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Sliding Mode Control of the Systems with Uncertain Direction of Control Vector

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Abstract

The Sliding Mode Control approach is used for a class of nonlinear systems

$$\dot{x} = f(t, x) + B(t, x)u$$

with uncertain matrix B. The control design is based on the partition of the extended system state space onto the cells with fixed control vector inside each cell. That results in multiple stabile equilibriums for the extended system. In general, these points are different for different values of B. Each equilibrium corresponds to the stability of the origin for the given system. If areas of attraction for the multiple stability points covers the entire state space the proposed control allows to stabilize the system even if the direction of the actuation is unknown.

I. Introduction

Traditionally the robustness properties of the control systems with sliding modes are exploited with respect to the additive perturbations. For a nonlinear system

$$\dot{x} = f(t, x) + B(t, x)[u + g(t, x)] \tag{1}$$

the sliding mode control provides the closed loop system invariant to the unknown function g, where g represents external disturbances or the model uncertainities. There are many practical control problems, where the disturbance influences the direction of the control actuation, i.e. the matrix $B(t, x)$. It is known [1] that the sliding mode control algorithms are robust with respect to variations ΔB if they are small enough so that the sliding mode existence condition is not violated. But it is not the case for large deviations of B, if, for example, it changes its direction for the opposite.

The paper presents a globally convergent sliding-mode control design for systems with uncertain direction of the control vector. The approach is based on the use of the periodic switching function. As a result the extended state space is partitioned onto cells with sliding manifolds as their boundaries. This construction allows to keep the system on the desired manifold even if the actuation direction is unknown and changes during the control process.

II. Problem Formulation

The general formulation of the problem is the following: for a nonlinear system

$$\dot{x} = f(t, x) + B(t, x)u, \tag{2}$$

where $x \in R^n$, $u \in R^m$, to design robust stabilizing control algorithm, which does not require the knowledge of the matrix $B(t, x)$. As it is always done in sliding mode control design, it is assumed that the objective is to steer the state of (2) to the manifold

$$\mathcal{M} = \{x \in R^n | S(x) = 0\}, \tag{3}$$

where $S(x) = col(s_1(x), \ldots, s_m(x))$ is a smooth function such that the system (2) constrained on the manifold $\mathcal{M}$ is stable.

As an example of the systems with variable direction of the control variable we can mention the torque control problem in the electric drives. The equations of many types of electric drives can be written as

$$\dot{I} = F(I, n) + B(\theta)u \tag{4}$$

$$J\dot{n} = T(I) \tag{5}$$

$$\dot{\theta} = n \tag{6}$$

where I is a vector of current, T is a torque, n is angular velocity and θ a position of the rotor, u is phase voltage vector, which is a control. For successfull implementation of the existing sliding-mode control laws [1, 3] the on-line values of $B(\theta)$, and therefore, θ are needed. But since the regulated torque variable T depends only on the current I, it seems possible to have the control algorithm not requiring θ for implementation, which will allow to design systems without expensive position sensors.

The other examles are mechanical systems in robotics, where the control variable is the magnitude of the applied force, while its direction depends on the positions, velocities and different external factors. In such systems, sometimes, it is undesirable or even impossible to use the information on the direction of this vector for control design, and it is preferred to have the control scheme, where these measurements are not needed.

III. The Control Design

The main idea of the control design is in partitioning the $\tilde{S}$-subspace ($\tilde{S} = col(\tilde{s}_1, \ldots, \tilde{s}_r) \in R^r$) of the extended system onto the cells with smooth boundaries. In a particular case, they may form an ε-grid

$$\mathcal{G} = \bigcup_{i=1}^{r} \bigcup_{k=0,\pm1,\ldots} \{\tilde{s}_i = \varepsilon k\}. \qquad (7)$$

Inside each cell the control is constant. Alternating control values along the grid, allows to obtain a set of stability points P_{st} for any B, under a nonsingularity condition. In contrast with traditional case, where the goal is $S(x) = 0$, the sliding mode will occur on $\tilde{S}(x) = const$. The steady state error, appearing, can be easily removed by using the dynamic compensator. The compensators based on sliding modes provide finite-time convergence of $S(x)$ to the origin.

In case of scalar control the uniform grid corresponds to the periodic switching function.

Consider the system:

$$\dot{x} = f(t,x) + b^T(t,x)u, \qquad (8)$$

where $u \in R^1$, $\mathcal{M} = \{x|s(x) = 0\}$ is a desired manifold $s(x) \in R^1$ and $b(t,x) = col(b_1(t,x), \ldots, b_m(t,x)$ is unknown. Let

$$G(x) = \frac{\partial s(x)}{\partial x} \qquad (9)$$

then

$$\dot{s} = G(x)f(t,x) + G(x)b^T(t,x)u. \qquad (10)$$

To obtain the sliding on the manifold $\mathcal{M}$ the control with periodic switcing function [2] is used

$$u = M_0 sgn \sin\left[\frac{\pi}{\varepsilon}(s(t) + \lambda\int_0^t sgn(s(\tau))d\tau)\right], \qquad (11)$$

where $\lambda > 0$.

Let

$$\sigma = s(t) + \lambda\int_0^t sgn(s(\tau))d\tau \qquad (12)$$

then

$$\dot{\sigma} = Gf + Gb^T M_0 sgn\left[\sin\left(\frac{\pi}{\varepsilon}\sigma\right)\right] + \lambda sgn(s). \qquad (13)$$

The second term in the right hand side of (13) is piecewise constant. In the neighbourhoods of the points

$$\sigma = k\varepsilon \qquad (14)$$

for even $k = 0, \pm2, \pm4, \ldots$ it has the form:

$$sgn\left[\sin\left(\frac{\pi}{\varepsilon}\sigma\right)\right] = sgn(\sigma - k\varepsilon), \qquad (15)$$

for odd $k = \pm1, \pm3, \ldots$

$$sgn\left[\sin\left(\frac{\pi}{\varepsilon}\sigma\right)\right] = -sgn(\sigma - k\varepsilon). \qquad (16)$$

Therefore, if the condition

$$|G(x)b^T(t,x)M_0| > |G(x)f(t,x)| + \lambda \qquad (17)$$

is fulfilled, the sliding occurs on one of the manifolds

$$\sigma = k\varepsilon \qquad (18)$$

for any sign of $G(x)b^T(t,x)M_0$.

The equation of the system in sliding mode can be obtaned by differentiating (18) using (12):

$$\dot{s} = -\lambda sgn(s). \qquad (19)$$

Therefore, the manifold $\mathcal{M} = \{x|s(x) = 0\}$ is reached in finite time interval. During the sliding motion the disturbance rejection property is preserved.

The control law (11) doesnot require the knowledge of $sign[G(x)b^T(t,x)]$. This sign can be different in different parts of the state space, which means that the system goes from one sliding manifold to another. But since the distance between the manifolds - ε can be chosen arbitrary small, and under the condition that $G(x)b^T(t,x) = 0$ does not coincide with the desired manifold, the equality (19) is violated only for a short period of time (it tends to zero when $\varepsilon \to 0$).

References

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