IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received December 21, 2019, accepted January 8, 2020, date of publication January 15, 2020, date of current version January 31, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2966772

Sliding Mode Control: An Incremental Perspective

GUO ZHANG[1,2], PING HE[1,3], HENG LI[3], YU TANG[2], ZUXIN LI[4], XING-ZHONG XIONG[2], WEI WEI[5], (Senior Member, IEEE), AND YANGMIN LI[6], (Senior Member, IEEE)

[1]School of Intelligent Systems Science and Engineering (Institute of Physical Internet), Jinan University, Zhuhai 519070, China
[2]Artificial Intelligence Key Laboratory of Sichuan Province, Sichuan University of Science and Engineering, Zigong 643000, China
[3]Department of Building and Real Estate, The Hong Kong Polytechnic University, Hong Kong
[4]School of Engineering, Huzhou University, Huzhou 313000, China
[5]School of Computer Science and Engineering, Xi'an University of Technology, Xi'an 710048, China
[6]Department of Industrial and Systems Engineering, The Hong Kong Polytechnic University, Hong Kong

Corresponding author: Ping He (pinghecn@qq.com)

This work was supported in part by the National Natural Science Foundation of China under Grant 11705122, Grant 61902268, and Grant 51575544, in part by the Fundamental Research Funds for the Central Universities, Jinan University, under Grant 12819026, in part by the Hong Kong Research Grants Council under Grant BRE/PolyU 152099/18E and Grant PolyU 15204719/18E, in part by the Natural Science Foundation of The Hong Kong Polytechnic University uner Grant G-YW3X, in part by the Key Research and Development Program of Shaanxi Province under Grant 2018ZDXM-GY-036, in part by the Shaanxi Key Laboratory of Intelligent Processing for Big Energy Data under Grant IPBED7, in part by the Sichuan Science and Technology Program under Grant 2019YFSY0045, Grant 2018GZDZX0046, and Grant 2018JY0197, in part by the Open Foundation of Artificial Intelligence Key Laboratory of Sichuan Province under Grant 2018RZJ01, in part by the Nature Science Foundation of Sichuan University of Science and Engineering under Grant 2017RCL52, and in part by the Zigong Science and Technology Program of China under Grant 2019YYJC03 and Grant 2019YYJC15.

ABSTRACT This study focuses on stabilization problem of a class of nonlinear systems. Generally, Lyapunov stability-based sliding mode technique is widely used to design controllers for nonlinear systems with uncertainties. In this paper, however, based on contraction property and sliding surfaces, the sliding mode control is suggested to provide incremental stability for nonlinear systems with uncertainties. The effectiveness of the method is illustrated by numerical simulations.

INDEX TERMS Sliding mode control, contraction theory, robust control, incremental stability.

I. INTRODUCTION

Stability theory plays an important role in system theory and engineering, including the well-known *equilibrium point stability* (EPS) and *input-output stability* (IOS), as well as the *incremental stability* (INS) that has a complicated development process. A simple explanation of the EPS is that all solutions starting near the equilibrium point close to this point [1]. IOS is a system stability property which can be examined from the external characteristics of the system [2]. INS is a stronger property comparing arbitrary trajectories with themselves, rather than with an equilibrium point or with a particular energy function. There are some evidences that EPS, IOS and INS are related [3]–[5]. Compared only on the concepts of EPS and INS stability, researchers more inclined to the stability of some particular solutions nearing the equilibrium points in the early years. However, in some cases it is more important to focus on the stability properties of all solutions independent of equilibrium points. Especially, the stability conditions of all solutions are more general when there are multiple equilibrium points in the systems, or when dealing with synchronization problems of complex networks. As another example, the construction of the energy function for some systems with physical properties might be easy, but it's hard to find a pattern to follow in the cases of the exceptions. The relationships (such as a distance) between trajectories exist objectively, in other words, incremental stability provides an analysis method for those unexpected situations.

To recall the history of incremental stability, an important concept is the Demidovich condition [6], [7], which provides sufficient conditions for the convergence of incrementally stable systems. A simple explanation of the Demidovich condition is that the system is convergent if all system trajectories converge to one trajectory on the whole time axis. In general, however, the reason for the explosive growth in the study of incremental stability is that Lohmiller and Slotine introduced the Riemannian metric into the control systems and defined the contraction properties (a generalization of the Demidovich condition) of incremental stability [8].

The associate editor coordinating the review of this manuscript and approving it for publication was Zheng Chen.

The contraction properties can be simply interpreted in Riemannian geometry as: requires the decrease of a distance, defined through a Riemannian metric, along trajectories. The number of applications of incremental stability has increased in the past decade. Examples in stability analysis of nonlinear systems [9]–[11], complex networks for time-delayed communications [12], concurrent synchronization [13], [14], switched networks [15], coupled identical nonlinear oscillators [16], construction of symbolic models [17], observer design [18]–[20], nonlinear control design [21]–[26].

From previous literature of authors' knowledge, where in terms of the control scheme, it involves feedback control [17], [23], [26], matrix inequality condition [24], [25], and backstepping design [14], etc. Motivated by above discussions, a control scheme combined sliding mode technology and incremental stability has not yet been investigated and still remains a big challenging issue. Another motivation is to expand the application of incremental analysis methods on uncertain systems. A common technique for processing uncertainties by contraction is the semi-contraction technique [27], which does not require accurate estimates for uncertain parameters, but the structure of the systems must be known. A newly developed technology called robust control contraction metrics [5] can guarantee robust stability of arbitrary trajectories via small gain arguments, but its calculations are complex and even require software assistance.

It is well known that the sliding mode control has a good robust performance for uncertain systems. In this technical note, by investigating the design method of incremental sliding control, the main contributions can be stated as following two aspects.

- Developed a sliding design method for second-order systems and provided controllers enforcing an incremental asymptotic stability and not an equilibrium point stability;
- Expand the application of incremental analysis methods on uncertain systems. The advantages are uncomplicated calculations and do not require a known structure;

However, the present technology is relative to the second-order nonlinear systems. In other words, in the case of higher-order [28]–[30], new technologies need to be developed. And, research on contraction analysis in finite-time control [31] is rare, one of the challenges in the future is the finite-time control, especially with an incremental sliding technology. Another challenge is the case of time-delayed systems [32], [33], whether the incremental sliding technology can be introduced.

The organisation of this paper is structured as follows. The concept of incremental stability and contraction are discussed in Section II. The incremental sliding mode problem is described in Section III. In Section IV, firstly, a sliding surface is designed for a second-order uncertain system. Secondly, a sliding control method with incremental stability is proposed. Thirdly, the case for interference of sliding surface is discussed. Then, the problem of chattering on switching delay is discussed. The results are described to verify the effectiveness of the proposed distributed control algorithm in Section V. Finally, some characterizations are pointed out in Section VI.

II. INCREMENTAL STABILITY AND CONTRACTION

Considering a manifold $\mathcal{M}$ and a system

$$\dot{x} = f(x, t), \tag{1}$$

where f is a nonlinear vector field which maps each $(t, x) \in \mathbb{R} \times \mathcal{M}$ to a tangent vector $f(t, x) \in T_x\mathcal{M}$.

Let $\mathcal{C} \subset \mathcal{M}$ and denote by $\psi_{t_0}(\cdot, x_0)$ the solution to (1) from the initial condition $x_0 \in \mathcal{M}$ at time t_0. According to [34], we can get following definition.

Definition 1: System (1) is *incremental asymptotically stability* in a positively invariant set $\mathcal{C} \subset \mathcal{M}$, if there exists a function $\alpha \in \mathcal{KL}$ such that for any $x_1, x_2 \in C$ and $t \geq t_0$,

$$\|\psi_{t_0}(t, x_1) - \psi_{t_0}(t, x_2)\| \leq \alpha(\|x_1 - x_2\|).$$

In the case $\mathcal{C} = \mathcal{M}$ we say that (1) is globally incrementally stable, or just incrementally stable.

Let (1) be a differential form

$$\delta\dot{x}(t) = \frac{\partial f(x, t)}{\partial x(t)}\delta x(t),$$

where $\delta x(t)$ denotes an infinitesimal displacement at a fixed time. According to [8], there exists following definition (*a contraction property*) and lemma.

Definition 2: The metric G is a contraction metric and β is a contraction rate, if there are a Riemann metric $\delta x^T G(x)\delta x$ and a strictly positive constant $\beta \in \mathbb{R}^+$ in (1), such that

$$\frac{d}{dt}(\delta x^T G\delta x) = \delta x^T\left(\frac{\partial f}{\partial x}^T G + G\frac{\partial f}{\partial x} + \dot{G}\right)\delta x < -\delta x^T \beta G\delta x,$$

when G is independent of state, it is called a flat contraction metric, which is similar to Demidovich condition.

Lemma 1: Given the system equations (1), any trajectory, which starts in a ball of constant radius with respect to the metric $G(x, t)$, centered at a given trajectory, remains in that ball. The distance of any trajectory within the ball is gradually shortened until it is unified into the given trajectory.

Remark 1 [8]: If $\lambda_{\max}(\beta G)$ is the largest eigenvalue of the symmetric part of the Jacobian $\frac{\partial f^T}{\partial x}G + G\frac{\partial f}{\partial x} + \dot{G}$, then

$$\|\delta x\| \leq \|\delta x_0\| e^{\int_0^t \lambda_{\max}(x)dt}.$$

Remark 2: The purpose of given the definition of INS is to distinguish it from EPS. We mainly use the concept of contraction properties in this paper. A detailed explanation of a contraction system is also an incremental stability system, can refer to [35].

III. PROBLEM FORMULATION

Considering a nonlinear system of the following form

$$\begin{cases} \dot{x} = f(x, t) + g(x, t)u, & \text{(2a)} \\ s = s(x, t), & \text{(2b)} \end{cases}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}$ is the control input with respect to state x, and $f(x, t)$ and $g(x, t)$ are unknown smooth nonlinear functions. The sliding variables s and $\dot{s} = \frac{ds}{dt}$ are assumed to be known.

Let $\dot{s} = \frac{ds}{dt}$ be a differential form

$$\delta\dot{s} = \frac{\partial\dot{s}}{\partial x}\delta x + \frac{\partial\dot{s}}{\partial u}\delta u. \tag{3}$$

Considering a contraction metric G and further to calculate (3) with Definition 2, it yields

$$\begin{aligned} \frac{d}{dt}(\delta s^T G\delta s) &= \delta\dot{s}^T G\delta s + \delta s^T \dot{G}\delta s + \delta s^T G\delta\dot{s} \\ &= \left(\frac{\partial\dot{s}}{\partial x}\delta x + \frac{\partial\dot{s}}{\partial u}\delta u\right)^T G\delta s + \delta s^T\dot{G}\delta s \\ &\quad + \delta s^T G\left(\frac{\partial\dot{s}}{\partial x}\delta x + \frac{\partial\dot{s}}{\partial u}\delta u\right) \\ &= \left(\frac{\partial\dot{s}}{\partial x}\delta x + \frac{\partial\dot{s}}{\partial u}\delta u\right)^T G\frac{\partial s}{\partial x}\delta x + \delta s^T\dot{G}\delta s \\ &\quad + \left(\frac{\partial s}{\partial x}\delta x\right)^T G\left(\frac{\partial\dot{s}}{\partial x}\delta x + \frac{\partial\dot{s}}{\partial u}\delta u\right). \end{aligned} \tag{4}$$

If there exists a control signal $u\big(x, \mathrm{sgn}(s), t\big)$ to cause (4) to shrink, we can get a conclusion $\delta x \to 0 \Rightarrow \delta s \to 0$, that is, system (2a) is incremental stable.

Remark 3: There are several well-known conventional Lyapunov methods that can be used to analyze reachability of sliding surface. However, it is not from an incremental perspective. The conjecture in equation (4) illustrates the possibility of incremental stability analysis for sliding mode surfaces.

IV. CONTRACTION ANALYSIS OF SLIDING SURFACE

This section will follow four sequences to illustrate the next work.

A. SLIDING SURFACE DESIGN

Considering a class of second-order systems, they can be described as

$$\begin{aligned} \dot{x}_1 &= h_1(x, t), \\ \dot{x}_2 &= h_2(x, t) + g(x, t)u + d(x, t), \end{aligned} \tag{5}$$

where $x \in \mathbb{R}^2$ is the state, $u \in \mathbb{R}$ is the control input. $h_1(x, t)$ is known smooth nonlinear function, $h_2(x, t)$ and $g(x, t)$ are unknown smooth nonlinear functions, $d(x, t)$ is a bounded interference.

Defining a sliding surface related to the state x, it yields

$$s = ax_1 + h_1(x, t), \tag{6}$$

where a is a positive constant. Considering a sliding surface that satisfies the following equation

$$\dot{s} = a\dot{x}_1 + \dot{h}_1 = ah_1 + \dot{h}_1 = y(x_1, x_2, u, t). \tag{7}$$

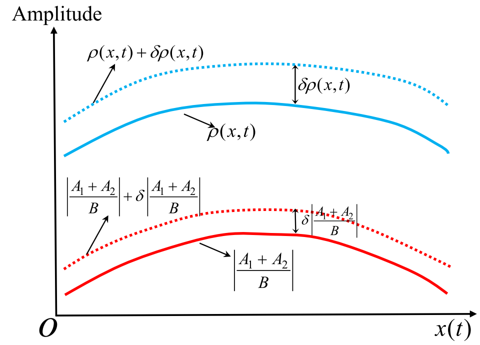


FIGURE 1. Graphical representation of Assumption 1.

Taking the differential form of system (7), it yields

$$\delta\dot{s} = A_1\delta x_1 + A_2\delta x_2 + B\delta u,$$

where $A_1 = \frac{\partial y}{\partial x_1}$, $A_2 = \frac{\partial y}{\partial x_2}$, $B = \frac{\partial y}{\partial u}$.

Remark 4: The reason of designed the sliding surface (6) for system (5) can be explained by a formula

$$\delta s = a\delta x_1 + \underbrace{\frac{\partial h_1}{\partial x_1}\delta x_1 + \frac{\partial h_1}{\partial x_2}\delta x_2}_{\delta\dot{x}_1}, \tag{8}$$

it can be see clearly that

$$\delta s = 0 \Rightarrow \delta\dot{x}_1 = -a\delta x_1.$$

In [8], equation (8) is a contraction case of linear time-varying system.

B. CONTROLLER DESIGN

Assumption 1: There exists a function $\rho(x, t)$ that satisfies the following inequalities

$$\left|\frac{A_1 + A_2}{B}\right| \le \rho(x, t), \tag{9}$$

$$\delta\left|\frac{A_1 + A_2}{B}\right| \le \delta\rho. \tag{10}$$

Remark 5: As shown in Figure 1, there are some details of sliding mode dynamics (7), it is that function $\left|\frac{A_1+A_2}{B}\right|$ with respect to the tangent line satisfies a bounded condition $\rho(x)$. And $\delta\left|\frac{A_1+A_2}{B}\right| \le \delta\rho$ can be explained as

$$\begin{aligned} \delta\left|\frac{A_1 + A_2}{B}\right| - \delta\rho &= \frac{\partial\left|\frac{A_1+A_2}{B}\right|}{\partial x}\delta x - \frac{\partial\rho}{\partial x}\delta x \\ &= \partial\left|\frac{A_1 + A_2}{B}\right| - \partial\rho \\ &\le 0. \end{aligned}$$

Theorem 1: Under Assumption 1, if there exists a function $\beta(x, t)$ greater than $\rho(x, t)$ to that satisfies the controller

$$u = \begin{cases} -\beta(x, t)\mathrm{sgn}(s), & B > 0, \\ \beta(x, t)\mathrm{sgn}(s), & B < 0, \end{cases}$$

$$\operatorname{sgn}(s) = \begin{cases} 1, & s > 0, \\ 0, & s = 0, \\ -1, & s < 0. \end{cases} \tag{11}$$

Then, the sliding mode dynamics (7) is shrink to zero, the second-order system (5) is a contraction system. ■

Proof: Consider the adjacent trajectories in the sliding surface $\dot{s} = a\dot{x_1} + \dot{x_2}$. According to Definition 2, the metric between the adjacent trajectories can be defined as $\delta s^T G \delta s$. Choose a flat contraction metric $G = I$, the change rate of $\delta s^T \delta s$ between the adjacent trajectories can be defined as

$$\begin{aligned} \frac{d}{dt}(\delta s^T \delta s) &= 2\delta s^T \delta \dot{s} \\ &= 2\delta s^T \left(\frac{A_1 \delta x_1 + A_2 \delta x_2}{B} \right) B + 2\delta s^T B \delta u. \end{aligned}$$

Since the increment δx is an infinitesimal displacement at a fixed time, the following inequality is obviously

$$\left| \frac{A_1 \delta x_1 + A_2 \delta x_2}{B} \right| \leq \left| \frac{A_1 + A_2}{B} \right|.$$

According to (9) in Assumption 1, we can get an inequality

$$\begin{aligned} \frac{d}{dt}(\delta s^T \delta s) &= 2\delta s^T (A_1 \delta x_1 + A_2 \delta x_2) + 2\delta s^T B \delta u \\ &\leq \begin{cases} 2|\delta s^T| \delta \rho B + 2\delta s^T B \delta u, & B > 0, \\ -2|\delta s^T| \delta \rho B + 2\delta s^T B \delta u, & B < 0. \end{cases} \end{aligned} \tag{12}$$

To take $\beta(x, t) = \rho(x, t) + \beta_0$, $\beta_0 > 0$, it yields

$$\delta u = \begin{cases} -\delta[(\rho + \beta_0)\operatorname{sgn}(s)], & B > 0, \\ \delta[(\rho + \beta_0)\operatorname{sgn}(s)], & B < 0. \end{cases} \tag{13}$$

To merge (12) and (13), it yields

$$\begin{aligned} \frac{d}{dt}(\delta s^T \delta s) &\leq \begin{cases} 2|\delta s^T| \delta \rho B - 2|\delta s^T| B \delta(\rho + \beta_0), \\ -2|\delta s^T| \delta \rho B + 2|\delta s^T| B \delta(\rho + \beta_0), \end{cases} \\ &= \begin{cases} -2|\delta s| B \delta \beta_0, & B > 0, \\ 2|\delta s| B \delta \beta_0, & B < 0. \end{cases} \end{aligned} \tag{14}$$

According to Lemma 1, since $\frac{d}{dt}(\delta s^T \delta s) \leq 0$, there exists a $\lambda_{\max}(s)$ that is uniformly strictly negative, it yields

$$\|\delta s\| \leq \|\delta s_0\| e^{\int_0^t \lambda_{\max}(s) dt}.$$

It is not difficult to see that the incremental sliding surface $\delta s = a\delta x_1 + \underbrace{\frac{\partial h_1}{\partial x_1}\delta x_1 + \frac{\partial h_1}{\partial x_2}\delta x_2}_{\delta \dot{x}_1} = 0$ at $t \to \infty$, that is, all trajectories of system (5) shrink to zero at $t \to \infty$. □

Remark 6: As described in the problem description in Section III, we performed a contraction analysis on the sliding surface. Note that we used the flat metric $G = I$ (independent of state), so G is implied during the derivation.

C. SLIDING SURFACE WITH UNCERTAINTIES

The Theorem 1 stabilized h_2 and g with uncertainties, the next is to stabilize h_1 with uncertainties. Considering the case that h_1 is affected by the bounded interferences $\omega(x, t)$ and the system (5) is changed to the following form

$$\begin{aligned} \dot{x}_1 &= h_1(x, t) + \omega(x, t), \\ \dot{x}_2 &= h_2(x, t) + g(x, t)u + d(x, t). \end{aligned} \tag{15}$$

As mentioned above, Theorem 1 used the standard sliding surface $s = ax_1 + h_1(x, t)$. However, the sliding surface fluctuated as the addition of $w(x, t)$, and it changed to

$$\hat{s} = ax_1 + h_1(x, t) + \omega(x, t).$$

Now, $\hat{s}$ is a disturbed sliding surface and its rate of change is

$$\dot{\hat{s}} = a\delta x_1 + \dot{x}_1 + \dot{w}(x, t). \tag{16}$$

Taking the differential form of system (16), it yields

$$\delta \dot{\hat{s}} = A_1 \delta x_1 + A_2 \delta x_2 + B \delta u + \frac{\partial \dot{w}}{\partial x_1} \delta x_1 + \frac{\partial \dot{w}}{\partial x_2} \delta x_2.$$

Theorem 2: If there exist a function $\hat{\beta}(x)$ satisfies the inequality

$$\left| \frac{A_1 + A_2}{B} \right| + \left| \frac{\frac{\partial \dot{w}}{\partial x_1} + \frac{\partial \dot{w}}{\partial x_2}}{B} \right| \leq \rho(x, t) + \rho_w(x, t) \leq \hat{\beta}(x, t), \tag{17}$$

where $\left| \frac{\frac{\partial \dot{w}}{\partial x_1} + \frac{\partial \dot{w}}{\partial x_2}}{B} \right| \leq \rho_w(x, t)$, then the sliding mode dynamics (16) is shrink to zero by a controller

$$u = \hat{u} = \begin{cases} -\hat{\beta}(x, t)\operatorname{sgn}(s), & B > 0, \\ \hat{\beta}(x, t)\operatorname{sgn}(s), & B < 0, \end{cases} \tag{18}$$

and the second-order system (15) is a contraction system. ■

Proof: Considering the adjacent trajectories in the sliding surface $\dot{\hat{s}} = a\dot{x}_1 + \dot{x}_2 + w(x, t)$. According to Definition 2, we also choose a flat contraction metric $G = I$, the change rate of $\delta \hat{s}^T \delta \hat{s}$ between the adjacent trajectories can be defined as

$$\begin{aligned} &\frac{d}{dt}(\delta \hat{s}^T \delta \hat{s}) \\ &= 2\delta(s + w)^T \delta(\dot{s} + \dot{w}) \\ &= 2\delta(s + w)^T \left(\frac{(A_1 + \frac{\partial \dot{w}}{\partial x_1})\delta x_1 + (A_2 + \frac{\partial \dot{w}}{\partial x_2})\delta x_2}{B} \right) B \\ &\quad + 2\delta(s + w)^T B \delta \hat{u}. \end{aligned} \tag{19}$$

Taking $\hat{\beta}(x, t) = \rho(x, t) + \rho_w(x, t) + \beta_0$ and to derive (19) to get an inequality

$$\begin{aligned} &2\delta(s + w)^T \delta(\dot{s} + \dot{w}) \\ &\leq \begin{cases} 2|\delta \hat{s}^T| \delta(\rho + \rho_w) B - 2|\delta \hat{s}^T| B \delta(\rho + \rho_w + \beta_0), \\ -2|\delta \hat{s}^T| \delta(\rho + \rho_w) B + 2|\delta \hat{s}^T| B \delta(\rho + \rho_w + \beta_0), \end{cases} \\ &= \begin{cases} -2|\delta \hat{s}| B \delta \beta_0, & B > 0, \\ 2|\delta \hat{s}| B \delta \beta_0, & B < 0. \end{cases} \end{aligned}$$

According to Lemma 1, since $\frac{d}{dt}(\delta\hat{s}^T\delta\hat{s}) \le 0$, there exists a $\lambda_{\max}(s)$ that is uniformly strictly negative, it yields

$$\|\delta\hat{s}\| \le \|\delta s_0\| e^{\int_0^t \lambda_{\max}(\hat{s})dt}. \tag{20}$$

So the sliding surface $\delta\hat{s} = a\delta x_1 + \frac{\partial h_1}{\partial x_1}\delta x_1 + \frac{\partial h_1}{\partial x_2}\delta x_2 + \frac{\partial w}{\partial x_1}\delta x_1 + \frac{\partial w}{\partial x_2}\delta x_2 = 0$, that is, all trajectories of system (15) shrink to zero at $t \to \infty$. □

Remark 7: The boundary of standard sliding surface s with an interference w can be expressed as

$$\|\delta\hat{s}_1\| = \|\delta s\| + \|\delta w\|.$$

A constraint $\left(\frac{\frac{\partial \dot{w}}{\partial x_1}\delta x_1 + \frac{\partial \dot{w}}{\partial x_2}\delta x_2}{B}\right)B \le \rho_w(x,t)$ in (17) is like to provide a bounded space to negatively determine the maximum eigenvalue $\lambda_{max}\left(\frac{\partial w}{\partial x_1}, \frac{\partial w}{\partial x_2}\right)$, such that

$$\|\delta\hat{s}\| \le \|\delta\hat{s}_0\| e^{\int_0^t \lambda_{\max}(\hat{s})dt},$$

it eventually forms incremental stability $\|\delta\hat{s}\| \to \|\delta s\| \to 0$.

D. REDUCTION OF CHATTERING

Although the above theorems showed that a sliding mode controller with incremental stability can be designed, but the symbol switching controller has a switching delay. It is generally known that zero-delay switching is difficult to implement in practical systems, and the delay causes the chattering of input. The disadvantage of chattering is obvious, it may reduce control accuracy of system, increase energy consumption, and other unfavorable factors. The conventional saturation function *sat* is applicable in this paper, it reserves enough reaction time for the control law u to reduction chattering. The chattering reduction control law of the system (5) can be rewritten as

$$u = \begin{cases} -\beta(x,t)\text{sat}(s/\kappa), & B > 0, \\ \beta(x,t)\text{sat}(s/\kappa), & B < 0, \end{cases}$$
$$\text{sat}(s/\kappa) = \begin{cases} s/\kappa, & |s/\kappa| \le 1, \\ \text{sgn}(s/\kappa), & |s/\kappa| \ge 1. \end{cases} \tag{21}$$

As shown in Figure 2, sat function is approximately sgn function in case of $\kappa \to 0$. To analyze the performance of the incremental sliding mode controller, we perform the following two-step analysis. The first is outside the boundary layer, that is, $|s| \le \kappa$. According to (14), we can get an inequality

$$\frac{d}{dt}(\delta s^T\delta s) \le \begin{cases} -2|\delta s|B\delta\beta_0, & B > 0, \\ 2|\delta s|B\delta\beta_0, & B < 0. \end{cases}$$

Therefore, as long as $|s(0)| \ge \kappa$, $\delta s(t)$ are strictly decreasing, until shrinking to set $\{|s| \le \kappa\}$ within a limited time, and then remain in it. The second is inside the boundary layer, that is,

$$\dot{x}_1 = -ax_1 + s, \; |s| \le \kappa. \tag{22}$$

utilizing a flat metric $\delta x_1^T G\delta x_1$ to check the contraction property in (22) and derive the rate of change, it yields.

$$\frac{d}{dt}(\delta x_1^T G\delta x_1) = 2\delta x_1^T(-aG)\delta x_1 + 2\delta x_1^T G\delta s. \tag{23}$$

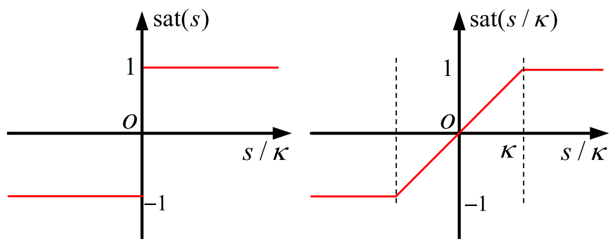


FIGURE 2. Sgn(s) and Sat(s/κ).

Since inside the boundary of sliding surface has $|s| \le \kappa$, we can get

$$-\kappa \le s + \delta s \le \kappa, \; |s + \delta s| \le \kappa.$$

Taking the boundary value of s, then δs should satisfy

$$-2\kappa \le \delta s \le 2\kappa, \; |\delta s| \le 2\kappa. \tag{24}$$

This also means (23) can be change to

$$\frac{d}{dt}(\delta x_1^T G\delta x_1) \le 2\delta x_1^T(-aG)\delta x_1 + 2|\delta x_1^T|G2\kappa. \tag{25}$$

After reaching the sliding surface, h_1 is close to a small value to ensure the stability of x_1, here, let $\triangle \approx \frac{\partial h_1}{\partial x_1}\delta x_1 + \frac{\partial h_1}{\partial x_2}\delta x_2 \approx 0$ in (8), we can get

$$-\triangle - 2\kappa \le a\delta x_1 \le 2\kappa + \triangle, \quad |a\delta x_1| \le 2\kappa + \triangle.$$

This also means (25) can be change to

$$\frac{d}{dt}(\delta x_1^T G\delta x_1) \le 2\delta x_1^T(-aG)\delta x_1 + \frac{4(2\kappa^T + \triangle^T)G\kappa}{a}. \tag{26}$$

Remark 8: Taking $\kappa = 0$, it is easy to see that x_1 will shrink to the boundary $\kappa = 0$, which is robust for $\dot{x}_2$. But $\kappa = 0$ is equivalent to a sgn function, the switching delay cause chattering to be inevitable.

Remark 9: Taking $\kappa > 0$ and a sufficiently large a, then (26) can be negative definite inside the boundary. Theoretically a larger gain a will have better convergence, it is shown as Figure 3 (noted that this figure comes from Example 2). Since a sat function inside the boundary no longer frequently switches, the contraction inside the boundary reduces the chattering.

Remark 10: Any trajectory within the boundary of the sliding surface can be think that to be restrict by κ, so the increment of any trajectories is limited from (24).

V. NUMERICAL SIMULATION

In this Section, two examples are performed to illustrate the advantages. Firstly, compared with sliding mode control based on Lyapunov stability theory, the advantages of sliding mode based on contraction are explained. Then, compared with the control contraction metrics technology, which illustrates the advantages of the sliding mode based contraction technology.

Example 1: (The variable-single pendulum [36]) A variable-single pendulum can be shown in Figure 4, R is the

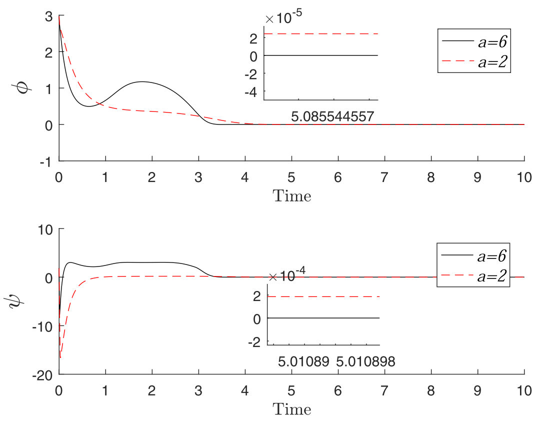


FIGURE 3. The effect under the changing of a.

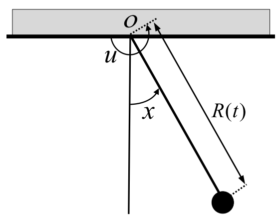


FIGURE 4. Sketch of pendulum in Example 1.

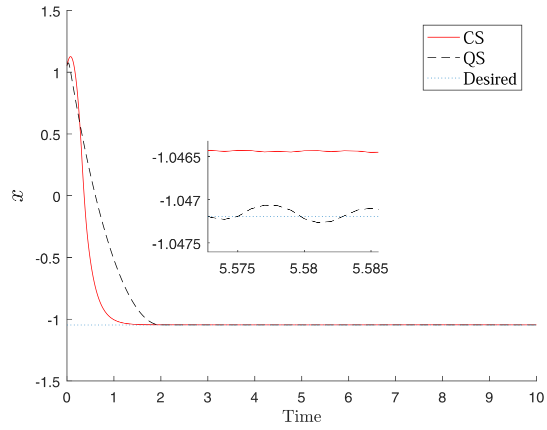


FIGURE 5. Pendulum: the state for θ.

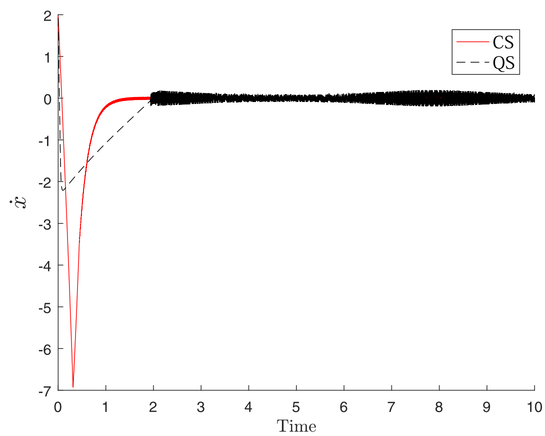


FIGURE 6. Pendulum: the state for $\dot{\theta}$.

distance from O to the mass m. x is the oscillation angle. The pendulum is driven by an engine installed on the top side, which is called control torque u. The mathematical model of the pendulum can be described by

$$\ddot{x} = -2\frac{\dot{R}}{R}\dot{x} - g\frac{1}{R}\sin(x) + \frac{1}{mR^2}u, \tag{27}$$

where $m = 1Kg$, $g = 9.8m/s^2$ is the gravitational constant, and $R = 1 - 0.2\sin(t)$. The task here is to design the control law u such that the oscillation angle x will track a given signal $x_c = \frac{\pi}{3}$.

Let $s = a(x - x_c) + \dot{x}$, to take the derivative of s yields

$$y = \dot{s} = a(x - x_c) - g\frac{1}{R}\sin(x) - 2\frac{\dot{R}}{R}\dot{x} + \frac{1}{mR^2}u, \tag{28}$$

to take the differential form of system (28) yields

$$\delta\dot{s} = A_1\delta x_1 + A_2\delta x_2 + B\delta u, \tag{29}$$

with

$$A_1 = \frac{\partial y}{\partial x} = -g\frac{1}{R}\cos(x) + a,$$
$$A_2 = \frac{\partial y}{\partial \dot{x}} = -2\frac{\dot{R}}{R},$$
$$B = \frac{\partial y}{\partial u} = \frac{1}{mR^2}.$$

Choosing $u = -\left(\left|\frac{(A_1+A_2)}{B}\right| + 20\right)\mathrm{sgn}(s)$ and considering a bounded interference $d(x, t) = 2\sin(x)$ into (27) yields

$$\ddot{x} = -2\frac{\dot{R}}{R}\dot{x} - g\frac{1}{R}\sin(x) + \frac{1}{mR^2}u + 2\sin(x),$$

and simulating with $a = 5$.

In order to illustrate the advantages of the proposed control method, we compared with the controller proposed in [37]. The so-called quasi-continuous 2-sliding mode algorithm in [37] is

$$u = -\big(4 + 5\bar{a}(x)\big)\frac{\lfloor\dot{\bar{s}}\rceil^4 + \beta_1^4\lfloor\bar{s}\rceil^2}{|\dot{\bar{s}}|^4 + \beta_1^4|\bar{s}|^2}, \tag{30}$$

where $\bar{s} = x - x_c$, $\bar{a}(x) = \frac{1}{2}|\dot{x}| + \frac{5}{4}g$, $\beta_1 = 1.5$ and $\lfloor x\rceil^\nu = |x|^\nu\mathrm{sgn}(x)$, $\forall \nu > 0$.

Numerical simulation for the variable-single pendulum as shown in Figure 5 to Figure 8, where CS denotes proposed method in this paper, QS denotes proposed method (30) in [37]. From Figure 5, it is clear that the reference signal can be tracked with a good dynamical performance by CS and QS. However, the convergence time of CS is $0.5s$ faster than QS, the tracking error only has 0.005 and it can be irrelevant almost. From Figure 6, it is clear that QS is about 0.05 higher than CS on the fluctuation value of state $\dot{x}$. From Figure 7, it is clear that the controllers have the property of sliding mode control, namely the chattering. However, the input overshoot of CS is 30 lower than QS. From Figure 8, it is clear that finite-time reachability of sliding mode dynamics, and the reach time of CS is $1.5s$ faster than QS.

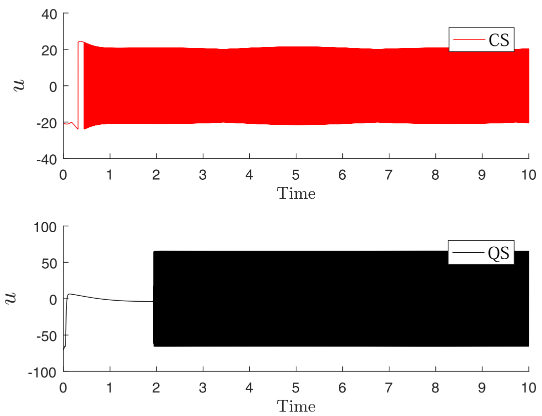


FIGURE 7. Pendulum: the input u.

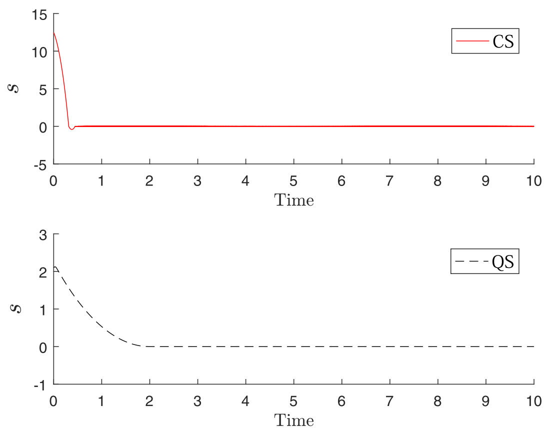


FIGURE 8. Pendulum: the sliding surface s.

Example 2 (The Moore-Greitzer Model [38]): A model of Moore-Greitzer was a simplified model of surge-stall dynamics based on a Galerkin projection of the partial differential equations on to a Fourier basis. The following reduced model of the surge dynamics was described as

$$\begin{aligned}\dot{\phi} &= -\varphi - \frac{3}{2}\phi^2 - \frac{1}{2}\phi^3,\\ \dot{\varphi} &= \phi + u,\end{aligned} \tag{31}$$

where u is the input as a sensor on φ. φ and ϕ are a measure of the mass flow and pressure rise in the compressor, under a change of coordinates. The source of difficulty is the nonlinearity $-\frac{3}{2}\phi^2 - \frac{1}{2}\phi^3$ which does not satisfy any global Lipschitz bound, and affects the dynamics of the variable ϕ, which is not directly controlled or measured.

To take $h_1 = -\varphi - \frac{3}{2}\phi^2 - \frac{1}{2}\phi^3$, $h_2 = \phi$, $g = 1$ and let $s = a\phi + h_1$, referring to the design steps in Example 1, it yields

$$\begin{aligned}A_1 &= -\frac{3}{2}a - \frac{3}{2}a\phi^2 + 3\varphi + 9\phi - 6\phi^3 + 3\phi\varphi\\ &\quad + \frac{27}{4}\phi^2 + \frac{15}{4}\phi^4 - 1,\\ A_2 &= -a + 3\phi + \frac{3}{2}\phi^2,\\ B &= -1.\end{aligned}$$

TABLE 1. CS compared to QS.

Term	CS	QS
Average Errors $x_{t>5}$	0.005	0.0002
Convergence time x	0.38s	1.8s
Average of absolute input (Pendulum)	20.7934	51.3449

TABLE 2. CS compared to CCM.

Term	C S	CCM
Average Errors $\phi_{t>5}$	0	0.05
Convergence time ϕ	3.2s	4s
Average of absolute input (Greitzer)	4.6494	0.4551

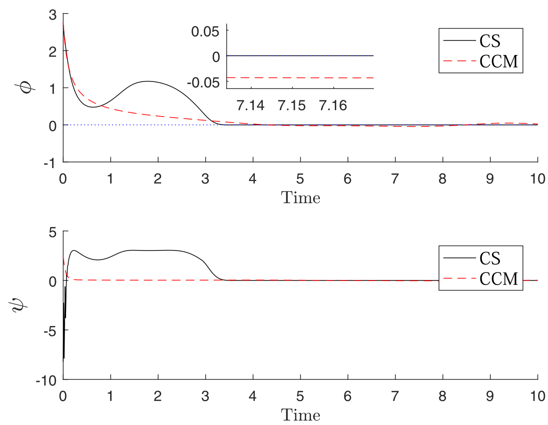


FIGURE 9. Greitzer: the state ϕ, φ.

Considering a bounded interference $w(x, t) = 2\sin(t)\varphi$ into (27), it is clear that $\left|\frac{\partial \dot{w}}{\partial \varphi}/B\right| \leq 2$. Then (31) is changed to

$$\begin{aligned}\dot{\phi} &= -\varphi - \frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 + 2\sin(t)\varphi,\\ \dot{\varphi} &= \phi + u.\end{aligned}$$

and simulating with $u = \left(\left|\frac{A_1 + A_2}{B}\right| + 8\right) sat(s)$.

In order to illustrate the advantages over other contraction methods, we compared with the contraction-based method in [25]. The so-called control contraction metrics (CCM) algorithm in [25] is

$$u = u^\star - \frac{1}{2}\rho(\phi, \varphi)W(\phi, \varphi)^{-1}B'(\phi, \varphi)\begin{bmatrix}\phi - \phi^\star\\ \varphi - \varphi^\star\end{bmatrix}, \tag{32}$$

where, $u^\star$, $\phi^\star$ and $\varphi^\star$ are target trajectories. To use Matlab sum-of-squares tools [39] to set up a two decoupled convex feasibility problems, that is, ρ and W.

Numerical simulation for the Moore-Greitzer model as shown in Figure 9 to Figure 11, where CS denotes proposed method in this paper (21), CCM denotes proposed method (32) in [25]. As shown in Figure 10, the tracking task can be completed by CS and CCM, but the CCM has a large error, about 0.04. As shown in Figure 9, it is clear that CS has a higher input overshoot, but it is stable in 0.2 seconds. As

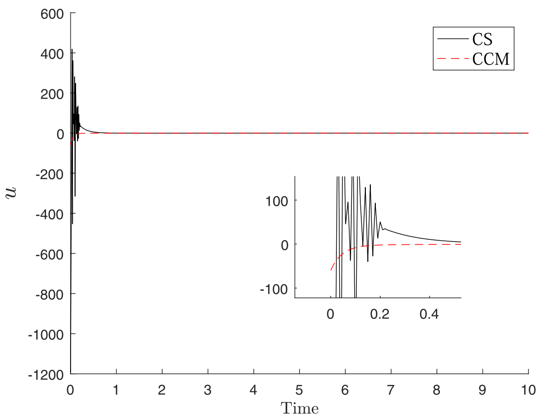


FIGURE 10. Greitzer: the input *u*.

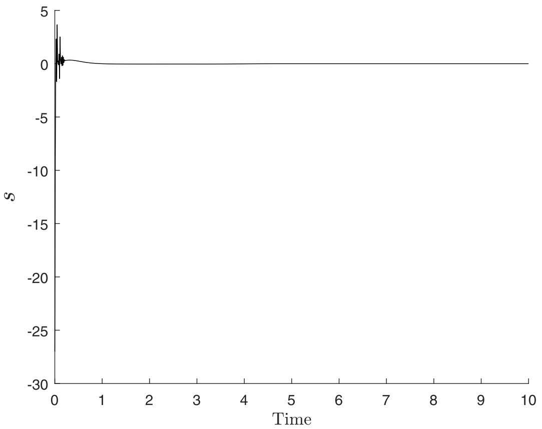


FIGURE 11. Greitzer: the sliding surface *s*.

shown in Figure 11, it is clear that finite-time reachability of sliding mode dynamics, and the reaching time is about 0.6s.

In summary, the proposed method (11) has two advantages, the first is a smaller control signal, the second are a faster reach time and a fast convergence time. The proposed method (21) has a extremely small error, although the initial input is large, it can be quickly stabilized. The performance index of above examples are shown in Table 1 and Table 2. It also illustrates the advantages of the method proposed in this paper.

VI. CONCLUSION

A methodology for incremental sliding mode controller with a simple structure for a class of second-order nonlinear uncertain systems is proposed. The controller is able to steer the initial trajectory of dynamic system with uncertainties to the given trajectory at a short time, the initial trajectory is generated in the contraction domain of the manifold s. There are several perspective generalisations of interest to be addressed in next researches among which dealing with high-order systems, time-delayed systems. New chattering reduction mechanism based on our method is also worth considering.

REFERENCES

[1] N. Rouche, P. Habets, and M. Laloy, *Stability Theory by Liapunov's Direct Method*, vol. 4. Cham, Switzerland: Springer, 1977.

[2] C. A. Desoer and M. Vidyasagar, *Feedback Systems: Input-Output Properties*, vol. 55. Philadelphia, PA, USA: SIAM, 1975.

[3] D. Hill and P. Moylan, "Stability results for nonlinear feedback systems," *Automatica*, vol. 13, no. 4, pp. 377–382, Jul. 1977.

[4] D. Angeli, "A Lyapunov approach to incremental stability properties," *IEEE Trans. Autom. Control.*, vol. 47, no. 3, pp. 410–421, Mar. 2002.

[5] I. R. Manchester and J.-J. E. Slotine, "Robust control contraction metrics: A convex approach to nonlinear state-feedback H_∞ control," *IEEE Control Syst. Lett.*, vol. 2, no. 3, pp. 333–338, May 2018.

[6] B. Demidovich, "Dissipativity of nonlinear system of differential equations," *Vestnik Moscow State Univ., Ser. Matem. Mekh.*, vol. 6, pp. 19–27, 1961.

[7] A. Pavlov, A. Pogromsky, N. Van De Wouw, and H. Nijmeijer, "Convergent dynamics, a tribute to Boris Pavlovich Demidovich," *Syst. Control Lett.*, vol. 52, nos. 3–4, pp. 257–261, Jul. 2004.

[8] W. Lohmiller and J.-J.-E. Slotine, "On contraction analysis for non-linear systems," *Automatica*, vol. 34, no. 6, pp. 683–696, Jun. 1998.

[9] J. Jouffroy and T. Fossen, "Tutorial on incremental stability analysis using contraction theory," *Model., Identificat. Control*, vol. 31, no. 3, pp. 93–106, 2010.

[10] F. Forni and R. Sepulchre, "A differential Lyapunov framework for contraction analysis," *IEEE Trans. Autom. Control.*, vol. 59, no. 3, pp. 614–628, Mar. 2014.

[11] E. M. Aylward, P. A. Parrilo, and J.-J.-E. Slotine, "Stability and robustness analysis of nonlinear systems via contraction metrics and SOS programming," *Automatica*, vol. 44, no. 8, pp. 2163–2170, Aug. 2008.

[12] W. Wang and J.-J. Slotine, "Contraction analysis of time-delayed communications and group cooperation," *IEEE Trans. Autom. Control.*, vol. 51, no. 4, pp. 712–717, Apr. 2006.

[13] Q.-C. Pham and J.-J. Slotine, "Stable concurrent synchronization in dynamic system networks," *Neural Netw.*, vol. 20, no. 1, pp. 62–77, Jan. 2007.

[14] B. Sharma and I. Kar, "Contraction theory-based recursive design of stabilising controller for a class of non-linear systems," *IET Control Theory Appl.*, vol. 4, no. 6, pp. 1005–1018, Jun. 2010.

[15] S. Zhai and X.-S. Yang, "Contraction analysis of synchronization of complex switched networks with different inner coupling matrices," *J. Franklin Inst.*, vol. 350, no. 10, pp. 3116–3127, Dec. 2013.

[16] W. Wang and J.-J.-E. Slotine, "On partial contraction analysis for coupled nonlinear oscillators," *Biol. Cybern.*, vol. 92, no. 1, pp. 38–53, Jan. 2005.

[17] A. Girard, G. Pola, and P. Tabuada, "Approximately bisimilar symbolic models for incrementally stable switched systems," *IEEE Trans. Autom. Control*, vol. 55, no. 1, pp. 116–126, Dec. 2009.

[18] W. Lohmiller and J.-J. Slotine, "On metric controllers and observers for nonlinear systems," in *Proc. 35th IEEE Conf. Decision Control*, vol. 2, Dec. 1996, pp. 1477–1482.

[19] W. Lohmiller and J.-J.-E. Slotine, "Contraction analysis of non-linear distributed systems," *Int. J. Control*, vol. 78, no. 9, pp. 678–688, Jun. 2005.

[20] D. Fiore, M. Coraggio, and M. di Bernardo, "Observer design for piecewise smooth and switched systems via contraction theory," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 2959–2964, 2017.

[21] M. Zamani and P. Tabuada, "Backstepping design for incremental stability," *IEEE Trans. Autom. Control.*, vol. 56, no. 9, pp. 2184–2189, Sep. 2011.

[22] A. Flores-Perez, I. Grave, and Y. Tang, "Contraction based adaptive control for a class of nonlinearly parameterized systems," in *Proc. Amer. Control Conf.*, Jun. 2013, pp. 2649–2654.

[23] S.-J. Chung and J.-J.-E. Slotine, "Cooperative robot control and synchronization of Lagrangian systems," in *Proc. 46th IEEE Conf. Decision Control*, 2007, pp. 2504–2509.

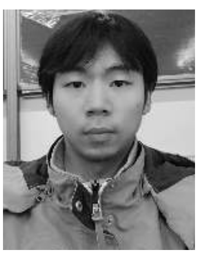
[24] I. R. Manchester and J.-J.-E. Slotine, "Control contraction metrics: Convex and intrinsic criteria for nonlinear feedback design," *IEEE Trans. Autom. Control.*, vol. 62, no. 6, pp. 3046–3053, Jun. 2017.

[25] I. R. Manchester and J.-J.-E. Slotine, "Output-feedback control of nonlinear systems using control contraction metrics and convex optimization," in *Proc. 4th Austral. Control Conf. (AUCC)*, Nov. 2014, pp. 215–220.

[26] I. R. Manchester, J. Z. Tang, and J.-J. E. Slotine, "Unifying robot trajectory tracking with control contraction metrics," *Robot. Res.*, vol. 2, pp. 403–418, Sep. 2017.

[27] J. Jouffroy and J.-J. Slotine, "Methodological remarks on contraction theory," in *Proc. 43rd IEEE Conf. Decision Control (CDC)*, vol. 3, Dec. 2004, pp. 2537–2543.

[28] S. Laghrouche, F. Plestan, and A. Glumineau, "Higher order sliding mode control based on integral sliding mode," *Automatica*, vol. 43, no. 3, pp. 531–537, Mar. 2007.
[29] Y. B. Shtessel, J. A. Moreno, and L. M. Fridman, "Twisting sliding mode control with adaptation: Lyapunov design, methodology and application," *Automatica*, vol. 75, pp. 229–235, Jan. 2017.
[30] C. Edwards and Y. Shtessel, "Adaptive continuous higher order sliding mode control," *IFAC Proc. Vol.*, vol. 47, no. 3, pp. 10826–10831, 2014.
[31] H. Zhao and Y. Niu, "Finite-time sliding mode control of switched systems with one-sided Lipschitz nonlinearity," *J. Franklin Inst.*, to be published, doi: 10.1016/j.jfranklin.2019.05.019.
[32] S. He, W. Lyu, and F. Liu, "Robust $\mathcal{H}^\infty$ sliding mode controller design of a class of time-delayed discrete conic-type nonlinear systems," *IEEE Trans. Syst., Man, Cybern. Syst.*, pp. 1–8, 2018, doi: 10.1109/TSMC.2018.2884491.
[33] S. Pandey, V. Dourla, P. Dwivedi, and A. Junghare, "Introduction and realization of four fractional-order sliding mode controllers for nonlinear open-loop unstable system: A magnetic levitation study case," *Nonlinear Dyn.*, vol. 98, no. 1, pp. 601–621, Oct. 2019.
[34] B. S. Rüffer, N. Van De Wouw, and M. Mueller, "Convergent systems vs. Incremental stability," *Syst. Control Lett.*, vol. 62, no. 3, pp. 277–285, Mar. 2013.
[35] J. W. Simpson-Porco and F. Bullo, "Contraction theory on Riemannian manifolds," *Syst. Control Lett.*, vol. 65, pp. 74–80, Mar. 2014.
[36] A. Levant, "Principles of 2-sliding mode design," *Automatica*, vol. 43, no. 4, pp. 576–586, Apr. 2007.
[37] S. Ding, J. Wang, and W. X. Zheng, "Second-order sliding mode control for nonlinear uncertain systems bounded by positive functions," *IEEE Trans. Ind. Electron.*, vol. 62, no. 9, pp. 5899–5909, Sep. 2015.
[38] F. K. Moore and E. M. Greitzer, "A theory of post-stall transients in axial compression systems: Part I–development of equations," *J. Eng. Gas Turbines Power*, vol. 108, no. 1, pp. 68–76, Jan. 1986.
[39] A. Papachristodoulou, J. Anderson, G. Valmorbida, S. Prajna, P. Seiler, and P. Parrilo, "SOSTOOLS version 3.00 sum of squares optimization toolbox for MATLAB," Oct. 2013, *arXiv:1310.4716*. [Online]. Available: https://arxiv.org/abs/1310.4716
[40] A. V. Pavlov, N. van de Wouw, and H. Nijmeijer, *Uniform Output Regulation Nonlinear Systems: A Convergent Dyn Approach* (Systems and Control: Foundations and Applications). Basel, Switzerland: Birkhäuser, 2006, doi: 10.1007/0-8176-4465-2.
[41] M. Krsti, I. Kanellakopoulos, and V. Petar, *Nonlinear And Adaptive Control Design*. Hoboken, NJ, USA: Wiley, 1995.
[42] W. Perruquetti and J.-P. Barbot, *Sliding Mode Control Engineering*. Boca Raton, FL, USA: CRC Press, 2002.



GUO ZHANG received the B.S. degree in electronic science and technology from the Nanyang Institute of Technology, Nanyang, China, in 2017. He is currently pursuing the M.S. degree in control engineering from the Sichuan University of Science and Engineering, Zigong, China. He is also a Research Assistant with Dr. Ping He's Research Group (System Optimization and Consensus), Jinan University, Zhuhai, Guangdong, China. His research interests include nonlinear systems, control theory and control engineering, and multiagent systems.

PING HE was born in Huilongya, Nanchong, Sichuan, China, in November 1990. He received the B.S. degree in automation from the Sichuan University of Science and Engineering, Zigong, Sichuan, China, in June 2012, the M.S. degree in control science and engineering from Northeastern University, Shenyang, Liaoning, China, in July 2014, and the Ph.D. degree in Electromechanical Engineering from Universidade de Macau, Taipa, Macau, in June 2017.

From December 2015 to November 2018, he was an Adjunct Associate Professor with the Department of Automation, Sichuan University of Science and Engineering. From August 2017 to August 2019, he was a Postdoctoral Research Fellow with the Emerging Technologies Institute, The University of Hong Kong, and also with the Smart Construction Laboratory, The Hong Kong Polytechnic University. Since December 2018, he has been a Full Professor with the School of Intelligent Systems Science and Engineering, Jinan University, Zhuhai, Guangdong, China. He has authored one book, and more than 40 articles. His research interests include sensor networks, complex networks, multiagent systems, artificial intelligence, control theory and control engineering.

Dr. Ping was a recipient of the Liaoning Province of China Master's Thesis Award for Excellence in March 2015, and the IEEE Robotics & Automation Society Finalist of Best Paper Award, in July 2018. He is also an Associate Editor of *Automatika*. He is the Reviewer Member for Mathematical Reviews of American Mathematical Society (Reviewer Number: 139695).

HENG LI was born in Hunan, China, in 1963. He received the B.S. and M.S. degrees in civil engineering from Tongji University, in 1984 and 1987 respectively, and the Ph.D. degree in architectural science from The University of Sydney, Australia, in 1993.

From 1993 to 1995, he was a Lecturer with James Cook University. From 1996 to 1997, he was a Senior Lecture with the Civil Engineering Department, Monash University. Since 1997, he was gradually promoted from Associate Professor to Chair Professor of construction informatics with The Hong Kong Polytechnic University. He has authored two books and more than 400 articles. His research interests include building information modeling, robotics, functional materials, and the Internet of Things. He is also a Reviews Editor of *Automation in Construction.*

Dr. Li was a recipient of the National Award from Chinese Ministry of Education, in 2015, and the Gold Prize of Geneva Innovation 2019.

YU TANG received the B.S. degree in electronic information engineering from the Sichuan University of Science and Engineering, Zigong, China, in 2017, where he is currently pursuing the M.S. degree in control engineering. His research interests include nonlinear systems, control theory, control engineering, multiagent systems, and fault tolerant control.

ZUXIN LI was born in Zhejiang, China, in 1972. He received the B.S. degree in industrial automation from the Zhejiang University of Technology, China, in 1995, the M.S. degree in communication and information system from Yunnan University, China, in 2002, and the Ph.D. degree in control theory and control engineering from the Zhejiang University of Technology, China, in 2008.

From May 2009 to March 2013, he was a Postdoctoral Research Fellow with the Institute of Cyber-Systems and Control, Zhejiang University, China. From August to November 2013, he was a Visiting Scholar with Dalhousie University, Canada. He is currently a Full Professor with the School of Engineering, Huzhou University, China. His research interests include networked control systems, robust control, estimation, prognostics, and health management.

XING-ZHONG XIONG received the B.S. degree in communication engineering from the Sichuan University of Science and Engineering, Zigong, China, in 1996, and the M.S and Ph.D. degrees in communication and information system from the University of Electronic Science and Technology of China (UESTC), in 2006 and 2009, respectively. In 2012, he completed a research assignment from the Postdoctoral Station of Electronic Science and Technology, UESTC. He is currently a Professor with the School of Automation and Information Engineering, Sichuan University of Science and Engineering. His research interests include wireless and mobile communications technologies, intelligent signal processing, the Internet-of-Things technologies, and very large-scale integration (VLSI) designs.

WEI WEI (Senior Member, IEEE) received the M.S. and Ph.D. degrees from Xi'an Jiaotong University, Xi'an, China, in 2005 and 2011, respectively. He is currently an Associate Professor with the School of Computer Science and Engineering, Xi'an University of Technology, Xi'an. He ran many funded research projects as a Principal Investigator and a Technical Member. His current research interests include the area of wireless networks, wireless sensor networks, image processing, mobile computing, distributed computing, and pervasive computing, the Internet of Things, and sensor data clouds. He has published around 100 research articles in international conferences and journals. He is a Senior Member of the China Computer Federation (CCF). He is an Editorial Board Member of *Future Generation Computer System*, IEEE Access, *Ad Hoc & Sensor Wireless Sensor Network*, *Institute of Electronics, Information and Communication Engineers*, and *KSII Transactions on Internet and Information Systems*. He is a TPC member of many conferences and a regular Reviewer of the IEEE Transactions on Parallel and Distributed Systems, the IEEE Transactions on Image Processing, the IEEE Transactions on Mobile Computing, the IEEE Transactions on Wireless Communications, the *Journal of Network and Computer Applications*, and so on.

YANGMIN LI (Senior Member, IEEE) received the B.S. and M.S. degrees in mechanical engineering from Jilin University, Changchun, China, in 1985 and 1988, respectively, and the Ph.D. degree in mechanical engineering from Tianjin University, Tianjin, China, in 1994.

He started his academic career, in 1994; he was a Lecturer with the Mechatronics Department, South China University of Technology, Guangzhou, China. He was a Fellow with the International Institute for Software Technology of the United Nations University (UNU/IIST), from May to November 1996; a Visiting Scholar with the University of Cincinnati, in 1996; a Postdoctoral Research Associate with Purdue University, West Lafayette, USA, in 1997. He was an Assistant Professor, from 1997 to 2001, an Associate Professor, from 2001 to 2007, a Full Professor, from 2007 to 2016, with the University of Macau. He is currently a Full Professor with the Department of Industrial and Systems Engineering, The Hong Kong Polytechnic University, Hong Kong. He has authored and coauthored 425 scientific articles in journals and conferences. His research interests include micro/nanomanipulation, compliant mechanism, precision engineering, robotics, and multibody dynamics and control.

Dr. Li is an Associate Editor of the IEEE Transactions on Automation Science and Engineering, *Mechatrionics*, IEEE Access, and the *International Journal of Control, Automation, and Systems.*