SMALL: A Strategy-Proof Mechanism for Radio Spectrum Allocation

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***Abstract*—With the growing deployment of wireless communication technologies, radio spectrum is becoming a scarce resource. Thus mechanisms to efficiently allocate the available spectrum are of interest. In this paper, we model the radio spectrum allocation problem as a sealed-bid reserve auction, and propose SMALL, which is a Strategy-proof Mechanism for radio spectrum ALLocation. Furthermore, we extend SMALL to adapt to multi-radio spectrum buyers, which can bid for more than one radio.**

I. Introduction

Radio spectrum is becoming a scarce resource due to the increasing deployment of wireless communication technologies. For historical reasons, much of the radio spectrum is statically allocated. The inefficiency of such an allocation is twofold. On one hand, the static allocation does not consider spatial and temporal variation of the spectrum. Large chunks of radio spectrum are left idle most of the time at a lot of places. On the other hand, many new wireless applications cannot find enough radio spectrum to operate on. Therefore, redistribution of idle radio spectrum is important to make a better utilization of the radio spectrum.

To redistribute radio spectrum, a natural way is to use auction, which is a process of buying and selling goods by offering them up for bid, taking bids, and then selling the item(s) to the highest bidder(s). Since 1994, the Federal Communications Commission (FCC) has conducted auctions of licenses for radio spectrum [2]. While FCC auctions target only large wireless applications, we consider small wireless application buyers, such as community wireless networks and home wireless networks. These small buyers can search for and reuse idle chunks of radio spectrum.

However, designing a practical spectrum auction mechanism has its own challenges. One of the major challenges is spatial reusability of the radio spectrum, which differentiate it from conventional goods. Spectrum buyers, who are within the interference range of each other, cannot use the same spectrum band simultaneously, while well-separated buyers can. Furthermore, the problem of finding the optimal spectrum allocation is NP-complete [1], [11]. Another major challenge, which is not limited only to spectrum auctions but applies to traditional auctions in general, is strategy-proofness (see Section II-B for the definition), which intuitively means that reporting true valuation as a bid maximizes one's payoff. Since the participants are rational and always want to maximize their own objectives, it is likely that the participants would strategically manipulate the auction, if doing so can benefit themselves. Therefore, truthfully behaving spectrum buyers can be discouraged from participating in the auction, if strategy-proofness is not guaranteed.

Recently, Zhou et al. proposed TRUST [13] and VERITAS [12] to support open auction-based spectrum redistribution. Both auction mechanisms achieve strategy-proofness. TRUST takes into account both buyers and sellers' valuation on the channels, and elegantly integrates double auction and radio spectrum allocation. TRUST enables spectrum reuse and can improve spectrum utilization. Unfortunately, to guarantee the strategy-proofness, TRUST has to sacrifice a good transaction, which includes a channel and a group of buyers. When TRUST is used, not all of the channels can be sold, and the number of sacrificed buyers grows almost linearly with the number of buyers. Furthermore, TRUST does not support the need from a buyer for multiple channels. Unlike TRUST, VERITAS does not sacrifice any good transaction, and provides the support for bidding multiple channels. But VERITAS does not consider seller's valuation of the channels, which may include the leasing expense of the channel. A channel may be sold at a price much lower than the seller's valuation, and thus the incentive of the seller to resell a channel may be hurt.

In this paper, we present a Strategy-proof Mechanism for radio spectrum ALLocation (SMALL). SMALL is a sealed-bid reserve auction mechanism, in which all bidders simultaneously submit sealed bids so that no bidder knows the bid of any other participant, and a channel may not be sold if the final bid is not high enough to satisfy the seller. SMALL supports radio spectrum reuse, bidding for multiple channels, and protects channel seller's incentive.

We make the following contributions in this paper:

- First, we model the radio spectrum allocation problem as a sealed-bid reserve auction, and design a novel auction mechanism, called SMALL, for single-radio spectrum auction. We prove that SMALL is a strategy-proof auction

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mechanism.

- Second, we extend SMALL to support multi-radio spectrum auction, and prove that the enhanced SMALL again achieves strategy-proofness.
- Finally, we claim that SMALL has some advantages compared with existing strategy-proof spectrum auction mechanisms. Compared with TRUST, SMALL sacrifices a much smaller number of buyers. The number of sacrificed buyers is bounded by the number of channels. SMALL does not sacrifice any channel, and thus all the channels can be sold, while TRUST has to sacrifice one channel. Compared with VERITAS, SMALL protects seller's incentive for selling channels, by introducing reserve prices.

The rest of this paper is organized as follows. In Section II, we present technical preliminaries. In Section III, we describe our spectrum auction mechanism — SMALL, and prove its strategy-proofness. In Section IV, we extend SMALL to support multi-radio spectrum auction. In Section V, we review related work. In Section VI, we draw conclusions and discuss future work.

II. Technical Preliminaries

In this section, we present our game model for the spectrum allocation problem, and review some useful solution concepts from game theory and mechanism design.

A. Game Model

We consider a static scenario in which there is a large wireless service provider, called "seller", who possesses a number of orthogonal spectrum channels and wants to lease out regionally unused channels; and there is a set of static nodes, called "buyers", such as WiFi access points, who want to lease channels in order to provide services to their users. A channel can be leased to multiple buyers, if these buyers can transmit simultaneously and receive signals with an adequate Signal to Interference and Noise Ratio (SINR). We model this problem as a sealed-bid reserve auction, in which all buyers simultaneously submit sealed bids so that no buyer knows the bid of any other participant, and a channel may not be sold if the final bid is not high enough to satisfy the seller. The objective of the auction is to efficiently allocate the channels to the buyers based on their bids, without violating interference conditions between the buyers.

We assume that the seller is trustworthy, and has a set $C = \{c_1, c_2, \ldots, c_m\}$ of orthogonal and homogenous channels to lease. Each channel can be simultaneously used by multiple non-conflicting buyers. The seller has a reserve price for each of the channels, denoted by $S = \{s_1, s_2, \ldots, s_m\}$. A reserve price can be an operating expense, if the seller put a channel on auction. A channel can be leased to one or a group of non-conflicting buyers if the sum of the bids is not lower than the reserve price. (We will define buyer group in Section III-A.)

We also assume that there is a set $N = \{1, 2, \ldots, n\}$ of buyers. Each buyer $i \in N$ only requests a single channel and has a valuation v_i on the channel. The channel valuation can be the revenue got by the buyer for serving her subscribers. (In Section IV, we will consider an extended model, in which buyers can be equipped with multiple radios and bid for multiple channels. The channel valuations are identical for multiple radios/virtual buyers, because the buyer can serve more subscribers or provide better service quality, when getting more channels.) The channel valuation v_i is a private information to the buyer i. It is also known as *type* in the literature. In the auction, the buyers simultaneously submit their sealed bids, denoted by $\{b_1, b_2, \ldots, b_n\}$, which are based on their types. The auction mechanism determines the set of winning buyers, channel allocation to the winners, and the charge of each winner. Denote the charge of a buyer $i \in N$ by p_i. Then we define the utility u_i of buyer i to be the difference between her valuation v_i on the channel and the charge p_i:

$$u_i = v_i - p_i.$$

We assume that the buyers are rational. The objective of each buyer is to maximize her own utility. A buyer has no preference over different outcomes, if the utilities are same to the buyer herself. We also assume that the buyers do not collude with each other.

In contrast to players' individual objective, the overall objective of the auction mechanism is to improve channel utilization and buyer satisfaction ratio. Here, channel utilization is the sum of allocated channels of all the winning buyers; buyer satisfaction ratio is the percentage of winning buyers in the auction. Furthermore, to avoid the buyers paying too high prices, a good auction mechanism should also be budget efficient, which means the overpayment, between buyers' total charge and sellers' total valuation/reserve price, should be small.

B. Solution Concepts

We review the important solution concepts used in this paper from game theory and mechanism design. First, we recall the definition of *Dominant Strategy*:

Definition 1 (Dominant Strategy [4], [7]): A dominant strategy of a player is one that maximizes her utility regardless of what strategies other players choose. Specifically, a_i is player i's dominant strategy if, for any $a_i' \neq a_i$ and any strategy profile of the other players a_{-i},

$$u_i(a_i, a_{-i}) \geq u_i(a_i', a_{-i}).$$

Before recalling the definition of *Strategy-proof Mechanism*, we define *direct-revelation* mechanism first. A direct-revelation mechanism is a mechanism in which the only actions available to players are to make claims about their preferences to the mechanism. In our channel auction, the strategy of a buyer $i \in N$ is reporting a bid $b_i = a_i(v_i)$, based on her actual channel valuation v_i. A direct-revelation mechanism is strategy-proof if it satisfies two conditions, *incentive-compatibility* and *individual-rationality*. Incentive-compatibility means reporting truthful information is a dominant strategy for each player. Individual-rationality means each player can always achieve at least as much expected utility from faithful participation as without participation. The formal definition of Strategy-proof Mechanism is as follows.

Definition 2 (Strategy-Proof Mechanism [6], [8]): A direct-revelation mechanism is strategy-proof if revealing truthful information is a dominant-strategy equilibrium.

III. Strategy-proof Radio Spectrum Allocation Mechanism — SMALL

In this section, we present our design of radio spectrum auction mechanism — SMALL, and prove its strategy-proofness.

A. Design of SMALL

SMALL is composed of three algorithms: buyer grouping, winner selection, and charge determination. Since the seller is a trustworthy authority, we let the seller perform the computation of the three algorithms.

1) Buyer Grouping: Since the channels can be spatially reused, SMALL divides the buyers into multiple non-conflicting groups, each of which can be assigned to a distinguished channel. To prevent the buyers manipulating the auction, the grouping need to be independent of the buyers' bids. Therefore, SMALL first constructs a conflict graph of the buyers. Any pair of buyers, who are in the interference range of each other, have a line connecting them in the conflict graph. Then buyer groups can be calculated by any existing graph coloring algorithm [9] which is independent of buyers' bids, such that no buyer can be in multiple groups. We note that the buyers cannot determine which group they are in by themselves, when the above grouping strategy is used. We denote the calculated buyer groups by $G = \{g_1, g_2, \ldots, g_l\}$.

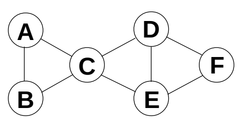

Fig. 1. A toy network with 6 buyers ($A - F$).

Figure 1 shows a toy network with 6 buyers ($A-F$). There are several grouping results, *e.g.*, $g_1 = \{A, D\}$, $g_2 = \{B, E\}$, and $g_3 = \{C, F\}$.

2) Winner Selection: We now determine an integrated group bid for each buyer group. A natural way to calculate the group bid is to simply add all the bids from the group members together. However, this way may allow the buyers to manipulate the group bid by reporting untruthful bids. Thus the strategy-proofness of the auction can be hurt. Therefore, to guarantee the strategy-proofness, we sacrifice the buyer with the smallest bid in each group, and define an integrated group bid σ_j for each group $g_j \in G$ as:

$$\sigma_j = (|g_j| - 1) \cdot \min\{b_k | k \in g_j\}.$$

By this way, the group bid is independent of valid members' bids (*i.e.*, the bids except the smallest one) in each group. Such a definition of group bid is reasonable, because the strategy-proofness can be guaranteed by sacrificing the buyer that makes the least contribution in a group. Then, we get a set of group bids $\Sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_l\}$.

Next, SMALL sorts the channels by reserve price in non-decreasing order and buyer groups by group bid in non-increasing order:

$$C' : s'_1 \leq s'_2 \leq \ldots \leq s'_m,$$

$$G' : \sigma'_1 \geq \sigma'_2 \geq \ldots \geq \sigma'_l.$$

Here each s'_i (σ'_j) corresponds to a unique reserve price in S (group bid in Σ). In the case of ties, the ordering is random, with each tied channel/group having an equal probability of being ordered prior to the other one.

Next, SMALL finds the maximal number of trades k s. t.

$$\sum_{i=1}^{k} s'_i \leq \sum_{i=1}^{k} \sigma'_i. \quad (1)$$

Finally, the winning groups are the first k buyer groups in G', and the first k channels in C' are leased to each of the corresponding winning groups. In each of the winning groups, the buyers, except the one with the smallest bid in that group, are winning buyers. In the case of ties, *i.e.*, more than one buyers report the smallest bid in the group, each tied buyer has an equal probability of being selected as a winning buyer.

Noting that exactly one buyer must be sacrificed for each channel leased, the total number of sacrificed buyers has an upper bound m, which is the number of channels. Since singleton groups cannot compete for channels, as their group bid would be zero, SMALL is more appropriate to be used in a radio spectrum auction with relatively large number of buyers scattered in a large area.

3) Charging: Each winning buyer $i \in g_j$ is charged an even share of her group bid, which is also equivalent to the smallest bid in the group:

$$p_i = \frac{\sigma_j}{|g_j| - 1} = \min\{b_k | k \in g_j\}.$$

In each winning group, we exclude the buyer with the smallest bid, and charge the others with the smallest bid, in order to make the charge be independent of winners' bids.

The seller collects all the payments:

$$q = \sum_{j=1}^{k} \sigma'_j. \quad (2)$$

We note that the auction is budget-balanced, which means that the total amount of the buyers' payments is equal to the total amount of the payments to be received by the seller [6].

Combining Equations (1) and (2), we get

$$q \geq \sum_{i=1}^{k} s'_i.$$

Therefore, the seller's profit is guaranteed. We note that we do not specify the algorithm for dividing the seller's revenue to each channel successfully leased. One of the possible ways is to divide the revenue proportionally to the channels' reserve prices.

In the next section, we will prove that buyers' truthfulness is also guaranteed.

B. Strategy-Proofness

Lemma 1: If SMALL is used, reporting the true channel valuation as a bid is a dominant strategy for each buyer.

Due to limitation of space, we do not present the proof in this paper.

From Lemma 1, we get that SMALL satisfies incentive compatibility. On one hand, we can see that each truthful buyer's utility is always ≥ 0. On the other hand, by not taking part in the auction, a buyer cannot get a channel and her utility remains to be 0. So participating is not worse than staying outside, which satisfies the individual rationality.

Since our mechanism satisfies both incentive compatibility and individual rationality, we have the following theorem:

Theorem 1: SMALL is a strategy-proof mechanism.

IV. Extension to Multiple Radios

In the previous section, we considered the scenario in which each buyer only has a single radio. In reality, some access points may be equipped with multiple radios. In this section, we extend our work to adapt to multiple radios having the same communication capabilities. A buyer with multiple radios can provide wireless services on multiple channels. So in the spectrum auction, a multi-radio buyer may bid for more than one channels. In the conflict graph, a r-radio buyer is represented by at most r virtual buyers inheriting the interference condition of their parent. Since the number of radios r on a buyer may be larger than that of the channels for sale m, we require that the number of virtual buyers for a r-radio buyer is $\min\{r, m\}$. The virtual buyers also have interference between each other. We assume that buyers have no preference over channels and they do not cheat about the number of radios. Considering that the buyer can serve more subscribers or provide better service quality, when getting more channels, we also assume that the channel valuations are identical for the virtual buyers. Hence, we let the virtual buyers share the same channel valuation and bid from her parent. The parent buyer's utility is the sum of the utilities got by her virtual child buyers. Since a buyer can have multiple virtual child buyers and report multiple bids, the previous auction mechanism cannot be directly applied here. In this section, first, we show an example, in which a multi-radio buyer can benefit by misreporting her bids. Then we present our enhanced SMALL to prevent misreporting when nodes have multiple radios.

A. Example: Multi-radio Buyer Can Benefit by Misreporting

Figure 2 shows a scenario, in which there are two channels and 6 buyers ($A - F$). The reserve prices of the channels are $s_1 = 3$ and $s_2 = 2$. The channel valuations are shown near the buyers. A line between two buyers indicates that they interfere with each other and cannot share the same channel. Among the buyers, D has two radios. Since buyer D has two radios, we duplicate D as D_1 and D_2, and connect them with a line. D_1 and D_2 inherit the interference condition from D.

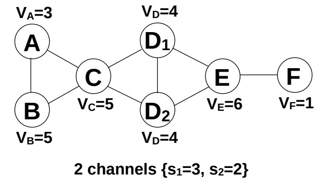

Fig. 2. Buyer D can get a higher utility by bidding $b'_D = 1.5$, when using SMALL.

Suppose the buyers are divided into 3 non-conflicting groups: $g_1 = \{A, D_1, F\}$, $g_2 = \{B, D_2\}$, and $g_3 = \{C, E\}$. If the buyers bid their true valuations, then the group bids are $\sigma_1 = 2$, $\sigma_2 = 4$, and $\sigma_3 = 5$. So the winning groups are g_2 and g_3, and the winning buyers are B and E. The utilities of B and E are

$$u_B = v_B - p_B = 5 - 4 = 1,$$

and

$$u_E = v_E - p_E = 6 - 5 = 1,$$

respectively, while the utilities of A, C, D, and F are 0.

But, buyer D can get a higher utility by unilaterally reporting a bid other than her true valuation. In particular, if D reports $b'_D = 1.5$, then the group bid of g_2 becomes $\sigma'_2 = 1.5$, while the other two remain unchanged. Consequently, the winning groups becomes g_1 and g_3,and the winning buyers are A, D_1, and E. The utilities of the winners are

$$u_A = v_A - p_A = 3 - 1 = 2,$$

$$u_D = u_{D_1} + u_{D_2} = v_D - p_{D_1} = 4 - 1 = 3,$$

$$u_E = v_E - p_E = 6 - 5 = 1.$$

We can see that D gets a higher utility by misreporting her channel valuation.

Therefore, the previous auction mechanism cannot be directly used when buyers have multiple radios.

B. Design of Enhanced SMALL

We observe that beneficial misreporting must result in that one radio of a node wins a channel, while the other one does not and holds the smallest bid in her group. Therefore, we propose an enhanced SMALL to prevent misreporting by eliminating the result which may be produced by misreporting. Since spectrum resource becomes relatively more scarce with the increased number of radios, we assume that all the channels can be sold in the multi-radio channel auction.

Algorithm 1 shows the pseudo-code of multi-radio winner selection algorithm in SMALL. In the algorithm, $grouping(N)$ and $winner(G, S, B)$ are the buyer grouping algorithm and the winner selection algorithm described in Section III-A1 and Section III-A2, respectively. The algorithm first groups the (virtual) buyers without violating interference

Algorithm 1 Multi-radio Winner Selection Algorithm

Input: A set of (virtual) buyers N, a set of bids B, and a set of reserve prices S.

Output: A set of winners W.

1: $G = grouping(N)$.
2: **repeat**
3: $\quad W = winner(G, S, B)$.
4: $\quad safe = TRUE$.
5: $\quad$ **for all** $g_j \in G$ **do**
6: $\qquad i = \underset{i \in g_j}{argmin}(b_i)$.
7: $\qquad$ **if** $\exists k \in W$ s.t. i and k belong to the same node **then**
8: $\qquad\quad G' = \{g_l | g_l \in G \wedge l \neq j\} \cup \{g_j - \{i\}\}$.
9: $\qquad\quad W' = winner(G', S, B)$.
10: $\qquad\quad$ **if** $W' \subseteq W$ **then**
11: $\qquad\qquad$ Remove i from G.
12: $\qquad\quad$ **else**
13: $\qquad\qquad$ Remove k from G.
14: $\qquad\quad$ **end if**
15: $\qquad\quad safe = FALSE$.
16: $\qquad\quad$ Break the **for** loop.
17: $\qquad$ **end if**
18: $\quad$ **end for**
19: **until** $safe == TRUE$
20: **return** W.

conditions. Then it iteratively eliminates one radio of a possible misreporting buyer from the buyer groups and recalculates the winner set (line 2-19). In an iteration, the algorithm picks the (virtual) buyer i with the smallest bid in each group, and checks whether there exists a winning (virtual) buyer k who shares the same parent with her. If such a pair (i, k) is found, the algorithm tests whether removing (virtual) buyer i will induce any new winner, by computing the winner set W' on G', which is the grouping if (virtual) buyer i is removed from G. If no new winner appears by removing (virtual) buyer i (*i.e.*, $W' \subseteq W$), then remove (virtual) buyer i from G. Otherwise, remove (virtual) buyer k from G. After eliminating all such (i, k) pairs, the algorithm outputs a misreporting-free winner set.

The enhanced version of SMALL still uses the charging algorithm described in Section III-A3. We note that the enhanced version of SMALL can also work in single-radio scenario.

Similarly, we get the following theorem.

Theorem 2: SMALL is a strategy-proof mechanism despite multiple radios.

Due to limitation of space, we do not present the proof in this paper.

V. Related Works

In this section, we review related works on channel allocation with selfish participants.

In an earlier work, Felegyhazi et al. [3] studied Nash Equilibria in a static multi-radio multi-channel allocation game. Later, Wu et al. [10] proposed a mechanism to make the multi-radio multi-channel allocation game converges to a much stronger equilibrium state, called strongly dominant strategy equilibrium, in which optimal system throughput is achieved.

The most closely related works are TRUST [13] and VERITAS [12], both of which are auction-based spectrum allocation mechanisms achieving strategy-proofness. TRUST considers both buyers and sellers' incentives, and elegantly integrate double auction and radio spectrum allocation. In contrast, VERITAS focus on spectrum buyers and support multiple needs of the buyers.

Another important related work on channel allocation game is [5], in which the authors proposed a graph coloring game model and discussed the price of anarchy under various topology conditions such as different channel numbers and bargaining strategies.

VI. Conclusions and Future Work

In this paper, we have modeled the radio spectrum allocation problem as a sealed-bid reserve auction, and proposed a strategy-proof radio spectrum allocation mechanism, call SMALL. As for future work, we are interested in designing similar simple mechanisms that can prevent collusion among multiple spectrum buyers.

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