Slow relaxation and compaction of granular systems

Granular materials are of substantial importance in many industrial and natural processes, yet their complex behaviours, ranging from mechanical properties of static packing to their dynamics, rheology and instabilities, are still poorly understood. Here we focus on the dynamics of compaction and its 'jamming' phenomena, outlining recent statistical mechanics approaches to describe it and their deep correspondence with thermal systems such as glass formers. In fact, granular media are often presented as ideal systems for studying complex relaxation towards equilibrium. Granular compaction is defined as an increase of the bulk density of a granular medium submitted to mechanical perturbation. This phenomenon, relevant in many industrial processes and widely studied by the soil mechanics community, is simple enough to be fully investigated and yet reveals all the complex nature of granular dynamics, attracting considerable attention in a broad range of disciplines ranging from chemical to physical sciences.

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Granular materials are ubiquitous in nature and are the second-most manipulated material in industry (the first one is water), encountered, for instance, in technological applications ranging from pharmaceutical, food, powders, mechanosynthesis and semiconductor industries up to geological granular flows, such as debris or pyroclastic flows and rock avalanches. Granular media, such as powders or sand, are very simple, made of discrete particles of size larger than 100 μm often interacting with each other only through dissipative contact forces. Without an external drive their kinetic energy is rapidly lost and they are thus referred to as non-thermal systems. Despite this deceptive simplicity, granular matter exhibits many complex behaviours, such as size segregation, formation of arches, convection rolls, pattern formation and dynamical instabilities[1–6]. Thus, although individual grains are solid, it is inappropriate to classify their collective properties as entirely solid-like or liquid-like. Currently, no rheological laws exist for such materials and their industrial manipulations are mainly based on empirical observations.

This progress article deals with a simple, yet fundamental, phenomenon in the physics of granular materials: granular compaction, that is, the fact that, for instance, under gentle shaking the packing of grains in a vessel slowly gets more and more compact. Compaction is related to both practical and basic scientific problems, as the quest for efficient packing or the investigation of fundamental theories to describe and predict general properties of granular packs and their dynamics. Compaction has been studied in different configurations, such as uniaxial or triaxial compression, which, for instance, are known to be less efficient than compaction under gentle mechanical excitation, such as tapping, shaking and shearing. Here, we focus on 'soft' compaction, that is, compaction without sintering or crushing, from the physicist's point of view, mainly restricting ourselves to the tapping and shearing mode.

What are the compaction mechanisms of grains subject to mechanical perturbations? What is the nature of compaction dynamics? Is it possible to predict the status attained by the system from a few control parameters? Here we discuss these issues, which, as a broader theoretical picture is emerging, have deep relevance in applications ranging from geophysics, to chemistry up to particle technologies[1,2,5].

In the next section we overview experimental results on relaxation laws, annealing processes and memory effects in granular compaction, and we point out its connections with glassy relaxation of thermal systems. We also recall experiments suggesting it is possible to introduce 'thermodynamic' parameters

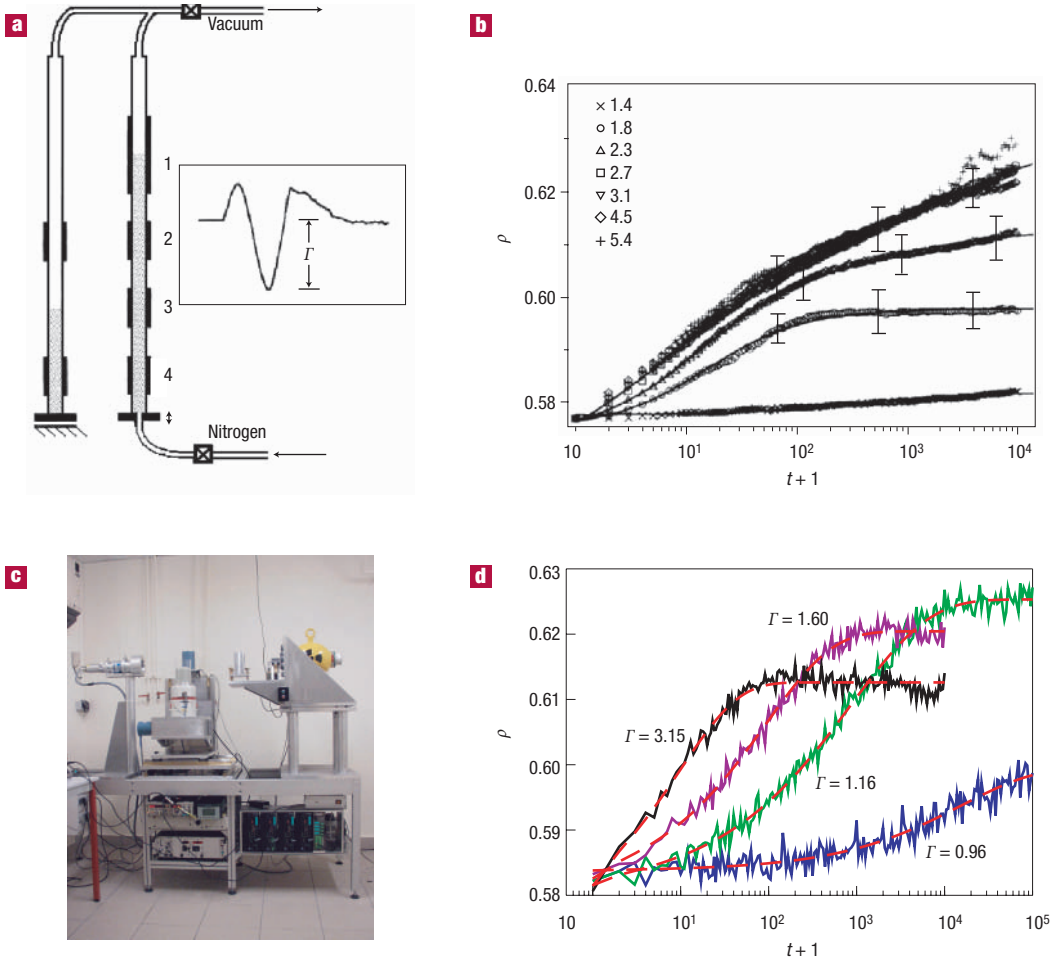

Figure 1 Experimental set-ups and density relaxation laws. **a**, Set-up for the experiments carried out in Chicago. The numbers 1–4 refer to capacitors used to measure the packing density (see ref. 9). **b**, Typical evolution of the packing fraction as a function of the number of taps, t, for experiments in Chicago for different values of the acceleration amplitude. Superimposed lines are logarithmic fits with equation (1). Each curve is an average of four to five separate runs, and the error bars represent the r.m.s. variation between runs (for clarity, just a few error bars are reported). **c**, Set-up for the experiments carried out in Rennes. **d**, Typical evolution of the packing fraction for the experiments in Rennes. Here dashed lines are stretched exponential fits with equation (2). Parts **a** and **b** reprinted with permission from ref. 9. Copyright (1995) by the American Physical Society.

to describe the status of the system. In the following section we briefly discuss theoretical models of compaction and Edwards' extension of statistical mechanics to granular media[7,8]. In this framework, we discuss current theories about the glassy phase of granular media and their deep connections to thermal glass formers. In the last section we will try to outline some of the open questions ahead.

EXPERIMENTAL RESULTS ON GRANULAR COMPACTION

DENSITY RELAXATION LAWS

The first quantity of interest in compaction is the packing fraction (or density) ρ, defined as the ratio of the volume of the grains to the total volume occupied by the packing. A few characteristic values of ρ for mono-sized sphere packings have to be noted. The maximal packing fraction achieved in a random packing of spheres (the so-called random close-packing fraction) is $\rho_{RCP} \approx 0.64$. This value is significantly lower than the maximal packing fraction obtained for face-centred-cubic (or hexagonal compact) packing ($\rho_{max} \approx 0.74$). Another limit is the so-called random loose packing corresponding to a mechanically stable packing with the lowest packing fraction ($\rho_{RLP} \approx 0.55$).

In a pioneer paper, Knight *et al.*[9] in Chicago first considered density relaxation laws in granular compaction. They recorded that, starting from a loose packing of beads confined in a very thin and tall tube (diameter 1.88 cm and 87 cm height, see Fig. 1a), a succession of vertical taps induces a progressive and very slow compaction of the system. They observed (see Fig. 1b) that the relaxation law can be well fitted by an inverse-logarithmic law, the so-called Chicago fit (resembling 'magnetic creep' of type-II supercondutors):

$$\rho(t) = \rho_f - \frac{\rho_0 - \rho_f}{1 + B \ln(1 + t/\tau)} \quad (1)$$

The only control parameter for the dynamics was found to be the ratio $\Gamma = a/g$ of the tap peak acceleration and gravity acceleration, that is, the fitting parameters: ρ_f the final packing fraction, ρ_0 the initial packing fraction, τ a characterisitc time, and B a fitting paramater, essentially depend on Γ. The small number of grains in a tube diameter (≈ 10) allows a local measurement of the packing fraction with a capacitive method and prevents any convection in the packing. Nevertheless, it induces strong boundary effects that may be responsible for crystallization (some packing-fraction values obtained are well above the random close-packing limit) visible in some of the Chicago group's experiments[10] and in the values of ρ_f obtained in ref. 9.

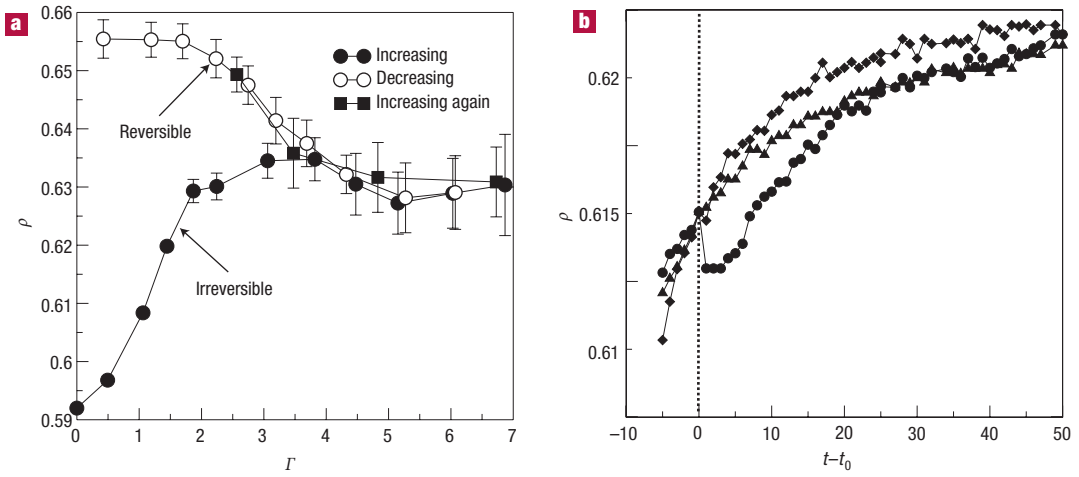

Figure 2 Annealing and aging during compaction. **a**, Annealing curve. The initial packing was prepared in a low-density initial configuration ($\rho \approx 0.59$) and then the acceleration amplitude Γ was slowly first increased (solid circles) and then decreased (open circles). At each value of Γ the system was tapped 10^5 times and Γ incremented by $\Delta\Gamma \approx 0.5$. The higher density branch (the upper one) is reversible to changes in Γ (see square symbols). Each curve is an average of separate experimental runs and the error bars represent the r.m.s. variations between runs. Reprinted from ref. 10, Copyright (1997), with permission from Elsevier. **b**, Time evolution of packing fraction for a system that was compacted to $\rho_0 = 0.613$ at time t_0 using three different accelerations: $\Gamma_1 = 1.8$ (circles), $\Gamma_0 = 4.2$ (triangles) and $\Gamma_2 = 6.3$ (diamonds). After the packing fraction ρ_0 was achieved ($t = 0$), the system was vibrated at acceleration Γ_0. The evolution for $t > t_0$ depended strongly on the previous history. Reprinted from ref. 18. Copyright (2000) by the American Physical Society.

More recently, Philippe and Bideau[11] carried out compaction experiments; in the following we will refer to these as the Rennes group's experiments. The vessel used is a 10-cm-diameter cylinder and 1-mm-diameter glass beads, which leads to 100 grains between the side walls. This restricts the boundary effects but, contrary to the Chicago group's experiments, allows convection. The packing fraction is measured using a γ-ray absorption set-up[11]. The relaxation laws obtained by these authors differ significantly from those obtained by Knight *et al.*[9], especially for the long-time behaviour. Indeed, whereas in previous experiments no clear steady-state was reached, this is definitely established in Rennes group's experiments, and may correspond to a dynamical balance between convection and compaction. The relaxation is better fitted by the Kohlrausch–Williams–Watts law (KWW law) — a stretched exponential:

$$\rho(t) = \rho_f - (\rho_f - \rho_0)\exp[-(t/\tau)^{\beta}] \qquad (2)$$

where ρ_f and ρ_0 correspond respectively to the steady-state and to the initial packing-fraction value. The adjustable parameters τ and β correspond here respectively to the relaxation time and to the stretching of the exponential. This characteristic timescale is found to be well described by an Arrhenius behaviour $\tau = \tau_0 \exp[\Gamma_0/\Gamma]$. Such a relaxation law is also found for strong glasses (the dimensionless acceleration Γ plays the role of the temperature).

Another quantity of interest is the final packing fraction obtained by the fit: ρ_f. Here again there exists a strong discrepancy between the Chicago group's experiments and the Rennes group's work. In the former case this packing fraction is found to increase with the tapping intensity, Γ, whereas it is found to decrease in the latter. These discrepancies are related to the fact that the Chicago experiments are performed in a region where the system is far from stationary, whereas the Rennes experiments are focusing on the stationary regime. This is likely to originate from the difference of confinement between the two experiments. Indeed, for the Chicago group's work, the strong boundary effects lead to order creation at least close to the side walls and to packing fractions higher than 0.64. On the contrary, all the final packing fractions obtained for glass beads and with a very low confinement by the Rennes group are below this value. Nevertheless, these results are affected by convection. Indeed, a significant change is observed in the dependence of ρ_f on Γ, which might correspond to different convective regimes[11,12]. Under a threshold $\Gamma_c \approx 2$, the final state of the free surface of the packing is an inclined plane and indicates a spontaneous breaking of symmetry. Above this value, the free surface takes a flat conical shape.

It should be pointed out that all the studies reviewed above deal with isotropic granular media. Of course, most actual granular materials are far from being isotropic, and the grain shape may modify the behaviour of the system during compaction. Villarruel *et al.*[13] have shown, using a Chicago set-up, that a nematic ordering can be observed for compaction of rods. Ribière *et al.* (ref. 14, and *ibid*, manuscript in preparation) carried out experiments of compaction of rice with the set-up in Rennes (low confinement). They did not observe such an ordering and obtained compaction characteristics similar to those obtained with glass beads. Note that the aspect ratio of the grains is probably an important parameter of the problem.

ANNEALING AND MEMORY EFFECTS

Further insight into the understanding of the nature of the relaxation can be gained by allowing the tap intensity to vary in time. Nowak *et al.*[10,15] (and later Philippe[16] with the Rennes set-up) have reported annealing experiments. Using the Chicago set-up they proceeded as follows: starting from a loose packing of grains, the material is tapped at a given intensity Γ for a given time t (10^5 taps). Γ is then modified and the compaction process continued for t taps (see Fig. 2a). The increase of Γ corresponds to an increase of the average packing fraction except for values larger than three for which a slow decrease can be observed. This slow decrease can be interpreted as void creation due to a too-large agitation. If Γ is then reduced, the

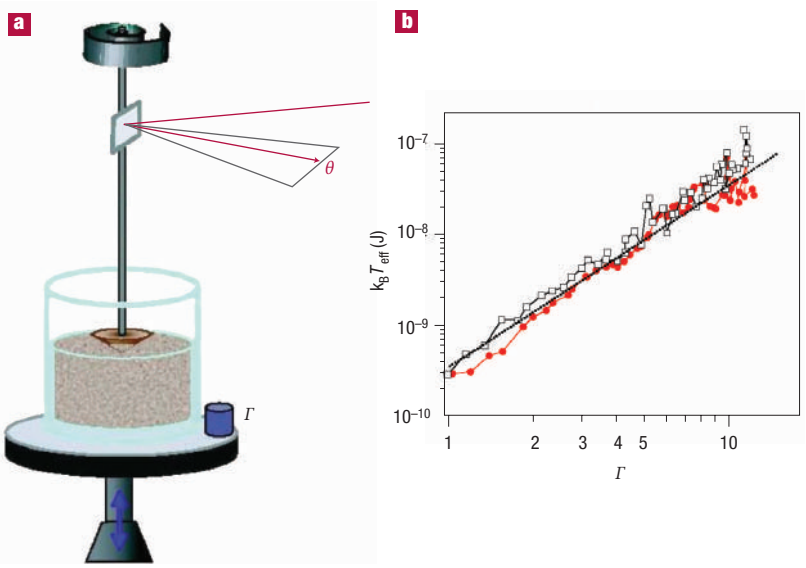

Figure 3 Experimental measurement of an effective temperature. a, Sketch of D'Anna's experimental set-up. A torsion oscillator is immersed in a granular material perturbed by external vertical tapping or shaking. **b**, With this apparatus, an 'effective temperature' can be measured for different oscillators, which is here plotted as a function of the vibration amplitude Γ (see ref. 20). Black squares: the oscillator used was conical. Red circles: the oscillator used was conical with a triple moment of inertia. Both **a** and **b** reprinted with permission from ref. 20.

Box 1: Fluctuation–dissipation relations

In usual statistical mechanics, at equilibrium, it is well known that the fluctuation–dissipation theorem holds, linking, for instance, the system average energy, E, to its fluctuations, ΔE^2, by the bath temperature. It is straightforward to verify that in Edwards' approach to granular media, similar relations are found where the bath is substituted by the 'configurational' temperature:

$$-\frac{\partial E}{\partial \beta_{\mathrm{conf}}} = \Delta E^2. \qquad (3)$$

Usefully, the integration of such an equilibrium fluctuation–dissipation relation may provide the value of β_{conf} from energy (or density) data measured at stationarity[35,42]: $\beta_{\mathrm{conf}}(E) = \beta^0_{\mathrm{conf}} - \int_{E_0}^{E} (\Delta E^2)^{-1}\, \mathrm{d}E$.

In thermal systems close to equilibrium, also dynamical fluctuation–dissipation relations (involving time-dependent quantities) hold, which are, in general, no longer valid far-from-equilibrium. Interestingly, in the aging dynamics of mean-field glassy models generalized out-of-equilibrium fluctuation–dissipation relations were discovered[46,47] where the role of the bath temperature, T_{bath}, is played by a 'dynamical temperature', T_{dyn}, equal for all slow modes. This scenario was later extended to aging granular media[23,38,40,44,48]. For instance, generalized Einstein diffusion relations are found between two far-apart times t and t_w ($t >> t_w$)[44,48]:

$$T_{\mathrm{dyn}} \frac{\delta\langle r(t) - r(t_w)\rangle}{\delta f} = \langle [r(t) - r(t_w)]^2 \rangle \qquad (4)$$

where $r(t)$ is a particle position at time t and f a constant small perturbing field coupled to it. Interestingly in systems aging in contact with an infinitesimal bath temperature, T_{conf} and T_{dyn} turn out to coincide[23,44,48] (as $T_{\mathrm{dyn}} \neq T_{\mathrm{bath}}$). In this way, out-of-equilibrium fluctuation–dissipation relations open a way to measure Edwards' configurational temperature in aging systems.

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packing fraction, rather than following reversely the previous curve, still increases. This new curve, contrary to the previous is reversible: the points corresponding to a new increase of Γ (black squares in Fig. 2a) still follow the curve. It should be pointed out that the reversible branch may be recovered after more than one cycle. Note that such a behaviour also exists for granular materials under cyclic shear[17]. From a practical point of view, varying the tap intensity allows very dense packings to be reached more rapidly. From a physicist's point of view, it allows the steady-state packing fraction (given by the reversible branch) to be reached for a given Γ, even if this Γ is very low. Note that these steady-state packing fractions are different from those obtained by the long time fit of Chicago logarithmic relaxation law. Such a distinction does not exist in the case of the Rennes group experiment, where the system is taken to a stationary state.

As mentioned previously, granular compaction has a glassy behaviour, so memory effects should also exist. For that aim, Josserand *et al.*[18] carried out experiments on the response function of a granular packing undergoing compaction to sudden perturbation of the tapping acceleration (equivalent to the temperature for a classic glassy system). Using the Chicago set-up, these authors drive a granular packing to the same packing fraction ρ_0 with three different accelerations Γ_0, Γ_1 and Γ_2. Then the system is tapped at the intensity Γ_0 and for short time its behaviour depends on the previous value of the tapping acceleration (see Fig. 2b). The data show a short-time memory effect: the future evolution of the packing fraction depends not only on its initial value but also on its history.

JAMMING IN GRANULAR MEDIA

We pointed out previously that these materials display deep common physical features with out-of-equilibrium systems such as glasses. This analogy can be surprising because granular materials are not thermal systems due to the irrelevance of their thermal energy in comparison with the energy needed to move a macroscopic grain. From a microscopic point of view this analogy is based on the idea that the geometry of the grains plays a major role, similar to 'geometrical frustration' in thermal glasses. Indeed, like glasses, a granular assembly can be trapped in a metastable configuration unless an external perturbation such as shear or vibration is applied. This has suggested the idea that granular media can be a good experimental system to apply out-of-equilibrium statistical mechanics. Note that, unlike thermal energy, mechanical agitation of grains is, in general, neither stochastic nor isotropic.

If this analogy is correct, a granular material submitted to mechanical perturbation should display a critical slowdown in its dynamics, as glasses freeze when they reach the glass transition. This behaviour, called jamming, was shown experimentally in a quantitative way by D'Anna and Gremaud[19,20]. They use 1.1-mm-diameter glass beads in a cylinder of 150 mm height and 94 mm diameter filled to a height of 130 mm (Fig. 3a). Using a shaker, the

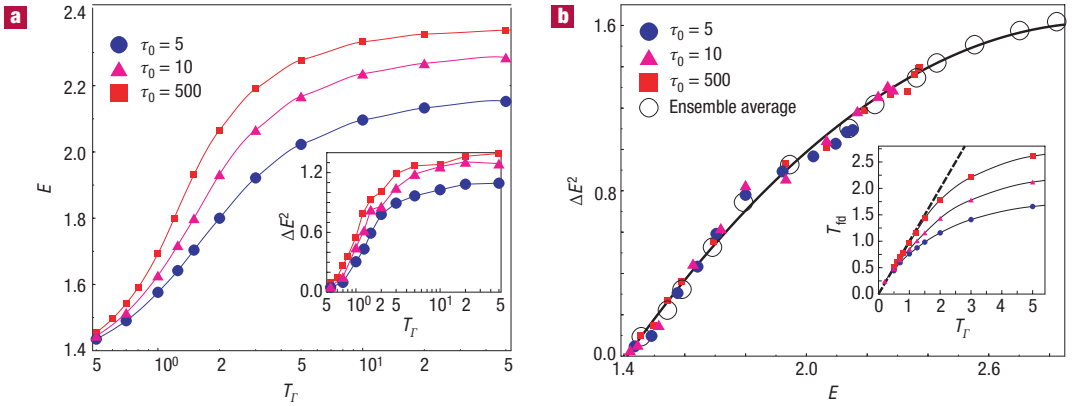

Figure 4 Temperature determination in a numerical model. a, The time average of the energy, $\bar{E}$, and (inset) its fluctuations, $\overline{\Delta E^2}$, recorded at stationarity during tap dynamics, as a function of the tap amplitude, T_Γ, in a three-dimensional lattice monodisperse hard-sphere model[35,42]. Different curves correspond to sequences of taps with different values of the duration of each single tap, τ_0. **b**, Time averages of energy fluctuations $\overline{\Delta E^2}$ plotted as function of the time average of energy $\bar{E}$ (refs 35,42). The filled circles, triangles and squares are the time averages obtained with the different tap dynamics shown in the left panel. The open circles are independently calculated ensemble averages according to Edwards' method[35,42]. The collapse of the data obtained with different dynamics shows that the system stationary states are characterized by a single thermodynamic parameter. The good agreement with the ensemble averages suggests Edwards' distribution is, in this case, an excellent approximation. Inset: The temperature T_{fd}, defined as the conjugate parameter to the energy, is plotted as function of T_Γ for $\tau_0 = 500$; 10; 5 Monte Carlo steps (from top to bottom). The straight line is the function $T_{fd} = T_\Gamma$.

whole is submitted to vertical continuous vibrations at the frequency f_s ($50\ \text{Hz} < f_s < 371\ \text{Hz}$) or to vertical taps. The perturbation intensity is measured using an accelerometer as the peak acceleration intensity normalized to the acceleration gravity $\Gamma = a/g$. To characterize the dynamics of the granular media the authors measure optically the motion of an oscillator (eventually submitted to an external torque) immersed in the media. The motion of this oscillator reflects, at least partially, the dynamical behaviour of the packing. The study of its power spectrum (and thus of the diffusivity) shows that the partially fluidized granular material displays a critical slowdown from a fluid state to a glassy state where — as for supercooled liquid — the diffusivity approaches zero[19]. The authors used the same set-up to try to define the equivalent of a temperature in this system. The fluctuation–dissipation theorem (see Box 1) allowed them to introduce an acceleration-dependent 'effective temperature' with the correct properties of a thermodynamic parameter (see Fig. 3b). In this respect, a quantitative analogy with thermal systems, such as glasses, appears, as further confirmed by the observations reported above.

THEORETICAL MODELS

We have seen that despite the fact that granular media may form crystalline packings, in most cases they are found at rest in disordered configurations, and when gently shaken, they exhibit a strong form of 'jamming'[9–11,15,19] (that is, an exceedingly slow dynamics) with deep connections to 'freezing' phenomena observed in thermal systems such as glass formers[21–23]. These observations suggested the idea of a unified description of 'jamming' in these different systems[22,24], whose precise nature is an important issue that is currently being debated[6]. To face these topics, it's necessary to solve another basic conceptual open problem, that is, the absence of an established theoretical framework to describe granular media. Edwards[7,8,25] proposed a statistical mechanics solution to such a problem for compact (that is, not 'fluidized') granular systems by introducing the hypothesis that time averages of a system, exploring its mechanically stable states subject to some external drive (for example, 'tapping'), coincide with suitable ensemble averages over its 'jammed states' (see Box 2).

MODELS FOR THE DYNAMICS OF COMPACTION

Before discussing Edwards' approach, it is interesting to mention that the slow compaction dynamics can be described by free-volume arguments as processes where a grain can jump into a hole in the pack only if the hole is empty. Mean-field-like dynamical models emphasizing these 'geometric frustration' mechanisms appear indeed to capture the essence of the dynamics[26,27]. Schematic lattice models of hard grains under gravity were introduced to describe dynamical and stationary properties of granular assemblies in further detail[22,28,29]. Numerical simulations of lattice models[21,30–35] and parking-lot models[27,36,37] show slow compaction dynamics. In the region of large tap amplitudes, the system density as a function of the number of shakes can be well fitted by exponentials. In the region of small tap amplitudes, the system gets 'jammed' and 'memory' and 'aging' phenomena are observed, along with 'annealing' effects as those described in previous experiments[9,10,15]. In this region, stationarity is typically not reached and logarithmic relaxation is found[21,27]. Correspondingly, the system relaxation time diverges approximately as an Arrhenius law[21], resembling experimental results[11,19], as the grains' diffusion coefficient goes to zero.

EFFECTIVE TEMPERATURES FROM FLUCTUATION–DISSIPATION RELATIONS

Interestingly, models of grain assemblies subject to a drive (that is, tap or shear) were also used to test Edwards' statistical mechanics approach[31,33–36,38–44]. In recent developments of glassy theory[45], a notion of 'effective dynamical temperature' can be introduced[46], based on the out-of-equilibrium extension of the fluctuation–dissipation theorem[47] (see Box 1). In granular media the 'glassy' region can be entered in the limit of very small 'shaking' amplitudes, that is, by letting the system age in contact with an almost zero bath temperature, T_{bath}. In this way, an effective temperature, T_{dyn} (see Box 1), was measured in a model for granular media[38]. In particular, within lattice gas 'glassy' systems, Barrat *et al.*[44,48] showed that the analogue of Edwards 'configurational temperature', T_{conf} (see Box 2), can be computed and, in the limit $T_{bath} \to 0$, it exhibits a very good agreement with T_{dyn}. The link between this dynamical

temperature and Edwards' temperature has been further studied in several numerical models of schematic 'tapped'[30,33,36,42] and 'aging'[34,38,44,48,49] systems or more realistic 'sheared' media[40].

In a different perspective, the validity of Edwards' approach can be tested, for any 'shaking' amplitude, by a technique based on dynamics as those used in experiments on granular compaction[35,39,42]. By some drive (such as 'tapping' or shearing) the system is allowed to explore the space of its blocked ('jammed') states. Once stationarity is reached, one has first to check that the system properties do not depend on the dynamical history, that is, that a genuine 'thermodynamic' description is indeed possible. In this case, a thermodynamic parameter characterizing the system, that is, a 'temperature' T_{fd}, can be defined by usual equilibrium fluctuation–dissipation relations (see Box 1). Then one could check whether time averages obtained using such dynamics compare well with ensemble averages as those proposed by Edwards (see Box 2). In such a case, the temperature T_{fd} coincides with Edwards' configurational temperature T_{conf}. This scenario was found to be consistent for simple lattice models of granular media[35,39,42,43].

THE STATIONARY STATES OF THE TAP DYNAMICS AND EDWARDS' AVERAGES

In particular, in a very schematic theoretical model of Chicago and Rennes experimental set-ups — a lattice model of tapped hard spheres under gravity — it was shown[35,42,43] by Monte Carlo simulations that Edwards' approach appears to hold to a very good approximation. In such a lattice gas model, grains are subject to dynamics made of a sequence of 'taps'[21]: a single 'tap' is a period of time, of length τ_0 (the tap duration), where particles can diffuse laterally, upwards (with probability p_{up}) and downwards (with probability $1 - p_{up}$). When the 'tap' is off, grains can only move downwards (that is, $p_{up} = 0$) and the system evolves until it reaches a blocked configuration. The parameter p_{up} has an effect equivalent to keeping the system in contact (for a time τ_0) with a bath temperature, $T_\Gamma \propto 1/\ln[(1 - p_{up})/p_{up}]$ (called the Monte Carlo 'tap amplitude').

The model Monte Carlo dynamics is apparently schematic (not to mention that many important material properties, such as friction or dissipation, are only superficially considered), but for this same reason it is fully tractable. During the tap dynamics, in the stationary regime, the time average of the energy, $\bar{E}$, and its fluctuations, $\overline{\Delta E^2}$, are calculated on the blocked states. Figure 4a shows $\bar{E}$ (main frame) and $\overline{\Delta E^2}$ (inset) as a function of the tap amplitude, T_Γ, in the model (for several values of the tap duration, τ_0). Sequences of taps, with same T_Γ and different τ_0, give different values of $\bar{E}$ and $\overline{\Delta E^2}$, thus T_Γ is not a correct thermodynamic parameter. On the other hand, when $\overline{\Delta E^2}$ is parametrically plotted as function of $\bar{E}$, the data collapse onto a single master function (see Fig. 4b) showing that the stationary states can be indeed characterized by a single thermodynamic parameter, β_{fd}, conjugated to the energy. T^{-1}_{fd} (or $T^{-1}_{fd} \equiv \beta_{fd}$) can be derived by integrating the usual equilibrium fluctuation–dissipation relation (see Box 1), and the functions $\beta_{fd}(\bar{E})$ or, for a fixed value of τ_0, $\beta_{fd}(T_\Gamma)$ obtained (inset of Fig. 4b)[35,42,43].

To test Edwards' hypothesis, time averages recorded during the taps sequences, $\bar{E}(\beta_{fd})$, must then be compared with ensemble averages, $\langle E\rangle(\beta_{conf})$, over Edwards' distribution equation (5). Figure 4b shows that the function $\langle\Delta E^2\rangle$ ($\langle E\rangle$), from the ensemble averages, collapses on the same master function of the time averaged data, $\overline{\Delta E^2}$ ($\bar{E}$), discussed before (notice that there are no adjustable parameters). This implies that, for the present model, Edwards' statistical mechanics holds to the current numerical accuracy and $T_{fd} = T_{conf}$.

Box 2: Edwards' statistical mechanics of granular media

In the statistical mechanics of powders introduced by Edwards[7,8,25] it is postulated that a system at rest (that is, not in its 'fluidized' regime) can be described by suitable ensemble averages over its blocked, 'jammed', states (related to 'inherent structures' of glass formers[35,42,59]). The probability, P_r, to find the system in its blocked state r, can be found under the assumption that these mechanically stable states have the same *a priori* probability to occur ('flat measure'). The knowledge of P_r has the conceptual advantage to substitute time with ensemble averages, as in thermodynamics.

P_r can be obtained[7,25,35,39,42] as the maximum of the entropy, $S = -\Sigma_r P_r \ln P_r$, under given macroscopic constraints. For instance, when the system volume is given $V = \Sigma_r P_r V_r$ (V_r is the volume of the blocked microstate r), one is lead to a Gibbs-like result: $P_r \propto e^{-Vr/\lambda X}$, where X is a Lagrange multiplier, called compactivity, enforcing the constraint and λ is a constant. Analogously, when the system energy, $E = \Sigma_r P_r E_r$, is given, that is, in the canonical ensemble, entropy maximization leads to:

$$P_r \propto e^{-\beta_{conf} E_r} \qquad (5)$$

where β_{conf} is a multiplier, called inverse configurational temperature, enforcing the energy constraint: $\beta_{conf} = \partial S_{conf} / \partial E$. Here $S_{conf} = \ln\Omega(E)$ is the 'configurational' entropy and $\Omega(E)$ the number of blocked states with energy E. Thus, summarizing, the system at rest has $T_{conf} = \beta^{-1}_{conf} \neq 0$, but no kinetic energy (that is, $T_{bath} = 0$).

Consider, for definiteness, a system of monodisperse hard spheres of mass m at rest in these blocked configurations. In Edwards' approach, by use of equation (5), we can write a generalized partition function in the canonical ensemble as[39,51]:

$$Z = \sum_{r\epsilon\Omega_{TOT}} \exp(-\mathcal{H}_{HC} - \beta_{conf} mgH) \cdot \Pi_r \cdot \qquad (6)$$

Here Ω_{TOT} is the system's whole configuration space, $\mathcal{H}_{HC}$ is the hard core term in the Hamiltonian (that is, the interaction between grains preventing overlaps), mgH is gravity contribution (H is particle height) and the factor Π_r is a projector on the space of 'blocked' states Ω: if r$\epsilon\Omega$ then $\Pi_r = 1$ else $\Pi_r = 0$.

Finally, it is important to stress that, in general, more than one thermodynamic parameter is needed to characterize the system[43]. Actually, a current matter of debate[6] is the determination of the range of validity of the above hypotheses, as well as the number of 'thermodynamic' parameters needed in different cases and their theoretical *a priori* individuation

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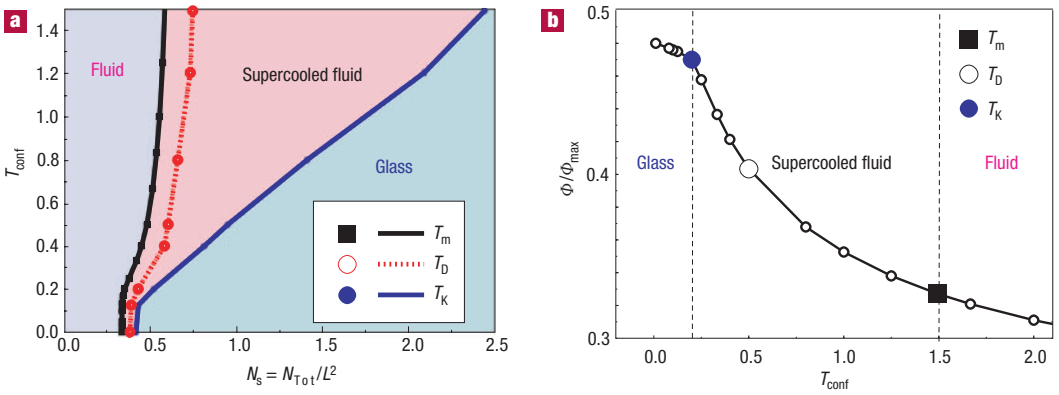

Figure 5 Phase diagram for a numerical model of granular material. a, The mean-field phase diagram of a monodisperse hard-sphere model of granular media[51] treated by Edwards' method is plotted in the plane of its two control parameters (T_{conf};N_s): T_{conf} is Edwards' 'configurational temperature' and N_s the average number of grains per unit surface in the box. At low N_s or high T_{conf}, the system is found to be in a fluid phase. The fluid forms a crystal below a melting transition line $T_m(N_s)$. When crystallization is avoided, the 'supercooled' (that is, metastable) fluid has a thermodynamic phase transition, at a point $T_K(N_s)$, to a granular glassy phase with the same structure found in mean-field theory of glass formers. In between $T_m(N_s)$ and $T_K(N_s)$ a dynamical freezing point, $T_D(N_s)$, is located, where the system characteristic timescales diverge. **b**, For a monodisperse hard-sphere model of granular media with a given number of grains (that is, a given N_s), the overall number density, Φ, calculated in mean-field approximation[51] is plotted as a function of T_{conf}; $\Phi(T_{conf})$ has a shape very similar to the one observed in the 'reversible branch' of 'annealing' experiments (see Fig. 2a) and Monte Carlo simulations. The location of the glass transition, T_K (filled circle), corresponds to a cusp in the function $\Phi(T_{conf})$. The passage from the fluid to supercooled fluid is T_m (filled square). The 'crystal' branch is not shown here. The dynamical crossover point T_D is found around the flex of $\Phi(T_{conf})$ and approximately corresponds to the position of a characteristic shaking amplitude Γ^* found in 'annealing' experiments where the 'irreversible' and 'reversible' branches approximately meet. **a** and **b** reprinted with permission from ref. 3.

We stress that no fluid-like convective motion of the grains[50], which might affect the general behaviours in real systems, are present in such a simple model.

More generally, the validity of Edwards' approach to non-thermal systems is a current matter of debate[6], where positive tests[31,33,38–40,42,44] are accompanied by more complex situations[34,36,41,43].

A MEAN-FIELD DESCRIPTION OF JAMMING

In Edwards' scenario, the nature of a system's properties and its 'glassy' region can be better understood by analytical calculation of the partition function, equation (6). This was accomplished in a Bethe–Peierls approximation for a monodisperse hard-sphere three-dimensional lattice system and its phase diagram derived[51], as shown in Fig. 5a in the plane (N_s; T_{conf}) (here N_s is the number of grains per unit surface, that is, the number of grains for one layer and T_{conf} the 'configurational temperature'). A phase transition is found from a fluid-like to a crystal phase at T_m. When crystallization is avoided, at a lower point, T_D, in the 'supercooled' fluid also a purely dynamical transition is present at a mean-field level, which in a real system is thought to correspond just to a dynamical crossover (see refs 52 and 53). Finally, at an even lower point, T_K, the supercooled fluid has a genuinely thermodynamic discontinuous phase transition to glassy phase. It is a currently debated issue whether T_K is non-zero in real systems[6]. The analytically calculated number density, Φ, plotted as a function of T_{conf} (in a system with a given N_s) in Fig. 5b, has a shape very similar to the one observed in tap experiments along the 'reversible branch' shown previously[10,11,15], or in Monte Carlo simulations[21,35].

The nature of this mean-field glass transition, obtained in the framework of Edwards' theory, is the same found in the glass transition of the p-spin glass and in other mean-field models for glass formers[23,52]: a discontinuous one-step replica symmetry-breaking phase preceded by a dynamical freezing point. These analytical results on one side show precise correspondences with experimental findings and, interestingly, on the other side may clarify the origin of the common 'glassy' features of granular media and thermal glass formers[21–23].

THE FUTURE OF GRANULAR COMPACTION

In this progress article we have reviewed recent important results on granular compaction, but many questions remain open ahead. The next experimental step is to understand what happens on the grain scale to packing microstructure. Experiments on 'memory' effects (see, for example, ref. 18) show the need for such information. As granular materials are opaque media, the main experimental difficulty is the determination of grain positions during the relaxation. Very interesting recent attempts were made to access such data: X-ray microtomography[54,55] and index-matching liquid imaging[56]. The experimental works in refs 54 and 55 are able to focus on structural quantities related to granular compaction (pore size distributions, pair correlation function ...) previously studied only numerically. In ref. 56 the authors found that during cyclic shearing of granular materials the grain motion is not just diffusive and exhibits a transient cage effect, similar to the one observed in glasses. They also started the exploration of the links between the macroscopic evolution of the packing fraction and the microscopic behaviour of grains. A study of the cage effect in two-dimensional granular media under cyclic shear can be found in a very recent work[57]. A current challenge is the generalization of these results to three-dimensional systems. These techniques can also shed light on a practical as well as on a theoretical issue: the role of the compaction mechanisms (tapping, shearing ...) on the system properties[58].

Finally, an important open issue is the theoretical foundation and experimental test of statistical mechanics approaches. In practice, the general validity of Edwards' scenario has just begun to be assessed and there are still many, crucial, open questions[31,33–36,38–43]. In some very schematic models, it turns out to be at least a good approximation, able to give a first reference framework to understand the physics of granular media and their deep connections with thermal systems such as fluids and glass formers. In this respect, the understanding of fluid-like motion[50], pattern formation[5], mixing/segregation transitions[3,4], will be of crucial importance. A deeper test of these theories and their consequences, the experimental determination of the described phase diagram of granular media[23], the important role of

fluid-like effects, are among relevant open research directions in this field.

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Competing financial interests

The authors declare that they have no competing financial interests.