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Smart Suspension Systems for Bridge-Friendly Vehicles

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ABSTRACT

In this paper, the effects of using semi-active control strategy (such as MR dampers) in vehicle suspensions on the coupled vibrations of a vehicle traversing a bridge are examined in order to develop various designs of smart suspension systems for *bridge-friendly vehicles.* The bridge-vehicle coupled system is modeled as a simply supported beam traversed by a two-degree-of-freedom quarter-car model. The surface unevenness on the bridge deck is modeled as a deterministic profile of a sinusoidal wave. As the vehicle travels along the bridge, the system is excited as a result of the surface unevenness and this excitation is characterized by a frequency defined by the speed of travel and the wavelength of the profile. The dynamic interactions between the bridge and the vehicle due to surface deck irregularities are obtained by solving the coupled equations of motion. Numerical results of a passive control strategy show that, when the lower natural frequency of the vehicle matches with a natural frequency (usually the first frequency) of the bridge and the excitation frequency, the maximum response of the bridge is large while the response of the vehicle is relatively smaller, meaning that the bridge behaves like a vibration absorber. This is undesirable from a bridge design viewpoint. Comparative studies of passive and semi-active controls for the vehicle suspension are performed. It is demonstrated that skyhook control can significantly mitigate the response of the bridge, while ground-hook control reduces the tire force impacted onto the bridge.

Keywords: Semi-active control, Dynamic interaction, Bridge friendliness, Deck irregularities, Magnetorheological (MR) damper

1. INTRODUCTION

Highway bridges have suffered a sharp decrease in service life, in part due to loading induced by heavy truck traffic occurring at levels in excess of those originally assumed by the designers. As a result, many bridges are approaching the end of their useful life and will require extensive repair and/or replacement unless other ways are found to reduce stresses and strains due to these loads and to sustain the safety of the bridges. There are roughly 600,000 bridges in the U.S. and statistics shows that one third of them are rated either functionally obsolete or structurally deficient. The Federal Highway Administration (FHWA) estimated the costs for refurbishment at about $6 billion/year for the next 25 years, but the TEA-21 budget is only at the level of $3 billion/year [1]. It is important, then, to be able to accurately predict both bridge and vehicle responses, in order to effect a reduction strategy such as semi-active control.

In order to extend the useful service life of bridges, some efforts have been taken for mitigating the vibration of bridges. The Center for Structure Control at the University of Oklahoma [2] has been engaged in a multiyear effort to develop an intelligent stiffener for bridges that can be retrofitted to existing bridges. In addition, the use of tuned mass damper [3] and magneto-rheological fluid damper [4, 5] has been examined as means to control the vibration of bridges as well. However, all these efforts of bridge control are based on equipping a device mounted on the underside of the bridge deck. An obvious disadvantage of this kind of bridge control strategy is the reduction of the vehicle height that is permitted to pass by under the bridge. Inspired by the work of Collop and Cebon [6] and Valášek et al. [7] on the design of *road-friendly* suspensions to minimize damages on road pavement, our goal here is to explore various designs of smart suspension systems for *bridge-friendly vehicles*; the suspension systems of these vehicles are tunable to minimize the impact loading and stress on the bridge. An initial attempt on the control formulation of such a dynamic interaction problem has been reported in [8]. It should be noted that the fundamental problem of bridge-vehicles interactions has generated interest worldwide, as evidenced by the recently concluded multinational DIVINE (**D**ynamic **I**nteraction Between **V**ehicles and **In**frastructure **E**xperiment) project [9]. This

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experimental study clearly shows that the interaction of the vehicle suspensions and bridge frequencies is critical and that current understanding on the interactions of *multiple* vehicles with a bridge is inadequate.

In the development of road-friendly truck suspensions, passive, active and semi-active control systems have been considered. A passive control system dissipates energy through sliding friction, shearing of viscous fluid or orificing of fluid in passive devices. The control force is developed as a result of the motion of the structure itself and therefore does not require external power. It is inexpensive, simple and reliable but has distinct performance limitation. The control force within an active control system is typically generated by electrohydraulic or electromechanical actuators that usually require large power sources. The active control system can provide better performance, but is more costly and is less reliable and robust due to complex design. Compromising cost and performance, a semi-active suspension system was proposed [10]. The initial semi-active system was based on “skyhook” scheme introduced by Karnopp [10]. The control policy was designed to modulate the damping force by a passive device to approximate the force that would be generated by a damper fixed to an inertial reference (so this is called “skyhook”). This device could only absorb vibration energy by a variable actuator, but requires low power (battery) to operate, and therefore have the flexibility of active systems and the reliability of passive systems. The skyhook damper was shown to be able to reduce the sprung mass vibration at low frequency [10, 11] and can therefore improve ride quality. By combining passive and skyhook dampers, it was shown that road damage can be reduced [13-15] and that the driving safety can be enhanced [16]. Another semi-active control policy was so-called ground-hook control, which was introduced by the motivation of developing an equivalent of skyhook for the reduction of dynamic tire-road force [17]. A modification by combining ground-hook with passive control was introduced to decrease road damage as well as ride comfort [18, 19]. The performance of three control policies, including the skyhook, groundhook and their hybrid, were compared experimentally [20]. It was shown that skyhook control generally improves the ride comfort, while the ground-hook control improves vehicle stability and reduces road damage. A hybrid of these policies may have improved effects on both the vehicle and the road.

Generally, the term “semi-active control system” can be used to refer to any policy in which the damping can be adjusted between a minimum and a maximum level. The modulation of semi-active dampers can be of two types. One is an on-off modulation damper, and the other is a continuous modulation damper. For the on-off damper, the damping is switched between the minimum and maximum level by means of a valve or solenoid. The switching can be performed based on the product of the absolute velocity of the mass and the relative velocity between the mass and the base. When the product is positive, the damper is switched “on” so a force is generated to reduce the vibration of the mass, when the product is negative, the damper is switched “off” so that no force is generated. In order to simplify the hardware implementation and reduce the cost, a number of simplified on-off control schemes have been developed. A comprehensive review of different on-off control schemes can be found in the paper by Wu and Griffin [21]. In the continuous damper, the damping levels are not limited to two states. They can be any values within two boundaries. Currently, there are two common types of hardware implementation for semi-active dampers. The first type is a hydraulic device in which the damping coefficient is changed by the modulation of the orifice area through which the fluid flows [22-24]. For the second type, the damping coefficients are varied by controlling the viscosity of the liquid (i.e., electrorheological or magnetorheological fluid) [25-27]. The MR fluid is the magnetic analog of ER fluid. MR-based devices have been mass-produced for commercial applications whereas ER-based devices remain at the prototype stage due to the superior performance of MR fluid. Regardless of whether MR or ER damper is used, both have been demonstrated to be promising due to their small power requirement, enhanced reliability and stability. Recently, these devices have been applied to vehicle [28-33] and bridge [4, 5] control.

As described above, there have been numerous studies on the semi-active control of vehicles and bridges, and the reduction of pavement damages. However, there is no systematic investigation on the simultaneous effects of the suspensions on the vibration of the bridge. Recent field tests have shown that the dynamic interaction of bridges with vehicles may be stronger than the interaction between vehicles and pavement [9, 34]; this coupling effect should be included in the analysis. Moreover, large response of the bridge occurs as a result of the surface deck roughness [35-38]. In this paper, the effects of using semi-active control strategy (such as MR dampers) in vehicle suspensions on the coupled vibrations of a vehicle traversing a roughness surface bridge are examined in order to gain insight into the various designs of smart suspension systems for *bridge-friendly vehicles.* Mathematical models are described in sections 2 and 3, and numerical results are presented and discussed in section 4.

2. BRIDGE-VEHICLE INTERACTION SYSTEM MODEL

Consider the problem of a vehicle traversing along a bridge with a constant speed v; see Figure 1. The bridge is modeled as a simply supported Euler-Bernoulli beam. The modeled vehicle is a quarter-car with two degree-of-freedom. This vehicle model allows us to consider a controller with variable damping.

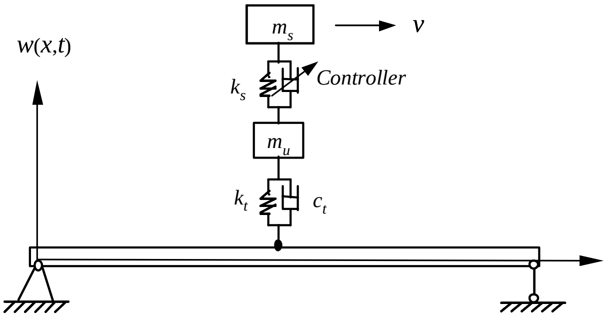


Figure 1. Schematic model of the bridge-vehicle interaction system.

The spung body mass of the vehicle, m_s, is supported by a suspension system, which can either be passive or semi-active, and has bounce motion z_s. The unsprung wheel mass, m_u, has hop motion z_u. The tire is modeled by a linear spring with stiffness k_t and a damper with coefficient c_t, and is assumed to be in contact with the bridge surface at all instants of time. The surface profile on the bridge deck is assumed to be rough and denoted by $r(x)$. In the numerical studies, the profile of $r(x) = A\sin(\frac{2\pi x}{b})$ is chosen. Then governing differential equation of the system is

$$\rho w,_{tt}(x,t) + Kw(x,t) = -\left[(m_s + m_u)g + F(t)\right]\delta(x - vt), \quad 0 \le x \le L,\ 0 \le t \le L/v, \tag{1}$$

where $w(x,t)$ is the transverse displacement of the continuum, $(\cdot),_{tt}$ denotes $\partial^2/\partial t^2$. ρ and K are positive-definite spatial differential operators representing the inertia and stiffness of the structure, respectively, and $\delta(x)$ is the Dirac-delta-function. For the beam model, ρ is the linear mass density of the beam and $K = EI\,\partial^4/\partial x^4$. The *dynamic* interaction force $F(t)$ is

$$F(t) = k_t\left[w(vt,t) - z_u(t) + r(vt)\right] + c_t\left[\frac{dw(vt,t)}{dt} - \dot{z}_u(t) + vr'(vt)\right], \tag{2}$$

and the governing equations of motion for the oscillator are

$$\begin{aligned} m_u\ddot{z}_u(t) &= F(t) - F_s, \\ m_s\ddot{z}_s(t) &= F_s, \end{aligned} \tag{3}$$

where F_s is the force supplied by the suspension system which is given by

$$F_s = k_s(z_u - z_s) + f_d. \tag{4}$$

In (4), the damping force f_d can either be fixed or tunable depending on the control strategies to be applied. Mathematical models for the passive and semi-active control strategies will be reviewed in the next section.

The solution for the displacement of the continuum can be represented as a series expansion in terms of the eigenfunctions $\varphi_n(x)$ of the distributed system [39]

$$w(x,t) = -\sum_{n=1}^{N}\varphi_n(x)\,q_n(t), \tag{5}$$

where the coefficients $q_n(t)$ are solutions to the coupled system of differential equations,

$$\begin{aligned} &\ddot{q}_n + \omega_n^2 q_n = \varphi_n(vt)(mg + F(t)), \quad 0 \le t \le L/v, \\ &F(t) = -k_t\Big(\sum_{n=1}^{N}\varphi_n(vt)q_n(t) + z_u(t) - A\sin\frac{2\pi vt}{b}\Big) \\ &\qquad - c_t\Big(\sum_{n=1}^{N}[v\varphi_n'(vt)q_n(t) + \varphi_n(vt)\dot{q}_n(t)] + \dot{z}_u(t) - \frac{2\pi v}{b}A\cos\frac{2\pi vt}{b}\Big), \end{aligned} \tag{6}$$

For simply supported beam model, the eigenfunctions and frequencies are

$$\varphi_i(x) = \sqrt{\frac{2}{\rho L}}\sin\frac{i\pi x}{L}, \quad \omega_i = \left(\frac{i\pi}{L}\right)^2\sqrt{\frac{EI}{\rho}}, \quad i = 1,\cdots,\infty \tag{7}$$

The coupled system of (3) and (6) is solved numerically by MatLab. It is noted that the parameter *vt* suggests that the system is excited both externally (due to bridge uneven profile) and parametrically (due to the coupling with the modes of the bridge). Since this dynamic interaction is limited to $0 \le t \le L/v$, we expect the transient response to govern the maximum response of the bridge [40]. Thus, in the numerical studies, we searched for the maximum response of the bridge within $0 \le t \le L/v$. Interest in the transient response also prohibits the use of many classical methods for the evaluation of the steady-state response.

3. CONTROL STRATEGIES

For a passive control, the damping force is a linear function of the relative velocity, which can be given by

$$f_d = c_s(\dot{z}_u - \dot{z}_s) \tag{8}$$

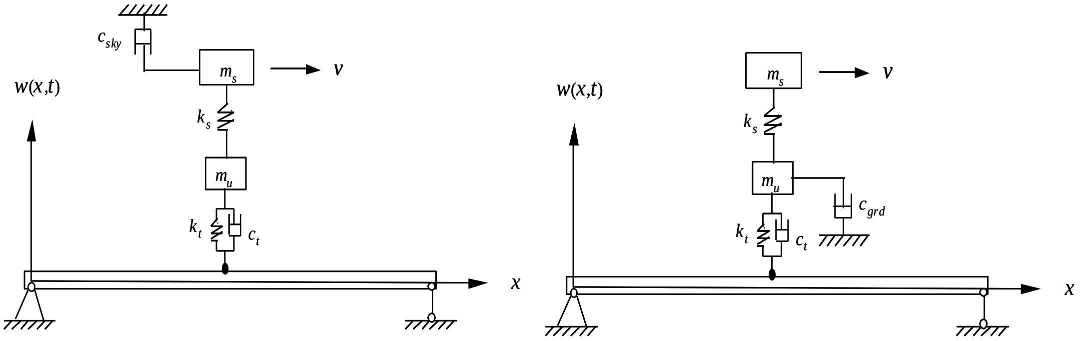
For semi-active control, it is tunable according to different strategies. The well-known skyhook control strategy can be described by the following equations

$$f_d = \begin{cases} c_{sky} z_s & \dot{z}_s(\dot{z}_s - \dot{z}_u) > 0 \\ 0 & \dot{z}_s(\dot{z}_s - \dot{z}_u) < 0. \end{cases} \tag{9}$$

The ground-hook control policy is given by

$$f_d = \begin{cases} c_{grd} z_u & \dot{z}_u(\dot{z}_u - \dot{z}_s) > 0 \\ 0 & \dot{z}_u(\dot{z}_u - \dot{z}_s) < 0. \end{cases} \tag{10}$$

The idealized configurations of both the skyhook and ground-hook controllers are depicted in Figure 2.



(a) skyhook (b) ground-hook

Figure 2. Idealized semi-active control configurations.

In both the skyhook and ground-hook dampers, a force is generated to reduce the absolute velocity of the masses, while the conventional passive damper exerts a force tending to reduce the relative velocity of the masses. The skyhook control is primarily designed to control the vibration of the sprung mass and the ground-hook control is designed to control the vibration of the unsprung mass. Both will supply power only when the relative velocity and the absolute velocity are of the same sign. That is, when the two velocities of the two masses are of opposite sign, the device supplies no force ideally. In practice, due to physical limitations of a damper, a damping coefficient of zero is not attainable as a lower limit and a small damping constant is used in the numerical studies.

The adjustment of the damping force can be realized by changing the orifice area through which the fluid flows in the hydraulic devices. Basic characteristics of electrorheological (ER) fluid and magnetorheological (MR) fluid offer more reliable semi-active control devices. The viscosity of the fluid increases with increased electric field or magnetic field applied across the fluid and therefore variable damping force can be achieved by varying the electric or magnetic field. The stability and fast response characteristics make the semi-active damper promising [26].

4. RESULTS AND DISCUSSION

Consider a simply supported girder bridge with a span length $L = 20$ m. The material properties of the bridge are chosen as: mass density, $\rho = 1.5 \times 10^4$ kg/m, Young's modulus, $E = 2 \times 10^{11}$ N/m^2, second moment of cross-section, $I = 0.048$ m^4. The first frequency of the bridge is at $\omega_1 = 20$ rad/s. The surface deck profile of the bridge is a sinusoidal wave, $r(x) = A\sin(\frac{2\pi x}{b})$, with $A = 0.02$ m, $b = 2$ m. The frequency of the external excitation due to the roughness profile is $\omega = \frac{2\pi v}{b}$. The masses of the body and the wheel are $m_s = 1.8\times10^4$ kg, $m_u = 2\times10^3$ kg, spring stiffness values are $k_s = 9.0\times10^6\ \mathrm{N\cdot m^2}$, $k_t = 3.6\times10^7\ \mathrm{N\cdot m^2}$. Note that one of the natural frequencies of the vehicle is $\omega_o = 20$ rad/s, which matches with ω_1. The damping parameter c_t is set to be zero.

4.1. Comparative study of different control strategies

Solving the coupled equations of (3) and (6) gives the responses of the system. Usually the maximum response of the bridge occurs at or around the mid-span of the bridge. Consider a bridge traversed by the vehicle at a normal speed of 20m/s (around 45mi/h). If no irregularity of the deck surface is considered, the mid-span response is shown by the solid curve in Figure 3. When the road roughness profile is included, the responses of the mid-span without control and with passive control are shown by the dashed and dotted curves, respectively. It is seen that the increment of the maximum mid-span response due to the roughness profile is about 66%. This increment of course has a direct relationship with the amplitude of the road profile (*A*). Moreover, when a passive damper with damping ratio of 0.1 is attached to the vehicle suspension, the maximum mid-span response of the bridge can be mitigated by about 20%. Therefore, tuning the suspension of the vehicle is a feasible way to control and reduce the maximum response of the bridge.

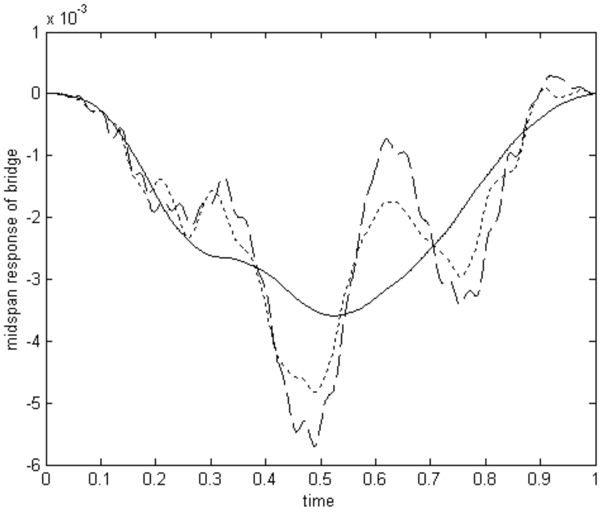


Figure 3. Response of the mid-span of the beam for the case of $v = 20$ m/s ; even surface (——), uneven surface without control (– – – –), uneven surface with passive control (------).

Effects of tuning suspensions in semi-active control strategies on the dynamics of the bridge-vehicle interaction system are shown in Figures 4 and 5. In these figures, the solid curves are the results for passive control, the dashed curves are the results for skyhook policy, and the dotted curves are that of the ground-hook strategy. Three sets of results are presented in Figure 4: mid-span response of the beam, body response and wheel response of the vehicle. Figure 5 shows dynamic interaction force (tire force), which is a critical parameter for examining damages to the surface profile. In Figure 4(a), the skyhook control reduces the maximum bridge response by about 20% compared to the passive control case. However, the ground-hook control leads to large amplitudes and therefore should not be considered for the control of the response of the bridge. For the body response in figure 4(b), the skyhook and ground-hook policies give almost the same response (amplitude-wise) and are preferred over the passive control. It is noted that while the skyhook semi-control significantly reduces the bridge response, it leads to large amplitudes in the wheel response and the dynamic interaction force. The ground-hook control is most preferable for reducing damages to road profiles, as have been demonstrated by previous work [20]. From these preliminary studies, it is clear that simple semi-active control leads to conflicting control criteria. Future work should incorporate optimal control formulations for designing the most bridge-friendly suspension systems, either semi-actively or some sort of hybrid combinations.

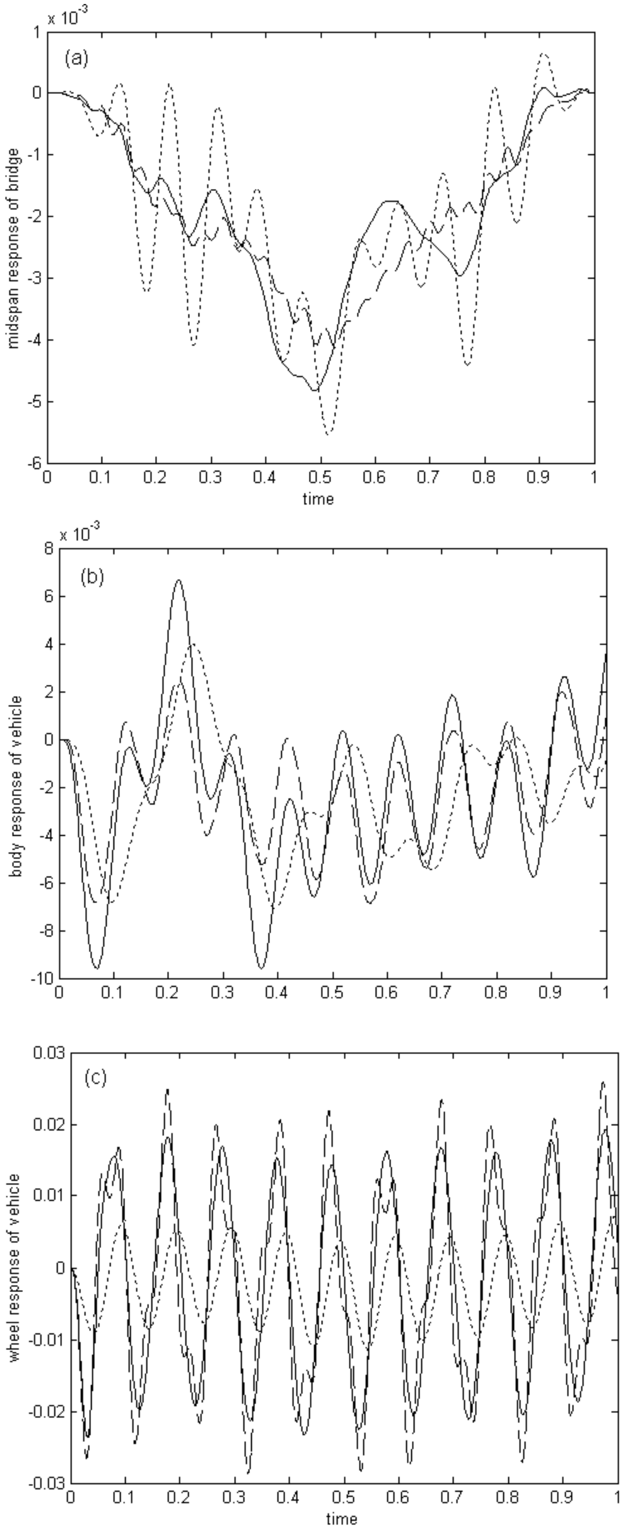


Figure 4. Response of the system with different control strategies for the case of $v = 20$ m/s ; passive (——), skyhook (– – – – –), ground-hook (------). (a) Mid-span response of the bridge, (b) body response of the vehicle, and (c) wheel response of the vehicle.

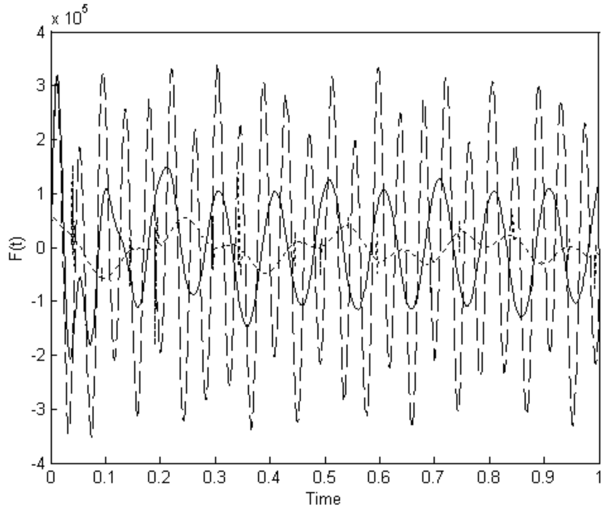


Figure 5. Dynamic interaction force with different control strategies for the case of $v = 20$ m/s ; passive (——), skyhook (– – – –), ground-hook (------).

4.2. Frequency matching phenomenon

For the problem of the dynamic interaction of the vehicle-bridge system under the excitation of the bridge deck roughness, there are three frequencies that may match with each other. These are the bridge which are chosen to be 20, 80, $\cdots$, rad/s. The natural frequencies of the vehicle depend on the suspension, body and wheel parameters, and for our numerical studies, the lower natural frequency has been chosen to be either 10 rad/s or 20 rad/s (match the first natural frequency of the bridge). The second natural frequency of the vehicle is much higher and do not match with the higher modes of the bridge. Note that the natural frequencies of the vehicle can be tuned by varying the spring stiffness. As the vehicle traverses at different speeds, the excitation frequency due to the roughness $\omega = \frac{2\pi v}{b}$ varies (which is like tuning the vehicle speed parameter).

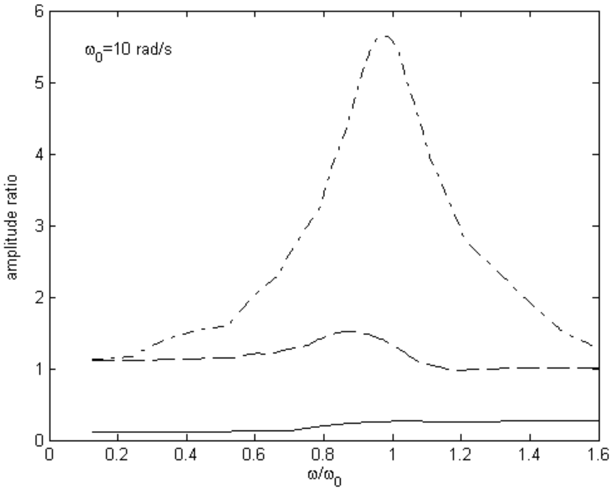


Figure 6. The transmissibility of the bridge and vehicle response with passive control (no frequency matching case); bridge (——), body (—·—·—), wheel (– – – –).

Figure 6 shows the response of the system when the frequencies of the vehicle do not match those of the bridge. In particular, the lower natural frequency ω_o is chosen to be 10 rad/s (which is far from the first bending frequency of the bridge). In Figure 6, the transmissibility is defined as the ratio of the response amplitude to the amplitude of the excitation (*A*). In both figures 6 and 7, passive control was considered. It is seen in figure 6 that the response of the body exhibits a resonance phenomenon when $\frac{\omega}{\omega_o} \approx 1$, as expected. Comparing Figure 6 with Figure 7, the case of frequency matching in which $\omega_o = 20\,\text{rad/s}$ (matches with the first natural frequency of bridge), it is seen that the bridge response is significantly larger when $\frac{\omega}{\omega_o} \approx 1$. In other words, the bridge behaves like a vibration absorber at this frequency matching condition. This is confirmed by the fact that the response of the vehicle reduced at $\frac{\omega}{\omega_o} \approx 1$. The amplitude of the response of the bridge is about 10 times higher than the case of no frequency matching. This is undesirable from a bridge design viewpoint. Note that the wheel response also increases when there is frequency matching.

Figure 8 presents similar results when a semi-active skyhook control strategy is adopted. The amplitude of the response of the bridge is reduced by 25%. However, the vibration absorber phenomenon remains. Therefore, more sophisticated control strategies are needed to effectively remove this vibration phenomenon.

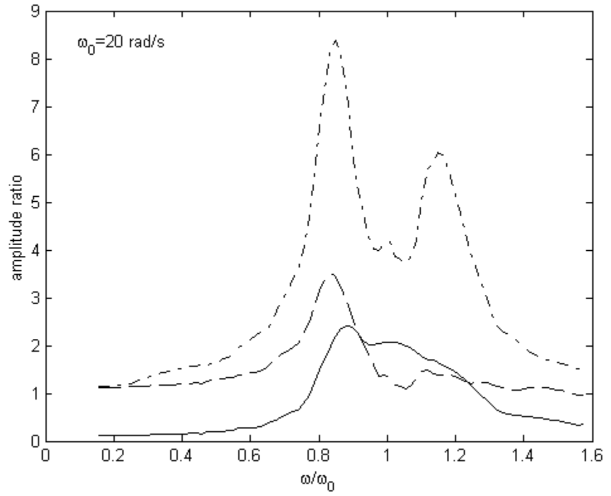


Figure 7. The transmissibility of the bridge and vehicle response with passive control (frequency matching case); bridge (——), body (—·—·—), wheel (– – – –).

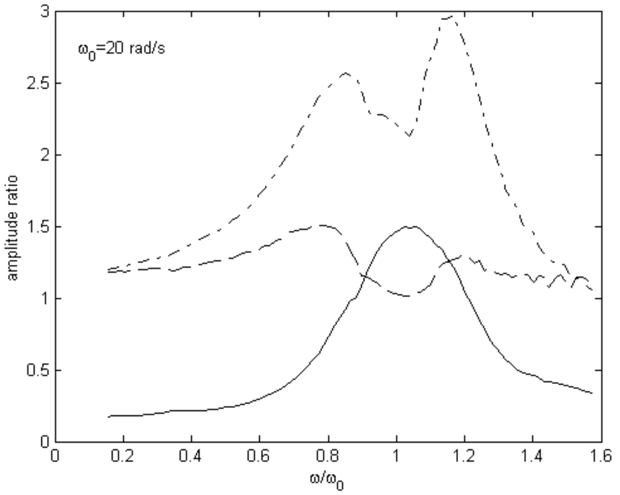


Figure 8. The transmissibility of the bridge and vehicle response with semi-active control (frequency matching case); bridge (——), body (—·—·—), wheel (– – – –).

5. SUMMARY AND CONCLUSION

Numerical studies on the effects of tuning suspension of a vehicle on the response of a bridge model were performed. The mathematical problem considered was a two-degree-of-freedom oscillator traversing a simply supported Euler-Bernoulli beam with surface roughness. Different control strategies of passive and semi-active (skyhook and ground-hook) were considered. Results are obtained to understand the effects of these controls on the coupled response of the vehicle and the bridge, and to demonstrate the advantages of developing smart suspensions for bridge-friendly vehicles. Key results of this study are summarized as follows.

1. Mitigation of the response of the bridge by tuning the parameters of the suspension of the vehicle is feasible.
2. Mitigation of the response of the bridge may conflict with reduction of tire forces and road damage, and ride comfort. The skyhook control strategy is generally effective for reducing the response of the bridge, but leads to large tire forces. However, the ground-hook policy is effective in reducing the tire force imparted onto the bridge.
3. When the natural frequency of the vehicle matches that of the bridge and the excitation frequency due to surface roughness, the bridge behaves like a vibration absorber.

Further studies are underway to: (1) develop optimal control formulations for effective reduction of bridge response and tire forces, and the vibration absorber phenomenon at frequency matching, (2) examine the problem of multiple vehicles interacting with the bridge in the framework that has been presented in this paper.

ACKNOWLEDGMENT

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