are drawn one at a time. Our system also supports real-valued feature representations using a numerically stable recognition algorithm. We present recognition results for hand-drawn electronic circuit diagrams. The results show that modeling temporal patterns at multiple scales provides a significant increase in correct recognition rates, with no added computational penalties.

Thesis Supervisor: Randall Davis
Title: Professor

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This thesis is dedicated to

my mother Hatice Sezgin
my father Fatin Sezgin
my sister Zerrin Sezgin

for caring about me more than I do...

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Chapter 1

Introduction

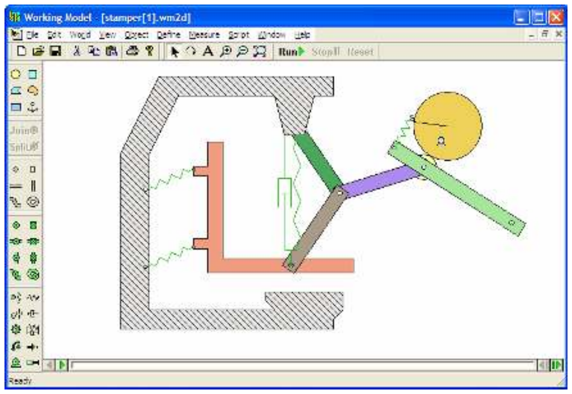
1.1 Why do sketch recognition?

Sketching is a natural modality of communication employed in a wide variety of settings. People sketch during early design and brainstorming sessions to guide the thought process and use sketches as a means of documentation[117]. We use sketching to convey thoughts that can not be put in words.

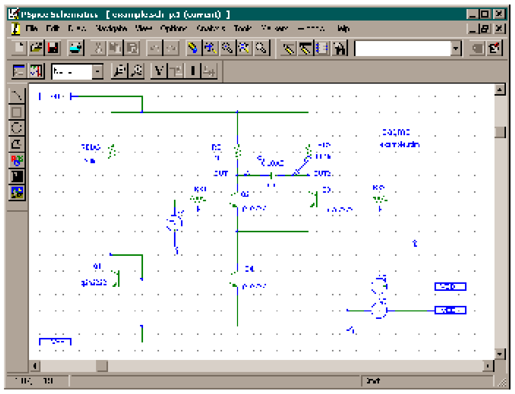
The increasing availability of pen based hardware such as PDAs and Tablet PCs makes capturing sketches easier than ever. Despite their ubiquity, suitability for certain HCI tasks, and the availability of supporting hardware, there is little computer support for sketching.

We believe making computers "sketch literate" will produce smarter, more natural user interfaces for a host of computer applications. Applications to benefit most from sketch based interfaces are the ones that use and manipulate graphical representations of objects. Fig. 1-1 shows examples of such commercial applications: a 2D rigid body physics simulator (Fig. 1-1.a), an analog circuit simulator (Fig. 1-1.b), a UML design tool (Fig. 1-1.c), and PowerPoint (Fig. 1-1.d).

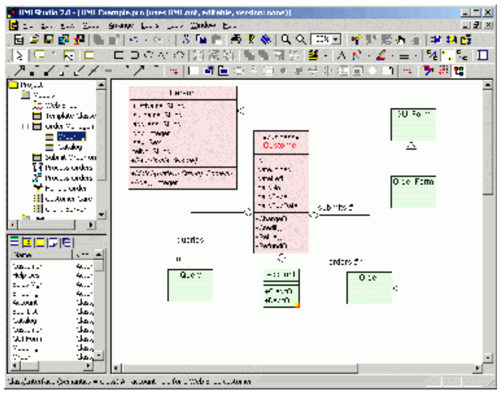
All of these applications currently share the same user interface paradigm based on mouse and keyboard input. Toolbars of the sort shown in Fig. 1-2 serve as the primary means of entering graphical information. In most cases, changing the default parameters of the graphical objects requires further keyboard and mouse input through menus and dialog boxes.



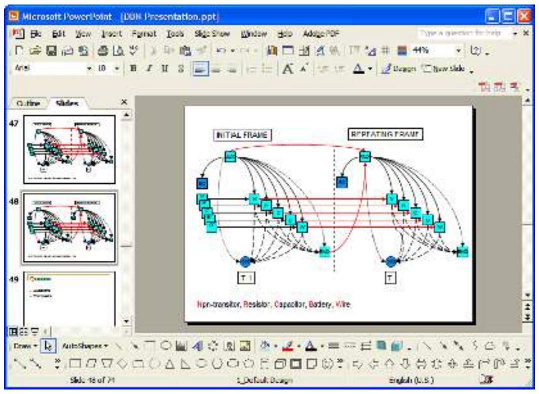
(a) Working Model 2D



(b) PSpice



(c) UMLStudio


Microsoft PowerPoint - [DBN Presentation.ppt]
INITIAL FRAME
REPEATING FRAME
Npn-transistor, Resistor, Capacitor, Battery, Wire

(d) PowerPoint

Figure 1-1: Examples of applications that use and manipulate graphical representations of objects. We believe such applications will benefit most from sketch based interfaces.

Our goal in building sketch recognition systems is to allow people to convey graphical information in the same way that they have done for thousands of years: simply by drawing. Imagine, for example, if an electrical engineer could draw the freehand circuit diagram in Fig. 1-3.a and have the computer automatically construct the PSpice model in Fig. 1-3.b. This is what we mean by sketch recognition and in this thesis, we show how it can be done by extracting as much information as it is practically possible from temporal patterns that appear during sketching.

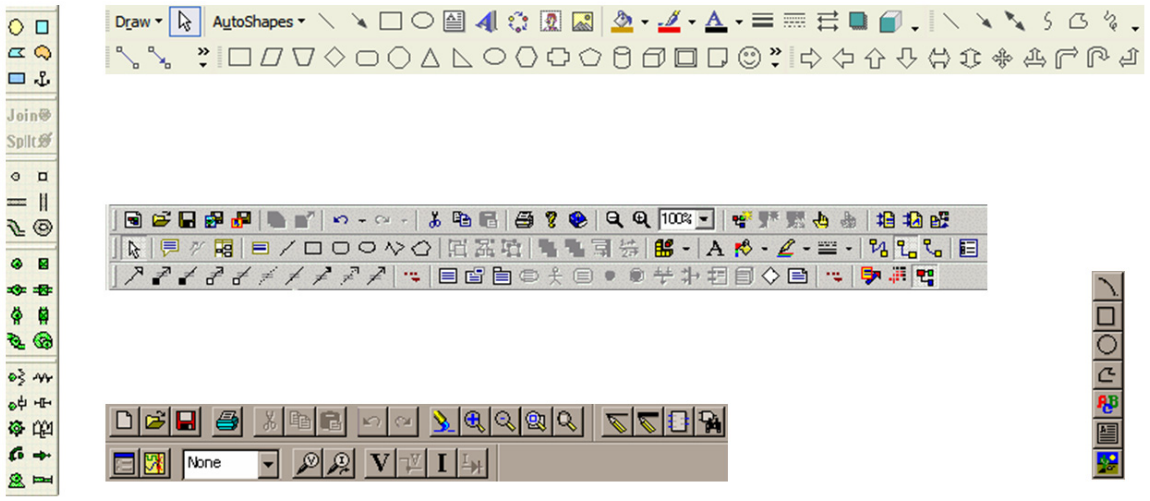
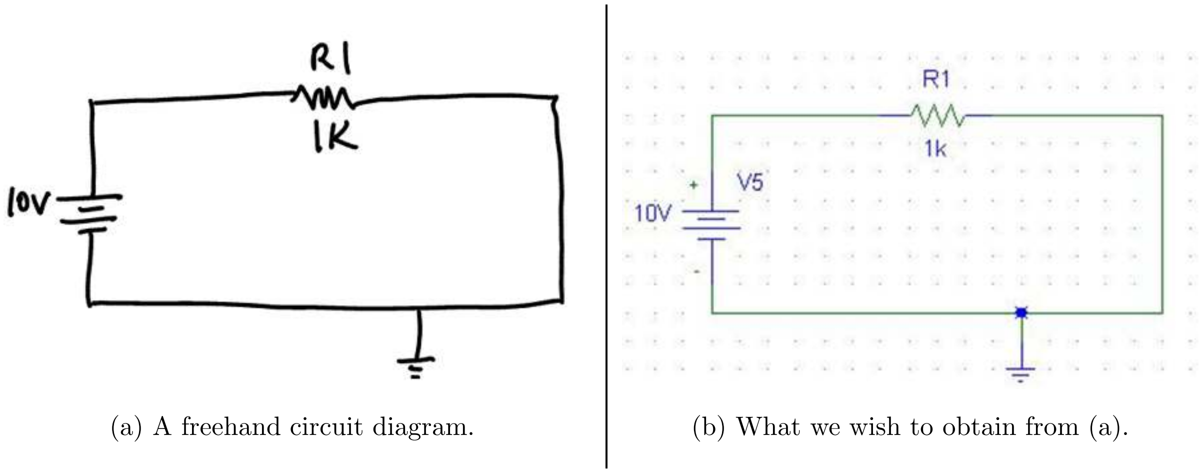


Figure 1-2: In most design software, toolbars serve as the primary means of entering graphical information. Here we see toolbars that are part of the user interface for the applications shown in Fig. 1-2.



(a) A freehand circuit diagram.

(b) What we wish to obtain from (a).

Figure 1-3: The goal of sketch recognition is to enable people to convey graphical information by drawing. Imagine if an electrical engineer could draw (a) and have the computer automatically construct the PSpice model in (b).

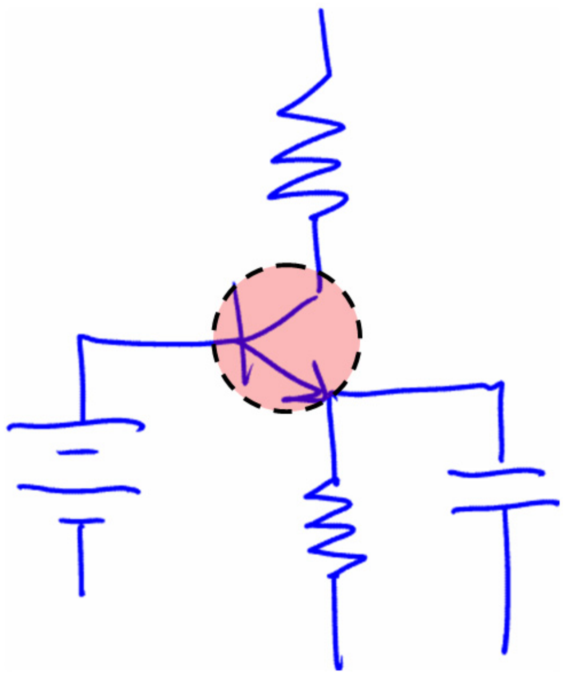


Figure 1-4: A circuit digram illustrating what we mean by a sketch. Our goal is to group strokes forming the same object together (segmentation) and determine what object they form (classification). In this case segmentation finds the circled grouping and classification indicates that they represent an *NPN transistor*.

1.2 What is sketch recognition?

The basic task of *sketch recognition* can be understood best by considering the input and the output.

By a *sketch*, we mean messy, informal hand-done drawings (e.g., Fig. 1-4). Specifically we are interested in recognizing sketches with objects of a symbolic nature that can be represented using structural descriptions, which have been the focus of the sketch recognition community [4, 107, 84, 45, 30].

It is worth emphasizing that we are interested in sketches that are drawn naturally as the user would draw on a piece of paper. This is unlike some systems that require users to draw each object using a single stroke, or pause after drawing each object to facilitate segmentation, thereby reducing the problem of sketch recognition to that of isolated symbol recognition.

We specifically refrain from forcing the user to sketch in a certain way or follow certain conventions during sketching. Consequently, our recognition algorithms will have to deal with unsegmented sketches with multi-stroke objects and drawing with interspersed objects (i.e., starting a new object before completing the current one).

Earlier we gave an informal definition of sketch recognition as the process of automatically converting freehand input into a representation which can be used by domain specific applications. A more formal definition of sketch recognition can be characterized in terms of three tasks:

- ***Segmentation:*** The task of grouping strokes so that those constituting the same object end up in the same group. At this point it is not known what object the strokes form. For example, in Fig. 1-4, the correct segmentation gives us fifteen disjoint sets of strokes (assuming each wire segment is a single object).

- ***Classification:*** Classification is the task of determining which object each group of strokes represents. For Fig. 1-4, recognition would indicate that the circled strokes represent an *NPN transistor*.

- ***Labeling:*** Labeling is the task of assigning labels to components of a recognized object (e.g., identifying the terminal with the current symbol in the NPN transistor in Fig. 1-4 as the emitter terminal).

Although the three tasks described above are conceptually separate, in any real sketch recognition scenario they may be intermixed. For example, it could be the case that a particular recognition architecture classifies objects and labels their constituent parts simultaneously. More likely, segmentation is done first, followed by classification and labeling.

1.3 Sketching is more than Graffiti

Graffiti is a single stroke gesture recognition scheme developed mainly for PDAs. One of the most popular gesture recognition methods for Graffiti-like input is described by Rubine [102]. Fig. 1-5 shows the Graffiti alphabet for punctuation and various symbols.

True sketch recognition is a different and much more challenging problem than recognizing the carefully prescribed strokes used in Graffiti. Unlike freehand drawing,

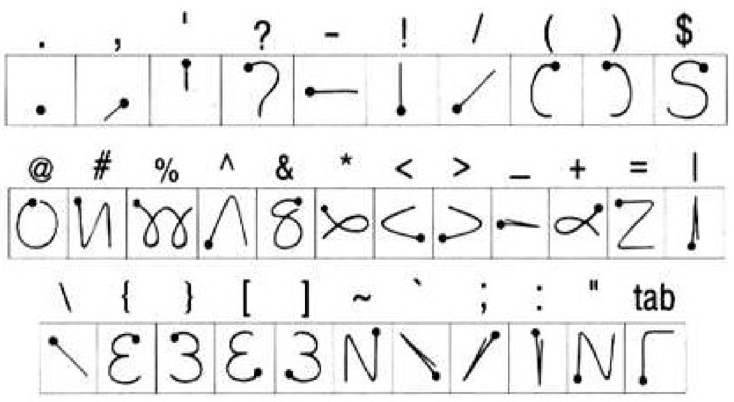


Figure 1-5: Gesture based Graffiti input scheme for PDAs. Input gestures must be drawn in a single stroke, starting at the heavy dot. Shapes of the some gestures do not look like the intended symbols (e.g., '%', '@').

the Graffiti alphabet comes with a number of conventions and restrictions that make it easy to recognize but unnatural. In particular:

- Symbols are specified by gestures that do not necessarily resemble the target shapes (e.g. '%', '@' in Fig. 1-5).
- Input gestures must be drawn in a single stroke.
- The gesture must start at a predetermined point (the heavy dots shown in Fig. 1-5).
- The starting point may affect classification because some symbols are specified with the same gesture but different starting points (e.g., [and { in Fig. 1-5).

These restrictions conflict with our goal of supporting freehand drawing where users can sketch without worrying about adhering to certain drawing guidelines.

1.4 Combinatorics makes sketch recognition hard

Treating sketches as images leads to recognition algorithms with exponential time complexities [84]: subgraph isomorphism-based methods and correspondence-based approaches have exponential time complexities. If we assume that we have m object classes and that each object model has k components, a simple calculation shows that in the worst case, recognizing an object surface with n strokes requires $\binom{n}{k}$ grouping operations and $k!$ constraint checking operations, yielding a total of $m\binom{n}{k}k!$ operations. In practice, the combinatorics get even worse because sketches are inherently noisy (e.g. due to digitization) and messy (e.g. sloppy drawing by the user).

Exponential time and space requirements are unacceptable for interactive sketch recognition, because giving users feedback about the recognition results is an essential part of sketch based interfaces[4]. Sketch recognition algorithms proposed so far either don't run in real-time [4], or make limiting assumptions about the domains (e.g., maximum object size, maximum number of strokes per object [108, 83, 85]). Furthermore most of these approaches have been demonstrated only in domains where objects are non-touching and hence segmentation is less of an issue compared to more challenging domains where objects may overlap or touch one another (e.g., box-connector type diagrams such as UML diagrams, family trees, circuit diagrams).

1.5 Using temporal patterns allows efficient sketch recognition

This thesis describes an approach to efficient sketch recognition. We show how treating sketching as an incremental and interactive process enables polynomial time recognition algorithms, by allowing us to exploit naturally appearing temporal patterns in sketching.

Current sketching systems are indifferent to who is using the system, employing the same recognition routines for all users. But user studies we have conducted indicate clearly that different users have different sketching preferences and styles, and that there is considerable value in being able to capture these different styles.

A major component of individual drawing styles is the temporal patterns seen during sketching. For example, when people draw stick figures, one frequently seen stroke-level pattern is a sequence consisting of a circular stroke, followed by a vertical line, followed by two pairs of positively and negatively sloped lines – respectively corresponding to the head, body, arms and legs of the stick-figure. It is these kinds of temporal patterns that can aid sketch recognition. Using temporal patterns allows

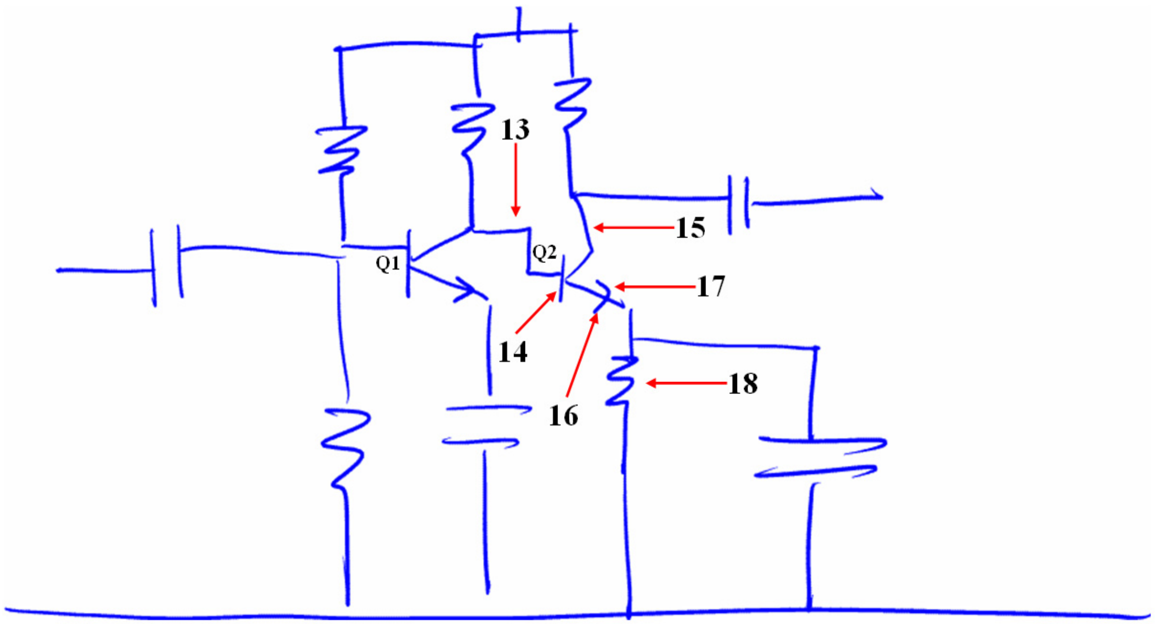


Figure 1-6: Circuit diagram for an amplifier. Stroke ordering for a fragment of the circuit is indicated by numbers.

us to work with one dimensional time series data, for which there are efficient, sound mathematical analysis methods.

Consider the circuit diagram in Fig. 1-6, showing an amplifier circuit and the stroke ordering for a fragment of the circuit of particular interest to us. Using the hierarchical probabilistic models that we present in this thesis, we can learn multiscale temporal patterns that naturally exist in the creation of such a sketch and interpret it under a second on a standard PC. Our hierarchical models capture knowledge of both common stroke orderings and common object orderings. Results of our evaluation involving circuit diagrams collected from eight participants show that modeling common object orderings reduces the number of recognition errors by 14%- 37%.

Furthermore our temporal model explicitly models interspersed drawing behavior and does not require the users to draw one object at a time. This means we can recognize sketches correctly even if the user starts drawing a new object before finishing the current one. For example, we can correctly recognize the circuit in Fig. 1-6 even though a wire (part of stroke #15) was drawn in the course of drawing the transistor **Q2**. Fig. 1-7 shows the interpretation of our system. Such interspersed drawing behavior poses serious challenges to a naïve temporal model for sketch recognition,

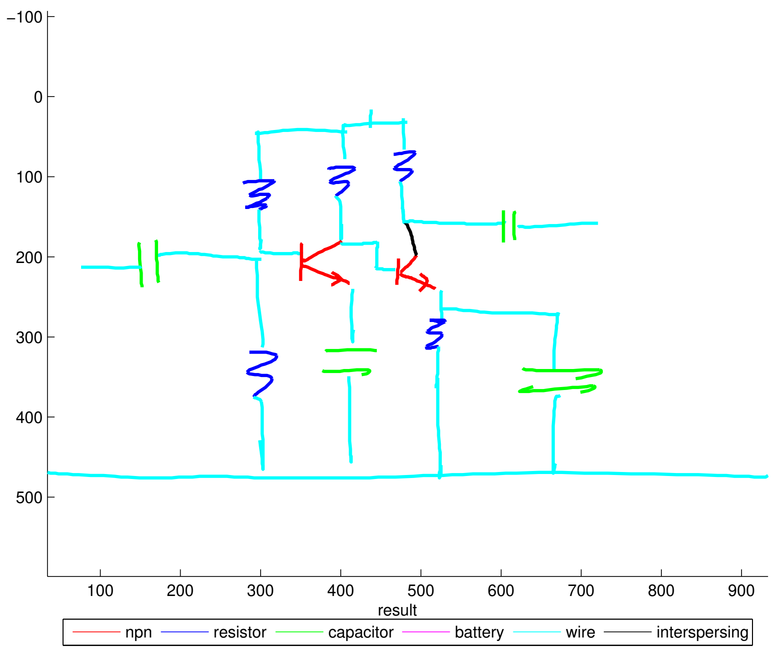


Figure 1-7: In this example, the user interspersed a wire connected to the collector of the transistor **Q2**. We present a model that can recognize objects correctly even in the presence of such interspersings and identify interspersed objects (indicated by coloring the interspersed wire in black instead of cyan). See Fig. A-1 for B&W printing.

but we show how the issue can be addressed remaining within the formal framework of Dynamic Bayesian Networks. We show that modeling interspersing reduces recognition errors by an additional 20%-37%.

Finally our recognition framework supports multi-object strokes (e.g., stroke #15 in Fig. 1-6 represents a wire and part of the transistor **Q2**), multi-stroke objects (e.g., **Q1** consists of four strokes), objects with a variable number of components (e.g., some resistors have fewer humps). We also support discrete and real-valued feature representations which allows us to encode our data using discrete encodings for categorical properties (e.g., a stroke being a circle vs. a line), and real-valued encodings for continuous features (e.g., length, orientation of a line). We report that a numerically stable belief propagation algorithm known as the Lauritzen-Jensen stable conditional Gaussian belief propagation algorithm should be used for modeling discrete and real-valued features.

1.6 Thesis roadmap

The next chapter summarizes properties of sketches and reports results from a user study that we conducted to measure the degree to which people use predictable stroke orderings when they sketch. Chapter three gives a formal description of the sketch recognition problem and introduces our model that uses hierarchical graphical models to model multiscale patterns in sketching while supporting interspersed drawing. Chapter three also introduces two baseline methods that we use in our evaluation. In chapter four, we report evaluation results comparing the performance of our model to the two baseline models. Chapters five and six summarize the related work and our main contributions. We conclude with future work.

Chapter 2

Properties of sketches

Here we briefly summarize properties of sketches because it is important to understand the nature of the data that we are dealing with.

2.1 Static properties of sketches

The static properties of sketches are those that would be present even if all we had was a scanned image of the sketch. They include image-like properties such as digitization noise and others that are specific to sketches, such as messiness.

Noise arises from digitization performed by a digitizing tablet, a scanner, or another imaging device. Each device has an inherent resolution and noise characteristics.

Messiness refers to phenomena such as overshooting, undershooting, messiness in the lines and jitters due to hand tremor. For example, in Fig. 2-1, stroke segments making the humps of the resistors don't always make the same angle, and some humps are relatively shorter although in an ideal resistor symbol, each segment is perfectly linear and they make the same angle. The missing connection between the capacitor in Fig. 2-1 and the wire above it is an example of undershooting – another kind of messiness. It is these kinds of phenomena resulting from the kinesthetic nature of sketching that makes sketch recognition a much harder problem than diagram recognition studied extensively by the document analysis community [114, 11].

Sketches are highly informal. Instances of the same icon may have varying aspect ratios and scales. Parts (subcomponents) of an object may have varying aspect ratios

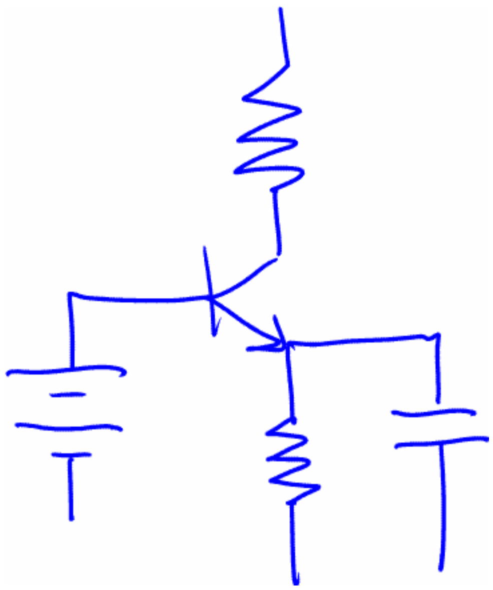


Figure 2-1: Example showing what we mean by a sketch. Note the messy, freehand nature of the diagram.

within the object. This high variability rules out popular transformation space search approaches that rely on affine properties of images.

The sketches we deal with are mostly iconic (e.g., a "light bulb" representing an idea, or a stick-figure representing a person). Their iconic nature makes it possible to have symbolic descriptions of sketches. There are also domains with highly non-iconic properties, such as "police sketches" or artistic renderings which fall outside the scope of this thesis (Fig. 2-2).

Most sketches can be broken down into compositions of simpler objects, and those objects can be further broken down into primitives such as lines, circles etc. For example, a simple sketch of a car can be broken down into the body, and the wheels; and the wheels can be further broken down into two circles representing the wheel and the axle.

2.2 Dynamic properties of sketches

One of the advantages of online sketching is that a sketch can be captured as it is constructed, making it possible to capture properties that may potentially aid recognition. Our stroke-based representation allows us to access stroke level information

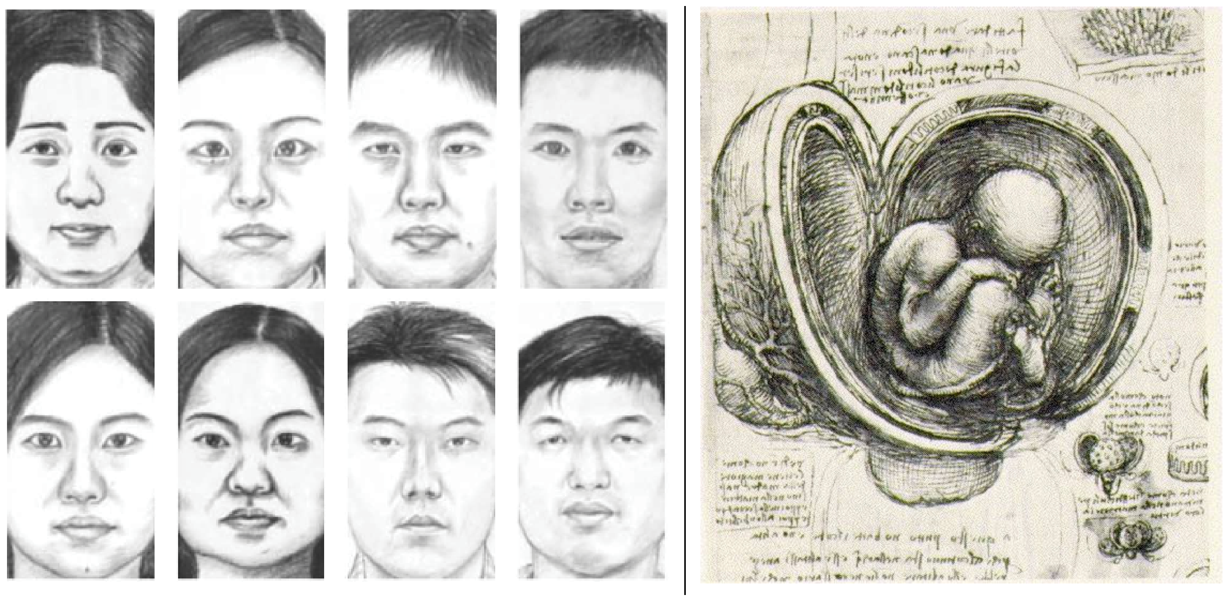


Figure 2-2: Police sketches (left) and artistic renderings. Recognition of such sketches fall outside the scope of this thesis. The police sketches were taken from [112] which describes a system for face sketch recognition. The *baby and the womb* drawing is by Leonardo da Vinci.

including what order the strokes came in, as well as timing information for individual points.

Because sketching is incremental (i.e, strokes are put on the sketching surface one at a time) the sketching surface will at almost every moment have on it a partially drawn object. Partially drawn objects are a common source of ambiguity in sketches. They can act as spurious strokes and confuse recognition algorithms. Partially drawn objects also raise the difficult issue of balancing the desire to generate all plausible recognition hypotheses and the desire to wait until there are enough components so the search is more constrained. The methods that we present explicitly address these issues by modeling the sketching process using a set of variables that, among other things, capture our belief about whether the user has done drawing the current object.

In a typical sketch recognition scenario, there is two way communication: information flows from the user to the computer via the strokes drawn and the editing operations, and from the computer to the user via computer's display of its interpretation of the strokes and editing operations. This interactive nature of sketching is an important source of knowledge when it comes to confirming the correctness of a

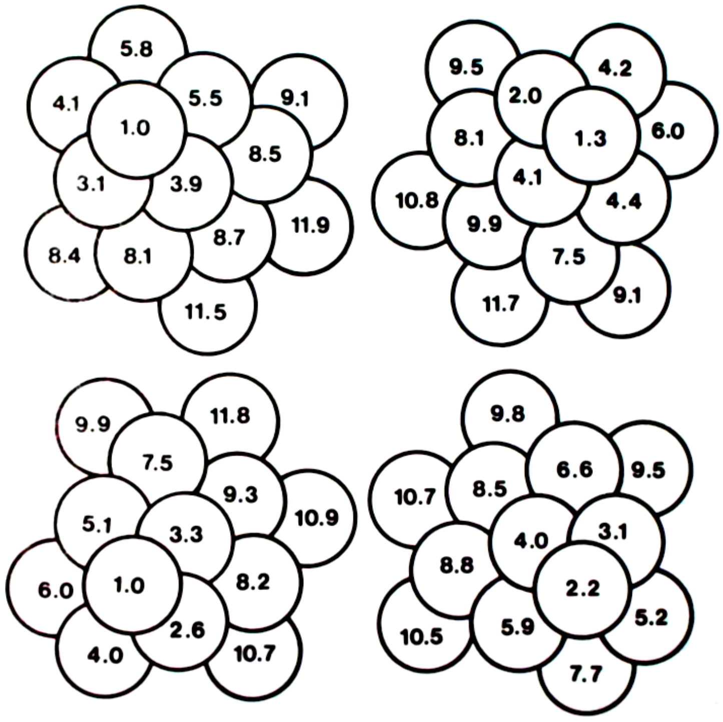


Figure 2-3: Results of the "grape clusters" experiment by van Sommers [119]. Users were asked to copy each cluster on paper. Numbers in each region show the average order in which the boundary was drawn and shows the strong effects of planning and anchoring.

system's interpretation. For example, the longer the user lets an interpretation exist, the more certain the system can be about its interpretation [6]. It is therefore important to have recognition algorithms that are fast enough to provide realtime feedback to the user.

Sketching is highly stylized in the sense that people have strong biases in the way they sketch (e.g., people draw enclosing objects first, use a left-to-right stroke ordering when drawing symmetric objects). There is psychological evidence attributing such phenomena to motor convenience, part saliency, hierarchy, geometric constraints, planning and anchoring [116, 119]. For example, Fig. 2-3 shows the results of the "grape clusters" experiment described by van Sommers [119] where the users were asked to copy four clusters on paper. Numbers in each region show the average order in which the boundary was drawn and shows the strong effects of planning and anchoring.

Existence of ordering patterns during drawing is significant from a recognition perspective because the regularities in sketching can be used for recognition. Because this forms the basis for our approach to recognition presented chapter 3, and we conducted user studies to validate this observation for domains of interest to us.

2.3 Temporal patterns appear to be common in sketching

We ran a user study to assess the degree to which people have sketching styles, i.e., a reasonably consistent stoke order used when drawing an item. For example, if one starts drawing a stick-figure with the head, then draws the torso, the legs and the arms respectively, we regard this as a style different from the one where the arms are drawn before the legs (see Fig. 2-5).

Our user study asked users to sketch various icons, diagrams and scenes from six domains. Example tasks included drawing:

- Finite state machines performing simple tasks such as recognizing a regular language.
- Unified Modeling Language (UML) diagrams depicting the design of simple object-oriented programs.
- Scenes with stick-figures playing certain sports.
- Course of Action Diagram symbols used in the military to mark maps and plans.
- Digital circuit diagrams that implement a simple logic expression.
- Emoticons expressing happy, sad, surprised and angry faces.

We asked 10 subjects to sketch three sketches from each of the six domains, collecting a total of 180 sketches. Requests were given to subjects in an arbitrary order to intersperse domains and reduce the correlation between sketching styles used in different instances of sketches from the same domain. Sketches were captured using a digitizing LCD tablet.

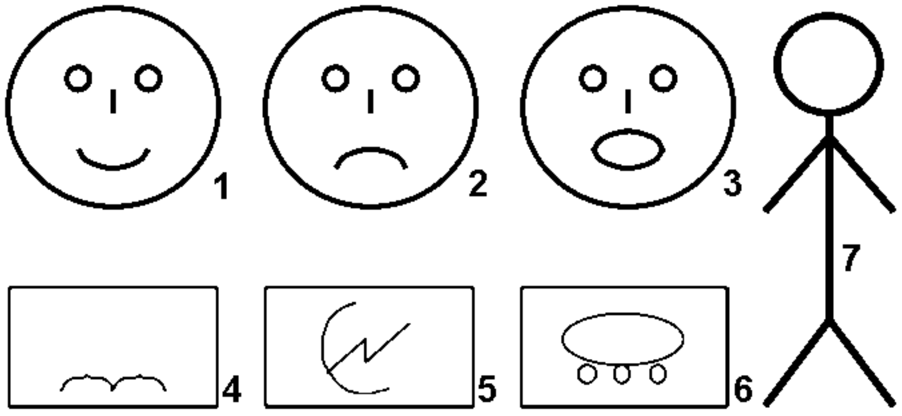


Figure 2-4: Examples of the figures that the users were asked to draw in the user study.

	Object id (from Fig.2-4)						
	1	2	3	4	5	6	7
Number of subparts	5	5	5	6	8	8	6
Max theoretical # of orders	120	120	120	720	40320	40320	720
Mean # of orders used	4	4	3	3	4	3	5

Table 2.1: A summary of the statistics from our user study for a subset of the symbols drawn by the users (shown in Fig.2-4). Note that 120=5!, 720=6! and 40320=8!. Users drew 30 or more examples of each object.

Table 2.1 shows statistics on drawing orders. The maximum possible drawing orders for each object shows the theoretical upper-bound. The table also shows the mean number of drawing orders per object for all users, rounded to the nearest integer. While in theory there are $n!$ ways of drawing an object with n subcomponents, as Fig. 2.1 shows, only a few are actually employed by users. This confirms our belief that people use predictable stroke orderings.

Our analysis of the sketches also involved constructing sketching style diagrams for each user. A *sketching style diagram* provides a concise way of representing how different instances of the same object are drawn. Nodes of the diagram correspond to partial drawings of an object. They are connected by arcs that correspond to strokes. Fig. 2-5 illustrates the sketching style diagram for the stick-figure example described above.

Our inspection of the style diagrams and the statistics revealed that:

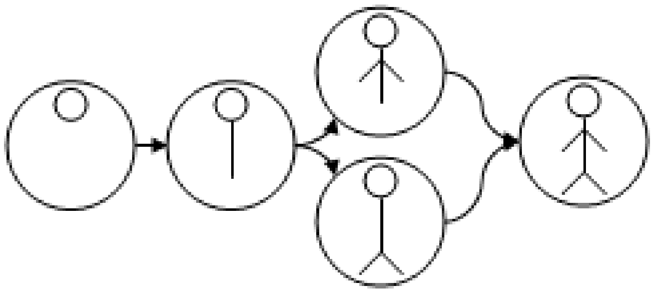


Figure 2-5: A sketching style diagram showing two ways of drawing stick-figures

- People sketch objects in a highly stylized fashion. In drawing the stick figure, for example, one of our subjects always started with the head and the torso, and finished with the arms or the legs (Fig.2-5).
- Individual sketching styles persist across sketches.
- Subjects prefer an order (e.g., left-to-right) when drawing symmetric objects (e.g., the two arms) or arrays of similar objects (e.g., three collinear circles).
- Enclosing shapes are usually drawn first (e.g., the outer circle in emoticons, or the enclosing rectangles in Fig. 2-4).

The user study confirmed our conjecture about the stylized nature of sketching in a number of domains that have received the attention of the sketch recognition community. In order to capitalize on this, we constructed a probabilistic model for learning temporal patterns in sketching and use them for sketch recognition.

Chapter 3

Approach

This chapter describes our approach to sketch recognition. Our model is designed to take advantage of the rich temporal patterns that exist in sketching while supporting interspersed drawing. To facilitate the explanation of this model and to serve as baseline systems, we also introduce two simpler models.

We start by discussing a toy example to illustrate the intuition behind using stroke ordering information for sketch recognition. We then describe properties that we want our models to have and give formal definitions of the terms we will use in the rest of this chapter.

3.1 The intuition

To make the basic intuition clear, we start with an over-simplified task scenario. Assume we have only two symbols to recognize: *skip-audio-track* and *stop*, and assume the user always draws them using the same stroke ordering, indicated by the numbers in Fig. 3-1-a. Finally, assume we need to recognize which of these symbols is present in a scene known to contain only a single instance of one of them (i.e., isolated object recognition).

Suppose the user draws the *stop* symbol as shown in Fig. 3-1-b. Assuming we can reliably recognize the individual strokes as lines and tell whether they are horizontal (**H**), vertical (**V**), negatively/positively sloped (**N**, **P**), we can look at the order in which the user drew the lines and classify the input as a *stop* symbol if we see the

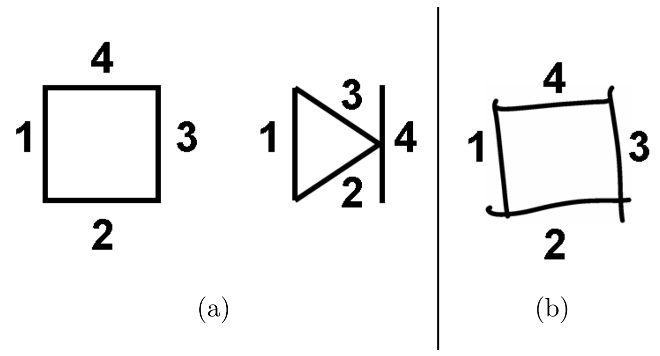


Figure 3-1: Symbols for *stop* and *skip-audio-track* (on the left), and a sketched *stop* symbol (right).

[**V, H, V, H**] ordering, and as a *skip-track* symbol for the [**V, P, N, V**] ordering.

The above approach works by encoding the user input to generate an *observation sequence* describing the scene (e.g. [**V, H, V, H**]), and comparing this sequence to its model of how the user is known to sketch. The result of the comparison is binary, indicating whether we have a match. This toy example shows how stroke ordering can be used for recognition in an over-simplified scenario.

3.2 Desired features of a model

We see the following properties as being important for models of sketch recognition.

3.2.1 Handling stroke-level patterns

As noted in chapter 2, stroke orderings used in the course of drawing individual objects naturally contain certain patterns. We call these **stroke-level patterns** because they capture the probability of seeing a sequence of *strokes* with certain properties when sketching a particular object. We want our sketch recognition algorithms to capture these patterns.

3.2.2 Support for multiple classes and drawing orders

We should be able to recognize multiple classes of objects. We also want to accommodate multiple drawing orders instead of just one. For example, it may be the case that sometimes the user sketches the *stop* symbol starting with a horizontal line. Furthermore, if the user prefers one drawing order more frequently than others, this should be accounted for as well. This requires using training and classification methods that can use such information.

3.2.3 Handling variations in encoding length

Users should be able to draw freely. For example, they should be able to draw the *stop* symbol using three strokes instead of four, or draw a resistor with five humps instead of six (thus generating an encoding of the input with only five observations instead of six).

3.2.4 Probabilistic matching score

We would like the result of matching an observation sequence against a model to be a continuous value reflecting the likelihood of using that particular drawing order for drawing the object. This is required if we are to have a mathematically sound framework for combining the outputs of multiple matching operations for scenes with multiple objects such that, among plausible interpretations, those corresponding to more frequently used orders are preferred.

3.2.5 Learning compact representations

In practice, different drawing orders will have similar subsequences. Ideally the system should learn compact representations of drawing orders from labeled sketch examples.

3.2.6 Rich feature representation

One of the steps in applying machine learning techniques to a problem is to decide on a set of features that are sufficiently expressive given the problem at hand. In sketch recognition, we deal with data that is most naturally described using geometric features such as the shape of a stroke segment, length and orientation of line segments, radii of circles etc. Some of these features are categorical (e.g., shape of a stroke segment can be `arc`, `line` etc.) and are best represented using discrete variables. Other features such as length and orientation are real-valued quantities and should be represented as such. Therefore our algorithms should be able to represent both discrete and real-valued features.

3.2.7 Handling object-level patterns

Another kind of temporal pattern present in online sketches is an **object-level pattern** that captures the probability of seeing a certain sequence of objects being drawn. Consider the domain of UML class diagrams drawn by software designers to describe inheritance relationships, generalizations, associations, etc. In this domain, when a designer draws a new class (indicated by a rectangle), it is natural to expect that the new object will be connected with an arrow to one or more of the objects drawn earlier. So in this domain, it is natural to have arrows drawn after a new class is created, which we define as an *object-level pattern*. It is also natural to expect that the kind of arrow will be different if the newly created class was a `final` class (because `final` classes cannot be extended).

It is easy to imagine that this sort of domain specific knowledge about object-level patterns could be a powerful addition to a UML diagram recognition system, and to sketch recognition systems in general. But not every domain may have patterns that are as well understood as they are for UML diagrams. And even if experts could identify such patterns, incorporating them into a recognition system would be a laborious task at best, considering all the ways in which various objects may combine together. We argue that a better way of incorporating object-level temporal patterns in a recognition framework would be to learn such features from data — along with stroke-level patterns — and use them in recognition. Therefore the ability to model object-level patterns is yet another feature we want.

3.2.8 Ability to handle interspersed drawing

During sketching, although people most often complete each object before starting a new one, as we show later they sometimes draw other objects before completing the current one. Such drawing behavior may severely hinder recognition unless the recognition algorithms deal with it explicitly. The ability to handle interspersed drawing is thus another feature that we desire.

3.3 Terminology and problem formulation

3.3.1 Terminology

We define a ***sketch*** $\mathscr{S} = S_1, S_2, ...S_N$ as a sequence of strokes captured using a digitizer, preserving the drawing order. [1] A ***stroke*** is defined as a set of time-ordered points sampled between pen-down and pen-up events during sketching.

Each stroke is broken into several geometric ***primitives*** such as line and arc segments as part of the preprocessing of the sketch, so let $\mathscr{P} = P_1, P_2, ...P_T$ be the sequence of time-ordered primitives obtained from $\mathscr{S}$. Because the preprocessing of a stroke may result in more than one primitive, the total number of primitives can be larger than the number of strokes.

We use ***segmentation*** to refer to the task of grouping together primitives constituting the same object. Given a set of classes $\mathscr{C} = \{C_1, C_2, ...C_n\}$ ***classification*** refers to the task of determining which object each group of primitives represents (e.g., a stick-figure or a rectangle). Segmentation produces K groups $G = G_1, G_2, ...G_K$, and classification gives us the labels for the groups $L = L_1, L_2, ...L_K$, $L_i \in \mathscr{C}$. Each group is defined by the indices of the primitives included in the group $G_i = \rho_1, \rho_2, ...\rho_n$ sorted in ascending order, so for cases where we don't allow interspersing $|G_i| = \rho_n - \rho_1 + 1$.

We define ***sketch recognition*** as the segmentation and classification of a sketch. A simplifying assumption in most sketch recognition systems is that a stroke can be part of only one object. Our definition of segmentation in terms of primitive groupings is more general than a definition based on stroke groupings and as long as primitives are not shared across objects it allows a stroke to be part of multiple objects (e.g., drawing a box and an arrow, or a resistors and a pair of wires in a single stroke (Fig. 3-2)).

Finally we use the term *observations* to refer to the sequence of features $\mathscr{O} = O_1, O_2, ..., O_T$ obtained from the primitives. We use $\boldsymbol{O_{G_i}}$ to refer to the sequence of observations corresponding to a group of primitives G_i.

[1] We will use the Matlab notation *begin* : *end* as a shorthand for a list of indices that begins with *start* and ends with *end* (i.e., 1:4 = 1 2 3 4). We will also use this notation in subscripts to denote a list of items (i.e., $S_{1:N} = S_1, S_2, ...S_N$).

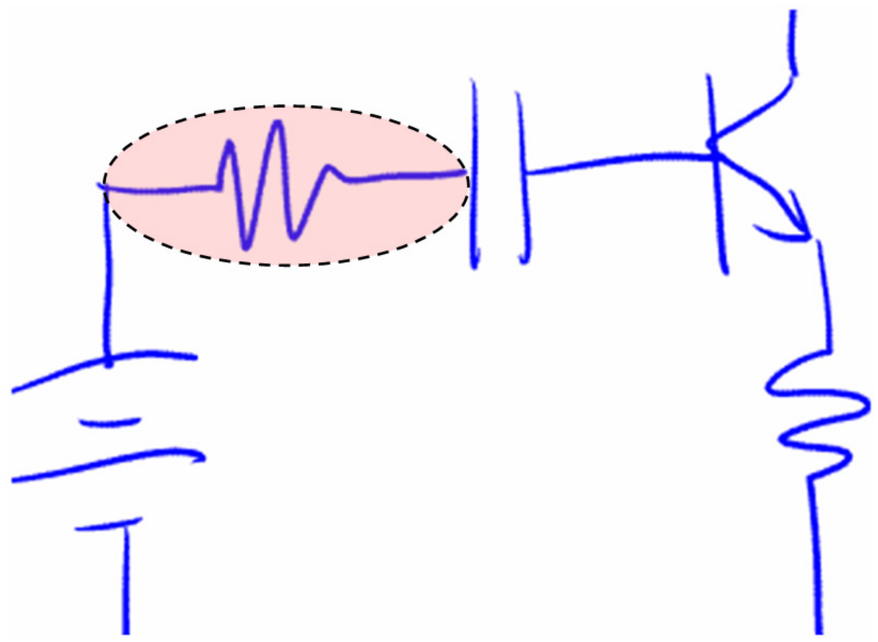


Figure 3-2: An example showing a multi-object stroke. The selected circuit fragment contains two wires and a resistor drawn using a single stroke.

3.3.2 Problem formulation

We consider three models. The first two serve as baseline methods in our evaluations and help introduce our multiscale-interspersing model. Our model can learn both object-level patterns and stroke-level patterns, and it explicitly models interspersed drawing.

Model 1: ***Flat model***

This model implements the first five features listed in section 3.2: It can learn compact representations of stroke ordering patterns for multiple objects, each of which can be drawn in more than one way, using a variable number of strokes.

Given a sequence of time-ordered primitives obtained from a sketch, the goal of this model is to find a segmentation and classification of the primitives that maximizes the likelihood of stroke-level patterns. Over all possible ways in which the primitives can be grouped, $\mathfrak{G}$, and all possible ways in which these groups can be labeled, $\mathfrak{L}(G)$, we want the grouping $G \in \mathfrak{G}$ and the labeling $L \in \mathfrak{L}(G)$ that maximizes the likelihood of the stroke-level observable features $\boldsymbol{O_{G_i}}$ for each group G_i given the model corresponding to its label ($\boldsymbol{\lambda_{L(G_i)}}$).

More formally, this can be written as a maximization problem, the solution to which gives us the segmentation and labeling of the input sketch.

$$\underset{G \in \mathfrak{G}, L \in \mathfrak{L}(G)}{\operatorname{argmax}} \prod_{i=1}^{K} P(\boldsymbol{O_{G_i}} | \boldsymbol{\lambda_{L(G_i)}}) \tag{3.1}$$

Model 2: *Multiscale model*

This model extends the flat model by adding the ability to have real-valued feature vectors and the ability to handle object-level patterns in addition to stroke-level patterns.

Given a sequence of time-ordered primitives obtained from a sketch, the goal of this model is to find a segmentation and classification of the primitives such that the likelihood of stroke-level patterns and object-level patterns is maximized simultaneously. Over all possible ways in which the primitives can be grouped, $\mathfrak{G}$, and all possible ways in which these groups can be labeled, $\mathfrak{L}(G)$, we want the grouping $G \in \mathfrak{G}$ and the labeling $L \in \mathfrak{L}(G)$ that:

- maximizes the likelihood of the sequence of labels $L_1, L_2, ..., L_K$ given our model for object-level patterns ($\boldsymbol{\lambda_{obj}}$), and
- maximizes the likelihood of the stroke-level observable features $\boldsymbol{O_{G_i}}$ for each group G_i given the stroke-level model corresponding to its label ($\boldsymbol{\lambda_{L(G_i)}}$).

The terms corresponding to the stroke-level and object-level patterns can be written together to obtain the following maximization problem, the solution to which gives us the segmentation and labeling of the input sketch.

$$\underset{G \in \mathfrak{G}, L \in \mathfrak{L}(G)}{\operatorname{argmax}} P(L_1, ..., L_K | \boldsymbol{\lambda_{obj}}) \prod_{i=1}^{K} P(\boldsymbol{O_{G_i}} | \boldsymbol{\lambda_{L(G_i)}}) \tag{3.2}$$

The expression above is maximized over the set of all groupings and their labelings ($\mathfrak{G}$ and $\mathfrak{L}$(G)).

Model 3: *Multiscale-interspersing model*

This model adds the ability to handle interspersed drawing to the multiscale model. The formulation of the recognition problem is the same as above, except now the indices of primitives in each group $\mathfrak{G}$ can have gaps (i.e., $|G_i| > \rho_n - \rho_1 + 1$).

$$\underset{G \in \mathfrak{G}, L \in \mathfrak{L}(G)}{\operatorname{argmax}} P(L_1, ..., L_K | \boldsymbol{\lambda_{obj}}) \prod_{i=1}^{K} P(\boldsymbol{O_{G_i}} | \boldsymbol{\lambda_{L(G_i)}}) \tag{3.3}$$

3.4 Choice of probabilistic model and input representation

All of the above formulations require maximizations over the set of all groupings and their labelings ($\mathfrak{G}$ and $\mathfrak{L}$(G)). As noted in Chapter 1, an exhaustive search of this space is computationally prohibitive.

Given that we are modeling sequential patterns, we make the assumption that both stroke-level and object-level patterns can be modeled as products of first order Markov processes. This allows us to efficiently compute maximum likelihood estimates, enabling us to both learn model parameters and do recognition efficiently.

We implement the flat model using a probabilistic model based on *Hidden Markov Models* which is somewhat easier to describe and much simpler to implement compared to the other two models. For the more complex models (#2 and #3), we use *Dynamic Bayesian Networks*. We now briefly review Hidden Markov Models and Dynamic Bayesian Networks mainly for the purposes of introducing notation. Excellent reviews of HMMs and DBNs can be found in Rabiner's HMM tutorial paper [99], and Kevin Murphy's book chapter on DBNs [89]).

3.4.1 Hidden Markov Models

An HMM $\lambda(A, B, \pi)$ is a doubly stochastic process for producing a sequence of observed symbols. An HMM is specified by three parameters A, B, π. A is the transition probability matrix $a_{ij} = P(q_{t+1} = j | q_t = i)$, B is the observation probability distribution $B_j(v) = P(O_t = v | q_t = j)$, and π is the initial state distribution. $Q = \{q_1, q_2, ...q_N\}$ is the set of HMM states and $V = \{v_1, v_2, ...v_M\}$ is the set of observations symbols.

Given an HMM $\lambda(A, B, \pi)$ and a sequence of observations $O = o_1, o_2, .., o_k$, we can efficiently determine how well each model λ accounts for the observations by computing $P(O|\lambda)$ using the Forward algorithm; compute the best sequence of HMM state transitions for generating O using the Viterbi algorithm; and estimate HMM parameters A, B and π to maximize $P(O|\lambda)$ using the Baum-Welch algorithm.

3.4.2 Dynamic Bayesian Networks

Bayesian networks encode the joint probability of a set of variables $Z = \{Z_1, ..., Z_n\}$ where the graphical structure of the network encodes the conditional dependencies among the variables. DBNs are extensions of Bayesian networks that model joint distribution of a set of variables over time by representing the conditional dependencies between the variables using a pair of Bayesian networks $(B_1, B_\rightarrow)$. B_1 defines the prior for the Z_i values at time $t = 1$, and $B_\rightarrow$ defines how variables at time $t+1$ relate to each other and to those from time t.

HMMs are quite expressive as statistical models for time series [17, 16]. They are also closely related to DBNs and it is possible to convert a DBN to an equivalent HMM and vice versa. For the multiscale model and the interspersing multiscale model, we choose to use a DBN representation because they allow parsimonious representations of generative processes.

3.4.3 The input

The input to our models is the observation sequence $\mathscr{O} = O_{1:T}$ obtained by computing features from corresponding primitives $\mathscr{P} = P_{1:T}$. For the first model, we use only discrete observations, while for the others we support both discrete and real-valued (continuous) observations which allows us to have richer features while avoiding discretization problems.

3.5 Flat model

As a reminder, the goal of this model is to segment and label an input sketch by finding the solution to the expression in Eq. 3.1, reproduced below:

$$\operatorname*{argmax}_{G \in \mathfrak{G}, L \in \mathfrak{L}(G)} \prod_{i=1}^{K} P(\boldsymbol{O}_{\boldsymbol{G}_i} | \boldsymbol{\lambda}_{\boldsymbol{L}(\boldsymbol{G}_i)}) \tag{3.4}$$

We solve this maximization problem by encoding the sketches using a very simple encoding scheme and modeling stroke orderings using HMMs, then combining the results from individual HMMs using dynamic programming implemented in the form of a shortest path algorithm. By using dynamic programming, we avoid the complexity of a naïve combinatorial search strategy.

3.5.1 Encoding

We encode strokes using the Early Sketch Processing Toolkit described in [105], which converts strokes into geometric primitives. We use a codebook of 13 symbols to encode the output of the toolkit, converting sketches into discrete observation sequences. Four codebook symbols encode lines: positively/negatively sloped, horizontal/vertical; three encode ovals: circles, horizontal/vertical ovals; four encode polylines with 2, 3, 4, and 5+ edges; one encodes complex approximations (i.e., mixture of curves and lines); and one denotes two consecutive intersecting strokes.

Because instances of the same object sketched in different styles may have encodings of different lengths, we formulated two frameworks for training and recognition that use fixed and variable length training examples respectively.

3.5.2 Modeling with fixed input length HMMs

Assume we have $\boldsymbol{n}$ object classes. Encodings of training data for class $\boldsymbol{i}$ may have varying lengths, so let $\boldsymbol{\psi_i = \{l_{i1}, l_{i2}, ...l_{ij}\}}$ be the distinct encoding lengths for class $\boldsymbol{i}$. We partition the training data into $\mathbb{K} = \sum_{i=1}^{n} |\psi_i|$ sets such that each partition has training data for the same object with the same length. Now we train $\mathbb{K}$ HMMs, one for each set, using the Baum-Welch method. Each class $\boldsymbol{i}$ is represented by $|\boldsymbol{\psi_i}|$ HMMs, and we have an inverse mapping that tells us which class each HMM represents.

Assume for a moment that our goal is to do isolated object recognition. Then we could compute $\boldsymbol{P(O|\lambda_i)}$ for each model $\boldsymbol{\lambda_i}$ using the Forward procedure with the observation sequence $\boldsymbol{O}$ generated by encoding the isolated object. $\boldsymbol{\lambda_i}$ with the highest likelihood would give us the object class. But isolated object recognition requires the input sketch to be presegmented, which is usually not the case, and segmentation is a part of the problem we are trying to solve. We achieve both segmentation and recognition by maximizing Eq. 3.4 while noting that for this model, we have assumed the user completes each object before moving to the next one (i.e., there is no interspersed drawing).

Given an observation sequence $\mathscr{O} = \{O_1, O_2, ..., O_T\}$, because we assume there is no interspersed drawing, we can say that prefixes $\mathscr{O}' = \{O_1, O_2, ..., O_{l_{ij}}\}$ of the observation sequence can be accounted for by $\mathbb{K}$ hypotheses — one for each of the $\mathbb{K}$ HMMs that we have. In other words, each HMM model offers a hypothesis that accounts for the first l_{ij} observations with a cost c. Here c is defined as the absolute value of the loglikelihood term obtained for the corresponding prefix $\mathscr{O}'$ and model λ_i written as $c = |log(P(\mathscr{O}'|\lambda_i))|$. Similarly for each of these cases, prefixes of the remaining portions of observation sequence $\mathscr{O}'' = \{O_{l_{ij}}, O_{l_{ij}+1}, ..., O_T\}$ can be accounted for by any of the $\mathbb{K}$ HMMs in a recursive fashion.

We keep assigning hypotheses and scores to subsequences of the observation sequence until we cover all of the observation sequence. Now our task is to find a sequence of hypotheses that account for the whole observation sequence with no gaps or overlaps, such that sum of costs for the hypotheses is minimum. This is essentially equivalent to finding the shortest path in a graph $G(V, E)$ where the nodes correspond to observations and arcs connecting the nodes representing observations O_s, O_d correspond to a hypothesis that accounts for the observations $\{O_s, O_{s+1}, ..., O_d\}$. Once we construct the graph $G(V, E)$, we can find the proper segmentation and labeling of the observations by computing the shortest path and determining which models are actually used in the shortest path.[2]

Here is a step-by-step description of how we build our graph. We will use a simple example with two classes (square and triangle) to facilitate our discussion. Fig. 3-3 shows examples of the training data and Fig. 3-4 shows the scene to be recognized. Note that for this example $\psi_{square} = \{3, 4\}$ and $\psi_{triangle} = \{2, 3\}$. Our graph $G(V, E)$ has $|O|$ vertices (V), one per observation, and a special vertex v_f denoting the end of observations (Fig. 3-5-a). Let k be the input length for model λ_i. Starting at the beginning of the observation O, for each observation symbol O_s, we take a

[2]The graph $G(V, E)$ that we build for segmentation should not be confused with the graphs that represent HMM topologies.

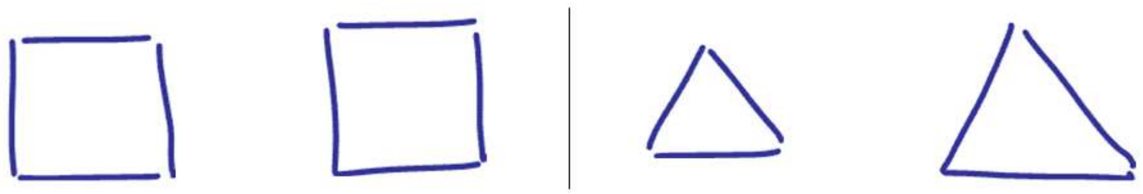


Figure 3-3: Training examples for the simple recognition problem that we will use to illustrate our HMM based recognition method. We have two classes (square and triangle). For simplicity assume the squares are drawn using three or four strokes, and the triangles are drawn using two or three strokes (i.e., $\psi_{square} = \{3, 4\}$ and $\psi_{triangle} = \{2, 3\}$).

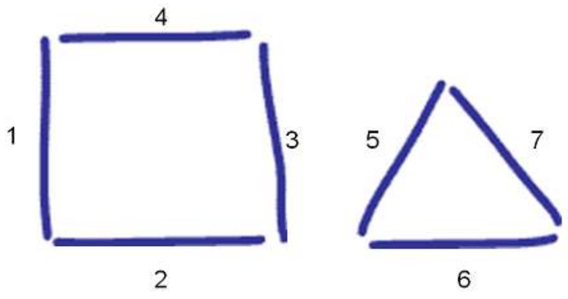
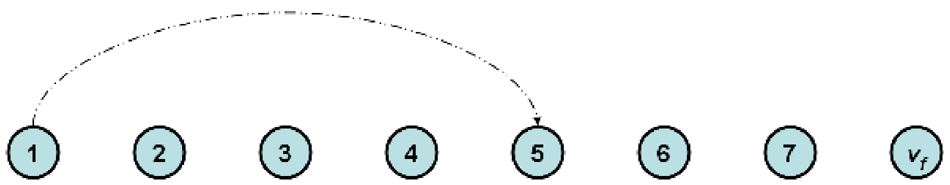


Figure 3-4: A simple scene consisting of a square and a triangle. Numbers indicate the stroke ordering.

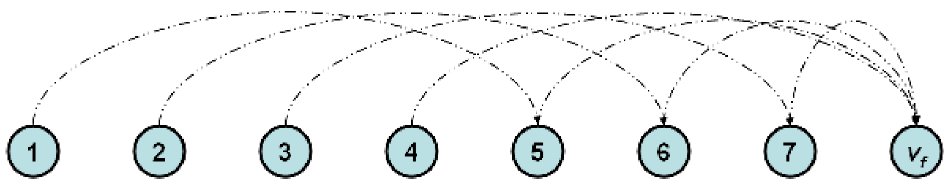
substring $O_{s,s+k}$ and compute the loglikelihood of this substring given the current model, $log(P(O_{s,s+k}|\lambda_i))$. We then add a directed edge from vertex v_s to vertex v_{s+k} in the graph with an associated cost of $|log(P(O_{s,s+k}|\lambda_i))|$ (Fig. 3-5-a). If the destination index $s + k$ exceeds the index of v_f, instead of trying to link v_s to v_{s+k}, we put a directed edge from v_s to the final node v_f (three rightmost edges in Fig. 3-5-b). We set the weight of the edge to $|log(P(O_{s,|O|}|\lambda_i))|$. Here $O_{s,|O|}$ is the suffix of O starting at index s. Adding these special edges from v_s to v_f for $s + k > |O|$ allows our segmentation and recognition algorithms to work even if the user hasn't completed drawing the current object, while preserving global consistency.

This feature is a major strength of our approach, allowing us to to do recognition when the scene is not yet complete — a major challenge in recognition that most other systems sidestep by requiring the user to aid segmentation either implicitly or explicitly. We complete the construction of G by repeating the above operation for all models (Fig. 3-5-c, Fig. 3-5-d).

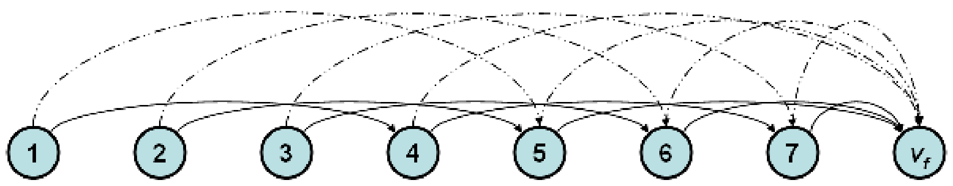
In the constructed graph, having a directed edge from vertex v_i to v_j with cost



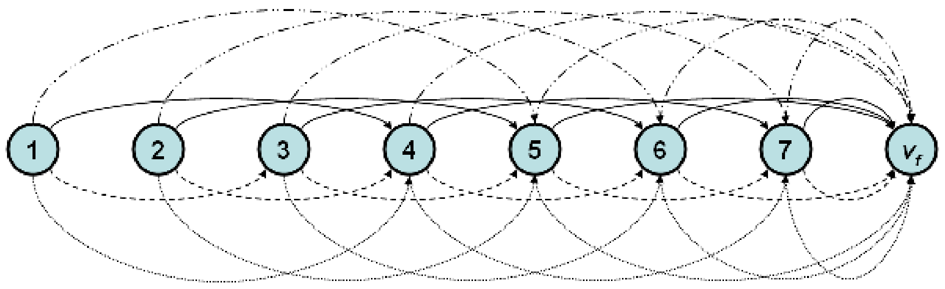
(a) The graph $\boldsymbol{G(V,E)}$ after the first edge for the square model with input length 4 is added.



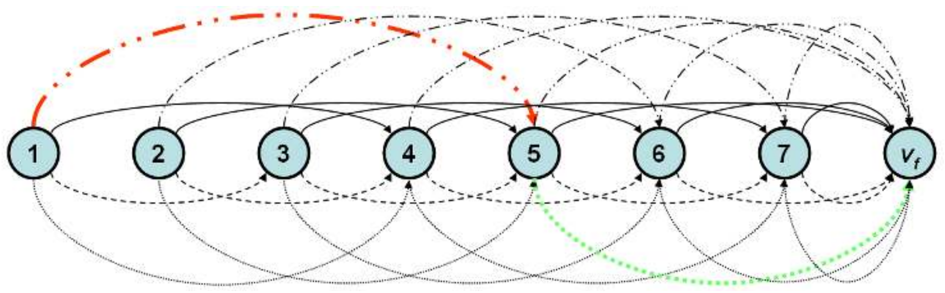
(b) After adding all the edges for square model with input length 4.



(c) After all the edges from the square models are added.



(d) Graph is completed after adding the edges for the triangle models.



(e) Shortest path in $\boldsymbol{G(V,E)}$ gives us the proper segmentation and classification (shown with bold arrows).

Figure 3-5: An illustration of graph construction for the sketch in Fig. 3-4 using HMMs with fixed input length.

$\boldsymbol{c}$ means that it is possible to account for the observation sequence $\boldsymbol{O_{i:j}}$ with some model with a loglikelihood of $\boldsymbol{-c}$. The constructed graph may have multiple edges connecting two vertices, each with different costs. By computing the shortest path from $\boldsymbol{v_1}$ to $\boldsymbol{v_f}$ in $\boldsymbol{G}$, we minimize sum of negative loglikelihoods, equivalent to maximizing the likelihood of the observation $\boldsymbol{O}$. The indices of the shortest path gives us the segmentation (Fig. 3-5-e). Classification is achieved by finding the models that account for each computed segment.

3.5.3 Modeling using HMMs with variable length training data

The formulation above makes the construction of the graph $\boldsymbol{G}$ easy because each HMM is trained using fixed-length data. At each step $\boldsymbol{s}$, we can easily compute the destination of the edge originating from the current vertex, $\boldsymbol{v_s}$, by adding the input length for $\boldsymbol{\lambda_i}$ to $\boldsymbol{s}$. One drawback of this method is that it requires an artificial partitioning of the training data for each model, dictated by the variations in description lengths for the same object. This artificial partitioning reduces the total number of training examples per model and prevents representing similar parts of different sketching styles with the same HMM graph fragment, which in turn reduces recognition accuracy and increases cumulative model sizes.

We avoid the artificial partitioning of the training data by grouping the data for all sketching styles together, and training one HMM per object class. After the training is over, for each model we also estimate the probability of ending at each state $\boldsymbol{q}$ of $\boldsymbol{\lambda_i}$ by getting the ending states for the training examples using the corresponding Viterbi paths. This information is used during recognition.

The graph $\boldsymbol{G}$ has the same number of nodes as the previous approach. We generate it by iterating over each model $\boldsymbol{\lambda_i}$, adding edges with the following steps: for each observation symbol $\boldsymbol{O_s}$, we take a substring $\boldsymbol{O_{s,s+k}}$ for each $\boldsymbol{k \in \psi_i}$. Next we compute the loglikelihood for the observation given the current model, $\boldsymbol{log(P(O_{s,s+k}|\lambda_i))}$, and add a directed edge from vertex $\boldsymbol{v_s}$ to vertex $\boldsymbol{v_{s+k}}$ in the graph with an associated cost of $\boldsymbol{|log(P(O_{s,s+k}|\lambda_i))|}$.

We augment each weight in the graph with a term that accounts for the probability that $\boldsymbol{O_{s,s+k}}$ is the encoding of a complete object. This is achieved by penalizing edges corresponding to incomplete objects, by testing whether the observation used for that edge puts $\boldsymbol{\lambda_i}$ in one of its final states using the ending probabilities estimated earlier. Segmentation and recognition is achieved by computing the shortest path in $\boldsymbol{G}$ as described above.

3.5.4 Advantages of the graph based approach

A nice feature of this graph-based approach using dynamic programming is that while the shortest path in $\boldsymbol{G}$ gives us the most likely segmentation of the input, we can also compute the next k-best segmentations using a k-shortest path algorithm[34]. A list of the n-best hypotheses can be used by a user interface that displays them for the user to choose from. This would serve as a means of correcting recognition errors as in speech recognition systems with n-best lists. It is also reasonable to believe that differing portions of the n-best hypotheses correspond to regions of ambiguity, and this information can be used by another algorithm for dealing with ambiguities.

Another nice feature of our approach is that the segmentation algorithm we use is invariant to how the value $\boldsymbol{log(P(O_{s,s+k}|\lambda_i))}$ is computed. Above, each $\boldsymbol{\lambda_i}$ was an HMM that captured temporal patterns but we could as well train two isolated object recognition models $\boldsymbol{\lambda_i^s}$ and $\boldsymbol{\lambda_i^t}$ using spatial and temporal features independently and build a recognition system that uses spatio-temporal information by using $\boldsymbol{log(P(O_{s,s+k}|\lambda_i^{s-t}))}$ instead of $\boldsymbol{log(P(O_{s,s+k}|\lambda_i))}$ where $\boldsymbol{\lambda_i^{s-t}}$ is the joint spatio-temporal model defined by a function $\boldsymbol{\lambda_i^{s-t} = f(\lambda_i^s, \lambda_i^t)}$.

3.6 Multiscale model

This model extends the flat model by adding the ability to have discrete and real-valued feature vectors and the ability to handle object-level patterns in addition to stroke-level patterns. Recall that the goal is to segment and label an input sketch by finding the grouping $G \in \mathfrak{G}$ and the labeling $L \in \mathfrak{L}(G)$ that maximizes the expression in Eq. 3.2, reproduced below for reference:

$$\underset{G \in \mathfrak{G}, L \in \mathfrak{L}(G)}{\operatorname{argmax}} P(L_1, ..., L_K | \boldsymbol{\lambda_{obj}}) \prod_{i=1}^{K} P(\boldsymbol{O_{G_i}} | \boldsymbol{\lambda_{L(G_i)}}) \tag{3.5}$$

Unlike the previous model, we solve the maximization problem by modeling the observations with a Dynamic Bayesian Network. It models the sketching process as a generative process and tracks its state using a distributed state representation that captures the belief about possible label assignments to each observation and their segmentation over time.

3.6.1 Encoding

One feature of this model is that it allows real-valued observations, and we take advantage of this feature by using real-valued geometric features of the input data. Once again we use the early sketch processing toolkit to preprocess each sketch and obtain a sequence of primitives $P_{1:T}$ (in this case simply line segments). For each primitive P_i, we obtain an observation vector O_t represented as a five-tuple $(\boldsymbol{l_t}, \boldsymbol{\Delta l_t}, \boldsymbol{\theta_t}, \boldsymbol{\Delta\theta_t}, \boldsymbol{sgn_t})$ where $\boldsymbol{l_t}$ is the length of P_i; $\boldsymbol{\Delta l_t}$ is relative length (l_t/l_{t-1}, 1 for $t = 1$); $\boldsymbol{\theta_t}$ is the angle with respect to the horizontal axis; $\boldsymbol{\Delta\theta_t}$ is the measure of relative angle between P_i and P_{i-1} given by the magnitude of the cross product $\vec{u} \times \vec{v}$ of vectors $\vec{u}$, $\vec{v}$, which in turn are length-normalized versions of P_i and P_{i-1} pointing in the direction of pen movement along each primitive. The only discrete observable $\boldsymbol{sgn_t}$ captures the direction that the stroke turns when moving from P_{i-1} to P_i. It is set to 0 for negative values of $\vec{u} \times \vec{v}$ and 1 for non-negative values (and for the observation at $t = 1$).

3.6.2 Model description

Our DBN-based model captures stroke-level and object-level patterns. The model graph has a subgraph that captures the stroke-level patterns. This part of the model conceptually corresponds to the individual object HMMs described in the flat model. For the sake of clarity of presentation, we present our model bottom up, starting with the stroke-level model.

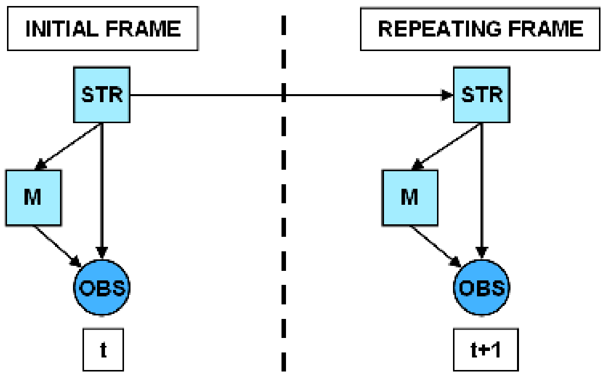


Figure 3-6: The dynamic Bayesian network representing the model for capturing stroke-level patterns. This fragment has a dual representation as a hidden Markov model with continuous observations and **M** mixtures.

The stroke-level model

For each object class to be recognized, we need a stroke-level model. The purpose of each stroke-level model is to compute the degree of agreement between a particular observation sequence and those sequences typically observed while drawing a particular class of objects.

The stroke-level models answer questions of the sort "what is the likelihood of seeing the observation sequence $\boldsymbol{O_{G_i}}$obtained from a group of primitives G_i under the rectangle model", or more formally what is $P(\boldsymbol{O_{G_i}}|\boldsymbol{\lambda_{rectangle}})$. This corresponds to the innermost likelihood term $P(\boldsymbol{O_{G_i}}|\boldsymbol{\lambda_{L(G_i)}})$ in expression 3.5. The corresponding model is shown in Fig. 3-6. It has a discrete node **STR** which captures the dynamics of the generative process corresponding to the patterns in the observations provided in the **OBS** variable. The **M** variable is a discrete mixture variable and allows us to represent the observations using mixtures of Gaussians. Each **STR** node is connected to the one in the next time slice, reflecting our choice of modeling stroke-level patterns as products of a Markov process.

The combined model

To get the final model capturing stroke and object-level patterns, we augment the stroke-level model by two nodes $\mathbf{OBJ_t}$ and $\mathbf{END_t}$ (Fig. 3-7). [3] $\mathbf{OBJ_t}$ is a multi-valued

[3]Our combined DBN model has a dual representation as a hierarchical HMM (HHMM) with continuous observables modeled using mixtures of Gaussians. We choose to present the DBN view

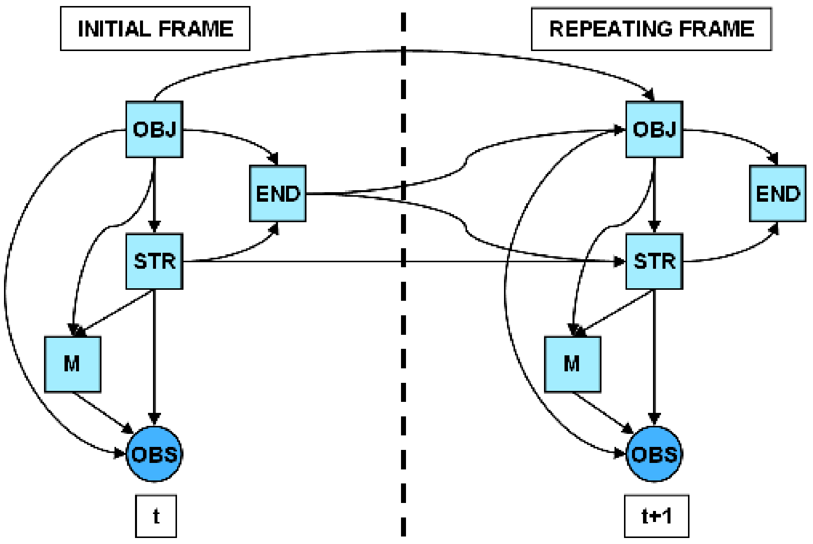


Figure 3-7: The dynamic Bayesian network representing the multiscale model.

discrete variable that holds our hypothesis of which object P_t is a part of. It can take as many values as the number of classes we can recognize. $\mathbf{END_t}$ reflects our belief about whether the user has just completed drawing object $\mathbf{OBJ_t}$ by drawing the primitive P_t.

The combined model defines a joint distribution over the variables from the stroke-level model, the sequence of object hypotheses $\mathbf{OBJ_{1:T}}$, as well as hypotheses on where each object begins and ends ($\mathbf{END_{1:T}}$). The joint probability over $\mathbf{OBJ_{1:T}}$ corresponds to $P(L_1, ..., L_K|\boldsymbol{\lambda_{obj}})$, the first term term in expression 3.5. Combined with the stroke-level likelihood term $P(\boldsymbol{O_{G_i}}|\boldsymbol{\lambda_{L(G_i)}})$ from above, the combined model allows us to compute the most probable values of $\mathbf{OBJ_{1:T}}$ and $\mathbf{END_{1:T}}$, which gives us the optimal segmentation and labeling of the input sketch.

The value of the **OBJ** node in each time slice is conditioned on the values of **END** and **OBJ** variables from the previous slice. This encodes our belief about the value of $\mathbf{OBJ_{t+1}}$ is based on the values of $\mathbf{OBJ_t}$ and $\mathbf{END_t}$. The justification for this dependence is that, we would expect the value of the **OBJ** node to change only if the user has just finished an object (i.e., $\mathbf{END_t}$=*true*), in which case the next object we would expect to see would depend on what object was just completed. Also note that the three new inter-slice arcs introduced into our model $\mathbf{OBJ_t} \rightarrow \mathbf{OBJ_{t+1}}$,

because DBNs generalize HHMMs and the DBN representation is more efficient.

$\mathbf{END_t}\rightarrow\mathbf{STR_{t+1}}$, and $\mathbf{END_t}\rightarrow\mathbf{OBJ_{t+1}}$, cross only a single slice boundary, thus our model remains first order Markovian, enabling training and recognition to be efficient.

In summary, the **OBJ** node keeps track of the class for the objects drawn over time and the joint distribution of the **OBJ** nodes over time gives us $P(L_1, ..., L_K| \boldsymbol{\lambda_{obj}})$, the first term in expression 3.5. In the combined model, the distribution of the observations is determined based on the value of the **OBJ** node given by $P(\mathbf{OBS_t}|\mathbf{OBJ_t},\mathbf{STR_t},\mathbf{M_t})$. The choice of which stroke-level process is activated is determined by the **OBJ** node because $\mathbf{STR_{t+1}}$ is conditioned on $\mathbf{STR_t}$ and $\mathbf{OBJ_{t+1}}$.

3.6.3 Implementation issues

There are two issues that need to be addressed in implementing the model described above, both related to the choice of the specific inference algorithm to be used.

As mentioned earlier, our model has a dual representation as a hierarchical hidden Markov model. Unfortunately, the original inference algorithm for HHMMs is both complicated and has time complexity $O(T^3)$ where T is the length of the observation sequence [36]. In our case T is the total number of primitives in a sketch, making the model impractical for situations where real-time feedback is required. By using the DBN representation we can apply efficient graphical model techniques that take linear time [92]. It is therefore essential to use the DBN representation, or convert any HHMM based representation to a DBN prior to inference and learning.

One important implementation issue is numerical instabilities that occur during training, due to the use of continuous observations. Bayesian networks that include continuous and discrete variables (mixed networks) usually represent the continuous variables as Gaussians or mixtures of Gaussians. The conventional belief propagation algorithm used for inference in these networks is the Lauritzen algorithm [75]. Unfortunately this algorithm is susceptible to numerical underflow. Although this has been documented in the machine learning literature [74, 90], it is not well known because it occurs rarely in practice. The problem appears in the form of singular matrices during inference and is hard to localize. In order to avoid this numerical instability, a specialized algorithm known as the Lauritzen-Jensen stable conditional Gaussian belief propagation should be used [74].

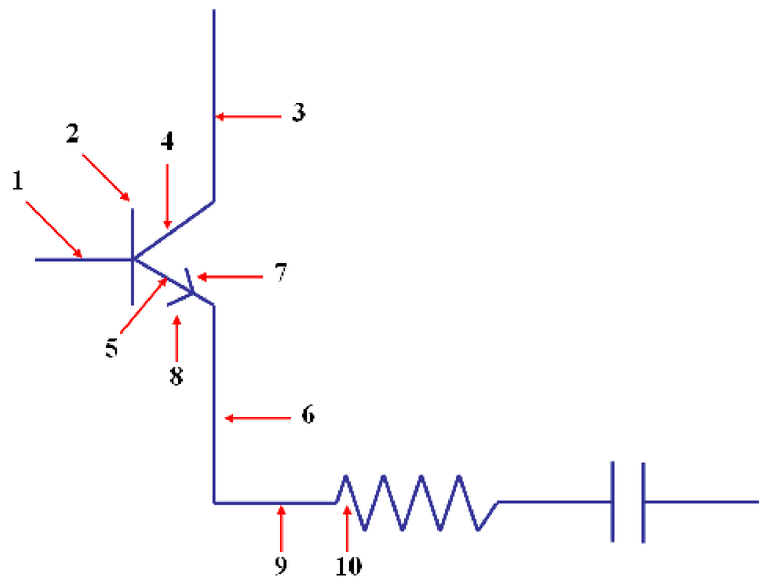


Figure 3-8: An example of interspersing: The user draws two other objects (wires #3 and #6) over the course of drawing the transistor. The drawing order is indicated by numbers.

Training

The goal of training is to estimate the parameters of the conditional probability distribution functions for each node in our DBN ($B_1, B_{\rightarrow}$) given a collection of labeled sketches as training data. We use parameter tying to ensure that the probability distribution $P(\mathbf{A}|Parents(\mathbf{A}))$ relating a node $\mathbf{A}$ and its parents is the same for each node A in B_1 and $B_{\rightarrow}$.[4]

During training we know what class each primitive belongs to and we also know when objects begin and end, so the values of the **OBJ** and **END** are observable and are supplied during training. In summary, for each labeled sketch, we use the values $\mathbf{OBJ_{1:T}}$, $\mathbf{END_{1:T}}$ and $\mathbf{OBS_{1:T}}$ to estimate the model parameters.

3.7 Multiscale-interspersing model

The goal of this model is to add the ability to handle interspersed drawing to the multiscale model in a principled way. By interspersing we refer to the situation where

[4]We add dummy parent nodes to the **OBJ** and **STR** nodes in B_1 to ensure that $P(\mathbf{OBJ}|Parents(\mathbf{OBJ}))$ and $P(\mathbf{STR}|Parents(\mathbf{STR}))$ can be represented using conditional probability tables (CPTs) with the same structure for B_1 and $B_{\rightarrow}$.

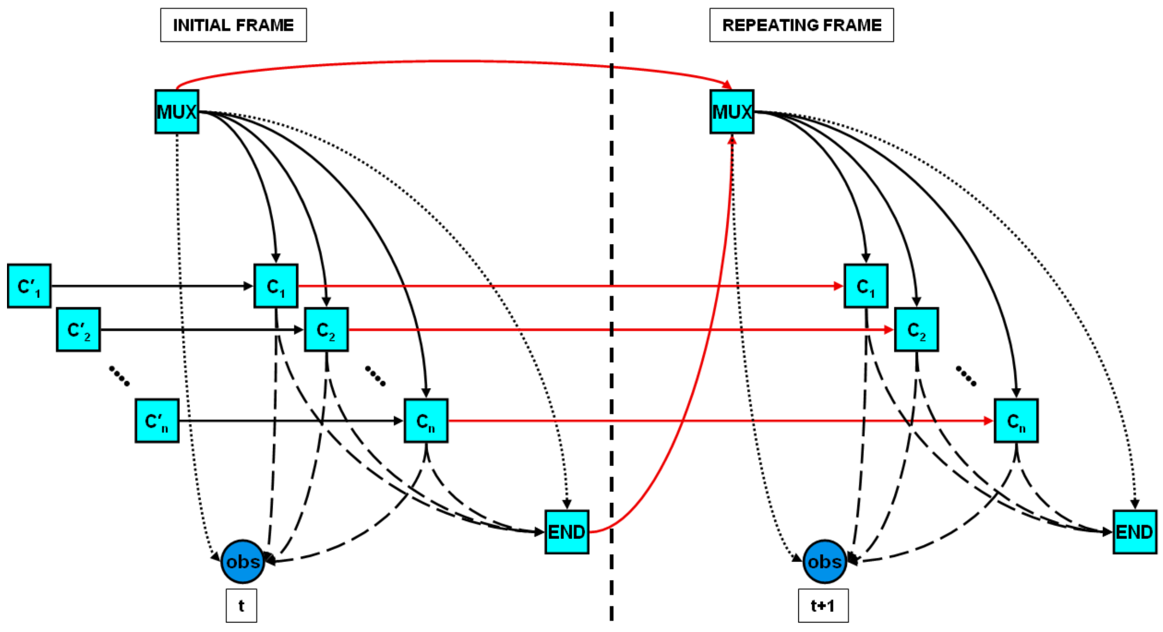


Figure 3-9: The dynamic Bayesian network representing our model that uses multi-scale patterns and handles interspersed drawing.

the user starts drawing one object but draws one or more other objects before it is completed. For example, Fig. 3-8 shows the case where the user draws the vertical part of the transistor (stroke #2), draws the wire connecting to the collector of the transistor (stroke #3), draws more of the transistor and another wire, and the transistor is actually completed after the current symbol is drawn. In this example two wires (#3 and #6) are interspersed with the transistor.

More formally, suppose we have two objects $\mathscr{A}$ and $\mathscr{B}$. Assume the proper grouping of primitives forming $\mathscr{A}$ and $\mathscr{B}$ are $G_{\mathscr{A}} = \rho_1, \rho_2, ...\rho_m$ and $G_{\mathscr{B}} = \rho'_1, \rho'_2, ...\rho'_n$. We say that $\mathscr{A}$ is interspersed with $\mathscr{B}$ if $\rho_1 < \rho'_i < \rho_m$ for $1 \leq i \leq n$ and $|G_{\mathscr{A}}| + |G_{\mathscr{B}}| = p_m - p_1 + 1$. Our model handles a more general case of interspersing where $\mathscr{A}$ can be interspersed with multiple objects.

Interspersing poses a challenge to both models presented earlier. Consider the multiscale model: it defines the state of the sketching process in terms of the current object being drawn $\mathbf{OBJ_t}$, the state of the generative process that captures the stroke-level patterns $\mathbf{STR_t}$, and two auxiliary states ($\mathbf{END_t}$ and $\mathbf{M_t}$). The scenario described above where object $\mathscr{A}$ is interspersed with $\mathscr{B}$ requires value of the $\mathbf{OBJ}$ node to change from $\mathscr{A}$ to $\mathscr{B}$ at time t_1 and back to $\mathscr{A}$ at time t_2 where $\rho_1 < t_1 < t_2 < \rho_m$. At time

t_2, the DBN should be able to continue from the state that it would normally assume without interspersing determined by $P(\ \mathbf{STR_t} \mid \mathbf{OBJ_t}= \mathscr{A}, \mathbf{STR_{t-1}}= s, \mathbf{END_{t-1}}= false)$ where $s = \mathbf{STR_{t_1}}$. However state of the **STR** node at time t_1 cannot be recovered because it is overwritten by $\mathbf{STR_{t_1+1}}$ determined by $P(\ \mathbf{STR_t} \mid \mathbf{OBJ_t}= \mathscr{B}$, $\mathbf{STR_{t-1}}= s, \mathbf{END_{t-1}}= false)$.

It is clear that in order to handle interspersing, state of the process for $\mathscr{A}$ must be saved for resuming later. It is also clear that regardless of the number of classes we support, the user can be drawing only one object at a given time, and only one object process is active at any given time. Based on these two observations, we devised a new model that handles interspersing. As before, we model the sketching process as a generative stochastic process and make use of the *switching parent* mechanism along with parameter tying to model interspersed drawing efficiently while remaining within the formal framework of Dynamic Bayesian Networks. Fig. 3-9 shows our model. We introduce the switching parent mechanism used in our model and then discuss the model in detail.

3.7.1 Switching parents

Our model contains new graphical notation (dotted and dashed arrows in Fig. 3-9) indicating conditional dependencies that change (switch) based on the value of a *switching parent*. The use of *switching parent mechanism* (also known as context specific independence or Bayesian multi-nets) allows us to represent conditional dependencies that change as a function of another node's value in an efficient fashion [19, 43, 56, 22]. Fig. 3-10 shows a simple example of switching parents. In this network **OBS** has two parents **P1** and **P2**, and a *switching parent* **MUX** that controls which one of **P1** or **P2** is activated. The semantics of the network is such that:

$$\begin{aligned} P(\mathbf{OBS}|\mathbf{P1},\mathbf{P2}) = \ & P(\mathbf{OBS}|\mathbf{P1},\mathbf{MUX}=1)P(\mathbf{MUX}=1)+ \\ & P(\mathbf{OBS}|\mathbf{P2},\mathbf{MUX}=2)P(\mathbf{MUX}=2) \end{aligned}$$

We use switching parents as an efficient mechanism for selecting the process corresponding to the active object in our dynamic model of sketching.

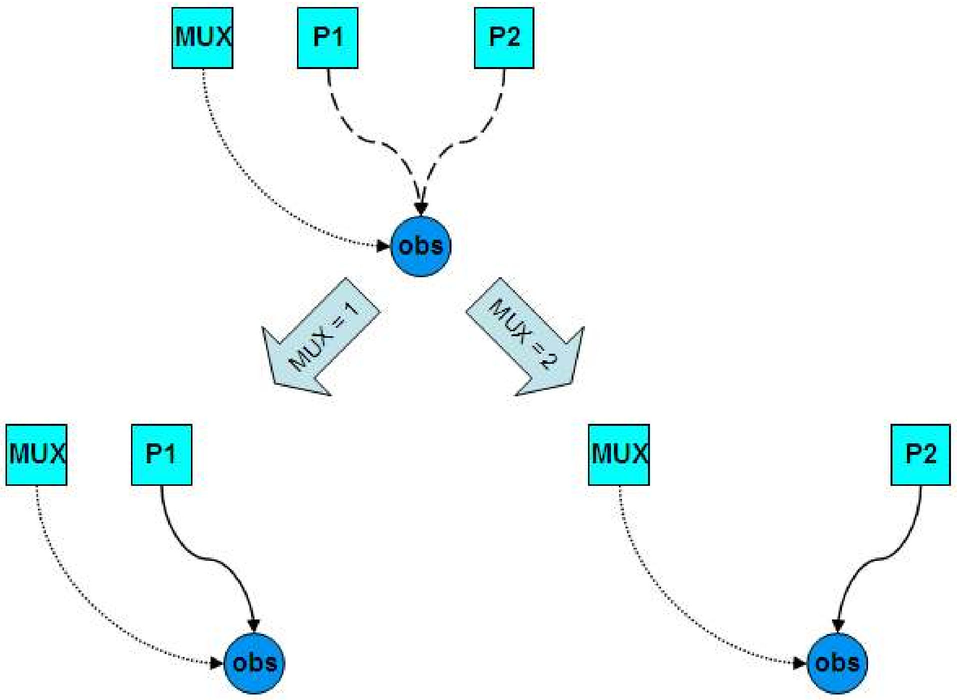


Figure 3-10: Example illustrating the switching parent mechanism.

3.7.2 Description of the model topology

The **MUX** node in our model tracks the state of the current object being drawn and the information of what objects are interspersed when interspersing occurs. Cardinality of the **MUX** node is equal to the sum of number the object classes and the number of objects that can be interspersed. The semantics of $\mathbf{MUX_t}= i$ is determined by the following:

$$\begin{aligned} 1 \leq i \leq |\mathscr{C}| &\Longrightarrow \text{drawing object } i,\ \mathbf{C_i} \text{ active,} \\ i > |\mathscr{C}| &\Longrightarrow \text{interspersing } \mathscr{A} \text{ and } \mathscr{B}, \text{ given by the function } \mathscr{F}_{int}(i) = \langle \mathscr{A}, \mathscr{B} \rangle. \end{aligned}$$

The function $\mathscr{F}_{int}(i)$ maps values of **MUX** to a pair of object classes $\langle \mathscr{A}, \mathscr{B} \rangle$. We construct it based on what interspersings exist in the data and which ones we choose to support.

For each object class that we support, the model has a node capturing the dynamics of the stroke-level features (nodes $\mathbf{C}_1$, $\mathbf{C}_2$, ... $\mathbf{C}_{|\mathscr{C}|}$, in Fig. 3-9). As before, features are provided in the **OBS** node and the **END** node is a binary node indicating the user is done drawing the current object. Nodes $\mathbf{C}'_1$, $\mathbf{C}'_2$, ... $\mathbf{C}'_{|\mathscr{C}|}$ are auxiliary nodes for ensuring that the form of the conditional probabilities for nodes $\mathbf{C}_i$ in the initial frame is the same as those in the repeating frame (e.g., $P_{B_1}(\mathbf{C}_i|Parents(\mathbf{C}_i)) = P_{B_\rightarrow}(\mathbf{C}_i|Parents(\mathbf{C}_i))$. Auxiliary nodes have the same cardinality as their children (i.e., $|\mathbf{C}'_i| = |\mathbf{C}_i|$).

Conditional dependencies for the initial frame

The **MUX** node in the initial frame has no parents and it has a prior that sums to 1 for values corresponding to object classes and 0 for values corresponding to object interspersings. For example a plausible choice for the prior values is to use a uniform distribution:

$$P(\mathbf{MUX_1} = i) = \begin{cases} 1/n & \text{if } 1 \leq i \leq |\mathscr{C}|, \\ 0 & \text{if } i > |\mathscr{C}| \end{cases}$$

The **OBS** node is conditioned on the **MUX** and one of the $\mathbf{C}_i$ nodes as determined by the value of **MUX**. This is an example of the switching parent mechanism. The distribution for **OBS** can be broken into two cases depending on if the user is currently interspersing an object of class $\mathbf{C}_j$ with $\mathbf{C}_k$ given by $\mathscr{F}_{int}(i) = \langle C_j, C_k \rangle$:

$$P(\mathbf{OBS}|\mathbf{MUX} = i, \mathbf{C}_1, \mathbf{C}_2, .., \mathbf{C}_n) = \begin{cases} P(\mathbf{OBS}|\mathbf{MUX} = i, \mathbf{C}_i) & \begin{array}{l}\text{if } 1 \leq i \leq |\mathscr{C}|, \\ \text{drawing an object of} \\ \text{type } \mathbf{C}_i\end{array} \\ P(\mathbf{OBS}|\mathbf{MUX} = i, \mathbf{C}_k) & \begin{array}{l}\text{if } i > |\mathscr{C}|, \\ \mathscr{F}_{int}(i) = \langle C_j, C_k \rangle, \\ \text{interspersing } \mathbf{C}_j \text{ with } \mathbf{C}_k\end{array} \end{cases}$$

The interspersing function $\mathscr{F}_{int}(i)$ mapping values of **MUX** to the range $1 \leq i \leq |\mathscr{C}|$ is provided prior to the training process based on the number of objects we would like to model and the kinds of interspersings present in the training data. Use of switching parents in this fashion also allows us to learn and share a single model for objects that are interspersed (i.e., instead of learning a *wire* model and an *interspersed wire* model, we learn a single *wire* model and reuse it). **END** also has the **MUX** node as its switching parent: $P(\mathbf{END}|\mathbf{MUX} = i, \mathbf{C}_1, \mathbf{C}_2, ..., \mathbf{C}_n) = P(\mathbf{END}|\mathbf{MUX} = i, \mathbf{C}_i)$.

Each $\mathbf{C}_i$ node in the initial frame is conditioned on the **MUX** and $\mathbf{C}'_i$ nodes. The prior for the $\mathbf{C}'_i$ nodes is represented by a sparse conditional probability table that sets $P(\mathbf{C}'_i = 1) = 1$.[5] This is our way of saying all stroke-level processes are at their beginning state when we enter the initial timeslice and based on the value of **MUX**, only one of these nodes updates its state using the inter-frame probability distribution $P_{B_\rightarrow}(\mathbf{C}_{i,t}|Parents(\mathbf{C}_{i,t})) = P_{B_\rightarrow}(\mathbf{C}_{i,t}|\mathbf{C}_{i,t-1},\mathbf{MUX_t})$ substituting the value of $\mathbf{C}'_i$ for $\mathbf{C}_{i,t-1}$. This allows the process selected by **MUX** to change its state from its default *begin* state to a state where it can generate the first observation $\mathbf{OBS_1}$. Using the probability distribution function $P_{B_\rightarrow}(\mathbf{C}_{i,t}|Parents(\mathbf{C}_{i,t}))$ learned over many examples in the first slice of our DBN is another example of parameter tying. $\mathbf{C}_i$ nodes that are not selected by the **MUX** node in the initial frame copy values from $\mathbf{C}'_i$.

Inter-slice dependencies

The **MUX** node at time t is conditioned on $\mathbf{MUX_{t-1}}$ and $\mathbf{END_{t-1}}$. Its value changes based on the object-level transition probabilities if $\mathbf{END_{t-1}}$ indicates that the current observation marks the beginning of a new object (not necessarily of a different class, but a different instance). The **MUX** node can change state even if the $\mathbf{END_{t-1}}$ is $false$ (i.e., $\mathbf{OBS_{t-1}}$ does not mark the end of an object). These cases correspond to interspersings and $P(\mathbf{MUX_t}{=}\ i|\mathbf{MUX_{t-1}}{=}\ j,\mathbf{END_{t-1}}{=}\ false) > 0$ only if have seen objects of type $\mathbf{C}_i$ being interspersed with another object in the training data.[6]

$\mathbf{C_{i,t}}$ nodes in the repeating frames are conditioned on the $\mathbf{MUX_t}$ and $\mathbf{C_{i,t-1}}$ nodes. The CPT for $P_{B_\rightarrow}(\mathbf{C_{i,t}}|Parents(\mathbf{C_{i,t}}))$ is estimated from the data subject to a few constraints that we specify prior to training in the form of deterministic CPTs [15]. Specifically, we require that:

[5]Note that we can get away with having only one auxiliary variable $\mathbf{C}'_i$ if all the $\mathbf{C}_i$ nodes have the same cardinality.

[6]For $1 \leq i, j \leq |\mathscr{C}|$ the conditional probability $P(\mathbf{MUX_t}{=}\ i|\mathbf{MUX_{t-1}}{=}\ j,\mathbf{END_{t-1}}{=}\ false)$ is 0 for $i \neq j$, and a non-zero value for $i = j$, thus it can be represented using a sparse conditional probability table.

$$P_{B_{\rightarrow}}(\mathbf{C_{i,t}} = c | \mathbf{MUX_t} = m, \mathbf{C_{i,t-1}} = c') = \begin{cases} 1 & \text{if } m \neq i, 1 \leq m \leq |\mathscr{C}|, c = 1 \\ 0 & \text{if } m \neq i, 1 \leq m \leq |\mathscr{C}|, c \neq 1) \\ 1 & \text{if } m > |\mathscr{C}|, c = c', \text{ and } \mathscr{F}_{int}(m) = \langle C_i, C_* \rangle \\ 0 & \text{if } m > |\mathscr{C}|, c \neq c', \text{ and } \mathscr{F}_{int}(m) = \langle C_i, C_* \rangle \end{cases}$$

These constraints ensure that if the user is drawing an object other than the one associated with the node $\mathbf{C_{i,t}}$, its state is reset to the begin state, and if the user has started interspersing an object of type C_i with any other object, the state of the $\mathbf{C_i}$ node is passed on to the next slice. This allows us to save the state of the process associated with C_i so that it can resume after the interspersing is over.

Choice of conditional dependencies for interspersing

A design question that needs consideration when building this model is whether or not the state transition probabilities for the $\mathbf{MUX_{t+1}}$ node depends on the value of $\mathbf{C_i}$ from time t. In the formulation above, we assumed that the point at which the user starts interspersing is conditionally independent of the state of the current object process (e.g., the probability of interspersing a wire is independent of how much of the transistor is drawn). This is appropriate for domains where the user behavior is consistent with this assumption. This may not be the case for domains where the beginning of an interspersing is more frequent for certain partial drawings of the current object. In such cases it would be more appropriate to add a conditional dependency arc from $\mathbf{C_i}$ at time t to $\mathbf{MUX_{t+1}}$ that switches on when the interspersing begins. Obviously when such a conditional dependence is introduced, more examples will be necessary to estimate model parameters without overfitting. If not enough training examples are supplied, interspersings that begin at states not covered in the training data will receive zero probabilities. This is a sparse data problem and can be handled using regularization techniques. A discussion of and an example of how this can be done using pseudo-counts and Dirichlet priors can be found in [91, 66].

3.8 Discussion

The design of each model we presented was guided by our observations about the domains that we studied including those mentioned in Chap. 2 (finite state machines, UML diagrams, Course of Action Diagrams, stick-figures and emoticons) and circuit diagrams which is our domain of choice for the evaluation chapter (Chap. 4). Among these, the circuit diagrams domain contained many instances of interspersed drawing. In the domains that we have studied, we never observed an object of the same type being interspersed, and our model doesn't handle such cases.

Chapter 4

Results

We report the performance of our multiscale-interspersing model on sketches from the circuit diagram domain to illustrate its performance in absolute terms, and to measure the incremental benefits of modeling object-level patterns and interspersing.

To measure the benefits of modeling object level patterns, we use as a baseline a version of the flat model extended with continuous features and compare its performance to the multiscale model that uses object-level patterns.

To test the benefits of modeling interspersing, we compare the correct recognition rates obtained using the multiscale model to those obtained using the multiscale-interspersing model.

4.1 Circuit data collection

Although anyone can copy a circuit diagram, we believe that professionals who design circuits think about and draw circuit diagrams differently from non-experts. In keeping with this, we collected circuit diagrams from participants studying electrical engineering at MIT, recruiting students who had recently taken the Microelectronic Circuits and Devices course (6.012).

We asked the participants to draw circuits from their course textbook, selecting examples that used a sufficiently expressive and challenging set of circuit components: NPN transistors, resistors, capacitors, batteries and wires. A total of eight participants contributed at least ten circuit diagrams each.

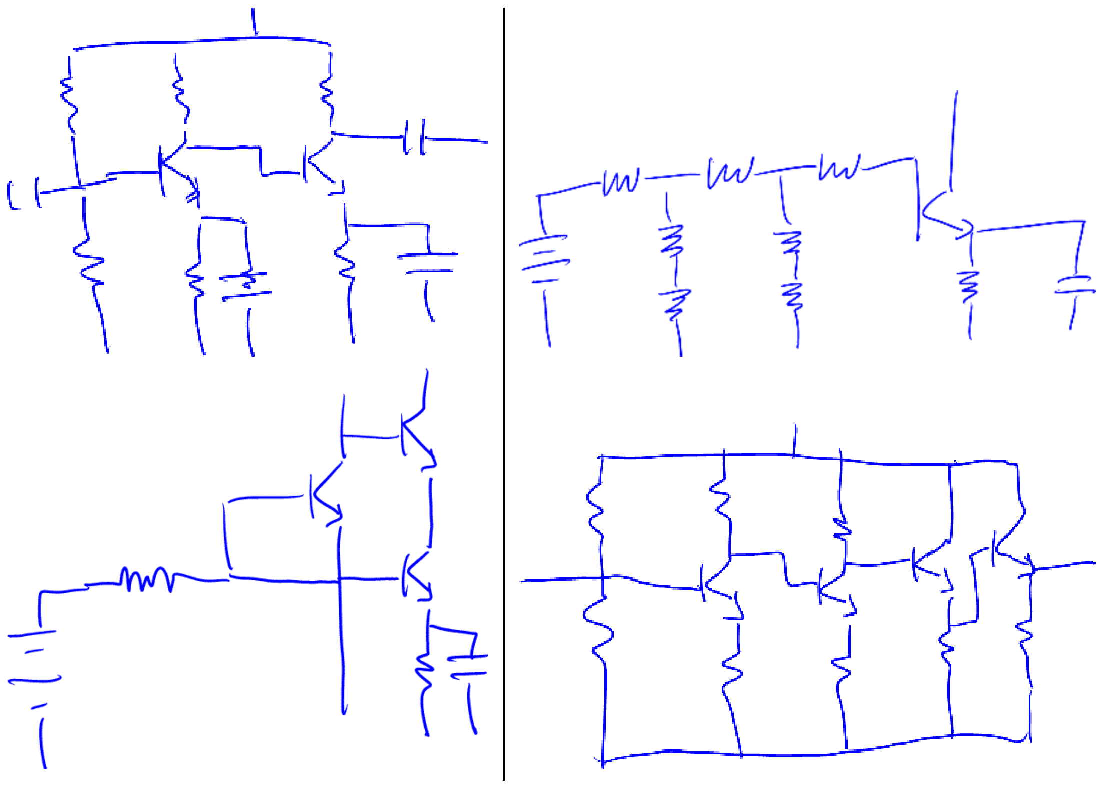


Figure 4-1: Examples of collected circuits.

We began by showing participants diagrams of the circuits to be drawn. To ensure participants got familiar with the circuits, we gave them some time to study each one until they understood how it worked. To ensure they understood it, we asked them the function of the circuit (e.g., an inverter circuit, an amplifier), then asked them to explain verbally how it worked. We then removed the textbook diagram and asked them to draw the circuit from memory on a Tablet PC, allowing them to consult the original circuit diagram if needed. Fig. 4-1 shows examples of circuits drawn by the participants. We manually annotated the sketches to be used as training and testing data in our experiments.

4.2 Generating an observation sequence

Both training and classification require converting a sketch to an observation sequence $O_{1:T}$. Using the early sketch processing toolkit, we first preprocess each sketch to obtain a sequence of primitives $P_{1:T}$ (which in this domain contained only line segments). Fig. 4-2 illustrates primitive extraction. The pen direction is indicated with arrows

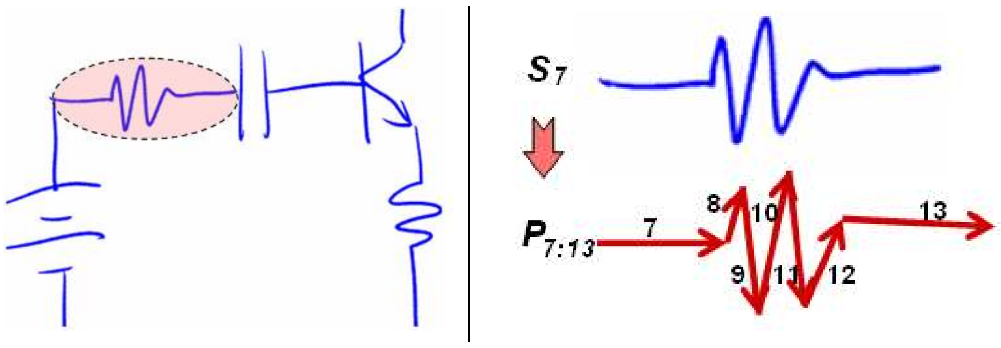


Figure 4-2: Conversion to the primitive sequence.

for each primitive.

For each primitive P_t, we obtain an observation vector O_t represented as a five-tuple $(\boldsymbol{l_t}, \boldsymbol{\Delta l_t}, \boldsymbol{\theta_t}, \boldsymbol{\Delta\theta_t}, \boldsymbol{sgn_t})$ where $\boldsymbol{l_t}$ is the length of P_t; $\boldsymbol{\Delta l_t}$ is relative length (l_t/l_{t-1}, 1 for $t = 1$); $\boldsymbol{\theta_t}$ is the angle with respect to the horizontal axis; $\boldsymbol{\Delta\theta_t}$ is the measure of relative angle between P_t and P_{t-1} given by the magnitude of the cross product $\vec{u} \times \vec{v}$ of vectors $\vec{u}$, $\vec{v}$, which in turn are length-normalized versions of P_t and P_{t-1} pointing in the direction of pen movement along each primitive. The only discrete observable $\boldsymbol{sgn_t}$ captures the direction that the stroke turns when moving from P_{t-1} to P_t. It is set to 0 for negative values of $\vec{u} \times \vec{v}$ and 1 for positive values or $t = 1$.

For our experiments, we create two datasets: "simple" and "mixed." The simple dataset contains only examples without interspersing, while the mixed data contains both the examples from the simple data set and examples where the user interspersed strokes during drawing.

4.3 Modeling object level patterns increases recognition accuracy

4.3.1 Experiment setup and quantitative results

For each user, we ran a series of hold-one-out experiments by getting the recognition rates for each sketch using a model trained on the remaining sketches. For each sketch, we also trained a baseline system that modeled only stroke-level patterns. The baseline system has the same computational complexity as our method and was obtained by assigning uniform transition probabilities for the object level:

$$P(\mathbf{OBJ_{t+1}}|\mathbf{OBJ_t},\mathbf{END_t}) = \begin{cases} 1/|\mathbf{OBJ}|, & \text{if } \mathbf{END_t} = true \\ 1 & \text{if } \mathbf{END_t} = false \text{ and } \mathbf{OBJ_{t+1}} = \mathbf{OBJ_t} \\ 0 & \text{if } \mathbf{END_t} = false \text{ and } \mathbf{OBJ_{t+1}} \neq \mathbf{OBJ_t} \end{cases}$$

In both models we used three Gaussian mixtures and set the number of states for the stroke-level models to 6. We trained them using simple data (because these models cannot handle interspersing) and tested them on the mixed data (because that is how the users eventually draw).

Table 4.1 shows the average correct recognition rates for each participant obtained using the baseline model and the multiscale model. The table also shows the average and maximum reduction values in the error rates in terms of percentages for each user. As seen in the table, on average, modeling object-level patterns reduced error rates between 14% and 37%, and allowed up to 65% of misrecognition errors to be corrected.[1]

A paired t-test for the values in Table 4.1 showed the difference to be statistically significant for $p \ll 0.05$ and 7 degrees of freedom. We believe such a decrease in the error would substantially improve users' satisfaction with the recognition system in an actual design setting.

4.3.2 Qualitative examples and discussion

We believe it is instructive to explore the kinds of errors avoided by modeling object-level patterns. Consider Fig. 4-3, an amplifier circuit, which also shows the stroke ordering for the fragment of the circuit diagram of particular interest to us. Two interpretations of this circuit are shown in Fig. 4-4.

Fig. 4-4-a shows the interpretation obtained by running the baseline method, which encounters four recognition errors. The capacitor **C2** is misrecognized as wires, most probably because of the hook hanging off one of the strokes. The NPN transistor **Q2** is misrecognized because the stroke indicating the current direction is noisy. Note

[1]A misrecognition is defined as computing a value for the object node that disagrees with the ground truth.

	Participant ID							
	1	2	3	4	5	6	7	8
μ_b	83.2	86.5	85.2	73.8	87.1	90.7	79.1	85.0
μ_m	89.4	89.4	87.3	77.4	89.8	93.0	84.6	88.2
Δerr	36.9	21.4	14.1	13.7	20.9	24.7	18.6	21.3
$max\Delta$	65.0	45	31.2	15.0	35.5	40.0	30.7	54.5

Table 4.1: Mean correct recognition rates for the baseline system (μ_b) and the multiscale model (μ_m) on sketches collected from 8 participants. The table also shows the percentage reductions in the error rates and maximum error reductions achieved for each user as percentages (Δ_{err} and $max\Delta$). On average, modeling object-level patterns always improves performance.

that the vertical part of this transistor (segment #11) and the wire connected to the emitter (segment #12) are grouped together and labeled as a capacitor. Inspection of the sketch reveals that these two strokes have roughly the same length and are sketched one after the other (shown by the numbers in Fig. 4-3). This easily leads to these strokes being mislabeled as a capacitor.[2] Finally, part of the resistor **R1** is misrecognized.

Fig. 4-4-b shows that two of these errors are fixed by the multiscale model. The multiscale model has learned that the chance of a resistor being followed immediately by a transistor, and capacitors being immediately followed by resistors with no wires in between is very small, so these interpretations are penalized. As a result, the two misrecognitions are avoided.

4.4 Handling interspersings increases recognition accuracy

4.4.1 Experiment setup and quantitative results

For participants who produced interspersed sketches (#1, #5, #6 and #7), we ran a series of hold-one-out experiments, comparing the recognition rate of the multiscale-interspersing model (implemented using features of `GMTK`[15, 18]) and the multiscale

[2]This is a result of the limitations of our ordered based recognition.

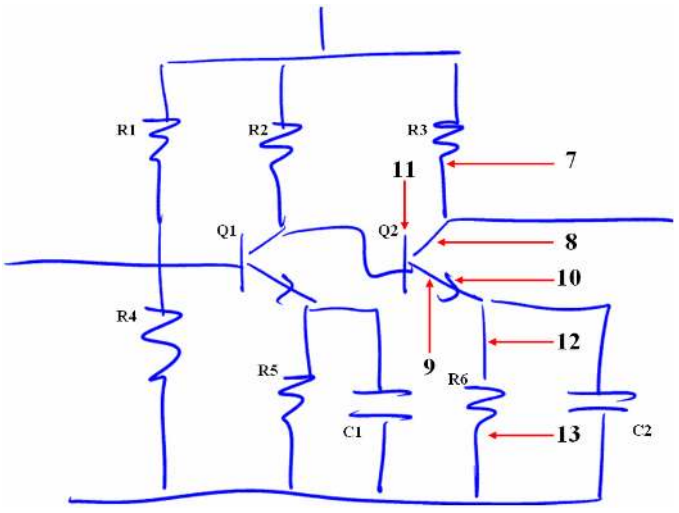


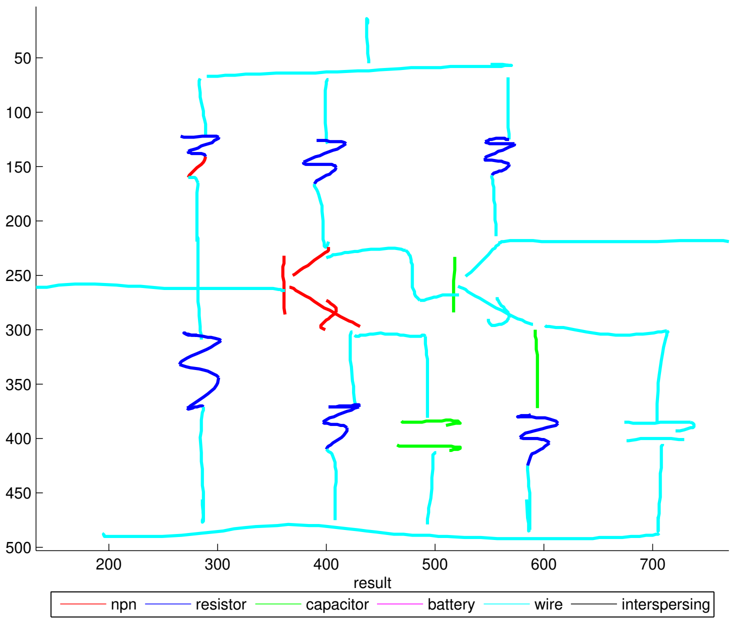
Figure 4-3: One of the circuits drawn by our study participants. Stroke ordering for a fragment of the circuit is indicated by numbers.

	Participant ID			
	1	5	6	7
μ_m	89.4	89.8	93.0	84.6
μ_{m-i}	92.9	92.2	95.6	87.7
Δerr	33.0	23.5	37.1	20.1
$max\Delta$	61.5	33.3	100.0	54.5

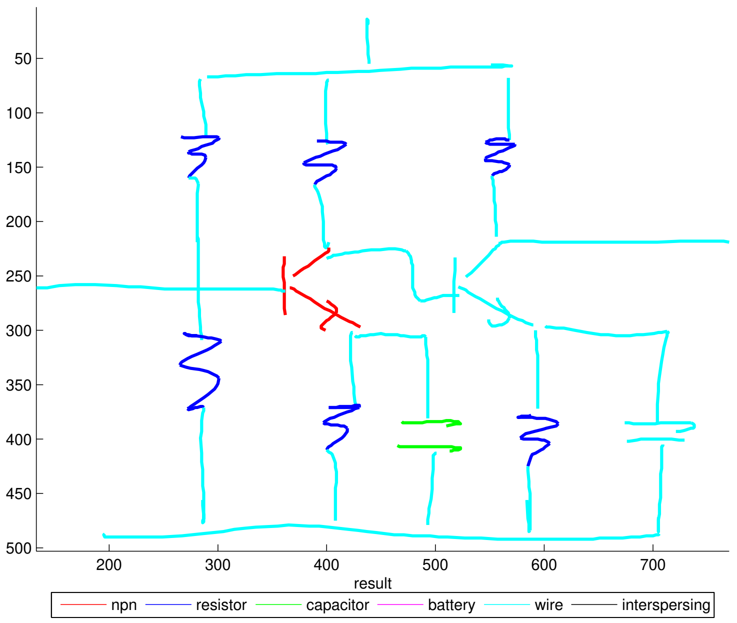
Table 4.2: Mean correct recognition rates for the multiscale-interspersing (μ_{m-i}) and the multiscale models (μ_m) for users who exhibited interspersed drawing behavior. The table also shows the percentage reductions in the error rates and maximum error reductions achieved for each user as percentages (Δ_{err} and $max\Delta$). On average, handling interspersing patterns always improves performance.

model. In both models we used three Gaussian mixtures and set the number of states for the stroke-level models to 6. The DBN representing the multiscale-interspersing model is shown in Fig. 4-5. We trained the multiscale model using simple data (because it cannot handle interspersing) and the multiscale-interspersing model using the mixed data. We tested both models using mixed data.

Table 4.2 shows the average correct recognition rates for each participant obtained using the multiscale model and the multiscale-interspersing model. The table also shows the average and maximum reduction values in the error rates in terms of percentages for each user. As seen here, on average, handling interspersing improves



(a) Interpretation of the baseline model.



(b) Interpretation of the multiscale model.

Figure 4-4: Examples of errors corrected using context provided by object-level patterns. There are four misrecognitions in (a). Note how two of these misrecognitions is fixed using knowledge of object level patterns (resistor in the upper left corner (**R1**) and the wire connected to the emitter of the misrecognized transistor). See Fig. A-2 for B&W printing.

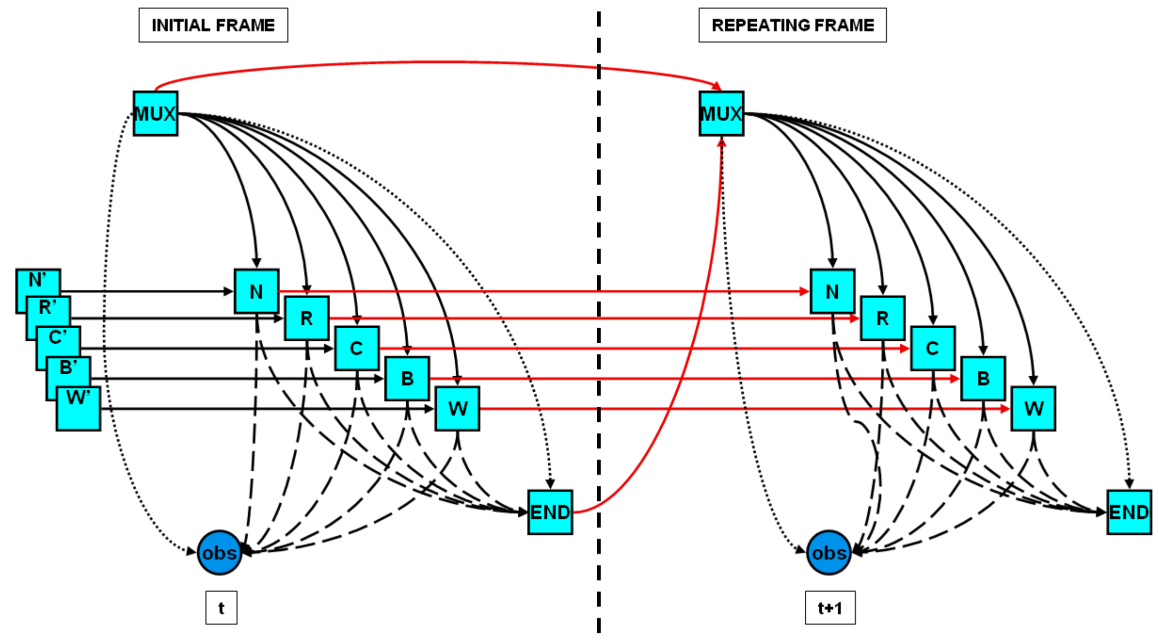


Figure 4-5: The dynamic Bayesian network representing the multiscale-interspersing model that handles interspersed drawing.

performance, and allows 20%-37% of misrecognition errors to be corrected.

A paired t-test for the values in Table 4.2 found the difference to be statistically significant for $p < 0.05$ and 3 degrees of freedom.

4.4.2 Quantifying errors per-interspersing

The percentages presented above do not show the full extent of the negative effects of not handling interspersing because the percentage of errors due to interspersing is diluted by the large number of other components successfully recognized. A more informative measure of the benefits of handling interspersing can be obtained if we look at the number of misclassified primitives per interspersed primitive.

In order to measure this, we ran both the multiscale model (the baseline) and the interspersing-multiscale model on a control group that contained ten sketches with no interspersing, and a test group that contained versions of these sketches with interspersings. The interspersed version of each sketch was obtained by moving one of the wires preceding/following a transistor back/forward in time. Our use of these examples ensures that we measure the misrecognition effects due only to interspersing.

We trained our baseline model with the non-interspersed data and supplied the

	Sketch ID										
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	μ
Total primitives	90	85	104	101	92	100	87	105	80	98	94.2
Added interspersings	2	2	4	4	3	3	2	2	2	4	2.8
Missed by the baseline	7	9	15	12	12	12	13	13	11	19	12.3
Missed by our model	3	3	4	1	0	0	2	2	2	3	2
Difference	4	6	11	11	12	12	11	11	9	16	10.3
Missed prim./intersp.	2	3	2.75	2.75	4	4	5.5	5.5	4.5	4	3.8

Table 4.3: This table shows the number of misrecognized primitives per interspersing. The first two row shows the number of total primitives in each sketch and the number of added interspersings. Next two rows show the number of primitives incorrectly recognized by the baseline and by our model. The next row shows the number of primitives that the baseline misses because it cannot handle interspersings and the last row is the number of primitives missed by the baseline per interspersing.

interspersing-multiscale model with the additional interspersed examples. Table 4.3 shows the number of misrecognized primitives per interspersing. The first two rows show the number of total primitives and the number of added interspersings per sketch. The table also shows the total number of primitives incorrectly recognized by the baseline and by our model. The last two rows show the total of primitives that the baseline misses because it cannot handle interspersings and the number of primitives missed by the baseline per interspersing. As seen in the table, a single interspersing causes as many as 16 primitives to be misclassified. Considering that a transistor has five primitives, this is worth about three transistors. When we normalize the number of misrecognized primitives per sketch by the number of interspersings we get about 2-6 misrecognized primitives per interspersing.

In the best case, the errors caused by each interspersing will require at least one correction by the user (e.g., when all the misrecognized shapes belong to the same shape). In the worst case, the errors may require as many corrections as the number of misclassified primitives, further showing the utility of modeling interspersing.

4.4.3 Qualitative examples and discussion

Fig. 4-6 shows an example illustrating how interspersed drawing causes misrecognitions if not handled properly. This example is particularly instructive because it

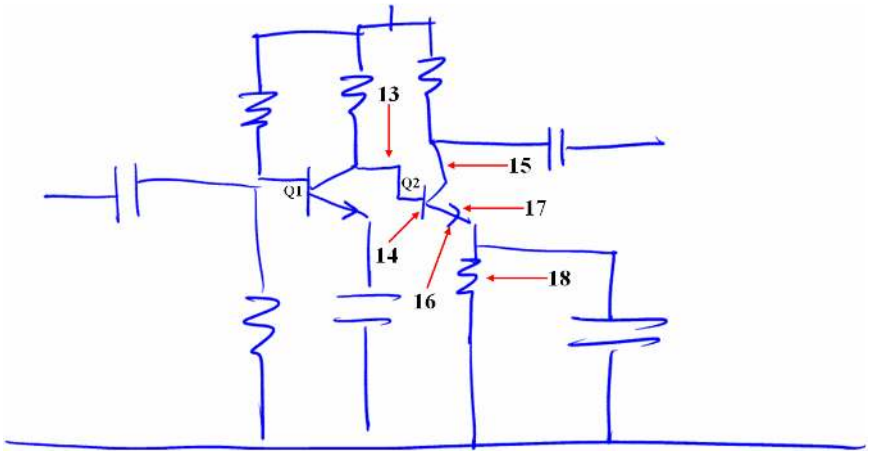


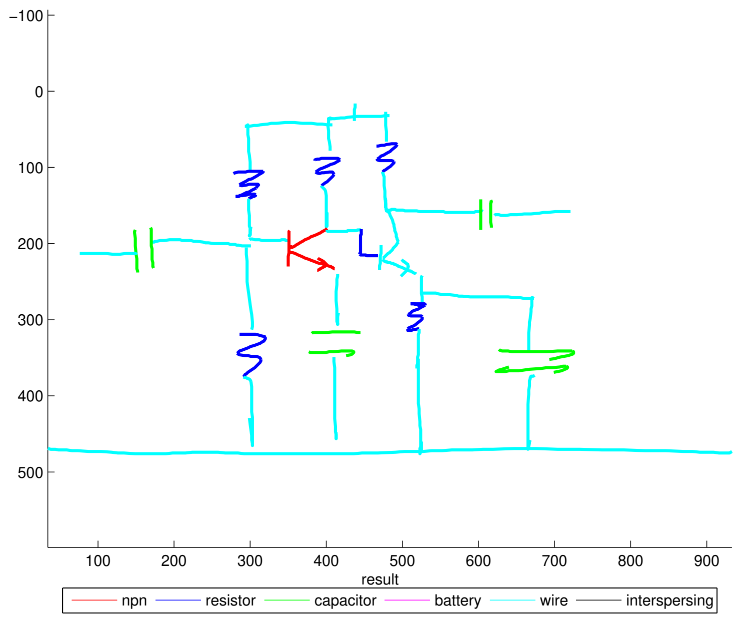
Figure 4-6: Another circuit drawn by our study participants. Stroke ordering for a fragment of the circuit is indicated by numbers.

shows that interspersing only a single primitive can lead to a cascade of misrecognitions. In this example, the user drew the collector of the transistor **Q2** and the wire connected to it using a single stroke (stroke #15, which is also an example of multi-object stroke).

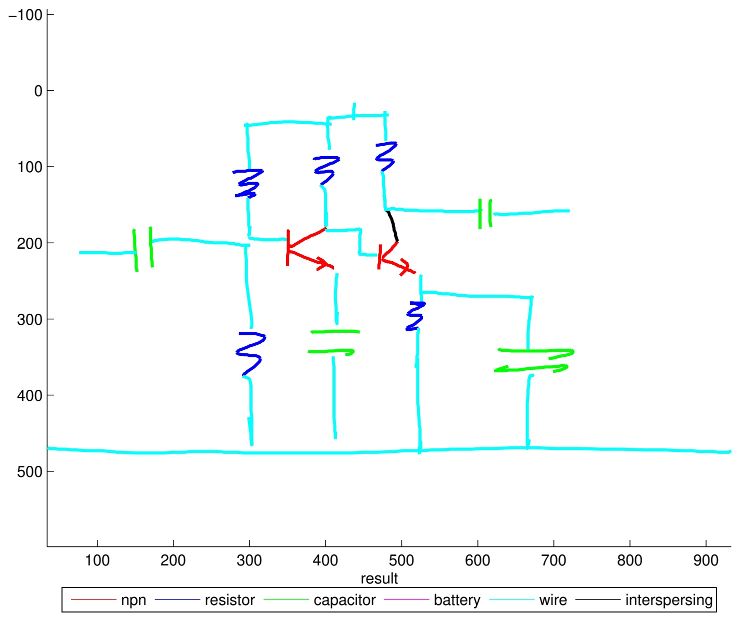
Two interpretations of the circuit are shown in Fig. 4-7. Fig. 4-7-a shows the interpretation obtained by running the multiscale model. In this figure, there are two recognition errors. **Q2** is misclassified as a set of wires, and the two wire segments connected to the base are misclassified as a resistor.

Fig. 4-7-b shows that both errors are fixed when we use the multiscale-interspersing model. The multiscale-interspersing model has learned that with probability 0.14 wires can be drawn in the course of drawing a transistor. This not only allows the transistor to be identified correctly but also helps the two wire segments to be classified correctly by using the knowledge that transistors very rarely proceed resistors, $P(\mathbf{OBJ_t}{=}\mathbf{NPN} \mid \mathbf{OBJ_{t-1}}{=}\mathbf{RESISTOR}) \approx 0$. This example shows the benefits of modeling both object-level patterns and interspersing.

The incremental benefits of using a more elaborate model may appear at times to be small (e.g. Fig 4-8). Nevertheless, correcting each misclassification requires effort on the part of the user and gets in the way of getting the job done. Hence we believe the error reduction rates we have demonstrated are significant in the context of a sketch-based user interface.

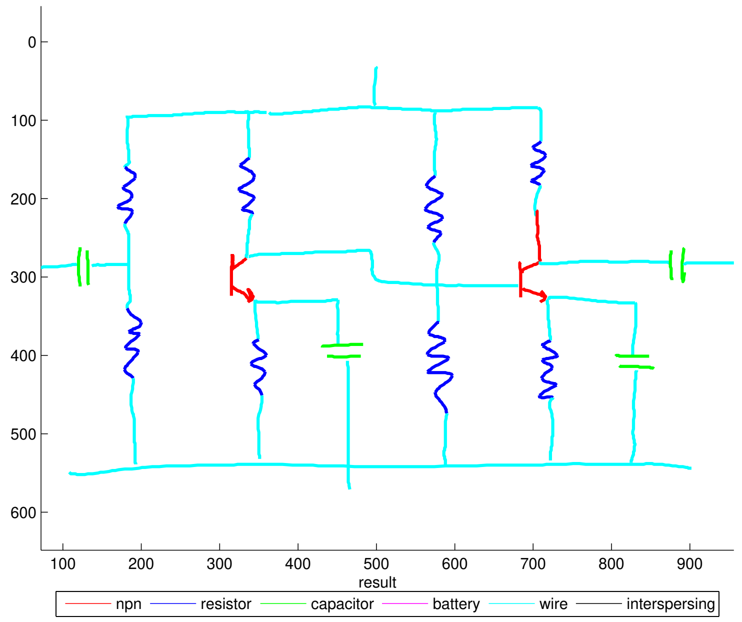


(a) Interpretation of the multiscale model.

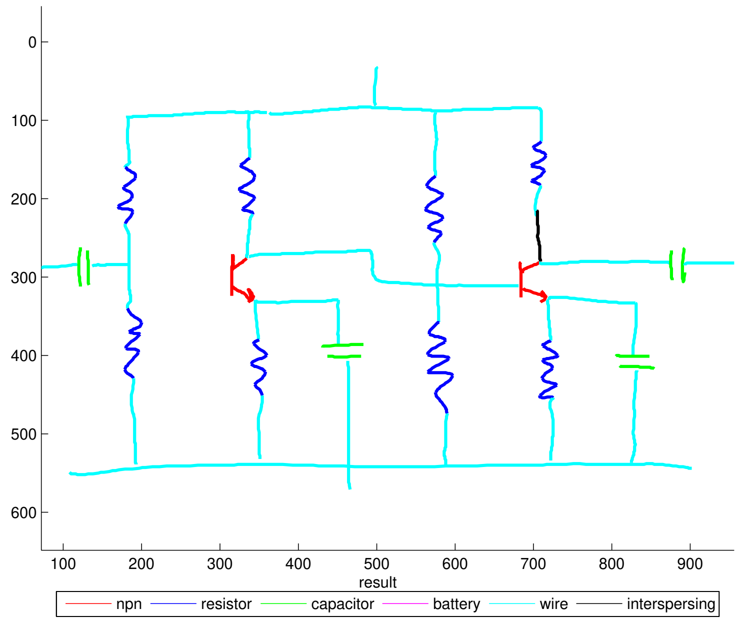


(b) Interpretation of the multiscale-interspersing model.

Figure 4-7: In this example, the user interspersed a wire connected to the collector of the transistor on the right. As a result the transistor is misclassified by the multiscale model (a). The multiscale-interspersing model identifies the wire interspersing (indicated by black) and correctly interprets the transistor (b). See Fig. A-3 for B&W printing.



(a) Interpretation of the multiscale model.



(b) Interpretation of the multiscale-interspersing model.

Figure 4-8: Another example of handling interspersing. In this case the errors caused by the interspersing is minor compared to 4-7. The interspersed wire is interpreted to be part of the transistor. The multiscale-interspersing model identifies the wire interspersing (indicated by black in b). See Fig. A-4 for B&W printing.

4.5 Summary

We have demonstrated that it is possible to recognize sketches using temporal patterns that naturally appear during sketching. This is particularly noteworthy because until recently, time-based graphical models were considered to be unsuitable for sketch recognition [5].

The results we have presented above show that using object-level temporal patterns increases recognition rates by incorporating contextual information about object orderings. We also showed that explicitly modeling interspersed drawing behavior in a principled way allows us avoid cascades of misinterpretations often caused by interspersings.

Finally, we note that the baseline models that we described are easier to implement compared to our main model. This gives designers the choice to work with easier to implement methods at the cost of sacrificing expressiveness of the model and recognition accuracy.

Chapter 5

Related work

Work in pen-based systems goes back to the 1970's [94, 57]. Although the potential impact of having computers interpret free-hand input was recognized early on, much of the early work suffered from ad-hoc recognition strategies and too many constants that need to be tuned. Nevertheless, early systems have been helpful in understanding what makes sketch recognition challenging. With the recent development of machine learning theory, well-founded statistical approaches to sketch recognition have been introduced. From a historical perspective, the goal of this thesis has been to contribute in this direction.

This chapter positions our work within the recent pen-based systems research and recognition systems. We discuss the related work under three groups:

- *Recognition systems* include speech, graphics and object recognition systems. These are relevant in the context of sketch recognition because in both cases the task is to interpret noisy signals, and approaches to recognition have a great deal in common.

- *Pen-based interfaces* are systems that aim to support natural interaction by supporting some degree of pen input. Although most of these systems process the pen input to some degree, the amount of recognition is too limited to call them sketch recognition systems. These systems focus mostly on the user interface aspects of pen-based interaction and recognition is often not their primary concern or the intended contribution.

- *Sketch recognition systems* are most relevant for the work we presented. We discuss sketch recognition systems under two categories, those that use spatial features and those that use temporal features.

5.1 Recognition systems

Other recognition problems are relevant to sketch recognition because many common themes appear in both cases such as noisy input, feature representation, choice of learning machinery.

5.1.1 Speech and handwriting recognition

Speech recognition systems model speech as sequence data. Use of statistical models for sequential data such as hidden Markov models has gained popularity after their success in speech recognition tasks [99]. Recently dynamic Bayesian networks have also been used for recognition. The advantage of DBNs is their ability to model dynamic processes using distributed state representations [15].

Handwriting recognition and online handwriting recognition work also use time based statistical models heavily [71, 98] although there is little work based on syntactic methods [86, 65, 70].

The handwriting recognition community has also come up with a large number of methods for recognizing single stroke or segmented pen input using HMMs and the single gesture recognition problem is mostly considered to be solved.

Both speech and handwriting recognition systems can take advantage of a vast number of robust preprocessing and segmentation methods (e.g., silence detection and word boundary detection). This reduces the computational cost of speech recognition [110] and results in increased recognition accuracy for handwriting recognition [25] systems because they can run individual classifiers on isolated characters. This is in contrast with sketch recognition where there is no segmentation strategy that will work for multiple domains at the signal level.

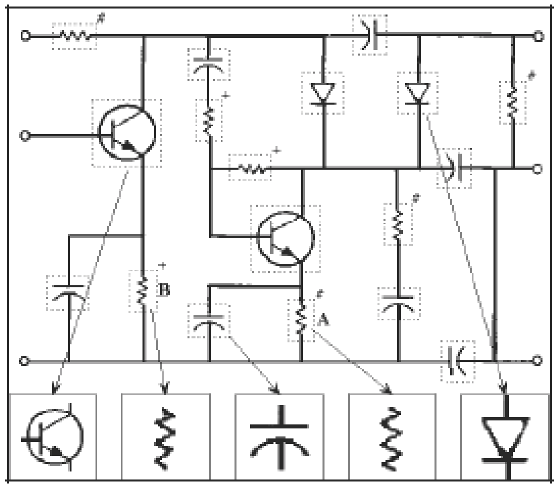


Figure 5-1: An example illustrating the kinds of input that graphics recognition systems accept. Here is a typical input for the graphics recognition system described in [82] and the processing of the system.

5.1.2 Graphics recognition

Graphics recognition aims to recognize computer generated diagrams [123, 82, 11, 114]. The input is a scan of a computer generated diagram (usually a CAD model). The output is either the vectorized version of the shapes, or a segmented and classified version of the input. The most important difference between graphics recognition and sketch recognition is the availability of clean and precise input for the graphics recognition task. Fig. 5-1 illustrates the kind of input that graphics recognition systems deal with. The figure shows the input for the graphics recognition system described in [82] and the processing of the system. Compared to sketch recognition the input to graphics recognition is cleaner and more precise, thus the recognition problem is better understood and it is easier to achieve high recognition rates using standard computer vision techniques. For the same reason, graphics recognition techniques don't transfer well to hand drawn diagrams.

5.1.3 Object recognition

Although the object recognition research is not relevant to the temporal model that we have presented, many sketch recognition algorithms are based on structural and syntactic methods (e.g., work in [4, 5, 107] and [84] use structural recognition strategies while [53, 52, 51] and [84] describe syntactic shape description languages).

Structural pattern recognition methods (also known as model-based or template-based methods) formulate the recognition problem based on implicit or explicit representations of objects in terms of their structure (i.e., components, sub-components and their relationships). In structural object recognition, *models* describe objects in terms of primitives and relationships between primitives (constraints). In the computer vision literature, models are also referred to as *templates*, and model features are referred to as *template slots* or *slots*. Primitives of the model are called *model features*, and primitives forming the scene are called *scene features*. Model and scene features need not be exactly the same. For example, 3D model edges may appear as 2D edges in the scene, but they are related by some function (translation, rotation, projection). The scene features include primitives originating from the objects in the scene that we would like to recognize, and may also contain *spurious features*. [1]

Syntactic approaches form a subset of structural approaches where the relationships are specified using a grammar [23, 113, 42]. Early work in in structural recognition is also related closely to CAD-based recognition [55, 60, 46, 10, 24, 39] and constraint satisfaction [115]. The syntactic methods on the other hand are related to visual languages two dimensional parsing [88]. In the sketch recognition community, [104] and [51] describe ways of compiling object descriptions to automatically generate recognizers. These techniques are more appropriate for graphics recognition tasks as outlined above because sketches are not clean and precise as required by these algorithms.

Other examples of object recognition algorithms include those that use constraints for efficient matching. The matching can be in the correspondence space [47, 81, 37, 38, 48] or in the transformation space [26].

Later work in computer vision intensively uses machine learning methods to learn statistical models from data. Some of the relevant work includes two-dimensional HMMs [77, 76] Bayesian networks [69, 78] and undirected graphical models for recognition that use Markov random fields [28] introduced by Geman and Geman [44].

As we describe in section 5.3, the main drawback of sketch recognition systems

[1]Spurious features are features originating from unknown objects or from errors made at the low level processing stage (i.e., edge detection, feature point detection).

based on computer vision techniques is the high computational cost of matching in presence of noise and imprecision that is specific to sketches. Although suggestions have been made to make matching more efficient by pruning the search space, such improvements always come with limiting assumptions.

5.1.4 Symbol recognition systems

Most sketch based systems assume that the users draw one object at a time (no interspersing) and specify the proper segmentation implicitly (e.g., by pausing for a certain amount of time between objects) or implicitly (e.g., pressing a button after drawing each object). These systems employ symbol recognizers to classify the already segmented objects. These systems are either gesture based or shape based. From a user's point of view, a shape based recognizer is usually preferable because it doesn't require the user to learn a prespecified gesture vocabulary.

The gesture recognizer described in [102] is perhaps the most popular method used for gesture based input. Its most severe limitation is the lack of support for multi-stroke gestures. The work by Apte et al. and Fonseca et al. describe ways of extending gesture based input to use multiple strokes [9, 87, 7, 29], but they still require isolated objects (e.g., no segmentation is performed).

Some of the shape based recognizers include the SVM-based active learning system developed by Sun et al. [111] and the recognition system being developed by Oltmans [95]. Oltmans' system uses a feature representation based on shape contexts originally developed by Malik et al. [13] and a learning framework based on SVMs to do recognition. Watanabe et al. describe a pattern representation and recognition scheme that uses data compression [67, 121]. Another symbol recognition system is based on Zernike moments described by Hse et al. [58]. In [64] Kara et al. describe a symbol recognizer using and image-based approach.

The shape based recognizers are appropriate for sketching interfaces where no segmentation is required. We believe requiring the user to actively participate in the segmentation process in ways described above interferes with the task at hand will result in unnatural interfaces. It is much more desirable to have a system that can do segmentation automatically such as ours.

5.2 Pen-based interfaces

5.2.1 Early processing systems

Systems that process pen input at the lowest level for the purposes of shape representation and feature point detection form the foundation of most sketch recognition systems. Work in [14, 103, 12, 105] are examples of such systems that specifically deal with freehand input. In addition, there is a wealth of work in the computer vision community that deal with issues of feature point detection, shape approximation and perceptual grouping [100, 79, 80]. Work in [35] and [105, 120] are examples of systems that try to detect regularities and symmetries in the input at the geometric-level and perform adjustments by mapping near-singular properties to singular properties. Our work uses the early sketch processing toolkit in [105].

5.2.2 Computer graphics and user interface applications

Systems that support pen-based interaction include computer graphics oriented systems that are intended for 2D and 3D rapid geometric design [35, 59, 124, 32, 97, 54]. These systems allow specifying geometric shapes and constraints using free-form ink and illustrate the potential of combining free-hand input with relatively little recognition to specify geometry in a design setting.

Other systems that support some form of pen input include [41], a gesture based system for entering music notation, the sketch-based GIS system by Blaser [20], and the Electronic Cocktail Napkin by [49, 50]. SILK [73] describes a gesture-based approach to interface design. Quickset [27] describes a multimodal interaction framework that supports gesture-like pen input.

Others have explored sketching in the context of multi-modal input fusion [40, 27, 2, 63, 96, 33, 31]. While [27] describes a general framework for multimodal fusion, [2] and [96] describe how speech can be combined with sketching and [33] investigates ways of using speech, gestures and sketching as complementary information sources in a multimodal interaction setting.

Although the systems described so far have some support for pen input, they have limited recognition capabilities and are thus not sketch recognition systems in the sense used in this thesis. A vast majority of these systems support only single stroke objects or gesture-like input. Most of them are built on the premise that the users know about the limitations of the system and will draw accordingly. Examples of this include having the user indicate appropriate segmentation by pausing or explicitly by clicking a “done” button. The lack of emphasis on recognition in these systems is balanced by their focus on the human-computer interface issues of sketch-based interaction.

There is also some work that does not present sketch recognition techniques but addresses general user interface issues [21, 61, 62, 40, 72]. Other work in [1] describes appropriate gestures selection strategies for gesture based systems. A sketch description language developed by Hammond investigates issues such as structural shape description, editing behavior and display options [53, 52, 51]. This body of work is relevant for future work on exploring general HCI issues as we describe below.

5.3 Sketch recognition systems

Work in sketch recognition can be divided into two groups based on whether it uses temporal features. We start with a review of the large body of work that does not use temporal features. These systems are complementary to our approach. Then we discuss those that use temporal features for recognition.

5.3.1 Use of spatial data for recognition

Early work that uses spatial constraints for sketch recognition includes work by Weisman and Muzumdar [122, 93] and the ASSIST system by Alvarado [3] focus on interpreting hand-drawn two dimensional sketches of mechanical devices. These systems have been demonstrated to be usable by experienced users (e.g., the developers).

The stick-figure interpretation system by Mahoney and Fromherz [84, 85, 83] uses a structural language for describing objects and a constraint-based matching approach. Their work is the first to investigate the computational complexity of model-based sketch recognition. They present heuristics such as grouping, anchors, salience and model bias for speeding up the matching. They also identify some of the messiness in sketches as being due to overshooting, undershooting, and failure of co-termination.

Given a sketch, their algorithm first generates an attributed graph representing the scene using the set of lines in the sketch and their relationships. Next, the scene graph is augmented by subgraphs corresponding to possible ambiguities in the interpretation of line relations. This augmentation operation – called *graph elaboration* – applies perceptual organization principles such as good continuation and proximity. Next, the algorithm looks for instances of the model graph in the scene graph by performing subgraph matching. This approach to sketch recognition can be thought of as an adaptation to sketches of the subgraph isomorphism approach, popular in computer vision, in a way that deals with the free-hand and noisy characteristics of sketches using perceptual organization principles. On the other hand, because the subgraph isomorphism treats the sketch as a static image, this approach does not capitalize on the dynamic properties of sketches such as the drawing order of strokes.

Shilman et al. describe a statistical, grammar based ink parsing algorithm for sketch recognition [107]. In this framework, the system designer describes objects with a declarative grammar and gives labeled training examples. The initial grammar requires predetermined constants for computing constraints between objects. A parser is generated using this information. During runtime, the parser looks at the sketching surface and generates a parse tree whose nodes correspond to alternate interpretations of the sketch. Next, Bayesian networks corresponding to interpretations at the parse tree nodes are generated. Most likely interpretations are chosen and expanded further, pruning nodes with probabilities less than a preset threshold. The authors report that their method outperforms a hand-coded parser for a domain with a relatively simple grammar. Having been inspired by language parsing, this approach is not built with the dynamic and interactive nature of sketching in mind.

Viola et al. describe a spatial recognition and grouping method for text and graphics [108, 106]. They use a discriminative classifier to classify connected subgraphs of a proximity graph constructed from the sketch. Each subgraph is classified as either making up an object or as an invalid stroke combination. They select a small subset of efficient features and perform an A^* search over connected subsets to find the optimal segmentation and recognition of all strokes on the page. In order to keep the search tractable they make some assumptions about the objects in the domain. For example, they assume that objects in their domain have no more than

8 strokes, or that the strokes constituting an object are within a certain distance. These assumptions seem to work for flowchart-like drawings where the connectors and objects are sufficiently separated from one another, but the practicality of these algorithms is yet to be demonstrated in general for domains where objects vary in size and shape where assumptions on the object size and scale may not hold.

Work by Alvarado describes a blackboard based architecture with a top-down recognition component based on dynamically constructed Bayesian networks that allows recovery from bottom-up recognition errors [4, 5, 6]. This system also uses a structural approach to recognition. A major strength of this system, which makes it unique with respect to the other recognition systems, is its ability to model low-level errors explicitly and use top-down and bottom-up information together to fix errors that can be avoided using context. This strength comes with a computational cost that prevents the system from running in real-time despite some limiting assumptions made and despite the use of efficient search strategies to prune the hypothesis space. In most cases the system takes several minutes to hours to process moderate size drawings. Also, although this system uses a probabilistic model based on dynamically constructed Bayesian networks, the object models have to be specified by an expert instead of learning them from data. One can also argue the other way saying that using expert knowledge is a strength of this system, but our experience seems to indicate that hand tuning probabilities is hard and time consuming even for experts, whereas creating moderate amounts of training data (as it was required in our case) is more straightforward and requires almost no expertise.

In [45], Kara et al. present a circuit diagram recognition system that does segmentation using a heuristic called “ink-density” and runs isolated symbol recognizers to generate an interpretation. Their system has as number of strengths such as fast recognition, support for multi-stroke objects and multi-object strokes and arbitrary stroke orderings within each object. On the other hand the segmentation algorithm used in this work does not handle interspersed drawing, so the user is required to finish an object before starting a new one. It is also not clear how well the segmentation algorithm based on ink density would work for other domains (e.g., UML diagrams or course of action drawings).

Finally Szummer et al. [68] describe a generative Bayesian framework for shape recognition. They use dynamic programming to achieve segmentation and classification simultaneously. The major strength of their approach is the ability to recognize messy drawings using single examples by assuming a Gaussian noise model. As was the case for some of the approaches described above, the authors make certain assumptions to keep the search for the optimal segmentation tractable. Specifically they assume that each subset of stroke fragments considered during segmentation contains no more than seven straight line segments. This might be a severe limitation in domains with moderately complex objects, such as the sloppy stick-figures considered by Mahoney et al. [84, 85, 83] or the course of action diagrams domain.

5.3.2 Use of temporal data for recognition

This thesis is the first to use temporal patterns for sketch recognition. Some of the related work includes the work in the handwriting recognition community where a large number of methods to recognize single stroke or segmented pen input using HMMs. In the sketch recognition community, work in [8] describes an HMM-based symbol recognizer that uses chain-code-like features to recognize isolated symbols. Our work does not assume segmented input and builds a joint model for complete sketches.

In [109], the authors present a sketch interpretation and curve refinement system that uses an HHMM setup. This work shares the same motivation as ours. In their work, they assume that the parameters of the object-level process (which they refer to as the scene level) is supplied by the user using a *semantic graph* representation. We learn the parameters of the stroke-level and object-level patterns from data, and use the more efficient DBN representation to avoid the $O(T^3)$ complexity of HHMMs. Furthermore, we support multi-stroke objects and real-valued continuous observations which allow using a rich set of features.

Chapter 6

Contributions

6.1 Contributions

We have presented a sketch recognition framework that exploits both stroke-level and object-level temporal orderings. Our framework allows learning object-level patterns from data, handles multi-stroke objects and multi-object strokes efficiently, supports continuous observable features and handles interspersed drawing.

A number of contributions make our work unique:

- Our work is the first to demonstrate the utility of stroke ordering information and the suitability of time-based graphical models for recognizing complicated sketches and is particularly noteworthy because until recently such models were considered to be unsuitable for sketch recognition [5].
- We showed how graphical models can be used to implement our model of temporal patterns efficiently using features such as switching parents, sparse representations and parameter tying. Our model interprets sketches with several dozen objects under a second on a standard PC, and as such is suitable for real-time sketch interpretation.
- We reported that a standard implementation of belief propagation, appropriate for simple conditional Gaussian belief networks, runs into numerical instabilities when used for inference in our model. We noted that the Lauritzen-Jensen stable

conditional Gaussian belief propagation algorithm should be used to avoid such numerical problems in recognition.

- We demonstrated that our hierarchical models capture knowledge of both common stroke orderings and common object orderings. Our evaluation involving circuit diagrams collected from eight participants shows that modeling common object orderings reduces the number of recognition errors by 14%- 37%.

- We also showed how a probabilistic temporal model can explicitly model interspersed drawing behavior. Interspersed drawing behavior poses serious challenges to a naïve approach of temporal sketch recognition model, but we address the issue remaining within the formal framework of Dynamic Bayesian Networks. Therefore we do not rely on the the assumption that users to draw objects one at a time. We showed that modeling interspersing reduces recognition errors by an additional 20%-37%.

- Our recognition framework allows strokes to be part of multiple objects as long as objects do not share primitives. We support multi-stroke objects and objects with variable number of components (e.g., resistors with variable number of humps).

- Finally we support discrete and real-valued feature representations that allows us to encode our data using discrete encodings for categorical properties (e.g., a stroke being a circle vs. a line), and real-valued encodings for continuous features (e.g., length, orientation of a line).

6.2 Discussion

We believe that our approach to modeling multi-scale patterns in presence of interspersing can be applicable to a number of other problems that deal with temporal data and use a sequence based representation. Fundamentally, the probabilistic model that we have presented is not specific to sketching data and it can handle any kind of sequence data where the Markov assumption and a representation based on dynamic Bayesian networks is appropriate.

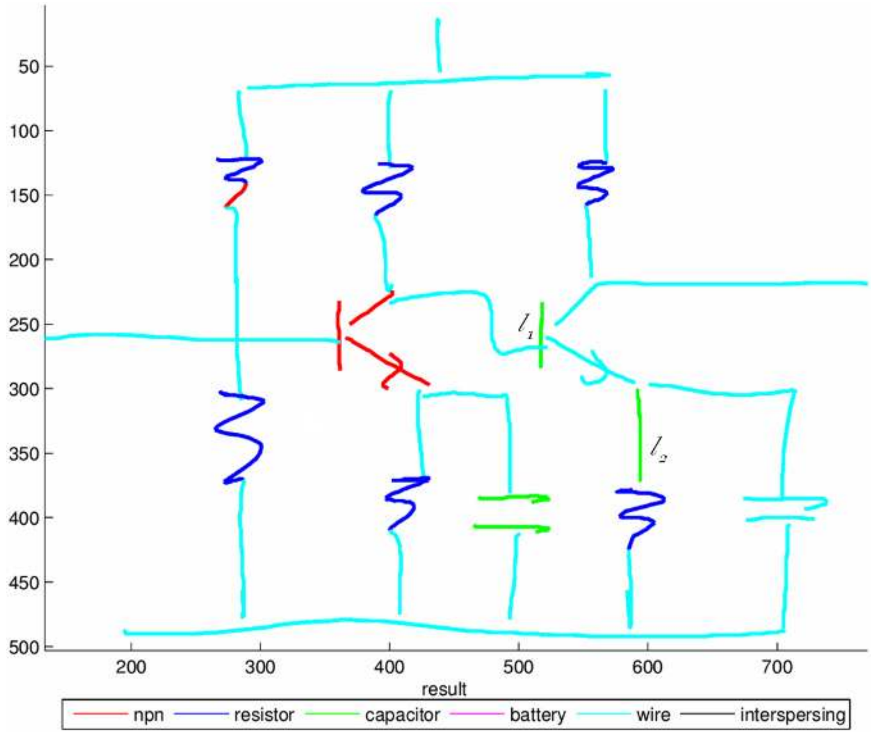


Figure 6-1: An example of strokes with the right temporal character but wrong shape being classified incorrectly (reproduced from Chap. 4). The two vertical lines (l_1, l_2) are offset from one another, but were classified to be a capacitor because they were drawn one after the other, and the temporal features extracted from them agree with the capacitor model that we learned. See Fig. A-5 for B&W printing.

It would be an interesting future work to apply our approach to problems where there are multiple sequence models and there models can be mixed in an interspersed fashion. Potential application areas may include activity recognition, meeting modeling, object tracking and plan recognition.

Our recognition framework also has a number of limitations that make interesting future research topics. Although our user studies and psychology research suggests that drawing is a highly stylized process and ordering phenomena is found in many domains [116, 119], our approach is not suitable for domains where stroke ordering is unpredictable.

Another limitation of our model is that currently it does not incorporate any spatial or geometric constraints beyond those used to encode stroke sequences, and as a result recognition is based strictly on temporal patterns and not shape. Therefore a sequence of strokes that has the right temporal character but the wrong shape can be classified incorrectly. For example, the two parallel strokes in Fig. 6-1 were classified as a capacitor, even though the strokes were offset from one another. We consider below how this limitation can be addressed by using shape based classifiers as external sources of information.

Chapter 7

Future work

Future directions for this research include exploring alternative approaches to some of the issues noted above, including other models for handling interspersing. There are also a number of problems that require further research such as finding ways of incorporating other knowledge sources into the decision process and exploring some of the user interface issues.

7.1 Alternative models of interspersing

As noted above, we framed our approach for handling interspersing based on our observations about the domains that we studied including those mentioned in Chap. 2 (finite state machines, UML diagrams, Course of Action Diagrams, stick-figures and emoticons) and circuit diagrams, which was our domain of choice for evaluation. In these domains, we never observed an object of the same type being interspersed (e.g., interspersing two transistors), and as a result our model doesn't handle such cases. It would be an interesting exercise to investigate the drawing patterns in other domains to see if any of them contain interspersings of the same object type. If that is indeed the case, then what is the best way of modeling such interspersings?

One possibility would be to stay within the framework we have introduced and add an extra stroke level model for the object class that has self interspersing. For example, assume we have a scenario where the user starts drawing another NPN transistor before finishing the current transistor. In this case our DBN model would

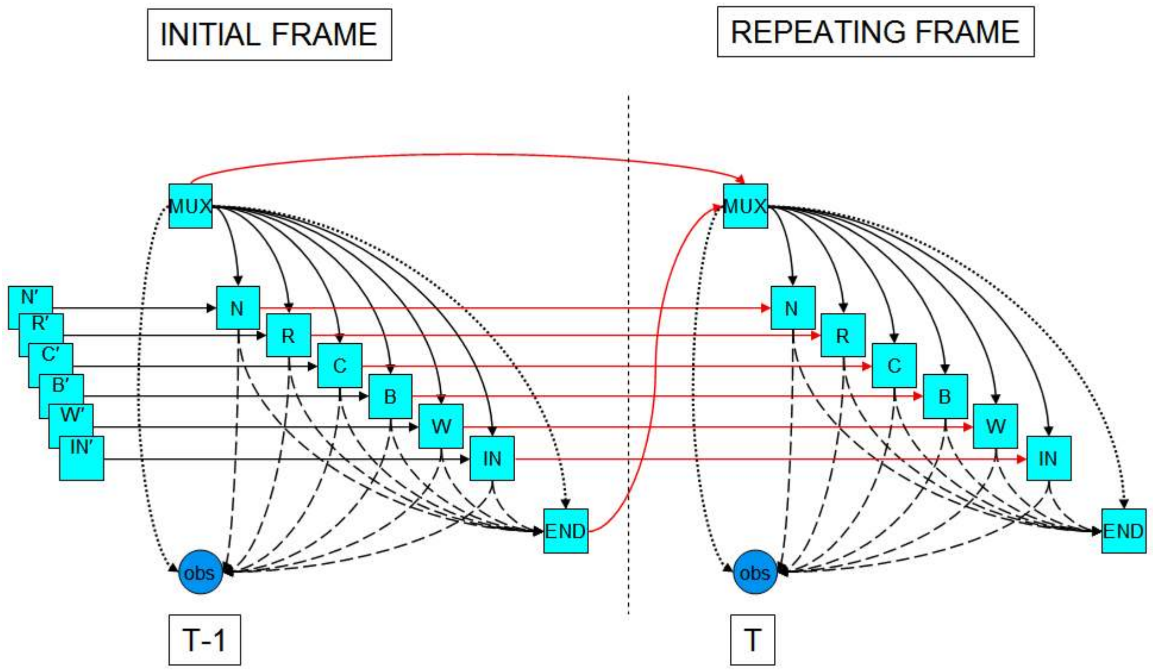


Figure 7-1: One plausible way of modeling self interspersing could be to duplicate the stroke level model for the self interspersing class. In this case, we assume a scenario where the user intersperses two transistors, so we have the **N** node as usual and an extra node **IN** to represent the stroke-level model for the interspersed NPN transistor.

have the **N** node as usual and an extra node **IN** to represent the stroke-level model for the interspersed NPN transistor (Fig. 7-1). The parameters that capture the stroke-level patterns would be the same for both models (i.e., we assume the user draws an interspersed transistor similar to a regular transistor). This would allow us to learn one stroke-level model for the **N** and **IN** nodes, and have a compact representation using parameter tying.

Another approach might be to have two separate multiscale models (as described in section 3.6) and a binary switching parent node that activates only one of these models at given time. The first multiscale DBN fragment would track the joint state of the generative process[1] without interspersing as usual, and the second fragment would get activated when there is an interspersing. The joint state of the first fragment would keep the information needed to resume the generative process once the interspersing is over. Obviously many interesting research issues need to be addressed

[1]In this case the joint state would include knowledge of what object is being drawn and the state of the stroke level generative process.

in such a framework including the tractability of inference.

7.2 Incorporating other knowledge sources

Another interesting direction to explore would be to find ways of incorporating information from external knowledge sources. The mechanism used to incorporate external information could be different based on the nature of the supplied information. For example, if the source of the information is the user (e.g., the user explicitly indicating the classification for a set of strokes), then we can take the information as ground truth and incorporate it in the inference mechanism as observed values.

7.2.1 Incorporating user feedback

Assume the user drew the circuit in Fig. 7-2 and the recognition system misclassified the circled strokes. Now, if the user selects these strokes and labels them explicitly as forming a capacitor, we can take this information as ground truth and inject it directly into our DBN model by making certain nodes in the unrolled DBN observed. Specifically, the **MUX** and the **END** nodes corresponding to the two strokes become observable (see Fig. 7-3). Because the the user identified the two strokes as a capacitor, the **MUX** nodes corresponding to these strokes become observable. Also, because there is no interspersing in our domain, it must be the case that the user has just finished drawing an object before the capacitor and will start a new object next (thus the three **END** nodes are now observed). Once the values of these nodes are injected into the unrolled DBN, we can perform inference which will potentially cause some of the other errors to be corrected once the effects of these observations are propagated to the neighboring nodes (indicated by dotted circles).

7.2.2 Incorporating information from external classifiers

If the external knowledge source is a potentially unreliable external classifier, then it should be treated as a noisy observation. One way of achieving this would be to use the information provided by the external classifiers as soft evidence [118] or virtual evidence [101] by having a child node associated with each **MUX** node (e.g., Fig 7-4).

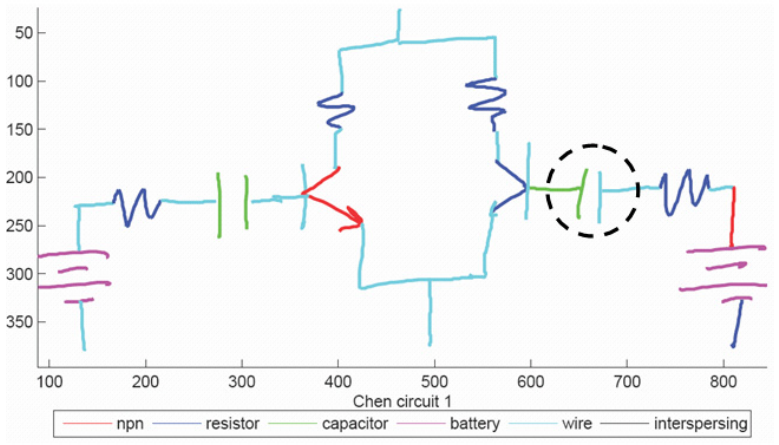


Figure 7-2: Suppose the user drew the above circuit diagram and the recognition system misclassified the circled capacitor. Now, if the user selects these two strokes and labels them explicitly as forming a capacitor, then we can take this information as being ground truth. This allows us to inject this information directly into our DBN model by making certain nodes in the unrolled DBN observed (see Fig. 7-3).

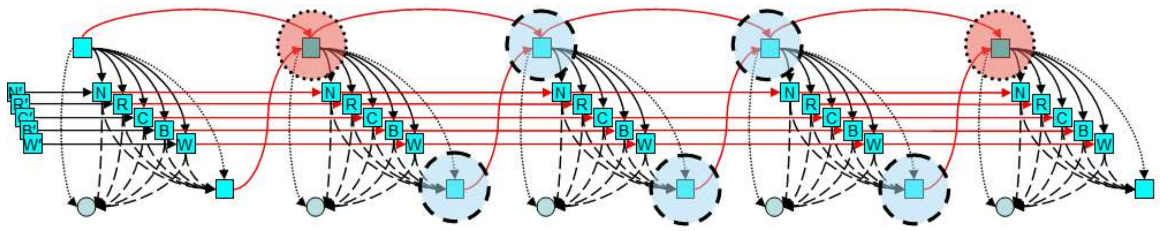


Figure 7-3: If the user explicitly corrects a misrecognition as describe in Fig. 7-2, then this makes certain nodes in the unrolled DBN observable (nodes circled with dashed lines). When we perform inference with the new observed values, the effects of the observations will propagate to the neighboring nodes (nodes circled with dotted lines).

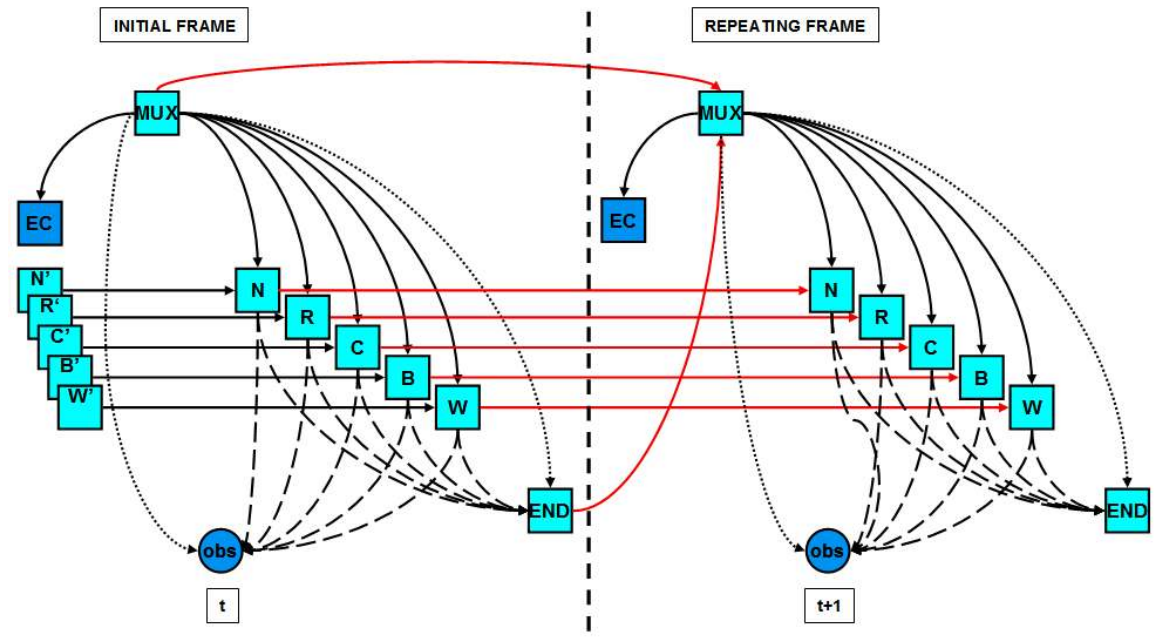


Figure 7-4: One way of incorporating information from external classifiers would be to use the provided information as soft evidence [118] or virtual evidence [101] by having a child node associated with each **MUX** node as shown here. The **EC** node carries information supplied by the external classifiers.

This would also serve as a step toward combining temporal and spatial/structural information for sketch recognition.

7.3 Incorporating spatial features

As mentioned in section 6.2, our temporal model does not incorporate any spatial or geometric constraints beyond those used to encode stroke sequences, and as a result recognition is based strictly on temporal patterns, not shape. Therefore a sequence of strokes that has the right temporal character but the wrong shape can be classified incorrectly. One way to remedy this shortcoming would be to combine spatial and temporal models. As mentioned in section 3.5.4, if there is no interspersing, an approach based on dynamic programming can be used to combine spatial and temporal constraints. A more challenging task is to find ways of combining two information sources when there is interspersing or detecting when the user is employing a novel stroke ordering such that the spatial features are given heavier emphasis. This is an interesting avenue that we plan to pursue.

7.4 User interface issues

Finally, in this thesis we focused entirely on recognition issues and did not address any user interface issues. Because our approach uses temporal features only, its failure modes may be different from those of a system that uses geometric constraints. It would be useful to investigate whether this is the case, and what the implications are from a user-interface point of view (e.g., do methods for error correction transfer to this new way of recognition).

Appendix A

Recognition results for non-color printing

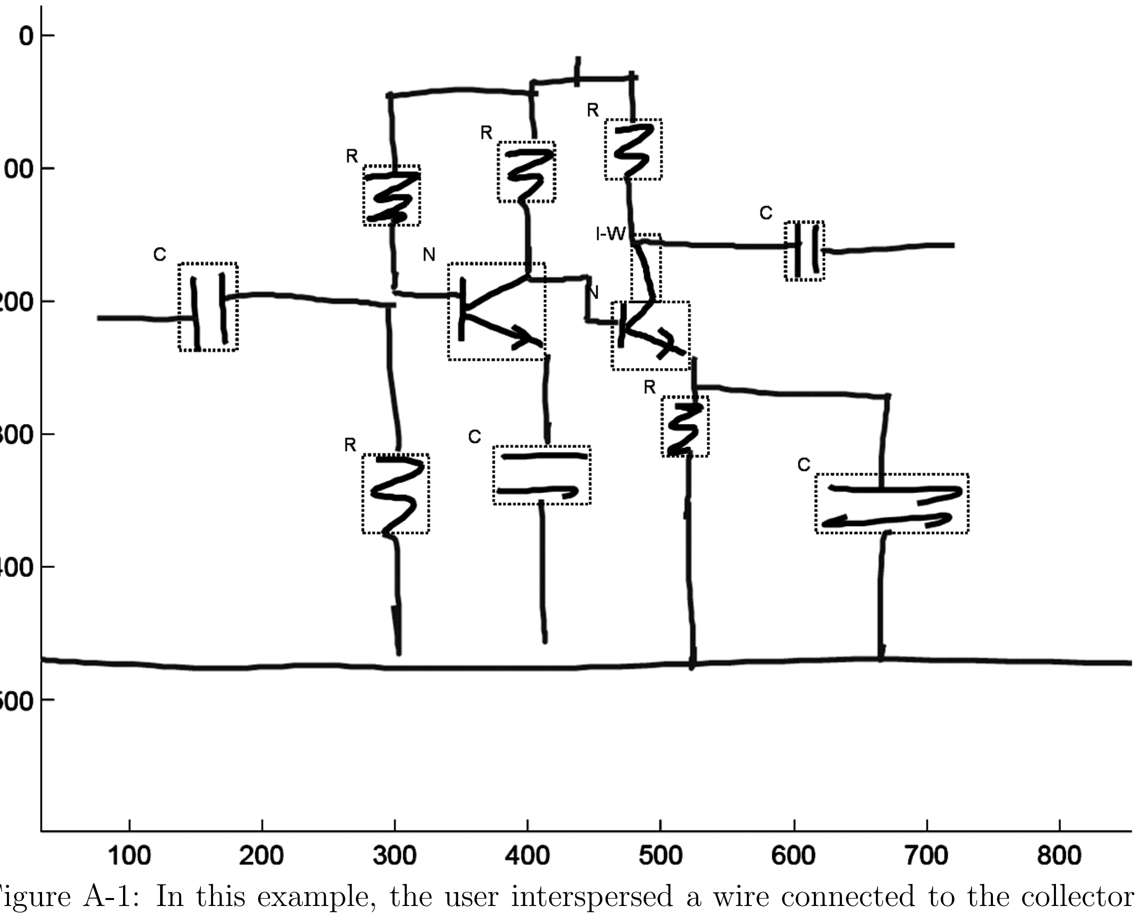
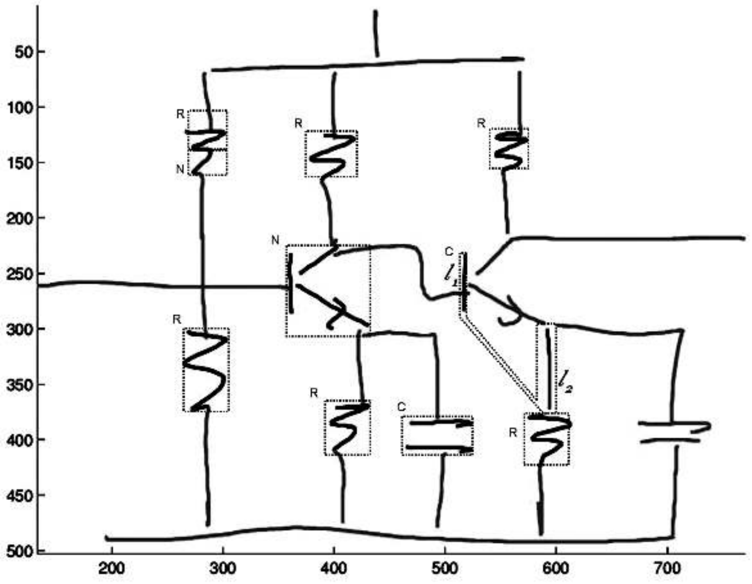
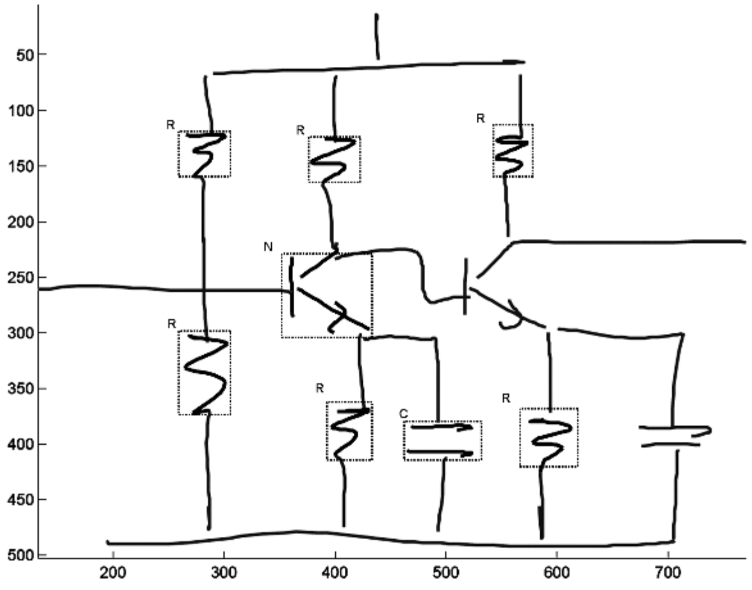


Figure A-1: In this example, the user interspersed a wire connected to the collector of the transistor **Q2**. We present a model that can recognize objects correctly even in the presence of such interspersings and identify interspersed objects (indicated by I-W).

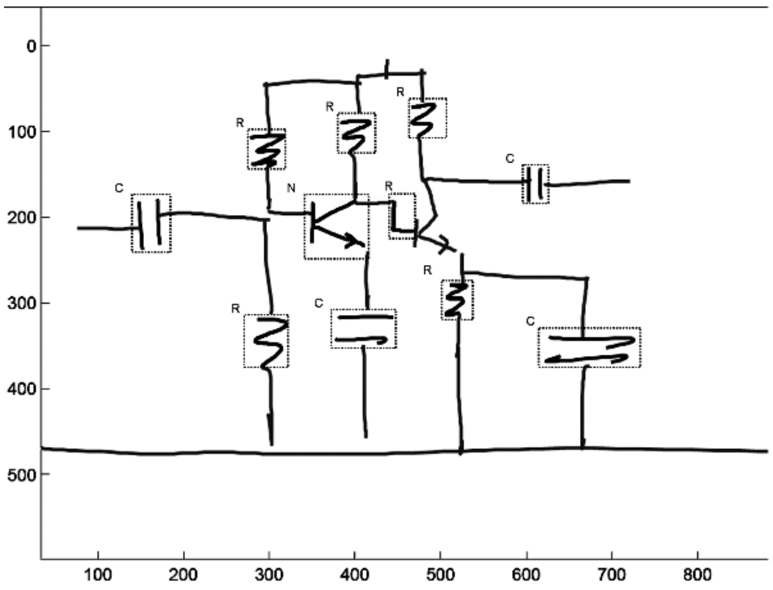


(a) Interpretation of the baseline model.

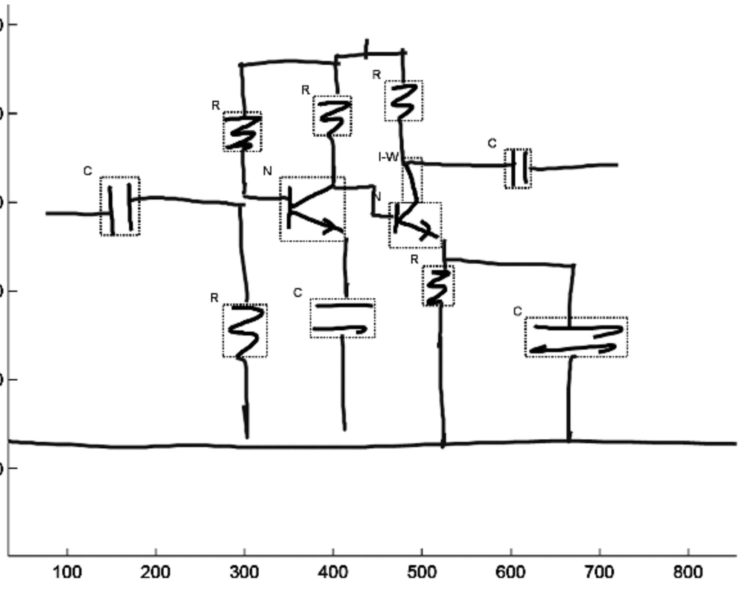


(b) Interpretation of the multiscale model.

Figure A-2: Examples of errors corrected using context provided by object-level patterns. There are four misrecognitions in (a). Note how two of these misrecognitions is fixed using knowledge of object level patterns (resistor in the upper left corner (**R1**) and the wire connected to the emitter of the misrecognized transistor).

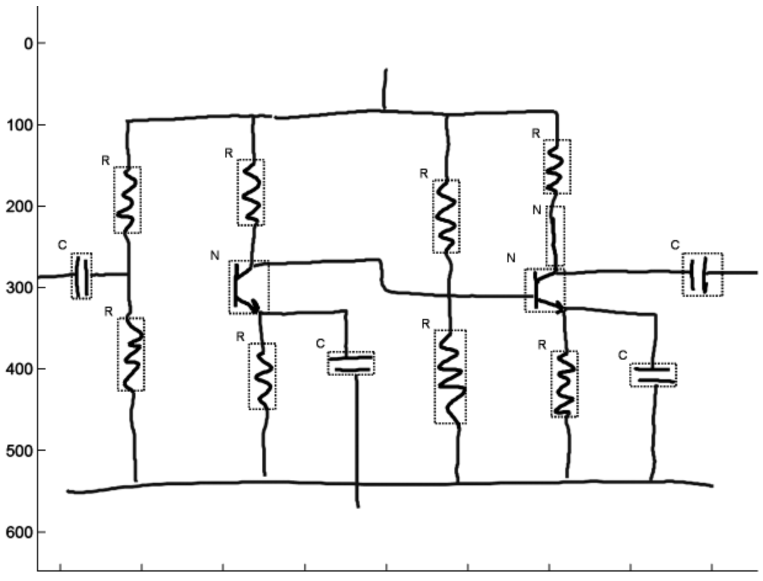


(a) Interpretation of the multiscale model.

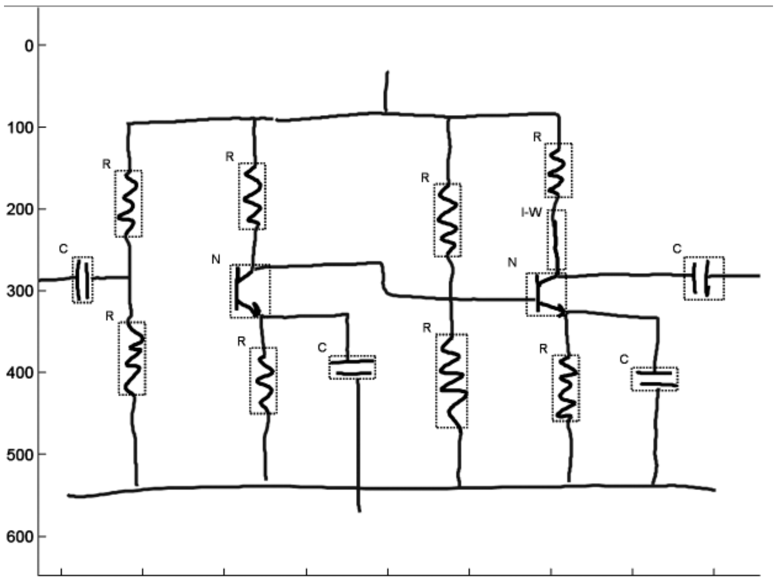


(b) Interpretation of the multiscale-interspersing model.

Figure A-3: In this example, the user interspersed a wire connected to the collector of the transistor on the right. As a result the transistor is misclassified by the multiscale model (a). The multiscale-interspersing model identifies the wire interspersing (indicated by I-W) and correctly interprets the transistor (b).



(a) Interpretation of the multiscale model.



(b) Interpretation of the multiscale-interspersing model.

Figure A-4: Another example of handling interspersing. In this case the errors caused by the interspersing is minor compared to 4-7. The interspersed wire is interpreted to be part of the transistor. The multiscale-interspersing model identifies the wire interspersing (indicated by I-W in b).

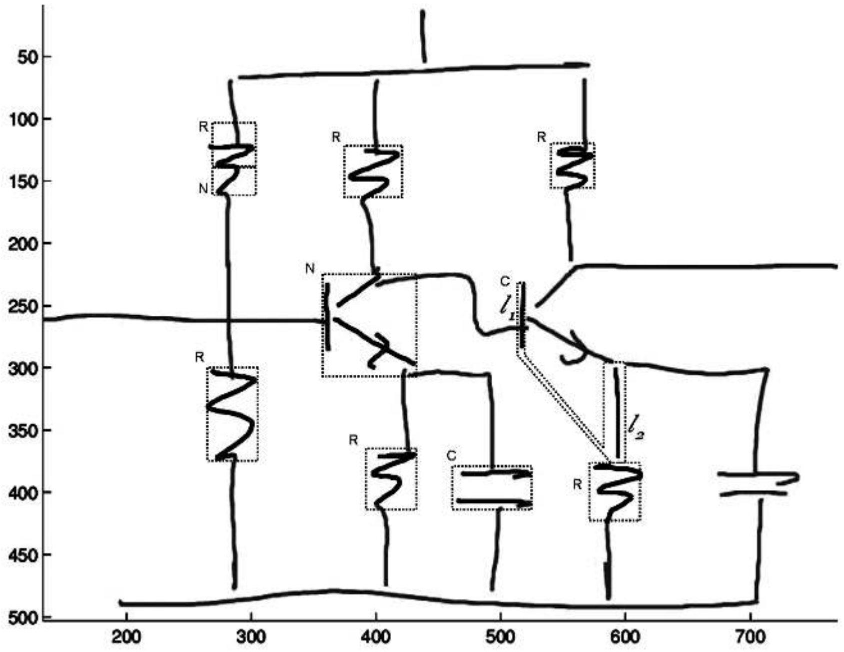


Figure A-5: An example of strokes with the right temporal character but wrong shape being classified incorrectly (reproduced from Chap. 4). The two vertical lines (l_1, l_2) are offset from one another, but were classified to be a capacitor because they were drawn one after the other, and the temporal features extracted from them agree with the capacitor model that we learned.

Appendix B

Symbols used

$\mathscr{S} = S_1, S_2, ...S_N$	A sketch, defined as a sequence of strokes.
$\mathscr{P} = P_1, P_2, ...P_T$	The sequence of time-ordered primitives obtained from $\mathscr{S}$.
$\mathscr{C} = \{C_1, C_2, ...C_n\}$	The set of classes.
$G = G_1, G_2, ...G_K$	The set of groups identified by the segmentation process.
$L = L_1, L_2, ...L_K$	Labels for the groups ($L_i \in \mathscr{C}$).
$G_i = \rho_1, \rho_2, ...\rho_n$	Indices of the primitives included in group G_i sorted in ascending order.
$\mathfrak{G}$	All possible groupings of the primitives in a sketch.
$\mathfrak{L}(G)$	All possible labelings of a group.
$\boldsymbol{O_{G_i}}$	The sequence of observations corresponding to a group of primitives G_i.
$\boldsymbol{\lambda_{L(G_i)}}$	Model corresponding to the label assigned to the primitives in group G_i.
$\lambda(A, B, \pi)$	A Hidden Markov Model.

A	Transition matrix for the HMM.
B	Observation matrix for the HMM.
π	Prior for the HMM.
$(B_1, B_\rightarrow)$	A dynamic Bayesian network.
B_1	Initial frame for the dynamic Bayesian network.
$B_\rightarrow$	Repeating frame for the dynamic Bayesian network.

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