Sliding-mode Control of Single-phase PWM Rectifier for DC Micro-grid Applications

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Abstract. DC micro-grid is a novel ideal of advanced electricity delivery infrastructure which actively integrates a variety of renewable sources, energy storages, and loads within a local network; this type of lumped system organization can offer a fast, high quality and survivable solution for future residential and industrial power network. DC micro-grids can be the building block of future smart-grid, which performs proper power management for the sources, storages and customers within, and interact with the smart-grid in an optimal manner. AC-DC converter as the only grid-end interface to the utility grid can achieve bi-directional power processing with high power factor and low harmonics, which ensures sustainable and economical operation of the DC micro-grid. In this paper, the rectification mode of the AC-DC interface (PWM rectifier) is studied; sliding mode control is used in PWM rectifier controller design; different sliding surface coordinate functions are evaluated for sliding-mode control in detail based on a derived large signal converter model; finally the sliding-mode controller in conjunction with a traditional PI controller are designed for current loop and voltage loop respectively. The proposed control system is implemented in a TMS320F2812 digital signal controller and tested with a five-port interface to DC micro-grid, satisfactory results are collected.

Keywords: DC micro-grid, Multi-port Power Electronic Interface, PWM Rectifier, sliding-mode control, load sharing

1. Introduction and Background

As the demand for electric power increasing daily, numerous efforts have been dedicated in building and improving current power systems, which are designed to deliver sufficient and reliable power to the customers. In addition to advanced generation technologies, developing new architecture of advanced electricity delivery system is one of the most promising ways to balance the power generation for local survivability. DC micro-grid is one of these advanced infrastructures which can seamlessly integrate different kinds of conventional and renewable energy sources, energy storages and loads within a local power network. Due to the self-evident advantages, micro-grids can be the building blocks for future Smart-Grid [1][2][3][4], harvesting, storing and dispatching high quality power to the local customers.

Environment energies are good supplement to existing fossil fuel based electric power system; however, those renewable sources are suffering from varying input, which can be directly translated into unregulated output voltages/currents. Multiple-input converter is a feasible solution to this problem; with an energy storage and renewable source connected to different input ports, the dynamic as well as steady state power variation can be easily compensated. Considering the facts addressed above, the concept of Multi-port Power Electronic Interface (MPEI) was brought up [5][6][7][8][9][10][11]. As indicated by the five-port implementation in Figure 1. A six-leg structure was used for MPEI which interfaces three renewable sources: fuel cell, solar panel, wind turbine (BLDC+rectifier), battery storage and single phase utility line. As indicated

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in Figure 1, the upper switches for unidirectional conversion (fuel cell, solar and wind) are disabled, only paralleled fast diodes are used. The ports interfacing with battery storage and ac load are both bidirectional.

There are three possible applications of this system: stand-alone AC generation, DC micro-grid by connecting MPEI's DC bus to the grid, AC micro-grid by connecting inverter/rectifier output to the grid. In this paper, we only discuss the opportunities of DC micro-grid integration using MPEI's DC-link as indicated in the figure. Due to the uncertain nature of renewable energy input, energy storage (such as battery) is used to compensate the dynamic fluctuation of input power from DC side. On the AC side, steady power input (usually limited to certain level) is required for balancing the power flow in the DC micro-grid as energy storage has finite capacity. Therefore, in this configuration, the grid-end interface is working under high power factor PWM rectification.

In this paper, the significance of DC micro-grid and its grid-end PWM rectifier are addressed and discussed; state-space technique with switching function is applied to obtain the large signal model of the PWM rectifier; different sliding surface coordinate functions are evaluated for Lyapunov stability; based on the evaluation results, a sliding mode controller is designed to control the rectifier input current, and a conventional PI controller is designed for rectifier output voltage. The rectifier system will be tested with the control algorithm implemented in a digital signal controller and experimental results will be demonstrated.
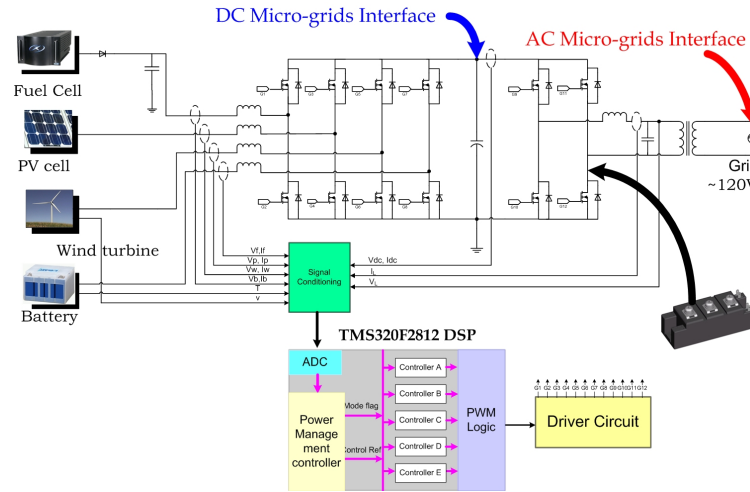

Figure 1. Multi-port Power Electronic Interface for DC&AC micro-grid

2. Modeling of a PWM Rectifier

The dynamics of PWM rectifier can be analyzed using proper switching functions d represents the status of upper switch in one phase leg. During the operation, the switch status on the same leg is complementary and switch on the diagonal position is of the same status.

$$d = \begin{cases} 0 & switch \quad off \\ 1 & switch \quad on \end{cases} \tag{1}$$

Assuming bi-polar PWM, the large signal model of the system can be derived by observing phase-leg terminal voltage/current [12]. The full-bridge can be simplified using small ripple assumption; the equivalent circuit can be modeled as in Figure 2.
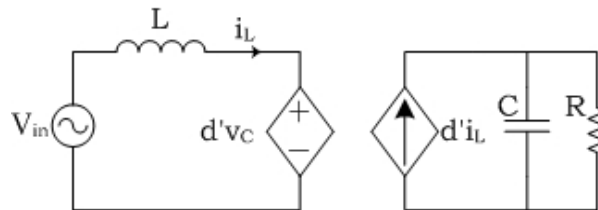

Figure 2. Large signal model of a single-phase PWM rectifier

By applying KCL and KVL and simple algebraic calculation, the large signal model of the single-phase PWM rectifier can be derived, where d is the only control variable as described in Equation (1)

$$\begin{cases} \dfrac{di_L}{dt} = -\dfrac{1-2d}{L} \cdot v_C + \dfrac{V_{in}}{L} \\ \dfrac{dv_C}{dt} = \dfrac{1-2d}{C} \cdot i_L - \dfrac{v_C}{RC} \end{cases} \tag{2}$$

Rewrite state space representation (2) into companion form,

$$\begin{cases} \dot{x} = f(x) + g(x) \cdot u \\ \quad y = h(x) \end{cases} \tag{3}$$

where $[x_1, x_2]=[i_L, v_C]$. $f(x)$ and $g(x)$ can be found as,

$$f(x) = \begin{bmatrix} -\dfrac{x_2}{L} + \dfrac{V_{in}}{L} \\ \dfrac{x_1}{C} - \dfrac{x_2}{RC} \end{bmatrix}, \quad g(x) = \begin{bmatrix} \dfrac{2x_2}{L} \\ -\dfrac{2x_1}{C} \end{bmatrix} \tag{4}$$

3. Sliding Surface Evaluation

Conventional design of PWM rectifier resorts to averaged small signal models on equilibrium operation point, which, however, is more susceptible to other load/source conditions. Sliding mode control can provide relative simple implementation and excellent dynamic response in spite of system operating point and load conditions. Therefore, for such switching circuits, sliding mode controller can be one of the best choices [13][14][15][16][17][18].

The first step toward designing a sliding-mode controller is to search for a sliding surface on which $h(x)=0$ is locally/globally satisfied by introducing proper control input d. Variation of h(x) can be found as,

$$\dot{h}(x) = \frac{\partial h}{\partial x} \cdot [f(x) + g(x) \cdot d] = 0 \Rightarrow$$
$$L_f h(x) + L_g h(x) \cdot d = 0 \tag{5}$$

Consider when sliding surface coordinate function $h(x)>0$ and $L_g h(x)>0$, therefore given $d \in \{0,1\}$, d have to be 0 to eliminate the trend of $h(x)$ to get further away from the sliding surface; in similar case when $h(x)<0$ and $L_g h(x)>0$, d have to be set to 1 to increase the trend of trajectory to pass the sliding surface. Same argument can be justified with we have $L_g h(x)<0$. According the analysis, the following inequality has to hold for sliding mode control to be applied properly to the system,

$$0 < -\frac{L_f h(x)}{L_g h(x)} < 1 \tag{6}$$

To operation the PWM rectifier circuit in Figure 1, one should note that switches on the same leg should not conduct at the same time, in which case the output capacitor will be short-circuited. Operation of diagonal switch will force current/voltage to increase or decrease, which corresponds to state $\{0,1\}$ of control input. By careful inspection, one can find that there is actually only one degree of freedom in control (d). And it is impossible to realize perfect tracking for both input current and output DC voltage. Therefore, it is desirable to check the accessibility of sliding surface $h(x)$ first [18].

Consider a proposed sliding surface coordinate function $h(x) = x_2 - \bar{x}_2$, which indicates the error dynamics of output voltage tracking. This renders,

$$L_f h(x) = \frac{\partial h}{\partial x^T} \cdot f(x) = \frac{x_1}{C} - \frac{x_2}{RC} \tag{7}$$

$$L_g h(x) = \frac{\partial h}{\partial x^T} \cdot g(x) = -\frac{2x_1}{C} \tag{8}$$

The equivalent control input is

$$d = -\frac{L_f h(x)}{L_g h(x)} = \frac{x_1 - \dfrac{x_2}{R}}{2x_1} \in (0,1) \tag{9}$$

where x_2/R represent output current, which is less than input current in this boost-type PWM rectifier.

The zero dynamics of the system can be found by setting x_2 to the steady state value $\bar{x_2}$.

$$\dot{x_1} = -\frac{x_2}{L} + \frac{V_{in}}{L} + \frac{2x_2}{L} \cdot d = \frac{1}{x_1} \cdot (\frac{V_{in}}{L} x_1 - \frac{\bar{x_2}^2}{RL}) \tag{10}$$

Upon choosing the Lyapunov function candidate as $V(x) = \frac{1}{2}(\frac{V_{in}}{L} x_1 - \frac{\bar{x_2}^2}{RL})^2$, we can reach,

$$\dot{V}(x) = \frac{V_{in}}{Lx_1} \cdot (\frac{V_{in}}{L} x_1 - \frac{\bar{x_2}^2}{RL})^2 \geq 0 \tag{11}$$

Therefore, the zero dynamics of the PWM rectifier with output voltage tracking is unstable!

One the other hand, choose the sliding surface coordinate function as $h(x) = x_1 - \bar{x_1}$, which is proposed as direct current control. This leads to

$$L_f h(x) = \frac{\partial h}{\partial x^T} \cdot f(x) = -\frac{x_2}{L} + \frac{V_{in}}{L} \tag{12}$$

$$L_g h(x) = \frac{\partial h}{\partial x^T} \cdot g(x) = \frac{2x_2}{L} \tag{13}$$

the equivalent input is found to be,

$$d = -\frac{L_f h(x)}{L_g h(x)} = \frac{x_2 - V_{in}}{2x_2} \in (0,1) \tag{14}$$

since in a boost-type rectifier, output voltage is always higher than input voltage.

The zero dynamics under the choice of new sliding coordinate function is,

$$\dot{x_2} = \frac{x_1}{C} - \frac{x_x}{RC} - \frac{2x_1}{C} \cdot d = \frac{V_{in}}{Cx_2} \bar{x_1} - \frac{x_2}{RC} \tag{15}$$

we can tell *the* new zero dynamics is a stable one simple by inspection, or pick up a Lyapunov function as $V(x) = \frac{1}{2}(x_2 - \bar{x_2})^2$,

$$\dot{V}(x) = -\frac{1}{RCx_2}(x_2 - \bar{x_2})^2 (x_2 + \bar{x_2}) \leq 0 \tag{16}$$

give the *steady*-state relationship between $\bar{x_1}$ and $\bar{x_2}$,

$$\bar{x_1} = \frac{\bar{x_2}}{RV_{in}} \tag{17}$$

Therefore, choosing the sliding surface coordinate function of $h(x) = x_1 - \bar{x_1}$ (the input current error) render asymptotically stable zero dynamics.

4. Sliding-Mode Controller Design

As desired by economical and survivable operation of DC micro-grid, the main tasks of PWM rectifier are:
- to achieve unity power factor and less harmonics;
- to stabilize DC-link voltage to feed load or downstream converters;
- to control the maximum power transferred into DC micro-grid.

Average-current mode (ACM) control is a control structure with cascaded current and voltage loop, where the outside voltage loop generates a programmed reference for inner current loop. In this case, the current is the directly controlled quantity, which coincides with the conclusion on stability in last section. The voltage is a indirectly controlled quantity, which, however, is still close-loop controlled. Therefore, using the control structure as shown in Figure 3 can meet the above system requirements. Therefore, under sliding mode current control, state x_2 (output DC voltage) exhibit asymptotically stable around equilibrium point $\bar{x_2}$.
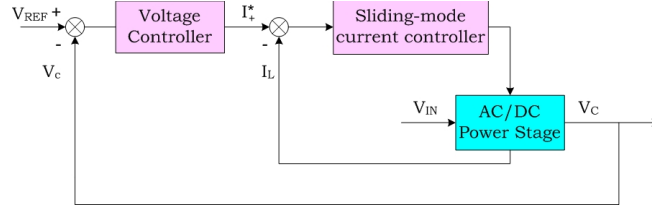
Figure 3. Control system for a single-phase PWM rectifier

The sliding-mode control law for the inner current loop is given as

$$d = \frac{1}{2}\left[sign(x_1 - \overline{x}_1) + 1\right] = \frac{1}{2}\left[sign(i_{error}) + 1\right] \tag{18}$$

The output voltage loop can be simplified given the input current loop is controlled properly. Assume perfect tracking of inner current loop [14], we have the steady state input current as,

$$i_L = I_L \sin(\omega t) \tag{19}$$

The error dynamics of current loop can be found by substitute $i_e = i_L - i_{ref}$ in to Equation (2), we can have,

$$\frac{di_e}{dt} = \frac{di_{ref}}{dt} - \frac{1-2d}{L} \cdot v_C + \frac{V_{in}}{L} \tag{20}$$

then the steady *sate* control input,

$$\overline{d} = \frac{(L\frac{di_L}{dt} - V_{in})\sin(\omega t) + \omega L I_L \cos(\omega t) + v_C}{2v_C} \tag{21}$$

substitute Equation (19) and (20) into (2), we can have reduced dynamic description for voltage loop,

$$\frac{dv_e}{dt} = \frac{-(L\frac{di_L}{dt} - V_{in})\sin(\omega t) - \omega L I_L \cos(\omega t)}{(v_e - V_{ref}) \cdot C} I_L \sin(\omega t)$$
$$- \frac{v_e - V_{ref}}{RC} \tag{21}$$

now the voltage error dynamics shows a highly nonlinear and time varying nature. Usually this kind of system can be solved using feedback linearization. However, the fictitious control input I_L forms quadratic term and the term of multiplication with its own derivation. The dynamic equation is of non-companion-form and undefined *relative* degree. Conventional feedback linearization based on companion form cannot be performed on this system. Try-and-error method is used to find the proper controller.

Since PWM rectifier output has to track a constant DC value, the PI controller is considered first. The proportional gain and integral gain is adjusted so that voltage controller and current controller act with different time constant to variations in order to avoid the interaction between two controllers.

5. Simulation and Experimental Results

Circuit simulation software PSIM is used in this paper. And a 1KW single-phase PWM rectifier with *200VDC* output is modeled. Based on the large signal model mentioned in Equation (2), we can come up with the circuit description mode of the rectifier system as show in Figure. 4(a). The close-loop system with inner sliding-mode current controller and external PI voltage controller is put under simulation as indicated in Figure 4(b). The upper and low band of *0.5A* is set for the current controller; for the voltage controller, let $K_p=8$ and $K_I=80$; the saturation block value is set based on the largest allowable current/power of the proposed converter circuit.
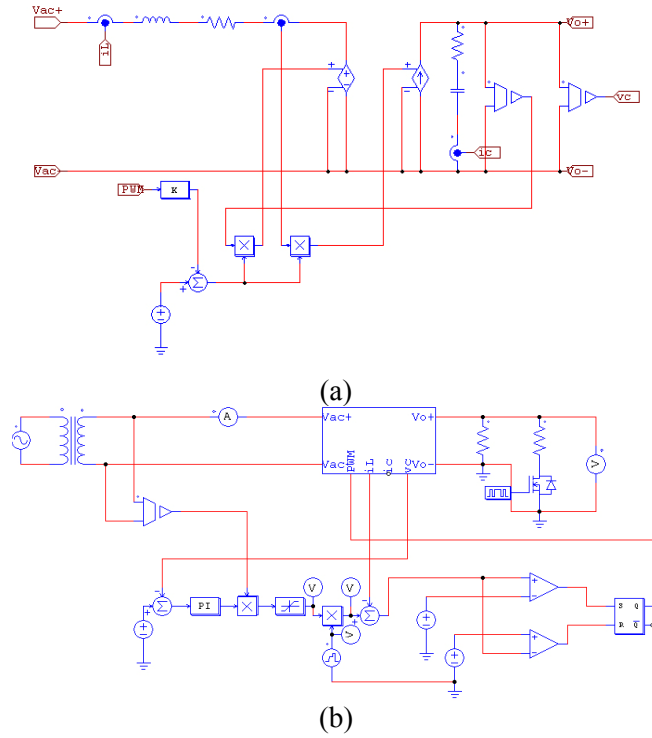
(a)

(b)

Figure 4. Simulation model: (a) large signal model of PWM
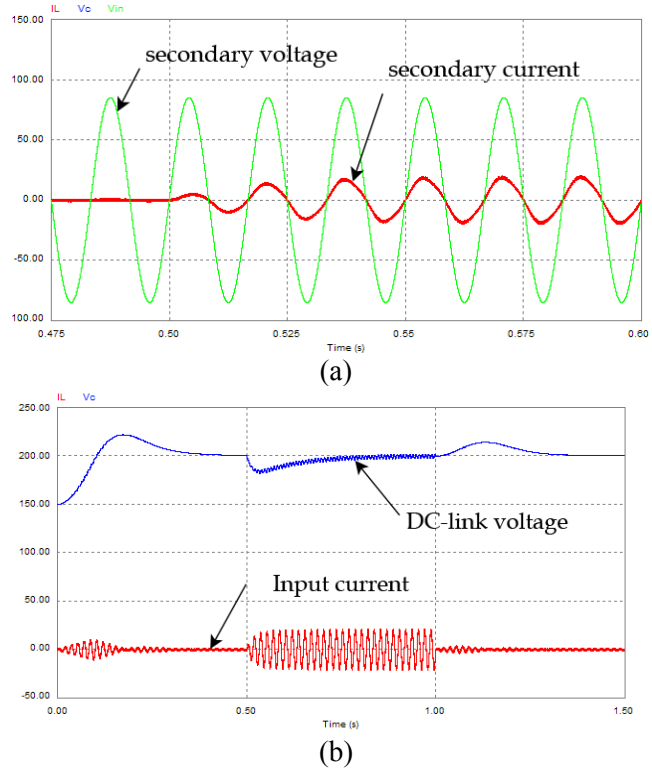rectifier, (b) control system implementation

(a)

(b)

Figure 5. Simulation results: (a) PWM rectifier start-up and
steady state, (b) PWM rectifier dynamic load

The simulation results are presented in Figure. 5. As expected, the voltage loop is sluggish comparing to fast reaction current loop; the DC-link voltage takes about *0.25sec* to settle on a 10% to 100% load step, however, only gives 10% overshoot during start-up phase. During all those transitions, the current is kept in phase with voltage under the sliding-mode controller.

The experimental test is performed on a five-port MPEI system, shown in [10], which takes five different sources/loads. The bus voltage for DC micro-grid system is chosen as *100VDC* considering the proper step-up ratio for the boost converters. Figure 6(a) shows the steady state operation of the PWM rectifier; during the steady state operation, the input current is tracking the input voltage closely and injecting controlled power into DC micro-grid bus. Figure 6(b) is a step-load test; in this test the PWM rectifier works as an inverter to supply an island load, solar, wind and battery are injecting power to the dc bus as the power sources; when a step load demand appears, the battery takes over the transient dynamics, maximum power tracking routines for solar and wind turbine are barely affected.
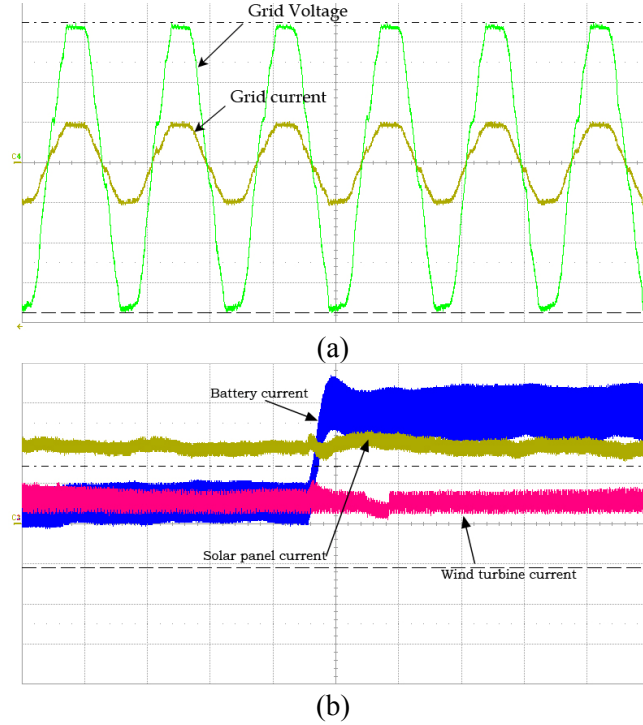

(a)

(b)

Figure 6. Experimental results: (a) Grid-tie voltage and current (Ch1:50V/div, Ch4:2A/div, 10msec/div, digital filtered), (b) Step load test with solar, wind and battery load sharing (Ch1:5A/div filtered, Ch2: 2A/div, Ch3:2A/div, 2sec/div)

6. Conclusion

PWM rectifier as the grid-end interface can provide high quality power balancing for DC micro-grid. In this paper, the significance of DC micro-grid and its grid-end PWM rectifier are addressed and discussed in detail; based on the small ripple assumption, a large signal model for the PWM rectifier is obtained; different sliding surface coordinate functions are evaluated and proved strictly for Lyapunov stability for controller design purpose; a sliding-mode controller is then designed to control the rectifier input current, and a conventional PI controller is designed for rectifier output voltage. The rectifier model and the control system are both implemented in simulation software and DSC hardware; the simulation and experimental results demonstrate the feasibility of the design.

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