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***Abstract*— A dynamical sliding-mode control is used to avoid different bit sticking problems present in a conventional vertical oilwell drillstring. The control goal of driving the rotary velocities of drillstring components to a constant positive value is achieved by means of this control. A discontinuous lumped-parameter torsional model of four degrees of freedom is considered. This model allows to describe drill pipes and drill collars behavior. The closed-loop system has two discontinuity surfaces. One of them gives rise to self-excited bit stick-slip oscillations and bit sticking phenomena. The other surface is introduced to accomplish the control goal despite variations in the weight on the bit (key to the dynamics) and other system parameters.**

***Index Terms*— Sliding motions, nonlinear control, oilwell drillstrings, dry friction, discontinuous systems.**

I. INTRODUCTION

Oilwell drillstrings are mechanisms that play a key role in the petroleum extraction industry. Failures in drillstrings can be significant in the total cost of the perforation process. These devices are complex dynamical systems with many unknown and varying parameters due to the fact that drillstring characteristics change as the drilling operation makes progress. The drillstring interaction with the borehole gives rise to a wide variety of non-desired oscillations. These oscillations are a major cause of drillstring component failures [1]–[3]. Permanent stuck bit and stick-slip at the bit are two phenomena particularly harmful. The first happens when the bit is unable to rotate, and the second causes the top-rotary system to move with a constant rotary speed, whereas the bit rotary speed varies between zero and up to six times the rotary speed at the surface.

Drillstrings complexity poses a modelling and a control problem. The model has to reproduce the most relevant phenomena arising in practice and has to be simple enough for analysis and control purposes. The control must deal with complex dynamics and be robust to operating conditions.

The great practical significance of oilwell drillstrings has interested some researchers. Several approaches have been used to treat the modelling and control problems. Most of them deal with the torsional behavior and the suppression of stick-slip oscillations. To keep a more simple analysis, lumped-parameter models have been proposed. Most of them are of one degree of freedom (DOF) [4], [5] and two DOF [1]–[3], [6]–[9]. For the control problem, there are also several solutions. For example, [4], [7], [10] proposed a vibration absorber at the top of the drillstring (referred to as soft torque rotary system). A classical PID control structure at the surface is used in [6], [11], [12]. More sophisticated techniques are used in [3] and [9] where a linear quadratic regulator and a linear H_∞ control are used, respectively.

Recently in [13], an analysis of bifurcations and transitions between several bit dynamics has been reported. In this work, the existence of a sliding motion on the discontinuity surface when the bit velocity is zero is shown to depend on the weight on the bit (WOB) and the torque given by the surface motor. By using the idea of introducing another discontinuity surface and forcing the system to evolve along this surface, in the present paper, a dynamical sliding-mode control is proposed to maintain the rotary velocities to a desired value without bit sticking phenomena. If the system trajectories reach this surface, they will enter in a sliding regime. On this surface, the rotary velocities will tend to the reference value and the bit rotary speed will follow the top-rotary system speed after a reasonable time, which can be adjusted by a proper selection of the two controller gains. Sliding-mode control has been effectively used in many practical control problems, see for example [14]–[16].

A lumped-parameter discontinuous torsional model with four DOF is considered. This model is a particular case of the generic n-dimensional model proposed in [13]. The model used here considers the drill pipe and drill collars dynamics. It is more general than the torsional lumped-parameter models of one and two DOF [1]–[9], [12] previously proposed. The bit-rock contact is modelled by means of a dry friction combined with an exponential decaying law, which introduces the discontinuity in the open-loop system.

In Section 2, a discontinuous torsional model for the drillstring including the bit-rock interaction is presented. Section 3 is based on previous results [13], which are very useful for the closed-loop system analysis. It relates bit sticking problems with the existence of a sliding motion when the bit velocity is zero. Section 4 proposes a dynamical sliding-mode control which overcomes the existing sliding

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motion when the bit angular velocity is zero and drives the rotary velocities to a desired value. The chattering problem when system trajectories move on the discontinuity surface is prevented. Simulations results are given in Section 5. Some implementation issues and controller advantages are also discussed. Conclusions are given in the last section.

II. Torsional model of a drillstring

Three main parts can be highlighted in a conventional vertical oilwell drillstring: 1) the surface rotating mechanism, 2) a set of drill pipes screwed one to each other, 3) the bottom-hole assembly (BHA), which consists of the drill collars, the stabilizers, a heavy-weight drill pipe and the bit (the cutting device). In this paper, the BHA, excepting the bit, will be considered as one block referred to as drill collars. The drill collars are stiffer than the drill pipes and the bit in order to prevent the drillstring from underbalancing.

Figure 1 depicts a simplified torsional model of the drillstring. It consists of four kinds of elements: 1) the top-rotary system (J_r), 2) the drill pipes (J_p), 3) the drill collars (J_l), 4) the bit (J_b). The inertias are connected one to each other by linear springs with torsional stiffness (k_t, k_{tl}, k_{tb}) and torsional damping (c_t, c_{tl}, c_{tb}). A viscous damping torque is considered at the top-drive system (T_{a_r}) and at the bit (T_{a_b}). A dry friction torque (T_{f_b}) is considered at the bit. The equations of motion are the following ones:

$$\ddot{\varphi}_r = -\frac{c_t}{J_r}(\dot{\varphi}_r - \dot{\varphi}_p) - \frac{k_t}{J_r}(\varphi_r - \varphi_p) + \frac{T_m - T_{a_r}(\dot{\varphi}_r)}{J_r}, \quad (1a)$$

$$\ddot{\varphi}_p = \frac{c_t}{J_p}(\dot{\varphi}_r - \dot{\varphi}_p) + \frac{k_t}{J_p}(\varphi_r - \varphi_p) - \frac{c_{tl}}{J_p}(\dot{\varphi}_p - \dot{\varphi}_l) - \frac{k_{tl}}{J_p}(\varphi_p - \varphi_l), \quad (1b)$$

$$\ddot{\varphi}_l = \frac{c_{tl}}{J_l}(\dot{\varphi}_p - \dot{\varphi}_l) + \frac{k_{tl}}{J_l}(\varphi_p - \varphi_l) - \frac{c_{tb}}{J_l}(\dot{\varphi}_l - \dot{\varphi}_b) - \frac{k_{tb}}{J_l}(\varphi_l - \varphi_b), \quad (1c)$$

$$\ddot{\varphi}_b = \frac{c_{tb}}{J_b}(\dot{\varphi}_l - \dot{\varphi}_b) + \frac{k_{tb}}{J_b}(\varphi_l - \varphi_b) - \frac{T_b(\mathbf{x})}{J_b}, \quad (1d)$$

with φ_i, $\dot{\varphi}_i$ ($i \in \{r, p, l, b\}$) the angular displacements and angular velocities of drillstring elements, respectively. T_m is the torque coming from the electrical motor at the surface. The actuator dynamics is not considered, and $T_m = u$, with u the control input. $T_{a_r} = c_r \dot{\varphi}_r$, with c_r the viscous damping coefficient; $\mathbf{x}$ is the system state vector defined as:

$$\mathbf{x} = (\dot{\varphi}_r, \varphi_r - \varphi_p, \dot{\varphi}_p, \varphi_p - \varphi_l, \dot{\varphi}_l, \varphi_l - \varphi_b, \dot{\varphi}_b)^T = (x_1, x_2, x_3, x_4, x_5, x_6, x_7)^T. \quad (2)$$

Finally, T_b is the torque on the bit:

$$T_b(\mathbf{x}) = T_{a_b}(x_7) + T_{f_b}(\mathbf{x}). \quad (3)$$

$T_{a_b} = c_b x_7$ approximates the influence of the mud drilling on the bit behaviour. $T_{f_b}(\mathbf{x})$ is the friction modelling the bit-rock contact, and is considered as a combination of the switch model [18] and the dry friction model in which a zero velocity band is introduced (Karnopp's model) [19]. Thus,

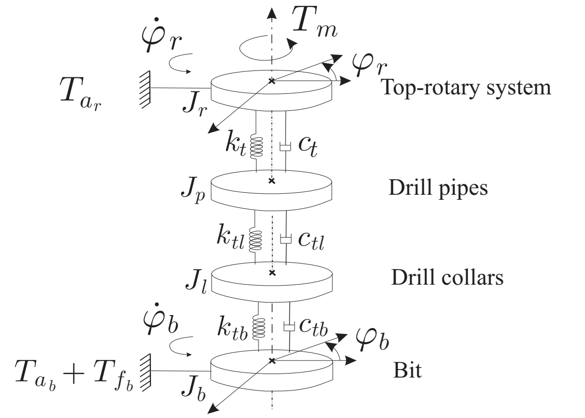

Fig. 1. Mechanical model describing the torsional behaviour of a conventional drillstring.

$$T_{f_b}(\mathbf{x}) = \begin{cases} T_{e_b}(\mathbf{x}) & \text{if } |x_7| < D_v,\ |T_{e_b}| \le T_{s_b} \\ T_{s_b} sgn(T_{e_b}(\mathbf{x})) & \text{if } |x_7| < D_v,\ |T_{e_b}| > T_{s_b} \\ W_{ob} R_b \mu_b(x_7) sgn(x_7) & \text{if } |x_7| \ge D_v \end{cases} \quad (4)$$

with $D_v > 0$, T_{e_b} the reaction torque, that is, the torque that the static friction torque $T_{s_b} = W_{ob} R_b \mu_{s_b}$ must overcome so that the bit moves, $R_b > 0$ is the bit radius and $W_{ob} > 0$ the WOB. $\mu_b(x_7)$ is the bit dry friction coefficient considered as,

$$\mu_b(x_7) = \mu_{c_b} + (\mu_{s_b} - \mu_{c_b}) e^{-\frac{\gamma_b}{v_f}|x_7|}, \quad (5)$$

with μ_{s_b}, $\mu_{c_b} \in (0,1)$ the static and Coulomb friction coefficients associated with J_b; $0 < \gamma_b < 1$ and $v_f > 0$. T_{e_b} is,

$$T_{e_b} = c_{tb}(x_5 - x_7) + k_{tb} x_6 - T_{a_b}(x_7). \quad (6)$$

Friction model (4) will be used for simulations. However, for simplicity's sake, the function

$$T_{f_b}(x_7) = W_{ob} R_b \mu_b(x_7) sgn(x_7), \quad (7)$$

will be used for analysis purposes. The exponential decaying behaviour of T_b coincides with experimental bit torque values and is inspired in the models given in [1], [6], [11].

Using (2), system (1) can be written as:

$$\begin{aligned} \dot{x}_1 &= \frac{1}{J_r}\left[-(c_t + c_r)x_1 - k_t x_2 + c_t x_3 + u\right], \\ \dot{x}_2 &= x_1 - x_3, \\ \dot{x}_3 &= \frac{1}{J_p}\left[c_t x_1 + k_t x_2 - (c_t + c_{tl})x_3 - k_{tl} x_4 + c_{tl} x_5\right], \\ \dot{x}_4 &= x_3 - x_5, \\ \dot{x}_5 &= \frac{1}{J_l}\left[c_{tl} x_3 + k_{tl} x_4 - (c_{tl} + c_{tb})x_5 - k_{tb} x_6 + c_{tb} x_7\right], \\ \dot{x}_6 &= x_5 - x_7, \\ \dot{x}_7 &= \frac{1}{J_b}\left[c_{tb} x_5 + k_{tb} x_6 - (c_{tb} + c_b)x_7 - T_{f_b}(\mathbf{x})\right], \end{aligned} \quad (8)$$

or in a compact form,

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u + \mathbf{T_f}(\mathbf{x}(t)), \quad (9)$$

where $\mathbf{A}$, $\mathbf{B}$ are constant matrices depending on system parameters and $\mathbf{T_f}$ represents the torque on the bit.

Notice that model (8) is a 7-dimensional discontinuous nonlinear system. The discontinuity is introduced by the bit-rock friction, which causes different complex dynamical phenomena, some of them are presented in Section 3.

III. Bit sticking phenomena and sliding motion existence in the open-loop system

Figure 2 shows a simulation of system (8). Parameters used for the simulation correspond to a real drillstring design [20] with a drill pipeline consisting of 130 drill pipes of 5 inches of outer diameter (OD), 4.408 inches of inner diameter (ID) and 9 meters of length, and a roller-cone bit of $6\frac{1}{2}$ (ID), $12\frac{1}{4}$ (OD) and 1.5 meters of length. Then,

$$\begin{aligned}
&J_r = 930\,kg\,m^2,\, J_b = 471.9698\,kg\,m^2,\, R_b = 0.155575\,m,\\
&J_p = 2782.25\,kg\,m^2,\, J_l = 750\,kg\,m^2,\, c_r = 425\,Nms/rad,\\
&k_t = 698.063\,Nm/rad,\, k_{tl} = 1080\,Nm/rad,\, \mu_{c_b} = 0.5,\\
&k_{tb} = 907.48\,Nm/rad,\, c_t = 139{,}6126\,Nms/rad,\, \mu_{s_b} = 0.8,\\
&c_{tl} = 190\,Nms/rad,\, c_{tb} = 181.49,\, Nms/rad,\\
&c_b = 50\,Nms/rad,\, D_v = 10^{-6},\, \gamma_b = 0.9,\, v_f = 1.
\end{aligned} \quad (10)$$

Together with these parameters, $W_{ob} = 97347\,N$, $u = 10\,kNm$ were used. The oscillations obtained for the drill pipe and drill collars associated with the stick-slip bit motion are in accordance with real drilltrings operation. Hence, model (8) appropriately describes stick-slip oscillations and other bit non-desired bit sticking situations.

By using an n-DOF model, in [13], it is shown that the existence of a sliding motion on a subset of the switching manifold is the cause of the presence of bit sticking phenomena. In [13], a range of (W_{ob}, u) is identified for non-desired bit situations to appear. These results are adapted for the model used in this paper and will be very useful for the discussion presented in Section 5.

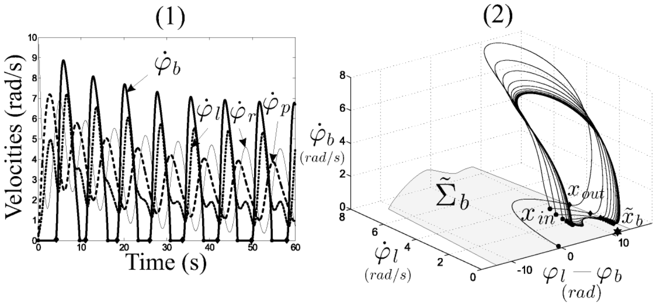

Fig. 2. Stick-slip in system (8): (1) angular velocities, (2) trajectory of the system in the space $(\varphi_l - \varphi_b, \dot{\varphi}_l, \dot{\varphi}_b)$. $\mathbf{x_{in}}$ (•) and $\mathbf{x_{out}}$ (♦) are the points at which the system trajectory enters and goes out of the sticking region. ★ quasi-equilibrium point ($\tilde{\mathbf{x}}_\mathbf{b}$), $\tilde{\Sigma}_b$ sliding region when $\dot{\varphi}_b = 0$.

Let Σ_b be the switching manifold and $\tilde{\Sigma}_b \subset \Sigma_b$ be the sliding region. For system (8), $\Sigma_b := \{\mathbf{x} \in \mathbb{R}^7 : x_7 = 0\}$, and the sliding region has the form,

$$\tilde{\Sigma}_b = \{\mathbf{x} \in \Sigma_b : |k_{tb}x_6 + c_{tb}x_5| < W_{ob}R_b\mu_{s_b}\}.$$

Three main kinds of bit dynamical behaviors can be identified: 1) stick-slip at x_7, that is, the trajectory enters and leaves repeatedly the sliding mode; 2) permanent stuck bit, i.e., $\mathbf{x}(t) \in \tilde{\Sigma}_b, \forall t$; 3) the bit moves with a positive constant velocity.

Let $\tilde{x}_b$ be the quasi-equilibrium point existing on Σ_b. In [13], it is also shown that the relative position of $\tilde{x}_b$ with respect to the boundary of $\tilde{\Sigma}_b$ plays a key role in the elimination of bit sticking problems. The bit will move with a constant positive velocity when $\tilde{x}_b$ is far away enough from the boundary of $\tilde{\Sigma}_b$, and this is accomplished when u is greater enough than $W_{ob}R_b\mu_{s_b}$.

IV. Sliding-mode-based controller

The control goal is to eliminate bit sticking phenomena and make the bit move with a desired constant velocity ($\Omega > 0$) established at the top-rotary system despite W_{ob} and bit-rock contact variations. This will be achieved by introducing a surface along which the system trajectories enter a sliding regime and the control goal is met ($x_7 \longrightarrow x_1$, $x_1 \longrightarrow \Omega$).

Let define the scalar function:

$$\begin{aligned}
\sigma_r(\mathbf{x},t) &= (x_1 - \Omega) + \lambda \int_0^t [x_1(\tau) - \Omega]\, d\tau + \\
&+ \lambda \int_0^t [x_1(\tau) - x_7(\tau)]\, d\tau = \\
&= (x_1 - \Omega) + \lambda(x_8 + x_9),\\
\dot{x}_8 &= x_1 - \Omega,\ \dot{x}_9 = x_1 - x_7,
\end{aligned} \quad (11)$$

with $\Omega > 0$ the desired velocity value and $\lambda > 0$. If σ_r is zero, x_7 will approach x_1 and x_1 will approach Ω, which is the control goal. For σ_r to approach zero, it is imposed that:

$$\dot{\sigma}_r = -\eta\, sgn(\sigma_r), \quad (12)$$

with η a constant to be chosen in order to have a sliding motion on $\sigma_r = 0$.

From (11) and (12), the following control is obtained,

$$\begin{aligned}
u = {} & c_t(x_1 - x_3) + k_t x_2 + c_r x_1 - \\
& - J_r\left[\lambda(x_1 - \Omega) + \lambda(x_1 - x_7) + \eta\, sign(\sigma_r)\right].
\end{aligned} \quad (13)$$

Control u is discontinuous, and σ_r divides the state space into two domains, thus:

$$u = \begin{cases} u^+ & \text{if } \sigma_r(\mathbf{x},t) > 0 \\ u^- & \text{if } \sigma_r(\mathbf{x},t) < 0 \end{cases} \quad (14)$$

with

$$\begin{aligned}
u^+ &= c_t(x_1 - x_3) + k_t x_2 + c_r x_1 - \\
&\quad - J_r\left[\lambda(x_1 - \Omega) + \lambda(x_1 - x_7) + \eta\right],\\
u^- &= c_t(x_1 - x_3) + k_t x_2 + c_r x_1 - \\
&\quad - J_r\left[\lambda(x_1 - \Omega) + \lambda(x_1 - x_7) - \eta\right].
\end{aligned} \quad (15)$$

Now, the stability of the controlled system will be proven. Useful relationships will be obtained in the process.

Proposition 1: Consider system (8) with control (13)-(11). If $\eta > 0$ then any trajectory of the system goes into a sliding motion on $\sigma_r(\mathbf{x},t) = 0$.

Proof: Since u was proposed in such a way that (12) is accomplished, it follows that $\sigma_r \dot{\sigma}_r < 0$, then, a sliding motion takes place when the system trajectory hits the surface $\sigma_r = 0$ [17].

Let us examine that σ_r becomes zero in a finite time interval t_s. Consider relation (12). Assume that $\sigma_r(\mathbf{x},t) > 0$ with $t_0 = 0$. Let t_s be the time needed for σ_r to become zero. Integrating (12) between $t = t_0$ and $t = t_s$, one yields to:

$$t_s = \frac{\sigma_r(\mathbf{x},t_0)}{\eta}.$$

Similarly, with $\sigma_r(\mathbf{x},t_0) < 0$:

$$t_s = -\frac{\sigma_r(\mathbf{x},t_0)}{\eta}.$$

Combining both cases, t_s leads to,

$$t_s = \frac{|\sigma_r(\mathbf{x},t_0)|}{\eta}.$$

Consequently, the bigger η is, the faster σ_r becomes zero; however, the higher the control effort is. ■

When the system trajectories move along $\sigma_r = 0$, they are governed by:

$$\dot{x} = f_s(x), \tag{16}$$

where f_s is a vector field tangent to the sliding surface,

$$\Sigma_r := \{\mathbf{x} \in \mathbb{R}^7, t \geq t_s : \; \sigma_r(\mathbf{x},t) = 0\}. \tag{17}$$

f_s can be calculated by means of the equivalent control method [14], [17] and has the form:

$$\mathbf{f_s}(\mathbf{x}) = \mathbf{A}\mathbf{x} + \mathbf{B}u_{eq}(\mathbf{x}) + \mathbf{T_f}(\mathbf{x})\big|_{T_{f_b} = T^+_{f_b}(\Omega)}, \tag{18}$$

with,

$$T^+_{f_b}(\Omega) = W_{ob}R_b\left[\mu_{c_b} + (\mu_{s_b} - \mu_{c_b})e^{-\frac{\gamma_b}{v_f}\Omega}\right].$$

The equivalent control u_{eq} is the solution for u of equation $\dot{\sigma}_r = 0$, that is:

$$u_{eq}(\mathbf{x}) = c_t(x_1 - x_3) + k_t x_2 + c_r x_1 - J_r\lambda\left[(x_1 - \Omega) + (x_1 - x_7)\right]. \tag{19}$$

The motion on Σ_r is determined by the stability properties of the quasi-equilibrium point of (16). Let $\tilde{\mathbf{x}}_\mathbf{r}$ be the quasi-equilibrium point of (16), which is on Σ_r. By making $f_s(\tilde{x}_r) = 0$, $\tilde{\mathbf{x}}_\mathbf{r}$ is obtained as,

$$\begin{aligned}
&\tilde{x}_{r,1} = \tilde{x}_{r,3} = \tilde{x}_{r,5} = \tilde{x}_{r,7} = \Omega,\\
&\tilde{x}_{r,2} = \frac{1}{k_t}h(\Omega),\; \tilde{x}_{r,4} = \frac{1}{k_{tl}}h(\Omega),\; \tilde{x}_{r,6} = \frac{1}{k_{tb}}h(\Omega),\\
&h(\Omega) = c_b\Omega + T^+_{f_b}(\Omega).
\end{aligned} \tag{20}$$

Proposition 2: The quasi-equilibrium point on Σ_r, $\tilde{\mathbf{x}}_\mathbf{r}$, given by (20) is asymptotically stable.

Proof: The following Lyapunov function is considered, which corresponds to the sum of the kinetic and potential energy of the system on Σ_r:

$$\begin{aligned}
V(\mathbf{x},\tilde{\mathbf{x}}_\mathbf{r}) = \frac{1}{2}\Big[&k_t(x_2 - \tilde{x}_{r,2})^2 + k_{tl}(x_4 - \tilde{x}_{r,4})^2 +\\
&+ k_{tb}(x_6 - \tilde{x}_{r,6})^2 + J_r(x_1 - \tilde{x}_{r,1})^2 + J_p(x_3 - \tilde{x}_{r,3})^2 +\\
&+ J_l(x_5 - \tilde{x}_{r,5})^2 + J_b(x_7 - \tilde{x}_{r,7})^2\Big].
\end{aligned}$$

The derivative of V along the trajectories of (16) is:

$$\dot{V}(\mathbf{x}) = -c_t(\Omega - x_3)^2 - c_{tl}(x_3 - x_5)^2 - c_{tb}(x_5 - x_7)^2.$$

Consequently, $\dot{V}(\mathbf{x}) \leq 0$. Due to the fact that $\dot{V}(\mathbf{x}) = 0$ only for $\mathbf{x} = \tilde{\mathbf{x}}_\mathbf{r}$, by LaSalle's invariance principle [21], $\tilde{\mathbf{x}}_\mathbf{r}$ is asymptotically stable. ■

It is interesting to notice that the quasi-equilibrium point when the system trajectories evolve on Σ_r depends on: 1) downhole characteristics, such as: bit geometric characteristics (R_b), bit-rock contact (friction characteristics), mud drilling viscosity characteristics (c_b); 2) desired velocity (Ω); 3) flexibility of drillstring components (k_t, k_{tl}, k_{tb}).

It is also interesting that according to relation $\sigma_r = 0$ when $t \geq t_s$, the trajectories will tend exponentially to $\tilde{\mathbf{x}}_\mathbf{r}$ with a time depending on $1/\lambda$, in addition to Ω and the bit dynamics, mainly, the bit-rock contact depending on W_{ob}, R_b and friction coefficients. Moreover, $|\sigma_r(\mathbf{x},t_0)|$ depends on λ and Ω. The higher λ or Ω is, the higher $|\sigma_r(\mathbf{x},t_0)|$ is and, consequently, the higher t_s is. Thus, λ and Ω also influence the convergence to the sliding motion. The more the sliding regime on Σ_r is delayed, the more oscillating the system is. Extensive simulations have been carried out, and the controlled system has a good performance for typical operation values of Ω and $0 < \lambda \leq 1$.

V. Simulation results and Implementation issues

A. Results for the controlled system

To illustrate the performance of controller (13)-(11), it will be applied to system (8) with parameters (10). The velocity reference value is considered as $\Omega = 12\,rad/s$, a typical value in drilling operations. Similar results are obtained with other values of Ω.

Due to changes in the function $sgn(\sigma_r)$ when σ_r is next to zero (because of imperfections in the implementation), the chattering problem (very fast switching) can appear. For this undesirable effect to dissappear, the sign function is substituted by a saturation function [14], [15] with the following form:

$$sat(\sigma_r/\delta) = \begin{cases} -1 & \text{if } \sigma_r(\mathbf{x},t) < \delta \\ \frac{\sigma_r}{\delta} & \text{if } |\sigma_r(\mathbf{x},t)| \leq \delta \\ 1 & \text{if } \sigma_r(\mathbf{x},t) > \delta \end{cases} \tag{21}$$

with $\delta > 0$ small enough.

Using (21), relation (12) takes the following form,

$$\dot{\sigma}_r = -\eta\, sat(\sigma_r/\delta), \tag{22}$$

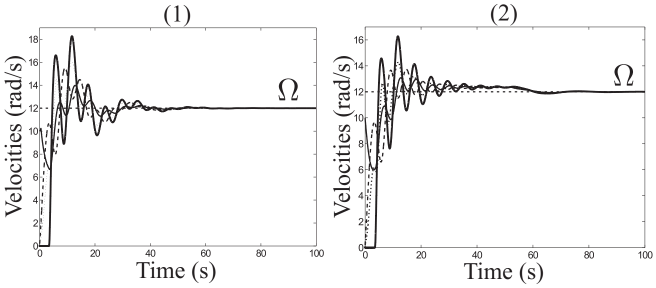

Fig. 3. Angular velocities for the closed-loop system with and without disturbance of controller terms: (1) controller (23)-(11) is applied to system (8), (2) controller (24)-(11) is applied to the system.

and controller (13) is rewritten as,

$$u = c_t(x_1 - x_3) + k_t x_2 + c_r x_1 - \\ - J_r\left[\lambda(x_1 - \Omega) + \lambda(x_1 - x_7) + \eta sat(\sigma_r/\delta)\right]. \tag{23}$$

It can be obtained that the time required for σ_r to be δ is $t_\delta = |\sigma_r(\mathbf{x}, t_0) - \delta|/\eta$ and σ_r will exponentially tend to zero for $t \geq t_\delta$. The dynamics on the sliding surface Σ_r does not change.

Figure 3.(1) shows the elimination of stick-slip oscillations depicted in Fig. 2 when control (23)-(11) is applied to system (8). Parameters (10) are used with $W_{ob} = 97347N$ and $\eta = 3$, $\lambda = 0.3$, $\delta = 0.001$. Notice that the control goal is met in a reasonable time interval.

To show the robustness properties of the controlled system under some parameters variation, instead of control (23), the following control is used,

$$u = \Delta_1 c_t(x_1 - x_3) + \Delta_2 k_t x_2 + \Delta_3 c_r x_1 - \\ - J_r\left[\lambda(x_1 - \Omega) + \lambda(x_1 - x_7) + \eta sat(\sigma_r/\delta)\right], \tag{24}$$

with Δ_i positive constants. Results for $\Delta_1 = 0.7$, $\Delta_2 = \Delta_3 = 0.8$ are shown in Fig. 3.(2). According to typical system parameters values, $0 \leq \Delta_i \leq 1$ covers a reasonable variation range of the parameters appearing in the control.

When (24) is applied to the system, it can be shown that there will still exist a sliding motion on $\sigma_r = 0$ and the control goal will be achieved, if the following relation is met,

$$|\phi(\mathbf{x})| < \eta, \tag{25}$$

with

$$\phi(\mathbf{x}) = (\Delta_1 - 1)\frac{c_t}{J_r}(x_1 - x_3) + (\Delta_2 - 1)\frac{k_t}{J_r}x_2 + (\Delta_3 - 1)\frac{c_r}{J_r}x_1.$$

Condition (25) can be almost always met for not so high values of η and maintaining the torque u low enough.

The control goal is achieved despite W_{ob} variations. In other words, the bit and the top-rotary system will reach the velocity reference value without depending on the bit situation (permanent stuck, stick-slip, or moving in a constant velocity different than Ω). This fact and the robustness under system parameters variations are remarkable properties of the controller proposed. In Figs. 3, a W_{ob} for which the bit presented stick-slip motion has been chosen. Nevertheless, similar results are obtained with other values of W_{ob}.

Figure 4 shows how σ_r behaves and control u changes with different values of η and when the controller parameters are subject to some disturbances. Parameters (10) are used with $W_{ob} = 97347N$, $\lambda = 0.3$, $\delta = 0.001$. The solid line corresponds to the behavior obtained with $\eta = 3$ and without introducing parameters disturbance. The dashed line represents the behavior with parameters disturbance using $\eta = 3$. If η is increased to 10 then the behavior is like the one represented by the solid line. This shows that despite controller parameters disturbance, the system performance is almost the same if η is high enough. It should be pointed out that under the presence of controller parameters disturbances, t_s is increased, however, the system response and control u are very similar to the nominal case (see Figs. 3.(2) and 4.(2)).

From Figs. 3 and 4, it can be also observed that the proposed sliding-mode control has the advantage that with relatively low gains (λ, η), the control goal is achieved with a good system performance. The settling time is approximately 50 seconds, which is acceptable taking into account the drillstring dynamics and that it is better avoiding failures than having smaller settling-time values. Furthermore, during the settling-time interval, the system does not enter the sticking region for a long time; this is not easily obtained with other classical controllers. For instance, consider a PID or PI controller with similar terms to the ones involved in control (13)-(11):

$$u = K_1\int_0^t\left[\Omega - x_1(\tau)\right]d\tau + K_2(\Omega - x_1) + K_3P(\mathbf{x}), \tag{26}$$

with $P(\mathbf{x}) = \int_0^t[x_7(\tau) - x_1(\tau)]\,d\tau$ or $P(\mathbf{x}) = x_7 - x_1$. Fig. 5.(1) shows the velocities of the system controlled by (26) with $K_1 = 30$, $K_2 = 10$, $K_3 = 20$, $W_{ob} = 97347N$. Fig. 5.(2) compares control (26) with control (23). Two disadvantages of the PID control can be observed from this figure. First, the controller gains (K_i) must be high enough for the bit not to enter in a permanent sticking situation. Second, during the settling time, the bit enters in the sticking region for a long time and the rest of velocities can reach negative velocities, which can generate important fatigue and problems along the drillstring components. The stuck-bit region is due to the fact that the control linearly increases until it reaches a value $u = u^*$ (u^* depends on W_{ob}), i.e., until the quasi-equilibrium

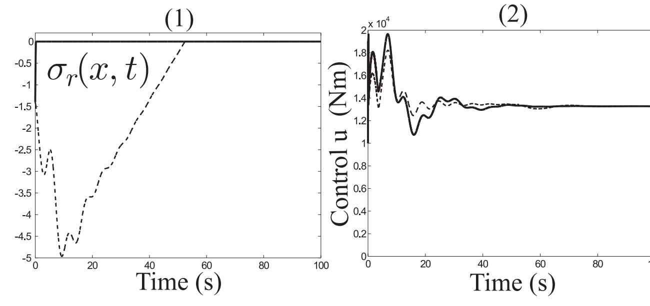

Fig. 4. σ_r and u under controller gains variations and disturbances in controller terms. ━ without parameters disturbance and $\eta = 3$; - - with parameters disturbance and $\eta = 3$.

$\tilde{x}_b$ is located far away from the sliding region $\tilde{\Sigma}_b$.

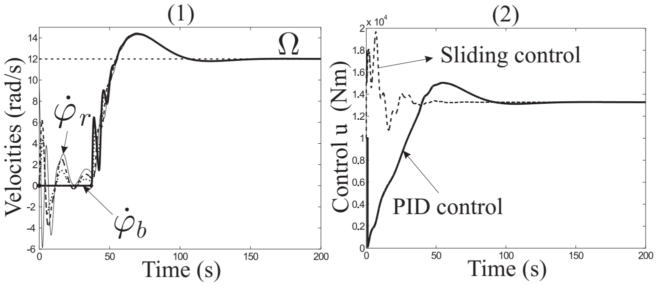

Fig. 5. System (8) response with a PID-type controller (26): (1) angular velocities, (2) PID control compared to the sliding-mode control (23) with $\lambda = 0.3$, $\eta = 3$.

B. In the oilwell: some remarks

Notice that excepting the bit speed, all other quantities needed in the controller can be easily measured or estimated. A key problem in any drilling operation is the measurement of downhole variables and the detection of bit sticking in the surface. There are several methods in order to estimate BHA and bit parameters, see for example the methods used together with TRAFOR system [22] or BHA measurement systems and intrumented bits (*Measure While Drilling* systems) [23].

If such tools were not available, a state estimator could be designed to estimate x_2, x_3 and the bit speed (x_7) based on model (8).

Finally, it must be pointed out that the introduction of a automatic controlled drilling system can be unfeasible due to the complexity of oilwell drillstrings and drilling practices. The proposed controller can be used off-line in order to develop operation recommendations and parameter selection methods to guide the driller to avoid bit sticking problems and to reach the control goal. The model and controller proposed can help the driller to design the well drilling profile with reference values for u, W_{ob} and Ω before starting the operation. For a combination of (W_{ob}, Ω), the torque u would be calculated to prevent non-desired bit phenomena from appearing.

VI. CONCLUSIONS

A dynamical sliding-mode control has been used to eliminate bit sticking phenomena in a multi-DOF system modelling a conventional vertical oilwell drillstring. In the closed-loop system, the angular velocities are driven to a desired reference value in spite of the presence of a dry friction modelling the bit-rock contact. The key idea of the controller is to introduce in the system a sliding surface in which the desired dynamics is accomplished. The control goal is achieved despite WOB variations (a key parameter to any drilling operation) and in the presence of stick-slip oscillations. Robustness under parameters variations has been also shown.

The control methodology proposed could be successfully applied to mechanical systems exhibiting stick-slip oscillations and dry friction described by similar models to the one studied in this work.

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