

## Slow Light in Degenerate Fermi Gases

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We investigate the effect of slow light propagating in a degenerate atomic Fermi gas. In particular we use slow light with an orbital angular momentum. We present a microscopic theory for the interplay between light and matter and show how the slow light can provide an effective magnetic field acting on the electrically neutral fermions, a direct analogy of the free electron gas in a uniform magnetic field. As an example we illustrate how the corresponding de Haas–van Alphen effect can be seen in a gas of neutral atomic fermions.

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The recent advances in trapping and cooling atoms has provided an excellent starting point for studying many different types of physical phenomena, ranging from fundamental atomic physics to cosmological aspects [1]. In this respect atomic Bose-Einstein condensates have attracted a lot of interest [2]. Recently several experimental groups have succeeded in trapping and cooling also fermions [3,4] well below the Fermi temperature. Fermi systems are well known from the study of electron properties in materials. Trapped atomic fermions are electrically neutral and a direct analogy between the magnetic properties of these systems and solid state phenomena is not necessarily straightforward. We suggest this problem can be circumvented if the properties of slow light is used, i.e., light with a group velocity as low as meters per second [5–7]. The coupling between the slow light and the atoms can give rise to some remarkable effects such as dragging of the light [8–10] and complete coherent freezing of the pulse [11–13]. In a similar manner the slow light should affect the atomic motion.

In this Letter we investigate the influence of slow light on the mechanical properties of a degenerate Fermi gas of atoms. The theory is fully microscopic and based on the explicit analysis of the quantum dynamics of atomic fermions coupled to the electromagnetic field. In particular we use slow light with an orbital angular momentum [14,15]. This allows us to introduce an effective magnetic field which acts on the electrically neutral fermions. As such we have a typical, often regarded as an academic, textbook scenario with free electrons moving in a constant magnetic field. This opens up the possibility to study phenomena well known from solid state and condensed matter physics, with all the benefits given by the trapped atoms where a range of experimental parameters such as atom-atom interactions, particle numbers, the shape of the trapping potential, etc., can easily be manipulated. In addition, using light as the effective magnetic field is going to be favorable since it is rather difficult to control real magnetic fields. As an example we show how the de Haas–van Alphen effect is obtained in a neutral cloud of fermions. Finally, we conclude by discussing some other

aspects and possibilities with slow light in degenerate quantum gases.

Light can be slowed down [5–7] by using the properties of electromagnetically induced transparency (EIT) [16–19], in which the group velocity of the light,  $v_g$ , is reduced by applying another beam, called the control beam. The beams act on the  $\Lambda$ -type atoms characterized by two hyperfine ground levels 1 and 2, as well as an electronic excited level 3, as depicted in Fig. 1. Initially the atoms occupy the lowest level 1.

The atoms are described in terms of the fermionic field operators  $\Psi_j(\mathbf{r}, t)$  representing the second-quantized wave function for the translational motion of atoms in the  $j$ th electronic state, with  $j = 1, 2, 3$ . The operator  $\Psi_j(\mathbf{r}, t)$  annihilates an atom positioned at  $\mathbf{r}$  characterized by the internal state  $j$ . In what follows, the spatial and temporal variables will be kept implicit in  $\Psi_j(\mathbf{r}, t) \equiv \Psi_j$ .

The atoms interact with two laser beams: A strong control laser drives the transition  $|2\rangle \rightarrow |3\rangle$ , whereas a weaker probe field is associated with the transition  $|1\rangle \rightarrow |3\rangle$  (see Fig. 1). The control laser has a frequency  $\omega_c$ , a wave vector  $\mathbf{k}_c$ , and a Rabi frequency  $\Omega_c = \Omega_c^{(0)} \exp(i\mathbf{k}_c \cdot \mathbf{r})$ , where  $\Omega_c^{(0)}$  is a slowly varying amplitude. The probe field, on the other hand, is characterized by a central frequency  $\omega_p = ck_p$ , a wave vector  $\mathbf{k}_p = k_p \hat{\mathbf{z}}$ , and a Rabi frequency

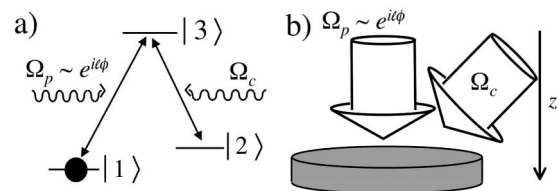


FIG. 1. (a) The level scheme for the electromagnetically induced transparency with the probe beam  $\Omega_p$  and control beam  $\Omega_c$ . (b) Schematic representation of the experimental setup with the two light beams incident on the cloud of atoms. The probe beam propagates in the  $z$  direction. The control beam can propagate parallel [12,13], perpendicular [5], or antiparallel to the probe beam.

$$\Omega_p = \Omega_p^{(0)} e^{i(\ell\phi + \mathbf{k}_p \cdot \mathbf{r})}, \quad (1)$$

where  $\Omega_p^{(0)}$  is a slowly varying amplitude. In writing Eq. (1) we have allowed the probe photons to have an orbital angular momentum  $\hbar\ell$  along the  $\mathbf{z}$  axis [14,15].

Introducing the slowly varying atomic field-operators  $\Phi_1 = \Psi_1 e^{i\omega_1 t}$ ,  $\Phi_3 = \Psi_3 e^{i(\omega_1 + \omega_p)t}$ , and  $\Phi_2 = \Psi_2 e^{i(\omega_1 + \omega_p - \omega_c)t}$ , and adopting the rotating wave approximation, one can write the following equations of motion:

$$i\hbar\dot{\Phi}_1 = -\frac{\hbar^2}{2m}\nabla^2\Phi_1 + V_1(\mathbf{r})\Phi_1 + \hbar\Omega_p^*\Phi_3, \quad (2)$$

$$i\hbar\dot{\Phi}_3 = \left(\epsilon_{31} - \frac{\hbar^2}{2m}\nabla^2\right)\Phi_3 + V_3(\mathbf{r})\Phi_3 + \hbar\Omega_c\Phi_2 + \hbar\Omega_p\Phi_1, \quad (3)$$

$$i\hbar\dot{\Phi}_2 = \left(\epsilon_{21} - \frac{\hbar^2}{2m}\nabla^2\right)\Phi_2 + V_2(\mathbf{r})\Phi_2 + \hbar\Omega_c^*\Phi_3, \quad (4)$$

where  $m$  is the atomic mass,  $V_j(\mathbf{r})$  is the trapping potential for an atom in the electronic state  $j$ ,  $\epsilon_{21} = \hbar(\omega_2 - \omega_1 + \omega_c - \omega_p)$ , and  $\epsilon_{31} = \hbar(\omega_3 - \omega_1 - \omega_p)$  are, respectively, the energies of the detuning from the two- and single-photon resonances,  $\hbar\omega_j$  being the electronic energy of the atomic level  $j$ .

It is noteworthy that the dissipation of the excited electronic state can be included into Eq. (3) replacing  $\epsilon_{31}$  by  $\epsilon_{31} - i\hbar\gamma_{31}$  and adding the appropriate noise operator. The second hyperfine state  $|2\rangle$  has usually a small decay rate  $\gamma_{21}$  which can therefore be omitted in the corresponding Eq. (4). Note also that the equations of motion (2)–(4) do not accommodate collisions between the ground-state atoms. This is legitimate for the degenerate Fermi gas in which  $s$ -wave scattering is forbidden and only weak  $p$ -wave scattering is present [3,20–22].

Since the probe field is much weaker than the control field ( $|\Omega_p| \ll |\Omega_c|$ ), depletion of the ground-state atoms is small. Furthermore, we assume that the two-photon detuning  $\epsilon_{21}$  is sufficiently small. Neglecting the terms with  $\Phi_3$ ,  $\nabla^2\Phi_3$ , and  $\dot{\Phi}_3$  in Eq. (3), one arrives at the adiabatic condition [16–19] relating  $\Phi_2$  to  $\Phi_1$  as

$$\Phi_2(\mathbf{r}, t) = -\zeta\Phi_1(\mathbf{r}, t), \quad (5)$$

where  $\zeta \equiv \zeta(\mathbf{r}) = \Omega_p/\Omega_c$ . The condition (5) implies that the control and probe beams have driven the atoms to the dark state  $|1\rangle - \zeta|2\rangle$  representing a special superposition between the two hyperfine ground states [16–19]. If the atoms are in the dark state, the resonant control and probe beams cannot populate the upper atomic level 3, as the two beams contribute destructively to the absorption process. This justifies neglecting the decay of the upper atomic level 3 in the equation of motion (3).

Equation (5) shows that the orbital angular momentum  $\hbar\ell$  of the probe field  $\Omega_p \sim e^{i\ell\phi}$  is transferred into the orbital angular momentum of the center of mass motion

for atoms occupying level 2. This goes along with a general rule saying that the exchange of the orbital angular momentum in the electric dipole approximation occurs exclusively between the light and the atomic center of mass motion [23]. The rule has been implicitly assumed in the initial equations of motion (2)–(4) containing no contributions due to exchange in the orbital angular momentum between the internal atomic states and the center of mass motion.

Consider now the influence of the slow light on the dynamics of the ground state atoms. Using Eqs. (4) and (5) one has:

$$\Phi_3(\mathbf{r}, t) = -\frac{1}{\hbar\Omega_c^*} \left[ \frac{\hbar^2}{2m}\nabla^2 + i\hbar\frac{\partial}{\partial t} - \epsilon_{21} - V_2(\mathbf{r}) \right] (\zeta\Phi_1). \quad (6)$$

The relationships (2) and (6) provide the following equation of motion for the field operator  $\Phi_1(\mathbf{r})$ ,

$$i\hbar\dot{\Phi}_1 = \frac{1}{2m} [i\hbar\nabla + \mathbf{A}_{\text{eff}}]^2 \Phi_1 + V_{\text{eff}}(\mathbf{r})\Phi_1, \quad (7)$$

where

$$\mathbf{A}_{\text{eff}}(\mathbf{r}) = i\hbar\zeta^*\nabla\zeta \equiv -\hbar|\zeta|^2\nabla S + i\frac{\hbar}{2}\nabla|\zeta|^2 \quad (8)$$

and

$$V_{\text{eff}}(\mathbf{r}) = V_1(\mathbf{r}) + (|\zeta|^{-2} - 2)\frac{|\mathbf{A}_{\text{eff}}|^2}{2m} + \hbar\omega_{21}|\zeta|^2 \quad (9)$$

are the effective vector and trapping potentials, with  $\hbar\omega_{21} = \epsilon_{21} + V_2(\mathbf{r}) - V_1(\mathbf{r})$ . Here the dimensionless function  $\zeta = e^{iS}\Omega_p^{(0)}/\Omega_c^{(0)}$  is characterized by a phase  $S = (\mathbf{k}_p - \mathbf{k}_c) \cdot \mathbf{r} + \ell\phi$ . In writing Eqs. (7)–(9), we made use of the assumption  $|\zeta|^2 \ll 1$ . Note that such an assumption is not essential in deriving Eqs. (7)–(9). By relaxing the condition  $|\zeta|^2 \ll 1$ , one arrives at  $\mathbf{A}_{\text{eff}} = i\hbar(1 + |\zeta|^2)^{-1}\zeta^*\nabla\zeta$ . In such a situation,  $V_{\text{eff}}$  also experiences modifications. This effect will be explored elsewhere.

It is instructive to note that  $\mathbf{A}_{\text{eff}}$  is generally non-Hermitian. The Hermitian contribution is due to the changes in the phase of  $\zeta$ , the non-Hermitian one being due to the changes in the amplitude. The non-Hermitian part of  $\mathbf{A}_{\text{eff}}$  can be eliminated by a pseudogauge transformation  $\Phi_1 = \Phi_1^{(0)} \exp[-|\zeta|^2/2]$ , where the transformed operator  $\Phi_1^{(0)}$  undergoes a unitary evolution. Since  $|\zeta|^2 \ll 1$ , one can neglect the small changes in the amplitude of  $\Phi_1$  making the operator  $\mathbf{A}_{\text{eff}}$  Hermitian. Note also that the probe field  $\Omega_p$  is considered to be an incident quantity not affected by the induced motion of the ground-state fermions. Consequently the probe field  $\Omega_p$  undergoes a usual propagation at a group velocity  $v_g \sim |\Omega_c|^2$  [16–19] in the  $z$  direction.

In this way, we can create an effective vector potential through the phase  $S$  of the incoming probe beam. The

experimental situation is schematically described in Fig. 1 where the incoming probe beam is of the form  $e^{i\ell\phi}$ . Suppose that the intensity of the control field does not vary considerably within the atomic cloud. If we consider copropagating control and probe beams and choose the intensity of the probe beam of the form  $|\Omega_p|^2 \sim r^2$  in the transversal plane, we obtain the effective vector potential

$$\mathbf{A}_{\text{eff}} \equiv -\hbar\ell|\zeta|^2\nabla\phi = \frac{\hbar\ell\alpha_0}{R^2}(y\hat{e}_x - x\hat{e}_y), \quad (10)$$

where  $\alpha_0 = |\zeta|^2 R^2/r^2$  is a ratio (typically  $\alpha_0 < 0.1$ ) between the intensities of the probe and the control beam at radius  $R$  of a cylinder in which the gas is contained. Such an external trap can be created by, for instance, high order Bessel beams [24,25]. It is interesting to note here that with this choice of light the effective vector potential (10) corresponds to a constant magnetic field in the direction opposite to the  $z$  axis, since we have the relation

$$\mathbf{B}_{\text{eff}} = \nabla \times \mathbf{A}_{\text{eff}} = -\frac{2\hbar\ell\alpha_0}{R^2}\hat{e}_z. \quad (11)$$

The strength of the effective magnetic field is given by the orbital angular momentum of the light and can be controlled by applying suitable phase and intensity holograms [15]. It is relatively straightforward to create and control high angular momenta of the order of several hundred  $\ell$  which consequently controls the effective magnetic field.

Equation (7) describes the effective dynamics of trapped noninteracting atoms obeying the Fermi-Dirac statistics. Substituting  $\Psi_1 = \Psi e^{iEt/\hbar}$  leads to the following eigenequation for the one particle solution  $\Psi$ :

$$\frac{\hbar^2}{2m} \left[ -\nabla^2 + \left( \frac{\ell\alpha_0}{R^2} \right)^2 r^2 + 2i \left( \frac{\ell\alpha_0}{R^2} \right) \partial_\phi \right] \Psi = E\Psi, \quad (12)$$

where the external trap  $V_1(r)$  is chosen such that  $V_{\text{eff}} = 0$ . After rescaling the radial coordinate,  $r = xR$ , and using the ansatz  $\Psi = \xi(r)e^{iq\phi}e^{ik_z z}$  we obtain the solution in the form of a confluent hypergeometric function

$$\xi(x) = x^{|q|} e^{-(\ell|\alpha_0/2)x^2} {}_1F_1 \left[ \frac{1+|q|}{2} - \left( \frac{\epsilon}{4|\ell|\alpha_0} + \frac{q}{2} \right), |q| + 1; |\ell|\alpha_0 x^2 \right], \quad (13)$$

$$F = N\eta - A \left( \frac{\epsilon_0^{3/2}}{24} + \frac{\pi^2}{3072} \frac{\theta^2}{\sqrt{\epsilon_0}} + \frac{4}{35} \epsilon_0^{7/2} + \frac{\pi^2}{96} \epsilon_0^{3/2} \theta^2 - \theta \frac{3}{4\pi} \sum_{s=1}^{\infty} \frac{\sqrt{s} \cos[\pi\epsilon_0 s - \frac{\pi}{4}] - \frac{\lambda}{\sqrt{s}} \cos[\pi\epsilon_0 s - \frac{3}{4}\pi]}{(-1)^s s^2 \sinh[2\pi^2 \theta s]} \right), \quad (17)$$

where  $A = (\sqrt{2m}/\hbar)(L_z/3\pi)(\hbar\Omega)^{3/2}$ ,  $\lambda = (2^{9/2}5/19\pi^{3/2})$  and  $\hbar\Omega = \hbar^2|\ell|\alpha_0/2mR^2$ . Using the free energy it is straightforward to calculate thermodynamic properties such as the specific heat

$$C = -T \frac{\partial^2 F}{\partial T^2}, \quad (18)$$

see Fig. 2, which consequently has an oscillating term as

where  $\epsilon = [E - (\hbar^2 k_z^2/2m)](2mR^2/\hbar^2)$ . The result presented in Eq. (13) differs from the standard Landau problem [26] for free electrons in a homogenous magnetic field in the sense that in our case we have to take into account the boundary conditions at  $r = R$  and the fact that with a particular choice of light beam we choose the form of the vector potential whereas in the Landau case the vector potential is not uniquely defined. It is interesting to note at this point the asymmetry induced by the vector potential. This is clearly seen in Eqs. (12) and (13) where the energy eigenvalue is shifted by  $2\ell\alpha_0 q$ . As such, Eq. (13) is rather intractable. We can, however, obtain analytical expressions for the eigenvalues in the limit  $|\ell|\alpha_0 \gg 1$ , where the energies are of the form

$$\epsilon_{n,q} = 2|\ell|\alpha_0(2n + |q| + q + 1), \quad (14)$$

with  $n = 0, 1, 2, \dots$  and  $q = \dots - 2, -1, 0, 1, 2, \dots$ . This is indeed the Landau result, since for  $|\ell|\alpha_0 \gg 1$  the boundary conditions play a less important role.

With the discrete energy levels given by Eq. (14) we are now in a position to calculate thermodynamic potentials, such as the free energy, as a function of the effective magnetic field. The free energy is defined as

$$F = N\eta - \int \frac{Z(E)}{e^{(E-\eta)/kT} + 1} dE, \quad (15)$$

where  $Z(E)$  is the number of states with energies less than  $E$ ,  $\eta$  is the chemical potential,  $N$  the number of atoms,  $k$  the Boltzmann constant, and  $T$  the temperature. In order to proceed we need an expression for the function  $Z(E)$  using the discrete energy levels in Eq. (14). In addition we assume a continuous atomic spectrum in  $z$  direction. The resulting function is

$$Z(E) = \frac{\sqrt{2m} L_z}{\hbar} \frac{1}{2\pi} \sum_{n,q} [E - \epsilon_{n,q}]^{1/2}, \quad (16)$$

where the sum is over  $n$  and  $q$  such that  $Z(E)$  is real and  $L_z$  is the length of the cloud in  $z$  direction. In the limit of  $\epsilon_0 = (\eta/\hbar\Omega) \gg 1$  and with the rescaled temperature  $\theta = (kT/\hbar\Omega)$  we calculate, after carefully examining the sum in Eq. (16) [27], the free energy which becomes

well. From Eq. (17) we see that for  $\theta \ll 1$  the specific heat is linear in  $\theta$  whereas the oscillating behavior stems from the  $1/\hbar\Omega$  dependence in the cos term.

In order to have a dominating oscillating part  $\theta$  must not be too large,  $\theta \lesssim 1$ , otherwise the oscillating term is exponentially damped. With present experimental cooling and trapping techniques temperatures of the order

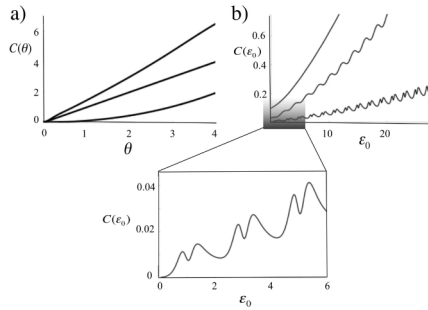


FIG. 2. (a) The specific heat as a function of the rescaled temperature  $\theta$ . The lowest line corresponds to  $\varepsilon_0 = 1$ , middle one to  $\varepsilon_0 = 10$ , and the top one to  $\varepsilon_0 = 30$ . (b) The specific heat as a function of the inverse effective magnetic field,  $\varepsilon_0 = \eta/\hbar\Omega$  where the three curves correspond to  $\theta = 0.1, 0.5$ , and  $1.0$  (top one). The inset shows a magnification of the region for small  $\varepsilon_0$  and for  $\theta = 0.1$ .

of  $\theta/\varepsilon_0 = T/T_F \sim 0.1$  are readily achievable. Hence an  $\varepsilon_0 = \eta/\hbar\Omega$  of the order of 1 would be preferable. For a homogeneous cloud in a cylindrical trap  $\varepsilon_0$  is given by  $\varepsilon_0 = [N(R/L_z)3\pi]^{2/3}/(\alpha_0\ell)$  which can become small if one considers a large aspect ratio trap and taking into account the fact that the term  $\alpha_0\ell$  can reach large values of the order of 100. In Fig. 2 we show the specific heat calculated from Eq. (18) where  $C$  is in units of  $k(\sqrt{2m\hbar\Omega}/\hbar)(L_z/2\pi)$ . In calculating  $C$  we used the exact Eq. (15) for  $F$  to show the full range of the  $\varepsilon_0$  dependence.

In this Letter we have shown how light with an orbital angular momentum can be used to create an effective magnetic field in a degenerate gas of electrically neutral atomic fermions. As an example of how the slow light can be used we calculated the free energy for the trapped degenerate fermions and found that the atomic gas shows a de Haas-van Alphen type behavior where oscillations in thermodynamic properties depend on the inverse effective magnetic field strength. There are other intriguing phenomena such as the quantum Hall effect which can be studied using cold fermionic gases and slow light with an angular momentum. In addition, if the collisional interaction between the atoms is taken into account slow light can be used to study the magnetic properties of a superfluid atomic Fermi gas [28]. Recent advances in spatial light modulator technology enables us to consider rather exotic light beams [29]. This will allow us to study the effect of different forms of vector potentials in quantum gases. In particular, the combined dynamical system of light and matter could give important insight into gauge theories in general. It is certainly tempting to push the analogy further and study phenomena from high energy physics in ultracold samples of atoms.

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