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Sliding mode control for a class of sub-systems with fractional order varying trajectory dynamics — Source link

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RESEARCH PAPER

SLIDING MODE CONTROL FOR A CLASS OF SUB-SYSTEMS WITH FRACTIONAL ORDER VARYING TRAJECTORY DYNAMICS

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Abstract

In this paper, a sliding mode control strategy is discussed for a class of nonlinear mechanical sub-systems with varying trajectory dynamics. The proposed class of sub-systems are represented in this simulation example by a two link robot actuator/manipulator. The fractional order is introduced in the setpoint definition as to represent changes in the desired trajectory of this sub-system. Furthermore, the same order is used to adapt the control law to the new dynamics. Uncertainties are introduced in the model used for the control law, hence robustness is intrinsic.

MSC 2010: 93C05, 93C95, 93B30, 93C40

Key Words and Phrases: fractional calculus, robot link, robustness, varying dynamic trajectory, sliding mode control, nonlinear systems

1. Introduction

Typical two link robot manipulators are often found in heavy duty industry, e.g. automotive assembly lines, agricultural harvesting machines [9, 1]. Due to varying product specification the reference trajectory may change dynamics and amplitude and calibration of the system along with re-tuning controller parameters are necessary to maintain optimal operation [23]. Often these systems are in fact part of complex processes, where sub-system interaction is present and safe operation must be ensured at all times by adapting the reference trajectory [28].

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Another class of applications where reference trajectory may change dynamics and amplitude is that of spacecraft dynamics and spacecraft rendez-vous. Orbital coordinates may be required to adapt to other values due to unexpected space drifts and winds, or obstacle avoidance maneuvers [13, 28, 10].

Medical applications such as radiotherapy for lung tumours also make use of robot link manipulators for positioning laser beam or near infra-red spectroscopy [14]. While at rest, the patient breaths during the treatment, hence the tumour changes position and shape along with the lung tissue [20]. This requires adaptation of the reference trajectory of the beam and accurate position control is of utmost importance.

A nice view upon the pioneers of fractional calculus is given in [27]. A comprehensive overview of tuning methods and applications of fractional order control is given in [25, 19, 16]. Discussion on stability in relay controlled systems is made in [3]. Gain adaptation in fractional order control has been discussed in [26]. A practical approach to implementing a fractional order control in a PLC for industrial use has been discussed in [11]. Design of sliding mode controllers for a class of fractional order chaotic systems has been proposed in [29, 18]. The systems under analysis were the fractional-order Chen system, the fractional order Lorenz system and a fractional order financial system. Numerical simulations supported the effectiveness of the proposed method. On the other hand, fractional order sliding mode controller with terminal convergence bound was proposed for a class of dynamical systems with uncertainty in [4]. Here the switching law contains fractional order differential operators and ensures finite stability of the closed loop system. Multivariable fractional order dynamics have been discussed in [17] and input and state delay problems have been tackled in [21].

A variety of applications have been employed for this type of control. Fractional order switching surfaces PI and PID control has been applied to DC-DC power converters [2] and PD type sliding surfaces have been applied in [5, 30]. Sliding mode control has been also applied to hexapod robot [22], and to rigid manipulators [8], for robotic systems with time-delays [12], and robot control [7]. An overview of other applications of fractional order sliding mode control is given in [6].

This paper proposes a sliding mode control algorithm in which varying reference trajectories are defined using fractional order dynamics. This information is used in the tuning of the control laws and simulation examples illustrate the effectiveness of the proposed methodology for this class of applications.

2. Model Description

Consider an n-joint robot as follows:

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) + \mathbf{F}(\dot{\mathbf{q}}) + \tau_{\mathbf{d}} = \tau, \qquad (2.1)$$

where $\mathbf{q} \in \mathbf{R}^n$ is the angle vector, $\mathbf{H}(\mathbf{q}) \in \mathbf{R}^{nxn}$ is the inertia matrix, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbf{R}^n$ denotes the centrifugal and coriolis forces, $\mathbf{G}(\mathbf{q}) \in \mathbf{R}^n$ is the gravity, $\mathbf{F}(\dot{\mathbf{q}}) \in \mathbf{R}^n$ is the frictional force, $\tau \in \mathbf{R}^n$ is the control moment, and $\tau_{\mathbf{d}} \in \mathbf{R}^n$ is the disturbance moment.

The characteristics of the kinetic model are ([9]):

- the kinetic model contains a higher number of elements and this depends on the number of robot joints;
- the model has a high degree of nonlinearity;
- there exists a strong interaction between the various sub-systems (i.e. joints);
- there exists model uncertainty and varying dynamics; these depend on the load and joint friction.

Properties of the model defined in (2.1) ([9, 24]):

- $\mathbf{H}(\mathbf{q})$ is a positive-definite symmetrical and bounded matrix; i.e. $m_1\mathbf{I} \leq \mathbf{H}(\mathbf{q}) \leq m_2\mathbf{I}$
- $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is bounded, i.e. $| \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) | \leq c_b(\mathbf{q}) \| \dot{\mathbf{q}} \|$
- matrix $\dot{\mathbf{H}} - 2\mathbf{C}$ is a skew-symmetric matrix, i.e. $\mathbf{x}^T(\dot{\mathbf{H}} - 2\mathbf{C})\mathbf{x} = 0$, with $\mathbf{x}$ a vector
- the measurable (known) disturbance is bounded by a positive constant $\| \tau_{\mathbf{d}} \| \leq \tau_M$.

The illustrative example used in this paper is given by a two joint robot manipulator. This is well in agreement with the real-life cases where position control is mainly achieved by accurate control of the last two joints, as in Figure 1.

The kinetic equation is simplified to:

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \tau, \qquad (2.2)$$

where $\mathbf{q} = [q_1 \quad q_2]$, $\tau = [\tau_1 \quad \tau_2]^T$ and

$$\mathbf{H} = \begin{bmatrix} \alpha + 2\epsilon\cos(q_2) - 2\eta\sin(q_2) & \beta + \epsilon\cos(q_2) + \eta\sin(q_2) \\ \beta + \epsilon\cos(q_2) + \eta\sin(q_2) & \beta \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} (-2\epsilon\sin(q_2) + 2\eta\cos(q_2))\dot{q}_2 & (-\epsilon\sin(q_2) + \eta\cos(q_2))\dot{q}_2 \\ (\epsilon\sin(q_2) - \eta\cos(q_2))\dot{q}_1 & 0 \end{bmatrix},$$

$$\mathbf{G} = \begin{bmatrix} \epsilon e_2\cos(q_1 + q_2) + \eta e_2\sin(q_1 + q_2) + (\alpha - \beta + e_1)e_2\cos(q_1) \\ \epsilon e_2\cos(q_1 + q_2) + \eta e_2\sin(q_1 + q_2) \end{bmatrix},$$

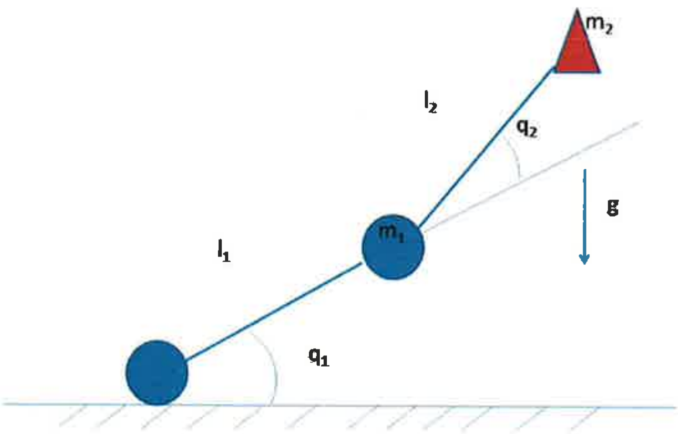

FIGURE 1. Schematic representation of the robot's last two links.

where α, β, ϵ and η are constants, with $\alpha = I_1 + m_1 l_{c1}^2 + I_e + m_e l_{ce}^2 + m_e l_1^2$, $\beta = I_e + m_e l_{ce}^2$, $\epsilon = m_e l_1 l_{ce} \cos(\delta_e)$, $\eta = m_e l_1 l_{ce} \sin(\delta_e)$. The numerical values of the robot joint elements are taken as:

$$\begin{array}{lllll} m_1 = 1kg & l_1 = 1m & l_{c1} = 1/2m & I_1 = 1/12kg & m_e = 3kg \\ l_{ce} = 1m & I_e = 2/5kg & \delta_e = 0 & e_1 = -7/12 & e_2 = 9.81. \end{array}$$

Let $\mathbf{a} = [\alpha\ \beta\ \epsilon\ \eta]^T$ and $\hat{\mathbf{a}}$ be its estimated values. We assume $\tilde{\mathbf{a}} = \hat{\mathbf{a}} - a$, since $\mathbf{a}$ is a constant vector and thus $\dot{\tilde{\mathbf{a}}} = \dot{\hat{\mathbf{a}}}$. This implies that we can estimate the matrices $\hat{\mathbf{H}}$, $\hat{\mathbf{C}}$ and $\hat{\mathbf{G}}$, respectively.

For our system, we do not know the values of $\mathbf{a}$. Denote by $\mathbf{q_d}$ the desired reference trajectory. The tracking error is given by:

$$\mathbf{e} = \mathbf{q_d} - \mathbf{q}. \tag{2.3}$$

Define

$$\dot{\mathbf{q}}_\mathbf{r} = \dot{\mathbf{q}}_\mathbf{d} + \mathbf{\Lambda}(\mathbf{q_d} - \mathbf{q}) \tag{2.4}$$

with $\mathbf{\Lambda}$ a positive diagonal matrix.

Making use of the dynamic regression matrix formulation from [13, 23], we have that:

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}}_\mathbf{r} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}_\mathbf{r})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_\mathbf{r}, \ddot{\mathbf{q}}_\mathbf{r})\mathbf{a} \tag{2.5}$$

and

$$\tilde{\mathbf{H}}(\mathbf{q})\ddot{\mathbf{q}}_\mathbf{r} + \tilde{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}}_\mathbf{r})\dot{\mathbf{q}} + \tilde{\mathbf{G}}(\mathbf{q}) = \mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_\mathbf{r}, \ddot{\mathbf{q}}_\mathbf{r})\tilde{\mathbf{a}}, \tag{2.6}$$

where

$$Y(q,\dot{q},\dot{q_r},\ddot{q_r}) = \begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} \\ y_{21} & y_{22} & y_{23} & y_{24} \end{bmatrix}$$

with:

$$y_{11} = \ddot{q_{r1}} + e_2\cos(q_1)$$
$$y_{12} = \ddot{q_{r2}} - e_2\cos(q_1)$$
$$y_{13} = 2\cos(q_2)\ddot{q_{r1}} + \cos(q_2)\ddot{q_{r2}} - 2\sin(q_2)\dot{q_2}\dot{q_{r1}} - \sin(q_2)\dot{q_2}\dot{q_{r2}} + e_2\cos(q_1+q_2)$$
$$y_{14} = 2\sin(q_2)\ddot{q_{r1}} + \sin(q_2)\ddot{q_{r2}} + 2\cos(q_2)\dot{q_2}\dot{q_{r1}} + \cos(q_2)\dot{q_2}\dot{q_{r2}} + e_2\sin(q_1+q_2)$$
$$y_{21} = 0; y_{22} = \ddot{q_{r1}} + \ddot{q_{r2}}$$
$$y_{23} = \cos(q_2)\ddot{q_{r1}} + \sin(q_2)\dot{q_1}\dot{q_{r1}} + e_2\cos(q_1+q_2)$$
$$y_{24} = \sin(q_2)\ddot{q_{r1}} - \cos(q_2)\dot{q_1}\dot{q_{r1}} + e_2\sin(q_1+q_2).$$

3. Sliding Mode Controller Design

In this paper, we propose the use of classical sliding mode control strategy as briefly introduced in the remainder of this section for the application defined by (2.5), [13]. The originality of our approach is not in the control algorithm itself, but in the definition of the reference trajectory, consequently used in the controller law. This approach is comparable in method with setpoint weighting proposed in [15, 28].

The sliding variable is given by

$$\mathbf{s} = \dot{\mathbf{e}} + \mathbf{\Lambda}\mathbf{e}. \tag{3.1}$$

Selecting the Lyapunov function

$$V(t) = \frac{1}{2}\mathbf{s}^T\mathbf{H}(\mathbf{q})\mathbf{s}, \tag{3.2}$$

we have that

$$\dot{V}(t) = \mathbf{s}^T[\mathbf{H}(\mathbf{q})\ddot{\mathbf{q_r}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q_r}} + \mathbf{G}(\mathbf{q}) - \tau]. \tag{3.3}$$

Hence, we can design the controller law as:

$$\tau = \hat{\mathbf{H}}(\mathbf{q})\ddot{\mathbf{q_r}} + \hat{\mathbf{C}}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q_r}} + \hat{\mathbf{G}}(\mathbf{q}) + \tau_\mathbf{s} \tag{3.4}$$

with $\tau_\mathbf{s}$ the design parameter for robustness. Making use of (3.2) and (3.3) it follows that:

$$\dot{V}(t) = \mathbf{s}^T[\tilde{\mathbf{H}}(\mathbf{q})\ddot{\mathbf{q_r}} + \tilde{\mathbf{C}}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q_r}} + \tilde{\mathbf{G}}(\mathbf{q}) - \tau_\mathbf{s}] = \mathbf{s}^T[\mathbf{Y}(\mathbf{q},\dot{\mathbf{q}},\dot{\mathbf{q_r}},\ddot{\mathbf{q_r}})\tilde{\mathbf{a}} - \tau_\mathbf{s}]. \tag{3.5}$$

We select

$$\tau_\mathbf{s} = \mathbf{k}sgn(s) + s = \begin{bmatrix} k_1 sgn(s_1) + s_1 \\ k_2 sgn(s_2) + s_2 \end{bmatrix}, \tag{3.6}$$

where $k_i = \sum_{j=1}^{4} \bar{Y}_{ij}\bar{\tilde{a}}_j$, with $i = 1, 2$. From (3.5) and (3.6) we obtain the control law.

Considering the desired reference trajectories as defined by:

$$q_{d1} = \omega^\gamma \sin(\omega^\gamma t),$$
$$q_{d2} = \omega^\gamma \sin(\omega^\gamma t). \tag{3.7}$$

This implies that if dynamics and amplitude are varying according to the effect of γ, the control law from (3.5) will adapt accordingly. However, if standard identification tools are used, they require nonlinear algorithms such as nonlinear least squares with gradient search, which depends strongly on initial values. Instead, starting from the initial value of *gamma*, an iterative procedure is employed within the sampling period:

- from the initial value, give two incremental values for γ with step increment of 0.1 and calculate the corresponding error on trajectory;
- from the initial value, give two decremental values for γ with step decrement of 0.1 and calculate the corresponding error on trajectory;
- select the lowest error and update its corresponding γ value;
- use the new γ value and apply control;
- at next sampling period, use the updated γ value and re-iterate.

This algorithm requires very few operations and can be easily implemented in execution elements, microcontrollers, PLCs, etc. It is also computationally modest and can be execute without any problem within the sampling period of the controller.

4. Results

In this analysis, we assume $\mathbf{\Lambda} = \mathbf{5} \cdot \mathbf{I}$ and an uncertainty in the model parameter estimation of 50%. To avoid chattering in the control effort, the saturated function is used instead of the switch function with $\triangle = 0.05$. These parameters do not change from these values in the next simulation tests. In all reported results, position units are in cm, speed units are in cm/s and control effort units in mV.

First, we test the system assuming we know perfectly the trajectory of the reference, for $\gamma = 0.1$. The results of the closed loop control are given in Figure 2 for the two controlled positions and two control efforts.

Second, we illustrate the efficacy of the recursive identification iterative algorithm for changing dynamics. In this scenario we assume that the initial value of $\gamma = 0.5$ and at time instant 2.5 seconds it changes to $\gamma = 0.75$. The results are given in Figure 3 for the output of the fist link, the second link and control effort. To make a realistic simulation, we introduce saturation at 500 mV in the upper and lower values of the control effort.

It can be observed that under all conditions, the control law is able to follow the reference trajectory without requiring re-tuning. This is in fact done automatically through the changes in the γ values.

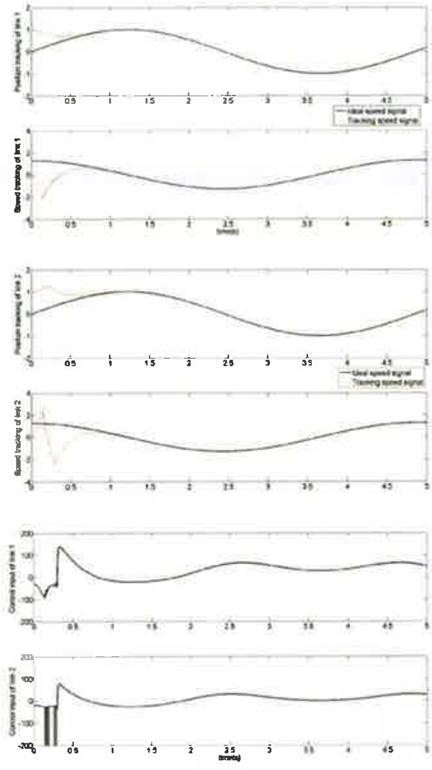

FIGURE 2. Simulation results for the ideal case when the reference trajectory is perfectly known, with a fractional order variable $\gamma = 0.1$. First joint (top), second joint (middle) and control effort (bottom).

As expected, the closed loop performance becomes optimal after the correct value for γ has been identified. Once this is done, then the controller has no difficulty to follow the dynamic reference trajectory.

Other reference trajectories are possible, such as multisine, step, ramp, etc. In these cases, the form of (3.7) has to be adjusted accordingly. However, the application envisaged in this study is for medical purposes, i.e. radiation of a lung tumour whereas the robot arm must follow the movement of the tumour in the lung tissue during breathing. Hence, reference trajectories of the sinusoidal form are within the scope of the paper.

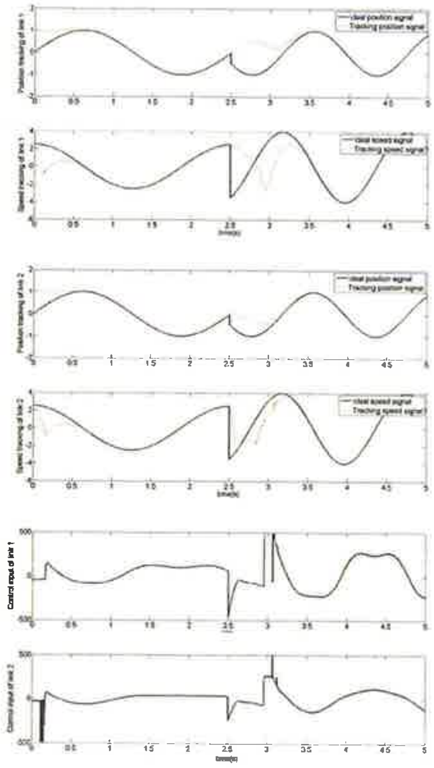

FIGURE 3. Output of the first joint (top), second joint (middle) and control effort (bottom) for changing value of gamma at instant 2.5 seconds.

5. Conclusions

In this paper, a sliding mode control of a dynamic reference trajectory is presented. The example used to illustrate the effectiveness of the control can be widely encountered in industrial and medical applications.

Next steps are taken to test this methodology in a real life robot for radiotherapy of lung tumour, whereas sinusoidal based reference trajectories are employed.

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References

[1] M. Aghababa, Optimal design of fractional-order PID controller for five bar linkage robot using a new particle swarm optimization algorithm. *Soft Computing, Methodologies and Application* (2015), First Online: 17 June 2015, Springer-Verlag; DOI: 10.1007/s00500-015-1741-2.

[2] A.K. Calderon, B. Vinagre, V. Feliu, Fractional order control strategies for power electronic buck converters. *Signal Processing* **86** (2006), 2803–2819; DOI: 10.1016/j.sigpro.2006.02.022.

[3] R. Caponetto, G. Maione, A. Pisano, M.R. Rapaic, E. Usai, Analysis and shaping of the self-sustained oscillations in relay controlled fractional-order systems. *Fract. Calc. Appl. Anal.* **16**, No 1 (2013), 93–108; DOI: 10.2478/s13540-013-0007-x; http://www.degruyter.com/view/j/fca.2013.16.issue-1/issue-files/fca.2013.16.issue-1.xml.

[4] S. Dadras, H.R. Momeni, Fractional terminal sliding mode control design for a class of dynamical systems with uncertainty. *Commun. Nonlinear. Sci. Numer. Simulat.* **17** (2012), 367–377; DOI: 10.1016/j.cnsns.2011.04.032.

[5] H. Delavari, R. Ghaderi, A. Ranjbar, S. Momani, Fuzzy fractional order sliding mode controller for nonlinear systems. *Commun. Nonlinear. Sci. Numer. Simulat.* **15** (2010), 963–978; DOI: 10.1016/j.cnsns.2009.05.025.

[6] M.O. Efe, Z.B. Given, D. Baleanu, J.A. Tenreiro Machado, *Fractional Order Sliding Mode Controller Design for Fractional Order Dynamic Systems.* Ser. New Trends in Nanotechnology and Fractional Calculus Applications, Springer (2010).

[7] C. Fallaha, M. Saad, Y. Kanaan, K. Al-Haddad, Sliding-mode robot control with exponential reaching law. *IEEE Trans. Ind. Inf.* **58**, No 2 (2011), 600–610; DOI: 10.1109/TIE.2010.2045995.

[8] Y. Feng, X. Yu, Z. Man, Non-singular terminal sliding mode control of rigid manipulators. *Automatica* **38**, No 12 (2002), 2159–2167; DOI: 0.1016/S0005-1098(02)00147-4.

[9] S. Kawamura, M. Svinin (Eds), *Advances in Robot Control: From Everyday Physics to Human-like Movements.* Springer-Verlag, Berlin-Heidelberg (2006).

[10] S. Ladaci, A. Charef, On fractional adaptive control. *Nonlinear Dynamics* **43**, No 4 (2006), 365–378; DOI: 10.1007/s11071-006-0159-x.

[11] P. Lanusse, J. Sabatier, PLC Implementation of a CRONE controller. *Fract. Calc. Appl. Anal.* **14**, No 4 (2011), 505–522; DOI: 10.2478/s13540-011-0031-7; http://www.degruyter.com/view/j/fca.2011.14.issue-4/issue-files/fca.2011.14.issue-4.xml.

[12] M. Lazaveric, Finite time stability analysis of PD$^{\alpha}$ fractional control of robotic time delay systems. *Mechanics Research Comm.* **33**, No 2 (2006), 269–279; DOI: 10.1016/j.mechrescom.2005.08.010.
[13] J. Liu, X. Wang, *Advanced Sliding Mode Control for Mechanical Systems.* Springer-Verlag, Berlin-Heidelberg (2012).
[14] T. Neicu, H. Shirato, Y. Seppenwoolde, S.B. Jiang, Synchronized moving aperture radiation therapy (SMART): Average tumour trajectory for lung patients. *Phys. Med. Biol.* **48** (2003), 587–598; PII: S0031-9155(03)57158-5.
[15] F. Padula, A. Visioli, Set-point weight tuning rules for fractional-order PID controllers. *Asian J. of Control* **15**, No 3 (2013), 678–690; DOI: 10.1002/asjc.634.
[16] I. Podlubny, Fractional-order systems and PID-controller. *IEEE Trans. Autom. Control* **44**, No 1 (1999), 208–214; DOI: 10.1109/9.739144.
[17] A. Pisano, M.C. Rapaic, Z.D. Jelicic, E. Usai, Sliding mode control approaches to the robust regulation of linear multivariable fractional-order dynamics. *Internat. J. of Robust and Nonlinear Control* **20**, No 18 (2010), 2045–2056; DOI: 10.1002/rnc.1565.
[18] A. Razminia, D. Baleanu, Complete synchronization of commensurate fractional order chaotic systems using sliding mode control. *Mechatronics* **23**, No 7 (2013), 873–879; DOI: 10.1016/j.mechatronics.2013.02.004.
[19] J. Sabatier, O. Agrawal, J. Tenreiro Machado (Eds), *Advances in Fractional Calculus.* Springer-Verlag, Berlin-Heidelberg (2007).
[20] Y. Seppenwoolde, H. Shirato, K. Kei Kitamura, S. Shimizu, M. van Herk, J.V. Lebesque, K. Miyasaka, Precise and real-time measurement of 3D tumour motion in lung due to breathing and heartbeat, measured during radiotherapy. *Int. J. Rad. Onc. Biol. Physics* **53**, No 4 (2002), 822–834; DOI: 10.1016/S0360-3016(02)02803-1.
[21] A. Si-Ammour, S. Djennoune, M. Bettayeb, A sliding mode control for linear fractional systems with input and state delays. *Commun. Nonlinear. Sci. Numer. Simulat.* **14**, No 5 (2009), 2310–2318; DOI: 10.1016/j.cnsns.2008.05.011.
[22] M. Silva, J. Tenreiro Machado, A. Lopez, Fractional order control of a hexapod robot. *Nonlinear Dynamics* **38** (2004), 417–433; DOI: 10.1007/s11071-004-3770-8.
[23] J.E. Slotine, W.P. Li, On the adaptive control of robot manipulators. *Int. J. Rob. Res.* **6**, No 3 (1987), 49–59; DOI: 10.1177/027836498700600303.
[24] J.E. Slotine, W.P. Li, *Applied Nonlinear Control.* Prentice Hall (1991).

[25] M.S. Tavazoei, Time response analysis of fractional-order control systems: A survey on recent results. *Fract. Calc. Appl. Anal.* **17**, No 2 (2014), 440–461; DOI: 10.2478/s13540-014-0179-z; http://www.degruyter.com/view/j/fca.2014.17.issue-2/issue-files/fca.2014.17.issue-2.xml.

[26] I. Tejado, H. HosseinNia, B. Vinagre, Adaptive gain-order fractional control for network-based applications *Fract. Calc. Appl. Anal.* **17**, No 2 (2014), 462–482; DOI: 10.2478/s13540-014-0180-6; http://www.degruyter.com/view/j/fca.2014.17.issue-2/issue-files/fca.2014.17.issue-2.xml.

[27] D. Valerio, J.T. Machado, V. Kiryakova, Some pioneers of the applications of fractional calculus. *Fract. Calc. Appl. Anal.* **17**, No 2 (2014), 552–578; DOI: 10.2478/s13540-014-0185-1; http://www.degruyter.com/view/j/fca.2014.17.issue-2/issue-files/fca.2014.17.issue-2.xml.

[28] B.M. Vinagre, I. Petras, I. Podlubny, Y.Q. Chen, Using fractional order adjustment rules and fractional order reference models in model reference adaptive control. *Nonlinear Dynamics* **29**, No 14 (2002), 269-279; DOI: 10.1023/A:1016504620249.

[29] C. Yin, S.-M. Zhong, W.-F. Chen, Design of sliding mode controller for a class of fractional order chaotic systems. *Comm. Non. Sci. Numer. Sim.* **17** (2012), 356–366; DOI: 10.1016/j.cnsns.2011.04.024.

[30] B. Zhang, Y. Pi, Y. Luo, Fractional order sliding mode control based on parameters auto-tuning for velocity control of permanent magnet synchronous motor. *ISA Transactions* **51** (2012), 649–656; DOI: 10.1016/j.isatra.2012.04.006.

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