Skin Friction of the Wind on the Earth's Surface.

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The mechanism by means of which momentum is transmitted to a solid surface, in order that it may exert a drag on a fluid flowing past it, is at present understood only very imperfectly. It seems certain, however, that the law of dynamical similarity is applicable to skin friction; if therefore it were possible to measure the tangential force exerted by the wind as it blows over a large tract of land, it should be equal to the skin friction on a similar small surface when subjected to the action of the very high wind which would correspond with the same value of lV/ν.*

In reducing the tract of land to a similar small flat plate, the trees and houses would be reduced to a mere roughness on the plate. It is to be expected therefore that, if the skin friction on unit area of the earth's surface be expressed in the form

$$F = \kappa\rho Q_s^2, \tag{1}$$

Q_s being the wind velocity near the surface and ρ the density of air, the constant κ will be the same as the constant which would be found in the laboratory by experimenting with a small, slightly roughened plate, if a sufficiently high value of lV/ν could be obtained. It should be noticed, however, that the velocity which should be compared with Q_s is the velocity close to the solid surface and not the general velocity of the air in the case of a flat plate, or the mean velocity over a cross section in the case of flow in a pipe.

In the case of a fluid flowing through a pipe, it has been shown by Stanton† that for high values of lV/ν the velocity, V, of the fluid near the wall is about 0·6 of the velocity in the middle, and that the mean velocity, V_0, is about 0·85 of the velocity in the middle, so that $V = 0{\cdot}7\,V_0$. It was found that the skin friction for the highest values of lV/ν which Stanton was able to obtain could be expressed by the formula $F = 0{\cdot}002\rho V_0^2$, which is equivalent to $F = 0{\cdot}004\rho V^2$. The constant κ of equation (1) must therefore be compared with 0·004.

The object of this paper is to show that it is possible to use observations of

* In this expression l represents a linear dimension of the system, V the velocity of the fluid, and ν the kinematic viscosity.

† See 'Collected Researches of the National Physical Laboratory,' vol. 9 (1913), Plate 1, opposite p. 6.

wind velocity at different heights above the ground in order to calculate the skin friction of the wind blowing over the earth's surface, that it can be expressed in the form $F = \kappa\rho Q_s^2$, where κ is a constant, and that the value of κ so calculated is 0·002 to 0·003 for the ground at Salisbury Plain, where the wind observations were made.

The existence of skin friction between the wind and the ground necessarily implies a transfer of momentum from the atmosphere to the surface of the earth, and the rate at which the layers of air near the surface must communicate this momentum to the ground in order that the observed distribution of wind velocity may be maintained has been calculated by the author.* If axes of x and y be chosen parallel and perpendicular to straight isobars, and if u and v be the components of wind velocity parallel to these axes, then the rates at which momentum parallel to the axes x and y leaves unit area of a layer of small thickness δz are $\mu \dfrac{d^2u}{dz^2}\delta z$ and $\mu\dfrac{d^2v}{dz^2}\delta z$ respectively, where the axis of z is vertical and μ is the coefficient of "eddy viscosity" whose value is calculated in the author's previous paper.† If F_x and F_y be the components of the skin friction force which acts on unit area of the ground,

$$F_x = \int_0^\infty \mu\frac{d^2u}{dz^2}\,dz = \mu\left[\frac{du}{dz}\right]_{z=0} \qquad \text{and} \qquad F_y = \mu\left[\frac{dv}{dz}\right]_{z=0},$$

since μ does not appear to vary much with height.‡ The total skin friction is evidently $F = \sqrt{(F_x^2 + F_y^2)}$.

It will be found from the equations given on pp. 15, 16, and 18 of "Eddy Motion" that

$$\left[\frac{du}{dz}\right]_{z=0} = \frac{2\tan\alpha}{1+\tan^2\alpha}\,Q_G\,\frac{\frac{3}{4}\pi+\alpha}{H_1} \qquad \text{and} \qquad \left[\frac{dv}{dz}\right]_{z=0} = \frac{2\tan^2\alpha}{1+\tan^2\alpha}\,Q_G\,\frac{\frac{3}{4}\pi+\alpha}{H_1},$$

where α is the angle between the surface wind and the gradient direction, Q_G is the gradient wind, and H_1 is the height at which the wind attains the gradient direction.

Hence
$$F = 2\mu Q_G \sin\alpha\,\frac{\frac{3}{4}\pi+\alpha}{H_1},$$

and from equation (1)

$$\kappa = \frac{F}{\rho Q_s^2} = \frac{2\mu}{\rho}\sin\alpha\,\frac{\frac{3}{4}\pi+\alpha}{H_1}\,\frac{Q_G}{Q_s^2}. \qquad (2)$$

* See "Eddy Motion in the Atmosphere," 'Phil. Trans.,' A, vol. 215 (1915).

† *Ibid.*, p. 21.

‡ *Ibid.*, p. 20.

The observations* on which the calculations given in "Eddy Motion" are based were divided into three classes:

(*a*) Light winds, for which the mean wind velocity near the ground† was 3·3 metres per second, and the mean gradient wind was 4·6 metres per second.

(*b*) Moderate winds, for which the mean surface and gradient winds were 5·9 and 9·1 metres per second respectively.

(*c*) Strong winds, for which they were 9·5 and 15·6 metres per second.

The value of κ will be calculated for each of these groups separately.

(a) *Light Winds.*

$\mu/\rho = 28\times10^3$,‡ $\alpha = 13°$,§ $H_1 = 600$ metres,‡ $Q_G = 460$, $Q_s = 330$.

Substituting these values in equation (2), it will be found that $\kappa = 0·0023$.

(b) *Moderate Winds.*

$\mu/\rho = 50\times10^3$,‡ $\alpha = 21\frac{1}{2}°$,§ $H_1 = 800$ metres,‡ $Q_G = 910$, $Q_s = 590$.

Hence $\kappa = 0·0032$.

(c) *Strong Winds.*

$\mu/\rho = 62\times10^3$,‡ $\alpha = 20°$,§ $H_1 = 900$ metres,‡ $Q_G = 1560$, $Q_s = 950$.

Hence $\kappa = 0·0022$.

Conclusions.

(1) The coefficient κ does not appear to increase or decrease with wind velocity, a three-fold increase in velocity corresponding with a nine-fold increase in skin friction. It appears therefore that the skin friction on the earth's surface is proportional to the square of the wind velocity.

(2) The actual values of the skin friction coefficient κ are of the same order of magnitude, but probably somewhat smaller than those found in the laboratory, being 0·002 to 0·003 as against the value 0·004 found for skin friction in a pipe. It appears therefore that approximately the same law of skin friction applies to small flat plates and pipes and to the friction of the atmosphere on the ground. The ratio of the values of lV/ν for the two cases is of the order 10^5. It certainly seems therefore that the "scale effect" on skin friction over this immense range of values of lV/ν is not large.

* Dobson, "Pilot Balloon Ascents at the Central Flying School, Upavon, during the year, 1913," 'Quart. Journ. Roy. Met. Soc.,' vol. 40, p. 124 (1914).

† At a height of 30 metres above the ground.

‡ See "Eddy Motion," p. 21.

§ See "Eddy Motion," Table II, p. 17.

(3) It is evident that if we had assumed originally that the skin friction of the air blowing over the earth's surface obeys the same law that it does in the case of the small scale models which are tested in the laboratory, the analysis given above might be used, in conjunction with the results given in the author's previous paper, to furnish an explanation of why it is that the wind near the ground is, on the average, about 0·7 of the gradient wind in the case of light winds, or about 0·6 of the gradient wind for strong winds.

Memorandum on the Kew Heliograph.

By Dr. C. Chree, F.R.S., Superintendent of the Kew Observatory.

(Presented to the Gassiot Committee, May 26, 1915.)

The Report of the Kew Committee, presented to the Council of the British Association June 27, 1855, describes the construction of the apparatus as follows :—

"The apparatus suggested by Sir John Herschel for photographing the spots on the sun's disc is progressing under the superintendence of Mr. Warren De la Rue. The Solar Photographic Telescope is promised by the maker complete in three months. . . . The diameter of the object glass is 3·4 inches, and its focal length 50 inches; the image of the sun will be 0·465 inch, but the proposed eyepiece will, with a magnifying power of 25·8 . . ., increase the image to 12 inches. . . . The object glass is under-corrected in such a manner as to produce the best practical coincidence of the chemical and visual foci. . . . It was originally intended to place the telescope in an observatory 12 feet in diameter, provided with a revolving roof. . . . It has, however, been found possible to somewhat alter the construction of the tube, so as to reduce its length sufficiently to allow of the telescope being placed under the dome of the Kew Observatory, which is only 10 feet in diameter."

The Report of the Kew Committee for 1856–57 contains the following statement :—

"On May 20, 1854, Benj. Oliveira, Esq., F.R.S., placed the sum of £50 at the disposal of the Council of the Royal Society, to be appropriated during that year in any manner the Council might consider most in harmony with the interests of science. Mr. Oliveira further stated that he might probably in future years offer a similar sum if the mode of its disposal appeared to