

SMOOTHNESS CONDITIONS AND FOURIER SERIES

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Abstract. We study the relationships between smoothness properties of a 2π -periodic function and the asymptotic behavior of its Fourier coefficients. The smoothness conditions of a function are expressed in terms of it being in the generalized Lipschitz classes.

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