

# Social Capital and Social Quilts: Network Patterns of Favor Exchange\*

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## Abstract

We examine the informal exchange of favors among the members of a society. We focus on situations where the interaction between any two individuals is insufficiently frequent to provide incentives to exchange favors over time, but where the social pressure of the possible loss of multiple relationships can sustain exchange. We characterize the network patterns of exchange that are robust in a sense that deleted relationships only result in a local loss of favor exchange. Such robustness necessitates “social quilts”: networks that are tree-like unions of completely connected subnetworks. More generally, with heterogeneity in agents’ costs and benefits from favors, robust networks are such that all links are “supported”: any pair of individuals exchanging favors must have a common friend. We show that favor exchange networks in 75 villages in rural India exhibit a frequency of this sort of support that is significantly higher than a standard ‘clustering’ measure. We also find some significant contrasts between support levels in favor networks and purely social networks.

Keywords: Social Networks, social capital, favor exchange, support, social quilts, renegotiation-proof

JEL Classification Codes: D85, C72, L14, Z13

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# 1 Introduction

Human beings rely on cooperation with others for their survival and growth. Although some forms of cooperation and behavior are enforced by social, religious, legal, and political institutions that have emerged throughout history, much of development, growth, and basic day-to-day functioning relies on a society’s ability to “informally” encourage cooperative behavior. This sort of informal enforcement of cooperation ranges from basic forms of quid-pro-quo (or tit-for-tat in the game theory parlance) to more elaborate forms of social norms and culture, all of which must function without enforceable contracts or laws.<sup>1</sup> Indeed, contracting costs are prohibitive for many day-to-day favors that people exchange, ranging from offering advice to a colleague, a small loan to a friend, or emergency help to an acquaintance. Such informal favor exchange and cooperative behaviors, in one sort or another, underly much of the literature on social capital.

Although there is a large literature on social capital, there is a paucity of work that provides careful foundations for how social structure relates to such favor exchange and cooperative behavior. Moreover, as we show here, favor networks do not exhibit the suggested patterns predicted by the previous literature that has considered network architecture. These points are related to each other since some standard network measures have emerged only loosely from the literature discussing the role of networks in fostering cooperation. In particular, the importance of social pressures on fostering cooperation has deep roots in the sociology literature including seminal work by Simmel (1950), Coleman (1988) and more recently by Krackhardt (1996), among others (see the literature discussion below). Standard measures of network clustering and transitivity have grown in part out of those works. Clustering measures examine the extent to which two friends of a given agent are friends of each other. In the data on favor exchange networks in Rural India that we examine here, clustering is on the order of ten to thirty percent. A puzzle emerges as to why one sees that level of clustering, and not some other higher level, and even whether clustering is really the appropriate measure for capturing social pressures. In contrast, the new concept of “support” that emerges from our theoretical analysis measures the number of pairs of friends that have some other friend in common. As we shall see in the data, support is several times higher than clustering, and indeed this distinction is consistent with the theory presented here.

To be specific, in this paper we provide a game theoretic foundation for social enforcement of informal favor exchange, and also examine network patterns of favor exchange from 75 rural villages. In particular, we consider settings where simple bilateral quid-pro-quo

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<sup>1</sup>In fact, the term “ostracism” (which has Greek origins based on a practice of banishments that originated in the Athenian democracy) has come to embody the idea of individuals cutting ties with members of society who do not perform properly.

enforcement is insufficient to sustain favor exchange. Some bilateral interactions may be infrequent enough that they fail to allow natural self-enforcement of cooperation or favor exchange. However, when such interactions are embedded in a network of interactions whose functioning can be tied to each other, then individuals can find it in their interest to cooperate given (credible) threats of ostracism or loss of multiple relationships for failure to behave well in any given relationship. We provide complete characterizations of the network patterns of favor exchange that are sustainable by a form of equilibrium satisfying two robustness criteria.

The setting that we examine is such that opportunities for one agent to do a favor for another agent arrive randomly over time. Providing a favor is costly, but the benefit outweighs the cost, so that it is efficient for agents to provide favors over time. However, it could be that the cost of providing a favor today is sufficiently high that it is not in an agent's selfish interest to provide the favor even if that means that he or she will not receive favors from that person again. Thus, networks of relationships are needed to provide sufficient incentives for favor exchange, and it may be that an agent risks losing several relationships by failing to provide a favor. We characterize the network structures that correspond to robust equilibria of favor exchanges. The criteria that we examine are twofold: first, the threats of which relationships will be terminated in response to an agent's failure to deliver a favor must be credible. Credibility is captured by the game theoretic concept of "renegotiation-proofness".<sup>2</sup> After an agent has failed to deliver a favor, that relationship is lost, but which additional relationships are lost in the continuation equilibrium, must be such that there is not another equilibrium continuation that all agents prefer to the given continuation. This sort of renegotiation-proofness rules out unreasonable equilibria such as the "grim-trigger" sort of equilibrium where once anyone fails to provide a single favor the whole society grinds to a halt and nobody provides any favors in the future. At that point, it would be in the society's interest to return to some equilibrium where at least some favors are provided. Renegotiation-proof equilibria can be complex, but have some nice intuitions underlying their structure as we explain in detail in the paper. The second criterion that we impose is a robustness condition that we call "robustness against social contagion." It is clear that to sustain favor exchange, an agent must expect to lose some relationships if the agent fails to deliver a favor. Those lost relationships can in turn cause other agents to lose some of their relationships since the incentives to provide favors change with the network structure. This can lead to some fragility of a society, as one agent's bad behavior can ripple through the society. The robustness against social contagion requires that the ripple effects of some

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<sup>2</sup>Although there are several definitions in the literature for infinitely repeated games, our games have a structure such that there is a natural definition which has an inductive structure reminiscent of that of Benoit and Krishna (1993).

agent’s bad behavior be confined to that agent’s neighbors and not propagate throughout the network.

The combination of renegotiation-proofness and robustness tie down a unique type of network configuration of favor exchanges that are possible. We call those configurations “social quilts.” A social quilt is a union of small cliques (completely connected subnetworks), where each clique is just large enough to sustain cooperation by all of its members and where the cliques are laced together in a tree-like pattern. One of our main theoretical results shows that configurations of favor exchange that are sustained in robust equilibria are precisely the social quilts. We then extend the model to allow heterogeneity in the cost and value of favors to various individuals. Under that extension, we prove that all robust equilibrium networks must exhibit a specific trait: each of its links must be “supported.” That is, if some agent  $i$  is linked to an agent  $j$ , then there must be some agent  $k$  linked to both of them. This is related to, but turns out to be quite different from, various clustering measures that are common in the social network literature.

With the theoretical underpinnings in hand, we then examine social networks in 75 villages in southern rural India.<sup>3</sup> These data are particularly well-suited for our study as they provide network structure for various favor relationships, and moreover have this for many separate villages. We are not aware of any other data set having these attributes. In particular, in these data we have information about who borrows rice and kerosene from whom, who borrows small sums of money from whom, who gets advice from whom, who seeks emergency medical aid from whom, and a variety of other sorts of relationships, as well as gps data. Using these data we can examine the networks of various forms of social interaction including specific sorts of favor exchange. In line with the theoretical predictions, we find that the number of favor links that have this sort of social support is in the range of eighty percent in these villages. Moreover, the level of support is significantly higher than what would arise if links were formed at random (even with some geographic bias to formation), and significantly higher than levels of clustering. We analyze various aspects of the levels of support and also find that it is significantly higher for favor relationships than other sorts of relationships.

Our research contributes to the understanding of informal favor exchange as well as social networks in several ways:

- We provide an analysis of repeated interactions where individual’s decisions are influenced by the network pattern of behavior in the community, and this provides new

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<sup>3</sup>Although we apply some of our findings to favor relationships in Indian Villages, such informal favor exchange is clearly not limited to developing countries. For example, a recent New York Times/CBS News poll (reported in the New York Times, December 15 2009) found that 53 percent of surveyed unemployed workers in the U.S. had borrowed money from friends or family as a result of being unemployed.

insights into repeated games on networks.

- Our model includes dynamic choices of both favor provision and relationship choices and provides new insights into the co-evolution of networks and behavior, and in particular into the phenomenon of ostracism.
- Our analysis suggests a new source of inefficiency in informal risk and favor sharing, showing why individuals may have to limit the number of relationships in which they take part.
- A by product of our analysis is an operational definition of social capital that is more specific and tighter than many existing definitions, and it makes tight predictions about how relationships in a society must be organized.
- We provide a new property of networks that we call “support” and show how this is distinguished from standard clustering measures.
- We examine data that include many sorts of interactions and cover 75 different villages, and find that the networks exhibit substantial and significant distinctions between our measure of support and standard measures of clustering.

## 1.1 Related Literature

As mentioned above, there is a large literature on social capital that studies the ability of a society to foster trust and cooperation among its members.<sup>4</sup> Although that literature is extensive and contains important empirical studies and many intuitive ideas, it has struggled in providing firm theoretical foundations and the term “social capital” has at times been used very loosely and as a result has lost some of its bite.<sup>5</sup> Part of the contribution of our paper is to provide an explicit modeling of how societies can enforce cooperative favor exchange and how this is linked to the social network structure within a society. In this way, our paper provides a very concrete definition of social capital that is embedded in three components: a notion of equilibrium that embodies notions of ostracism and social expectations of individual behaviors, implications of this for resulting social network structure, and individual payoffs from the resulting behaviors.

Coleman (1988) discusses closure in social networks, emphasizing the ability of small groups to monitor and pressure each other to behave. Here we provide a new argument

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<sup>4</sup>For example, see Homans (1958), Loury (1977), Bourdieu (1986), Coleman (1988, 1990), Woolcock (1998), Dasgupta (2000), Putnam (1993, 1995, 2000), Glaeser, Laibson, and Sacerdote (2002) Guiso, Sapienza, and Zingales (2004), Tabellini (2009), among others.

<sup>5</sup>See Sobel (2002) for an illuminating overview and critique of the literature.

for, and a very specific variety of, closure. Here a specific form of minimal clique structures emerge because of a combination of renegotiation-proofness and a local robustness condition, rather than for informational, monitoring, or pressuring reasons. Minimal sized cliques offer both credible threats of dissolving in the face of bad behavior, and in terms of minimal contagion for a society. Our analysis also formalizes this in terms of support and contrasts it with clustering.

The most closely related previous literature in terms of the theoretical analysis of a repeated game on a network is a series of papers that study prisoners' dilemmas in network settings, including Raub and Weesie(1990), Ali and Miller (2009), Lippert and Spagnolo(2009), and Mihm, Toth, Lang (2009).<sup>6</sup> In particular, Raub and Weesie(1990) and Ali and Miller (2009) show how completely connected networks shorten the travel time of information which can quicken punishment for deviations of behavior. Although cliques also play a prominent role in some of those papers, it is for very different reasons. In those settings, individuals do not have information about others' behaviors except through what they observe in terms of their own interactions. Thus, punishments only travel through the network, and the main hurdle to enforce individual cooperation is how long it takes for someone's bad behavior to come to reach their neighbors through chains of contagion. This builds on earlier work by Greif (1989), Kandori(1992), Ellison (1994), Okuno-Fujiwara and Postlewaite(1995) among others, who studied the ability of a society to sustain cooperation via threats of contagions of bad behavior. Our analysis is in a very different setting, where individuals have complete information. The quilts in our setting emerge because they *do not* lead to large contagions but instead compartmentalize the damage from an individual's defection. Moreover, the quilts consist of minimal sized cliques because only those sorts of implicit punishments are immune to renegotiation.

Haag and Lagunoff (2004) provide another reason favoring small cliques: heterogeneity. In their analysis large differences in preferences can preclude cooperative behavior, and so partitioning a group into more homogeneous subgroups can enable cooperative behavior which might not be feasible otherwise. Although our reasoning behind cliques comes from very different reasons, when we examine heterogeneous societies we do find assortativity in who exchanges favors with whom. Here, the reasoning is not because of direct reciprocity considerations, but because robustness requires balanced cliques and so agents need to have similar valuations of favors in order for their cliques to be critical. In this way, we provide new insights into homophily, where relationships of agents are biased towards others who

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<sup>6</sup>Other studies of network structure and cooperative or various forms of risk-sharing behavior and the relationship to social network structures include Fafchamps and Lund (2003), De Weerd and Dercon (2006), Bramoullé and Kranton (2007), Bloch, Genicot, and Ray (2007, 2008), and Karlan, Mobius, Rosenblat and Szeidl (2009).

have similar characteristics in terms of their values and arrival rates of favors.

Finally, our analysis of the data not only provides support for the support measure, but also uncovers significant differences between different sorts of networks. Differences between the network structure of various sorts of relationships is something that might be expected based on the different ways in which links might form across applications (e.g., see Jackson (2008)) and here we add a new angle to this understanding, finding statistically distinct patterns of support in various sorts of favor and social networks. These suggest some interesting questions for future research.

## 2 A Model of Favor Exchange

### 2.1 Networks, Favors, and Payoffs

A finite set  $N = \{1, \dots, n\}$  of agents are connected in a social network described by an undirected<sup>7</sup> graph. Given that the set of agents or nodes  $N$  is fixed throughout the analysis, we represent a network, generically denoted  $g$ , simply by the set of its links or edges. Let  $g^N$  be the set of all links (so the set of all subsets of  $N$  of size 2), and let  $G = \{g \mid g \subset g^N\}$  be the set of all possible networks. For simplicity, we write  $ij$  to represent the link  $\{i, j\}$ , and so  $ij \in g$  indicates that  $i$  and  $j$  are linked under the network  $g$ . We write  $g - ij$  to denote the network obtained from  $g$  by deleting a link  $ij$ . For an integer  $k$ ,  $0 \leq k \leq n(n-1)/2$ , let  $G_k$  be the set of all networks that have exactly  $k$  links, so that  $G_k = \{g \in G : |g| = k\}$ .

The neighbors of agent  $i$  are denoted

$$N_i(g) = \{j \mid ij \in g\}.$$

We follow a convention that rules out self-links, and so all agents in  $N_i(g)$  are distinct from  $i$ . The degree of agent  $i$  in the network  $g$  is the number of his or her neighbors denoted by  $d_i(g) = |N_i(g)|$ .

Time proceeds in discrete periods indexed by  $t \in \{0, 1, \dots\}$  and in any given period, there is a chance that an agent will need a favor from a friend or will be called upon to grant a favor to a friend. In particular, an agent  $i$  who is connected to an agent  $j$  (so that  $ij \in g$ ) anticipates a probability  $p > 0$  that  $j$  will need a favor from  $i$  in period  $t$  and a probability  $p$  that  $i$  will need a favor from  $j$ . It is assumed that at most one favor will be needed across all agents in any given period, and so we require that

$$n(n-1)p \leq 1.$$

we allow the sum to be less than one to admit the possibility that no favor is needed in a given period.

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<sup>7</sup>This is not necessary for the analysis, and we comment later on possible extensions to directed networks.



This is a proxy for a Poisson arrival process, where the chance that two favors are needed precisely at the same moment is 0. By letting the time between periods be small, the chance of more than one favor being called upon in the same period goes to 0. Thus, when applying the model it is important to keep in mind that periods are relatively small compared to the arrival rate of favors.

A restriction of this formulation is that  $p$  does not depend on the network structure. More generally, the chance that  $i$  needs a favor from  $j$  will depend on many things including how many other friends  $i$  has. We characterize the equilibrium networks for the general case in Section 5. We begin with the current case since it more clearly provides the basic intuitions, but the results have very intuitive analogs for the general case that are easy to describe once we have presented the simpler case.

Doing a favor costs an agent an amount  $c > 0$  and the value of the favor to an agent is an amount  $v > c$ . “Favors” can embody many things including asking for advice, to borrow some good, to borrow money, or to perform some service. The important aspect is that the value of a favor to one agent exceeds the cost, so that it is ex ante Pareto efficient for agents to exchange favors over time. However, we examine settings where it is impossible (or too costly) for agents to write binding contracts to perform favors whenever called upon to do so. This applies in many developing countries, and also in developed countries where it is prohibitively costly and complex to write complete contracts covering the everyday sort of favors that one might need from friends. Thus, we examine self-enforcing favor exchange.

Agents discount over time according to a factor  $0 < \delta < 1$ . Thus, if there were just two agents who always performed favors for each other, then they would each expect a discounted stream of utility of

$$\frac{p(v - c)}{1 - \delta}.$$

The more interesting case from a network perspective is the one that we examine, where

$$c > \frac{\delta p(v - c)}{1 - \delta}.$$

In this case, favor exchange between two agents in isolation is not sustainable. When called upon to perform a favor, the agent sees a cost that exceeds the future value of potential favor exchange (in isolation) and so favor exchange cannot be sustained between two people alone, but must be embedded in a larger context in order to be sustained. Sustaining favor exchange between two individuals requires a high enough frequency of arrival coupled with a high enough marginal benefit from a favor and sufficient patience. In a marriage, there are generally sufficiently many opportunities for each spouse to help the other out with some task or need that bilateral favor exchange can be sustained. However, in other contexts, where such needs are rarer - such as a need to borrow cash due to an emergency, or a need for medical advice, etc., one might need a multilateral setting to sustain favor exchange.

A society is described by the profile  $(N, p, v, c, \delta)$ .

## 2.2 The Game

The favor exchange game is described as follows.

- The game begins with some initial network in place, denoted  $g_0$ .
- Period  $t$  begins with a network  $g_{t-1}$  in place.
- Agents (simultaneously)<sup>8</sup> announce the links that they are willing to retain:  $L_i \subset N_i(g_{t-1})$ . The resulting network is  $g'_t = \{ij \mid j \in L_i \text{ and } i \in L_j\}$ .
- Let  $k_t$  be the number of links in  $g'_t$ . With probability  $2pk_t$  need for a (single) favor arises and with probability  $1 - 2pk_t$  there is no need for a favor in the period. If a favor is needed, then it could apply to any link in  $g'_t$  with equal likelihood and then go either direction. If a favor is needed, then let  $i_t$  denote the agent called upon to do the favor and  $j_t$  the agent who needs the favor, where  $i_t j_t \in g'_t$ .
- Agent  $i_t$  chooses whether or not to perform the favor. If the favor is performed then  $i_t$  incurs the cost  $c$  and agent  $j_t$  enjoys the benefit  $v$ . Otherwise no cost or benefit is incurred.
- The ending network, denoted  $g_t$ , is  $g'_t - i_t j_t$  if the need for a favor arose and it was not performed, and is  $g'_t$  otherwise.

People make two sorts of choices: they can choose with whom they associate and they can choose to do favors or not to do favors. Opportunities for favor exchange arise randomly, as in a Poisson game, and people must choose whether to act on favors as the need arises. Choices of which relationships to maintain, however, can be made essentially at any time. In the model this is captured by subdividing the period into link choices and favor choices, so that agents have a chance to adjust the network after any favor choice, as well as before any potential favor arises.

This structure embodies several things. First, favor relationships can either be sustained or not. Once a favor is denied, that relationship cannot be resuscitated. Thus, at any point in time an agent's decision is which relationships to maintain. This simplifies the analysis in that it eliminates complicated forms of punishments where various agents withhold favors from an agent over time, in order to punish an agent, but then eventually revert to providing

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<sup>8</sup>Given the equilibrium refinements that we use, whether or not the link choices are simultaneous is effectively irrelevant.

favors. There is a way in which this is an intuitive and natural behavioral assumption. It can be motivated on various behavioral (e.g., emotional) or pro-social grounds and effectively it acts as a sort of refinement of the set of all possible punishments that might occur, as it requires that one of the ostracizing agents be the one who failed to get the favor. This simplification allows us to gain a handle on sustainable network structures as the problem is already very complex (as will become clear shortly), and it appears that much of the intuition carries over to the more flexible case, but that is a subject for further research. As will be clear, this approach generates quite a rich, natural, and interesting set of conclusions.

Second, we do not consider the formation of new links, but only the dissolution of links. This embodies the idea that the formation of new relationships is a longer-term process and that decisions to provide favors and/or ostracize an agent can be taken more quickly and are shorter term actions. It is important to note that we cover the case where society starts with the complete network, so we do not a priori restrict the links that might be formed, and so our results do make predictions about which networks can be formed/sustained in a society. The important wedge that we impose is that an agent who has lost a relationship cannot (quickly) replace it with a newly formed one.

One other aspect of the model is important to mention. Agents do not exchange money for favors. At least hypothetically, that all favor exchange could simply be monetarized. Of course we do not see this in reality, as when a colleague asks to borrow a book we would not charge her or him a rental fee; but that observation does not explain why we do not charge our friends and acquaintances for every little favor that we perform. One explanation is a behavioral one: that monetarizing favors would fundamentally change the way in which people perceive the relationship, and this explanation is consistent with people no longer viewing a monetarized relationship as a long run relationship. For more discussion of this point see Kreps (1997). The specifics of why all favors are not monetarized is outside of our scope, and our starting point is one of simple favor exchange.

For now, we consider a complete information version of the game, in which all agents observe all moves in the game at every node. We discuss more limited information variations in section 7.1.

An agent  $i$ 's expected utility from being in a network  $g$  that he or she expects to exist forever<sup>9</sup> is

$$u_i(g) = \frac{d_i(g)p(v - c)}{1 - \delta}$$

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<sup>9</sup>This applies at any point within the period other than at the very instant at which the agent is called to receive or produce a favor.

If an agent  $i$  is called to do a favor to  $j$  and chooses to perform the favor and expects a network  $g$  to be played in perpetuity thereafter<sup>10</sup> then he or she expects a utility of

$$-c + \delta u_i(g).$$

Similarly if agent  $i$  is called to receive a favor from agent  $j$  and expects to receive the favor and then anticipates a network  $g$  to be played in perpetuity thereafter then his or her expected discounted stream of utility is given by

$$v + \delta u_i(g).$$

For ease of expression, we assume that the discounting parameter  $\delta$  enters the agents' calculations between the announcement stage and the favor stage in any given period.<sup>11</sup>

We note that although we work with favor exchange as a building block in our model, the analysis here directly extends to supporting cooperation more generally, and the same results apply to the play of a prisoners dilemma between agents, or other forms of trust and cooperation games with free-rider or short-term deviation challenges.

## 2.3 Equilibrium

In this setting, any network of favor exchange  $g$  can be sustained in perpetuity as a pure strategy subgame perfect equilibrium as long as

$$c < d_i(g) \frac{\delta p(v - c)}{1 - \delta}$$

for every agent  $i$ . One way in which this is sustained is by a sort of “grim-trigger” strategy where all relationships are sustained and favors are provided as long as no agent refuses a favor, and once any favor is denied then all agents delete all their links and never expect to receive any favors again in the future. Thus, if each agent has enough relationships that he or she might lose, then favor exchange can be sustained. So, for instance, if

$$c < (n - 1) \frac{\delta p(v - c)}{1 - \delta}$$

then the complete network with the most efficient favor exchange could be sustained.

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<sup>10</sup>after the following period's link choices,

<sup>11</sup>This is purely for expository ease as it slightly simplifies the expressions for the critical utility levels for different behaviors, but does not alter the basic structure of the arguments or conclusions. Effectively, it ensures that whenever an agent is either making a decision of which links to announce or whether to follow through on a favor, any potential future favors that might be influenced by the decision are discounted. If discounting happens after the favor period, then when making link choices agents would not discount one round of future favors, but when making favor decisions they would. This simply makes sure that all future favors are discounted in the same way. It is also convenient to begin with an announcement phase, but again this is not essential to the conclusions.

### 2.3.1 Renegotiation-Proofness

While the above conclusion offers some optimism regarding a society’s ability to efficiently sustain favor exchange, it rests upon drastic threats that are not always credible. If for some reason a favor was not performed and some link disappears, a society might wish to reconsider its complete dissolution. Indeed, the idea that if some person in a society acts selfishly and fails to provide a favor, the whole society collapses and no favors are ever exchanged again is drastic and unrealistic. This sort of observation is not unique to this setting, but has been an issue in repeated games for some time (e.g., see the discussion in Bernheim, Peleg and Whinston (1987)). The basic problem is that if agents have some chance to communicate with each other (and perhaps even if they cannot), then when beginning some phase of equilibrium which involves low payoffs, if there is some other *equilibrium* continuation, in which all agents are better off, then they have a strong incentive to change to the play that gives them all better payoffs. Even though this sort of “renegotiation” problem with many sorts of equilibria is well known, it is rare for researchers to do more than to acknowledge it and forge ahead. The reason for this is that properly accounting for renegotiation becomes quite complicated, especially in infinite settings where it is not even clear how to define equilibrium in the face of renegotiation (e.g., see Farrell and Maskin (1989), Bernheim and Ray (1989), and Abreu, Pearce and Stacchetti (1993)). Thus, there are few analyses of renegotiation-proofness outside of some of the original papers working out the definitions.

Our setting has a nice structure that makes it relatively easy to provide a natural definition of renegotiation-proofness and to characterize such equilibria. Before moving to the formal definitions, we present an example that illustrates the ideas.

**EXAMPLE 1** *The Logic of Renegotiation-Proofness*

Let there be 4 agents and consider a case such that

$$2\frac{\delta p(v-c)}{1-\delta} > c > \frac{\delta p(v-c)}{1-\delta}.$$

Here, no link is sustainable in isolation, since the value of providing a favor  $c$  is greater than the value of the future expected stream of giving and receiving favors:  $\frac{p\delta(v-c)}{1-\delta}$ . However, if an agent risks losing two links by not performing a favor, then links could be sustainable depending on the configuration of the network, since  $c < 2\frac{p\delta(v-c)}{1-\delta}$ .

In this case, note that networks where each agent has exactly two links, for example,  $g = \{12, 23, 34, 41\}$ , can be sustained as a subgame perfect equilibrium. If any agent ever fails to perform a favor, then a link will be lost. For example, suppose that 1 fails to deliver a favor to 2, and so the link 12 is lost. At this point, agent 1 only has one relationship left: 14. It is now clear that agent 1 will never perform future favors for 4 and so the link

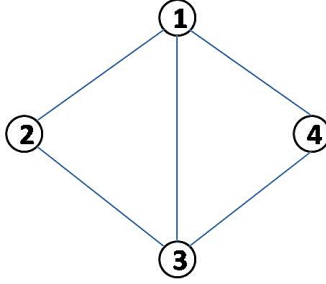


Figure 1: A five link network that is not sustained as a renegotiation-proof equilibrium

14 is effectively useless as well. The same is true of 23. Iterating on this logic, there is no subnetwork that could be sustained as a subgame perfect equilibrium. As such, an agent realizes that if he or she fails to provide a favor, then that will lead to a collapse of all favors and so the network of favor trading is sustained in equilibrium, as failing to provide one favor induces a loss of two relationships. So, starting with such a minimal network there is no difficulty with renegotiation, as following any deviation from the prescribed favor exchange the equilibrium continuation is unique. So, favor exchange sustaining this network is renegotiation-proof as an equilibrium (to be defined shortly).

The problematic subgame perfect equilibria come with  $k = 5$  or more links. Consider the network  $g' = \{12, 23, 34, 41, 13\}$  as pictured in Figure 1. Agents 1 and 3 each have three links and agents 2 and 4 have two links. There is a subgame perfect equilibrium sustaining this network: if any link is ever cut, then all agents cut every link in the future. However, there is no renegotiation-proof equilibrium sustaining this network. To see this, suppose that agent 1 is called upon to do a favor for agent 3. If agent 1 does not do the favor, then the resulting network is  $g' = \{12, 23, 34, 41\}$ . Note that  $g$  is sustainable as a subgame perfect equilibrium as argued above (and in fact is sustainable as part of a renegotiation-proof equilibrium). The logic is now that if  $g$  is reached, then it will be sustained rather than having agents delete all links, since it is not credible for agents to destroy these links as they are all strictly better off sustaining  $g$  than going to autarchy. Thus, when reaching  $g$ , in the absence of some exogenous commitment device, the agents' previous threat to delete all links lacks credibility. As a result of this, agent 1 can cut the link 13 and still expect the network  $g$  to endure, and so this is the unique best response for agent 1 and so  $g'$  is not sustainable as an equilibrium if we require that continuations not be Pareto dominated by another (renegotiation-proof) equilibrium continuation.  $\diamond$

We define *renegotiation-proof networks* to be networks that are sustainable via pure strategy subgame perfect equilibria. It is easiest to define the networks directly, in a way that simultaneously implicitly defines renegotiation-proof equilibrium and explicitly tracks

the networks that are sustainable via those equilibria.

We define the set of pure strategy renegotiation-proof equilibria inductively.<sup>12</sup> The induction operates via the number of links in the starting network. In terms of notation, it will be useful to keep track of the set of all networks that have exactly  $k$  links and can be sustained in perpetuity as part of a pure strategy renegotiation-proof equilibrium if we start at that network.

We let  $RPN_k$  denote the set of *renegotiation-proof networks* with  $k$  links.

- Let  $RPN_0 = \{\emptyset\}$
- Let  $RPN_k$  denote the subset of  $G_k$  such that  $g \in RPN_k$  if and only if beginning with  $g_0 = g$  implies there exists a pure strategy subgame perfect equilibrium<sup>13</sup> such that
  - on the equilibrium path  $g$  is always sustained (all favors are performed and all links are maintained), and
  - in any subgame<sup>14</sup> starting with some network  $g' \in G_{k'}$  with  $k' < k$  if  $g'$  is played in perpetuity with some probability<sup>15</sup> in the continuation then  $g'' \in RPN_{k''}$  for some  $k''$  and there does not exist any  $g''' \subset g'$  such that  $g''' \in RPN_{k'''}$  and  $u_i(g''') \geq u_i(g'')$  for all  $i$  with strict inequality for some  $i$ .<sup>16</sup>

The definition is inductive, since the logic of renegotiation-proofness requires that a network sustained in some continuation not be Pareto dominated by any other continuation that itself is renegotiation-proof. The logic is inherently self-referential, and this is what generally provides difficulties in identifying an unambiguously “correct” definition in an infinite setting. Here, despite our infinite horizon, we can define renegotiation-proofness cleanly since relationships can be severed but not resuscitated and so there is a natural induction on the number of links.

We say that a network  $g$  is *renegotiation-proof* or a *renegotiation-proof network* if there exists some  $k$  such that  $g \in RPN_k$ .

As a further illustration of the definition, let us return to Example 1 and characterize all of the renegotiation-proof networks.

## EXAMPLE 2 *Renegotiation-Proof Networks*

<sup>12</sup>Our analysis concentrates on pure strategy equilibria. As will become clear, considering mixed strategy equilibria would not add much to the insights regarding sustainable network structures.

<sup>13</sup>Pure strategy requires that agents use pure strategies at all nodes on and off the equilibrium path.

<sup>14</sup>This includes subgames starting at any node, not just beginning of period nodes.

<sup>15</sup>Even though agents use pure strategies, nature randomly recognizes favors, and so there can be some randomness in a continuation path.

<sup>16</sup>Note that this condition implies that in any subgame starting with a network  $g' \in RPN_{k'}$ ,  $g'$  is played in the continuation.

Let there be 4 agents and consider a case as in Example 1 such that

$$2 \frac{\delta p(v - c)}{1 - \delta} > c > \frac{\delta p(v - c)}{1 - \delta}.$$

Here,  $RPN_1 = \emptyset$  since no isolated links are sustainable.

Similarly,  $RPN_2 = \emptyset$  since any agent who only has one link will never perform a favor.

$RPN_3 = \{g = \{ij, jh, ih\} : \text{for some distinct } h, i, j\}$ . Thus, triads are sustainable, since if any agent severs a link, then that will lead to a two-link network which is not sustainable, and so becomes an empty network. Thus, not performing a favor leads to an empty network, and so it is a best response to perform a favor, anticipating favors by other agents.

$RPN_4 = \{g = \{ij, jh, hl, li\} : \text{for distinct } h, i, j, \ell\}$ . This is an obvious extension of the logic from three-link networks.

Following the argument in Example 1 it is easy to check that there are no five-link renegotiation-proof equilibria. Thus  $RPN_5 = \emptyset$ .

Next, note that  $RPN_6 = \emptyset$  as well. To see this, note that if some agent  $i$  deletes a link  $ij$ , then a continuation must result in a pure strategy renegotiation-proof equilibrium, which would be either a triad, four link network (with a cycle), or the empty network. The remaining four link network that has a cycle Pareto dominates any other continuation. Thus, if an agent  $i$  severs a link  $ij$ , then the agent is sure that he or she will still have two links in the continuation and so only loses one link.  $\diamond$

### 3 Characterizing Renegotiation-Proof Networks

In this section we provide a complete characterization of renegotiation-proof networks. Before providing the complete characterization, however, we provide some intuitive sufficient conditions that give insight into the structure of renegotiation-proof networks.

Let  $m$  be the whole number defined by

$$m \frac{\delta p(v - c)}{1 - \delta} > c > (m - 1) \frac{\delta p(v - c)}{1 - \delta}. \quad (1)$$

It is clear that there is at most one such  $m$  and that  $m \geq 1$ .

We work with the generic case, ignoring exact equality on either side above. The parameter  $m$  captures how many relationships, each with a future value of  $\frac{\delta p(v - c)}{1 - \delta}$ , an agent has to risk losing in the future in order to have incentives to perform a favor today at the cost of  $c$ .

Throughout the remainder of the paper, the definitions will all be relative to  $m$ , and so we take it to be fixed and defined by (1) and omit explicit mention of it in some of the definitions.

We begin with a formal statement of the proposition on subgame perfect equilibria that motivates our analysis of renegotiation-proof equilibria.



**PROPOSITION 1** *A network is sustainable in perpetuity on the equilibrium path of a subgame perfect equilibrium of the favor exchange game if and only if each agent has either 0 links or at least  $m$  links.*

The proof of Proposition 1 is obvious and thus omitted.

Now we examine renegotiation-proof networks.

### 3.1 Critical Networks and Renegotiation-Proofness

Before proceeding to the complete characterization of renegotiation-proof networks, we examine a natural class of renegotiation-proof equilibria.<sup>17</sup> These sufficient conditions provide an intuitive look at equilibrium structure and help motivate the complete characterization.

Let

$$G(m) = \{g \mid \forall i, d_i(g) \geq m \text{ or } d_i(g) = 0\}$$

denote the set of networks in which each agent has either at least  $m$  links or 0 links. So  $G(m)$  is the set of networks sustainable as subgame perfect equilibria. Note that any renegotiation-proof network must be in  $G(m)$  and since any network sustained in any off-equilibrium path continuation must also be a renegotiation-proof network it must also be contained in  $G(m)$ .

Following the argument above, one way to build a sustainable network is to offer proper incentives for sustaining favors: if an agent deletes a link in a network or fails to provide a favor, it is sufficient that the agent expect to lose at least  $m$  links in the sequel. That is captured by the following definition.

We say that a network  $g$  is  *$m$ -critical*, if

- $g \in G(m)$
- for any  $i$  and  $ij \in g$ , there is no subnetwork  $g' \subset g - ij$  such that  $d_i(g') > d_i(g) - m$  and  $g' \in G(m)$ .

As  $m \geq 2$  will generally be a given in the analysis (although we consider different levels in some examples), we simply omit its reference and use the term “critical” in what follows.

An easy way to build a critical network is to have each agent have exactly  $m$  links. However, we remark that criticality does not require all agents to have exactly  $m$  links. It only requires that any possible continuation equilibrium after some agent fails to provide a favor be such that the agent lose at least  $m$  links. To see how this can allow agents to have more than  $m$  links, consider Figure 2, which pictures such a critical network for a case with  $m = 3$  where agent 1 has four links.

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<sup>17</sup>Additional classes of renegotiation-proof equilibria are discussed in the supplementary appendix.

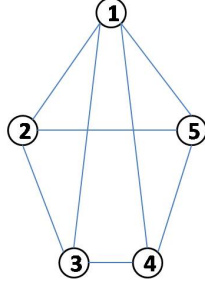


Figure 2: A critical network for  $m = 3$ , but where agent 1 has 4 links.

There is no proper nonempty subnetwork in which all agents who still have links, have at least 3 links each. Thus, if agent 1 (or any other agent) severs any link, the entire network would have to collapse since any proper subnetwork that is nonempty will have some agent with fewer than 3 links, and thus that agent will not have incentives to provide favors. Therefore, if agent 1 fails to provide a favor on some link, then agent 1 would lose all four links.

Critical networks provide an important and nonempty class of networks that are renegotiation-proof. In the sense of Proposition 2, they are a foundational class of networks.

**PROPOSITION 2** *Any nonempty network  $g \in G(m)$  contains a nonempty critical network, and any critical network is renegotiation-proof.*

The last part of Proposition 2 follows from Theorem 1, but we mention the idea behind how this can be proven directly. In order to prove this, we need to show that there exists a pure strategy renegotiation-proof equilibrium such that for any  $i$  and  $ij \in g$ , if  $i$  is called to grant a favor to  $j$  and refuses the favor,  $i$  must lose at least  $m$  links in any continuation network  $g'$ . Renegotiation-proofness requires that the continuation  $g'$  be a subnetwork  $g' \subset g - ij$  and be sustainable as a pure strategy renegotiation-proof equilibrium and so it must be in  $G(m)$ . By the definition of criticality, it then follows that any such  $g' \in G(m)$  be such that  $d_i(g') \leq d_i(g) - m$  (and there always exists at least one such  $g'$  since the empty network is in  $G(m)$ ), and so provides the necessary incentives.

To see that criticality is not necessary for renegotiation-proofness, consider the network in Figure 3: let  $m = 2$  and  $g = \{12, 23, 13, 14, 15, 45, 26, 27, 67\}$  is a tree union of three triads. This is not critical since if 1 cuts the link 12, then all agents in the sub-network still have at least 2 links. Nonetheless, we shall see that it is renegotiation-proof below.

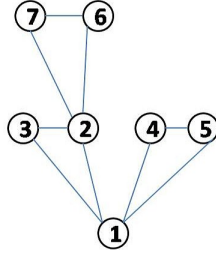


Figure 3: A non-critical network, but still renegotiation-proof.

### 3.2 A Complete Characterization of Renegotiation-Proof Networks: Transitively Critical Networks

We now turn to the complete characterization of renegotiation-proof networks.

Let  $D(g)$  denote the profile of degrees associated with a network  $g$ :

$$D(g) \equiv (d_1(g), \dots, d_n(g)).$$

Write  $D(g) > D(g')$  if  $D(g) \geq D(g')$  and  $D(g) \neq D(g')$ .

We define *transitively critical* networks as follows.

Given a whole number  $m$  satisfying (1), let  $TC_k(m) \subset G_k$  denote the set of transitively critical networks with  $k$  links.

- Let  $TC_0(m) = \{\emptyset\}$ .
- Inductively on  $k$ ,  $TC_k(m) \subset G_k$  is such that  $g \in TC_k(m)$  if and only if for any  $i$  and  $ij \in g$ , there exists  $g' \subseteq g - ij$  such that  $g' \in TC_{k'}(m)$ ,  $d_i(g') \leq d_i(g) - m$ , and there is no  $g'' \in TC_{k''}(m)$  such that  $g'' \subset g - ij$  and  $D(g'') > D(g')$ .

Even though this is also an inductive definition (not surprisingly, given that renegotiation-proof equilibria are so defined), it does not involve any incentive descriptions and is effectively an algorithm that can identify equilibria directly from  $m$ . In fact, we use the equivalence set forth by Theorem 1 to build a computer program that calculates renegotiation-proof networks. In the supplementary appendix we present a table with the number of non-isomorphic transitively critical networks for small values of  $n$  and  $m$ , along with renderings of some of these networks.

**THEOREM 1** *A network is renegotiation-proof if and only if it is transitively critical.*

Although one might expect the proof to be straightforward given that both definitions are inductive, there are some subtleties and challenges in proving Theorem 1. The main one is

that there are many strategy profiles that may hypothetically sustain a collection of networks in a subgame perfect manner, in a way such that the collection satisfies the self-consistency property demanded by renegotiation-proofness. The issue then is that to show that some network is not renegotiation-proof we must be sure that none among the large number of the different strategy profiles that could sustain it, actually work. Moreover, showing that some network is renegotiation-proof involves first showing that some other networks are not in the set. The way in which we tackle this difficulty is by arguing that we can avoid the vast set of potential equilibria that could be used to sustain networks and can focus on a nicely behaved set of strategy profiles. The details appear in the appendix.

## 4 Robustness

We now turn to our criterion of robustness. The idea is that a network is robust if it relies only on local “damage” due to a failure to provide a favor, rather than more global sorts of damage. That is, failure to provide a favor will require some lost links and there is a question of how far that loss of links propagates. We begin with a simple observation.

**OBSERVATION 1** *If (1) holds for  $m \geq 2$ ,  $g$  is a renegotiation-proof network, and  $ij \in g$ , then  $g - ij$  is not a renegotiation proof network.*

The observation follows since otherwise the continuation from  $i$  or  $j$  failing to do each other a favor would only result in the loss of one link, and so they would not do each other favors and  $g$  would not be sustainable.

The important implication of the observation is that beginning from a network that is renegotiation-proof, if a link is deleted then the network will necessarily further degrade in terms of what is sustainable. There may be different ways in which things could degrade. Here is where the idea of robustness comes into play. Robustness against social contagion seeks to minimize the extent to which the loss of links propagates beyond the original deviator(s) in the network.

**EXAMPLE 3** *Robustness*

Suppose that (1) holds for  $m = 2$ , and there are  $n = 9$  agents. Figure 4 lists two renegotiation-proof networks: one network is a single cycle containing all agents,  $\{12, 23, 34, 45, \dots, 91\}$ ; the other network is a 2-“quilt”,  $\{12, 23, 13, 34, 45, 35, 46, 47, 74, 58, 59, 89\}$ .

Note that if any link is deleted from the first network, then it completely collapses. If any link is deleted from the second network, only two other links are deleted and they are limited to a local neighborhood of the original link that is deleted.◊

The second network in Example 3 is more “robust” than the first one in the sense that the damage by the deletion of a link is more “local” in a sense that we now discuss.

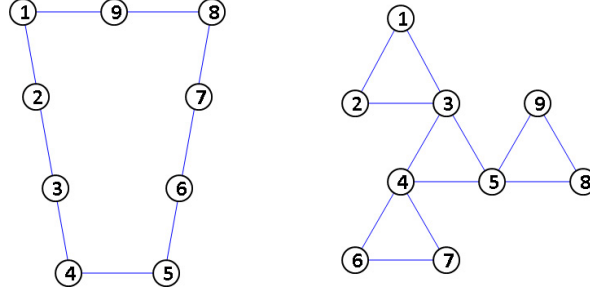


Figure 4: Two renegotiation-proof networks when  $m = 2$  and  $n = 9$ : a non robust one and a robust one.

## 4.1 Robustness Against Social Contagion

We say that a network  $g$  is *robust against social contagion* if it is renegotiation-proof and sustained as part of a pure strategy subgame perfect equilibrium with  $g_0 = g$  such that in any subgame continuation from any renegotiation proof  $g' \subset g$ , and for any  $i$  and  $ij \in g'$ , if  $i$  does not perform the favor for  $j$  when called upon, then the continuation leads to  $g''$  such that if  $h\ell \notin g''$  then  $h \in N_i(g') \cup \{i\}$  and  $\ell \in N_i(g') \cup \{i\}$ .

Robustness requires that a network be sustainable as part of an equilibrium such that in any continuation starting from some renegotiation proof (sub-)network, if some link is deleted then the only other links that are deleted in response must only involve the agent deleting the link and his or her neighbors. In a well-defined sense this localizes the damage to society.<sup>18</sup>

We now characterize the networks that are robust against social contagion, or robust for short.

### 4.1.1 Social Quilts

The networks that are robust against social contagion have a particular structure to them that is described as follows.

An  $m$ -*clique* is a complete network with  $m + 1$  nodes so that every node has exactly  $m$  links.  $m$ -cliques are an important class of critical networks.<sup>19</sup>

<sup>18</sup>An alternative approach would be to explicitly allow nodes or links to fail with given probabilities, and then to look for networks that are still sustainable as equilibria in the face of such failures. If failure probabilities are high enough, then that will result in the definitions that we examine. However, for lower failure rates, other equilibria would also be sustainable. As the contagion effects of such probabilities become intractable quickly in a network setting, we work with the definitions as presented here, but it could be interesting to investigate other approaches.

<sup>19</sup>Note that a clique  $g$  with  $m + 2$  nodes (each having  $m + 1$  links) is not renegotiation-proof. To see this, suppose the contrary and have some  $i$  delete a link  $ij$ . In order for this not to be a valid deviation, it must

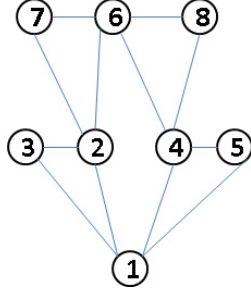


Figure 5: A union of  $m$ -cliques that is not an  $m$ -quilt.

We can build robust renegotiation proof networks by putting cliques together as long as we do not end up generating any cycles involving more than  $m + 1$  nodes when we construct the network out of more than one clique. Thus, a social quilt is a “tree union” of networks.<sup>20</sup>

We say that a network  $g$  is an  $m$ -quilt if  $g$  is a union of  $m$ -cliques<sup>21</sup> and there is no cycle in the network involving more than  $m + 1$  nodes.

Figure 3 shows a 2-quilt. The following example shows a non-tree union of cliques that is not a social quilt and is not robust against social contagion.

**EXAMPLE 4** *A union of  $m$ -cliques that is not an  $m$ -quilt and is not robust.*

Let  $m = 2$  and consider the network  $g = \{12, 23, 13, 14, 15, 45, 26, 27, 67, 46, 68, 84\}$  as in Figure 5. It is a union of four linked 2-cliques (triads) and any two of these cliques intersect in at most one node. However, it is not a 2-quilt since there is a simple cycle  $C = \{12, 26, 64, 41\}$  involving 4 nodes which is more than  $m + 1$ . The presence of this cycle makes it not robust against social contagion:  $g' = \{12, 26, 64, 41\}$  is a subnetwork of  $g$  and is a critical network and so is renegotiation-proof, however  $g'$  is not robust since any deleted links leads to its total collapse and so more than local contagion. It then follows from the definition of robustness,

be that  $i$  loses all links in the continuation, so that the continuation is such that there are at most  $m + 1$  agents in the network and each has just  $m$  links. This is Pareto dominated by a network  $g'$  with all  $m + 2$  agents such that all agents have  $m$  links except for possibly one agent. There is such a network that is a subset of  $g - ij$  (which is such that all but  $i$  and  $j$  have  $m + 1$  links and  $i$  and  $j$  each have  $m$  links). It also follows that  $g'$  is critical and thus renegotiation-proof. Thus, we have a contradiction.

<sup>20</sup>A union of several networks  $g_1, \dots, g_K$  is a *tree union* if the networks can be ordered in a way  $g_1, \dots, g_K$  such that successive unions

$$U_1 = g_1, \dots, U_k = U_{k-1} \cup g_k, \dots, U_K = \bigcup_{k=1 \dots K} g_k$$

are such that each additional network has at most one node in common with the preceding union:  $|N(U_{k-1}) \cap N(g_k)| \leq 1$ .

<sup>21</sup>That is, for any  $ij \in g$  there exists a subnetwork  $g' \subset g$  with  $ij \in g'$  and such that  $g'$  is an  $m$ -clique.

which requires that any subnetwork that could be reached and sustained in a continuation be robust itself.◊

Here are some useful properties of  $m$ -quilts, where  $m > 1$ :

- There are no bridges.
- The removal of a link does not change the distance between any two nodes except the two losing the link, and that distance increases just by 1.
- The removal of any link increases the diameter by at most 1, so there are no “long-distance” links.

**THEOREM 2** *A network is robust against social contagion if and only if it is a social quilt.*

Given our previous discussion of critical networks, it is a simple extension to see that social quilts are renegotiation-proof, and the critical cliques limit contagion to be local in nature. The subtle and difficult part of the proof of Theorem 2 is in showing that only social quilts are robust. For example, why is a complete network not robust? This requires an involved argument, which draws upon both the renegotiation-proofness and the local aspect of punishments. Roughly, the intuition is as follows. First, any robust network must contain some cliques, as an agent who cheats must lose some number of links, which must all be local. In terms of continuation equilibria, any smallest sustainable subnetwork of a given network must be a clique. This follows since any deviation must lead to the loss of all its links since it is the smallest, and by locality the agents must all be neighbors. Moreover, it must be of minimal size by renegotiation-proofness as otherwise the society could renegotiate to keep a minimal sized clique which would contradict this being the smallest sustainable subnetwork. The proof then works by using some graph theoretic reasoning to show that any network that is not a social quilt has some subnetwork that is a critical network, and hence a smallest sustainable subnetwork, but is not a clique. Thus, if a network is not a social quilt, there is some way in which it could be broken down so that the eventual contagion in a last stage of destruction would necessarily be non-local.

## 4.2 The Relative Number of Robust Networks Compared to Subgame Perfect Equilibria

We now present some results that contrast the set of subgame perfect networks with the set of robust networks. Whereas almost all networks are subgame perfect equilibria, a fraction going to 0 of networks are robust. Thus robustness is a very discriminating refinement of the set of equilibrium networks providing pointed predictions.

PROPOSITION 3 *Fix  $m$  and let  $n$  grow.*

- *The fraction of networks that are sustainable as subgame perfect equilibria goes to 1.*
- *The fraction of social quilts (and thus robust networks) goes to 0.*

## 5 Asymmetric Payoffs

Before we examine data concerning favor exchange settings, we extend the model to allow for asymmetries in payoffs. Given the heterogeneity in characteristics of agents in the societies we examine, it is clear that they may face different costs and benefits from favor exchange, and so this extension is needed to provide predictions to take to the data.

In particular, suppose that the probabilities of favors, and their values and costs are specific to relationships. Moreover, let the probability that  $i$  needs a favor from  $j$  depend on the degree of agent  $i$ ,  $d_i(g)$ . Suppose that doing a favor for an agent  $j$  costs an agent  $i$  an amount  $c_{ji} > 0$  and the value of the favor to an agent  $i$  from an agent  $j$  is an amount  $v_{ij}$ . Let  $p_{ij}(d_i(g))$  denote the probability that  $i$  needs a favor from  $j$ . Moreover, this also allows us to discuss directed networks, as  $p_{ij} = 0$  and  $p_{ji} > 0$  suggest that only  $j$  needs favors from  $i$  and  $i$  never needs favors from  $j$ .

Agents discount over time according to a factor  $0 < \delta_i < 1$ . Agents' expected utilities are similarly as that in the symmetric case. An agent  $i$ 's expected future utility from being in a network  $g$  where all favors are provided in perpetuity is

$$u_i(g) = \delta_i \frac{\sum_{j \in N_i(g)} p_{ij}(d_i(g))v_{ij} - p_{ji}(d_j(g))c_{ji}}{1 - \delta_i}.$$

Thus, if agent  $i$  currently provides a favor to agent  $j$  with  $ij \in g$ ,  $i$ 's current expected discounted utility stream is  $-c_{ji} + u_i(g)$ , whereas if agent  $i$  is called to receive a favor from agent  $j$  it is  $v_{ij} + u_i(g)$ .

Let us consider settings such that

$$c_{ji} > \delta_i \frac{p_{ij}(d_i)v_{ij} - p_{ji}(d_j)c_{ji}}{1 - \delta_i}. \quad (2)$$

for each  $ij$  and  $d_i > 0$  and  $d_j > 0$ . Thus, no relationship is sustainable on its own.

Our definitions of renegotiation-proof networks and robustness are exactly as previously stated.

A link  $ij \in g$  is *supported* if there exists agent  $k$  different from  $i$  and  $j$  such that  $ik \in g$  and  $jk \in g$ .

So a link is supported if it is part of a triad. Support is a necessary condition for robustness in the general heterogeneous case where bilateral relationships are not sustainable by themselves.





Figure 6: The contrast between support (left) and a standard clustering or transitivity measure (right).

**THEOREM 3** *If (2) holds and a network  $g$  is robust against social contagion, then all links in  $g$  are supported.*

Support is an important prediction since it differs from standard clustering measures, as illustrated in Figure 6. For example, it is possible that a standard measure of clustering<sup>22</sup> of a network is close to 0 while support is close to or even equal to 1.<sup>23</sup> In fact, as we shall see below, the support measure in the observed networks is quite high while standard clustering measures are much lower.

## 5.1 A Special Heterogeneous Case

An interesting case that generalizes the homogeneous case and yet is not as fully general as the heterogeneous case examined above is one where agents may have idiosyncratic values and costs to favors  $v_i$  and  $c_i$ , and discount factors  $\delta_i$ , but where these values are not dependent upon to whom agents are linked and also where the favor probabilities are not agent dependent. In that case, each agent is characterized by his or her own  $m_i$  such that<sup>24</sup>

$$m_i \frac{\delta_i p(v_i - c_i)}{1 - \delta_i} > c > (m_i - 1) \frac{\delta_i p(v_i - c_i)}{1 - \delta_i}. \quad (3)$$

For this case, our previous results have analogs. The definition of social quilts is slightly more complicated, but intuitively related to the previous definition. We provide a full characterization in the Supporting Appendix.

<sup>22</sup>See Jackson (2008) for various definitions of clustering and transitivity.

<sup>23</sup>For example, consider an agent  $i$  who has many friends:  $N_i(g) = (j_1, k_1, j_2, k_2, \dots, j_M, k_M)$  such that  $j_m$  and  $k_m$  are linked to each other for each  $m$  but such that there are no other relationships between the friends. The support measure here is 1 since every link is part of a triad. Yet, the clustering for  $i$  is very small:  $\frac{M}{M(M-1)/2}$ , which simplifies to  $\frac{2}{M-1}$  and goes to 0 as  $M$  grows.

<sup>24</sup>Again, we rule out indifference.

## 6 Support vs. Clustering in Favor Networks in Rural India

Along with the degree distribution, and the distribution of distances between nodes, the clustering coefficient is one of the standard network statistics most often reported in network analysis. As mentioned in the introduction, it has often been thought that clustering coefficients reflect the ability of social groups to address social dilemmas.

Our analysis has suggested a different and new measure, support, as being the necessary condition for a society to enforce cooperation, at least in the robust manner that we have defined here. This is a distinct measure, and a close scrutiny reveals that clustering and support are conceptually very different. Clustering is a property of the neighborhood of an agent while support is an edge property. While networks with very high levels of clustering will necessarily display a high fraction of supported links the converse is not true. For example, the social quilt on the right hand side of Figure 4 has a support measure of 1 (all of its links are supported) and yet only has an overall clustering of  $1/2$  (as only half of the pairs of links  $ij$  and  $ik$  lead to a completed triad -for instance only  $1/3$  of node 3's pairs of neighbors are linked to each other). More generally, in any society in which agents tend to have multiple disjoint sub-neighborhoods (different cliques of friends), for instance their colleagues at work, the geographic neighbors at home, their hobby friends, etc., the fraction of supported links would be close to 1, while the overall clustering could be very low.

Given that the theory predicts support to be high, but does not predict the same for clustering, we investigate whether this is true empirically. In particular, we now analyze a large data set of a variety of networks that include various forms of favor exchange. We find that support is quite high and significantly higher than the clustering coefficient.

As mentioned in the introduction we have data that are particularly well-suited for our study: they document network structures for a variety of different sorts of relationships, including various sorts of favor exchange, and moreover have this for many separate villages. We are not aware of any other data set having these attributes.

### 6.1 Description of the Data

The data are from 75 rural villages in Karnataka, an area of southern India within a few hours from Bangalore. The average population per village is 926.48. The survey was designed as part of a study of the deployment of a micro-finance program (see Banerjee et al. (2010)).

Only half of the households were surveyed, which could bias our support measures (downwards) as we discuss below. Households were selected by a stratified random sample in order to control for selection biases; with households stratified by religion (Hindu, Muslim, Chris-

tian) and also by geographic sub-locations based on a full census of the villages that was conducted just prior to and in conjunction with the survey.

Each surveyed individual was asked to name the people that he or she has various sorts of relationships with. The relationships that were queried in the survey are listed in Table 1.<sup>25</sup>

Relationships in Survey	
Codename	Question in Survey
Friends	Name the 5 non-relatives whom you speak to the most.
Visit-go	In your free time, whose house do you visit?
Visit-come	Who visits your house in his or her free time?
Borrow-kerorice	If you need to borrow kerosene or rice, to whom would you go?
Lend-kerorice	Who would come to you if he or she needed to borrow kerosene or rice?
Borrow-money	If you suddenly needed to borrow Rs. 50 for a day, whom would you ask?
Lend-money	Whom do you trust enough that if he or she needed to borrow Rs. 50 for a day you would lend it the him or her?
Advice-come	Who comes to you for advice?
Advice-go	If you had to make a difficult personal decision, whom would you ask for advice?
Medical-help	If you had a medical emergency and were alone at home, whom would you ask for help in getting to a hospital?
Relatives	Name any close relatives, aside those in this household, who also live in this village. Plus people in the same household.
Temple-company	Whom do you go to temple with?

Table 1: The Contents of the Survey

There are several potential sources of measurement error in these data. First, not all people were surveyed and so there are missing nodes and links in the data set.<sup>26</sup> Without

<sup>25</sup>In the borrowing and lending relationships, fifty Rupees are roughly a dollar and the per capita income in the areas surveyed is currently on the order of three dollars per day or less, although a precise income census is not available.

<sup>26</sup>Nodes in the networks that we construct from the surveys are individuals who were surveyed. Surveyed individuals could name non-surveyed individuals as friends, relatives, etc., but we omit such links unless

any particular selection of which nodes are missing (given the random selection of households), the missing data would bias the measure of support downwards, since support looks at observed links  $ij$  and then asks whether  $i$  and  $j$  have a common neighbor. If that neighbor is missing from the data, then support can be underestimated. Second, there are the usual measurement issues with survey data: people may forget to mention some of their connections, people get fatigued by interviews, and the survey did not allow individuals to name more than five or eight other people depending on the categories (although the cap was reached in a negligible number of cases).<sup>27</sup>

Out of the 12 survey questions described in 1 the caps were only ever reached in those questions with codenames Visit-come, Borrow-money, Lend-money and Relatives. The caps were 8, 5, 5 and 8 respectively, and they were reached in less than 0.6 in 10000, 2 in 10000, 0.5 in 10000 and 2 in 10000 of the total number of surveys<sup>28</sup>.

We provide some analysis of measurement error in the supplementary appendix. Thus, in building networks, we say that agent  $i$  borrows money from agent  $j$  if and only if either agent  $i$  reports borrowing money from  $j$  or agent  $j$  reports lending money to agent  $i$ , and we do the same with respect to lend-money, borrow-kerorice and lend-kerorice, visit-come and visit-go and advice-come and advice-go relationships. Note that these are all directed relationships. In the case of friends, medical-help, relatives and temple-company we define that agent  $i$  has a relationship of the type in question with agent  $j$  if and only if at least one of them acknowledge so. One can also work with other variations on these definitions, and in the supplementary appendix we report on some of these variations, which do not significantly alter the conclusions.<sup>29</sup> The supplementary appendix contains a variety of statistical measures of these networks.

## 6.2 Measuring Support and Clustering

The data include a variety of types of relationships. The way in which we categorize relationships is captured in Table 2.

In our definitions of these relationships, although we call certain relationships “hedonic,” it could be that those relationships involve various sorts of interactions, including things like favor exchange or risk-sharing, and so forth. Indeed, as we shall see, although we find

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both individuals were surveyed. Thus, the networks we work with are sub-networks of the true networks.

<sup>27</sup>Note that the questions were worded in ways to avoid basic perception issues that are associated with questions such as “who are your friends?” Based on wording that is more explicit about particular interactions (borrowing rice, asking for medical advice, etc.) the relationships are more concrete.

<sup>28</sup>The Supplementary Appendix shows the cumulative distribution of the number of reported relations for each relationship type.

<sup>29</sup>In particular, we re-analyze all of the data where we exclude “relative” links, among other things. Whether or not relatives are included does not significantly change the results.

$$All = \left\{ \begin{array}{l} \text{Relatives} \\ \text{Temple-company} \\ \text{Hedonic} = \left\{ \begin{array}{l} \text{Visit-go \& Visit-come} \\ \text{Friends} \end{array} \right. \\ \text{Favors} = \left\{ \begin{array}{l} \text{Physical Favors} = \left\{ \begin{array}{l} \text{Borrow-money \& Lend-money} \\ \text{Borrow-kerorice \& Lend-kerorice} \end{array} \right. \\ \text{Intangible Favors} = \left\{ \begin{array}{l} \text{Advice-come \& Advice-go} \\ \text{Medical-help} \end{array} \right. \end{array} \right. \end{array} \right.$$

Table 2: Network Definitions

*In relationship a&b, i and j are related if and only if they are related in a and in b.*

more support in various “favor” relations, there is still a high level of support among the “hedonic” relationships.

In order to capture the possible combinations of various relationships, we enrich our measures of support and clustering. In particular, we can ask whether relationships of one type are supported through relationships of another type, and analogously for clustering, whether a pair of neighbors of a given node in a network of one type have a relationship of a second type.

To this end, we define the support of a network  $g'$  relative to another network  $g$  as  $S(g', g)$ ,

$$S(g', g) = \frac{\sum_{ij \in g'} 1_{\{\exists k, ik \in g, kj \in g\}}}{\sum_{ij \in g'} 1}$$

which is the proportion of links in  $g'$  whose nodes have common neighbors in  $g$ . Similarly we can define the clustering of a given network  $g'$  relative to some other network  $g$ ,  $Clus(g', g)$ , as

$$Clus(g', g) = \frac{\sum_{ij \in g', ik \in g'} 1_{\{jk \in g\}}}{\sum_{ij \in g', ik \in g'} 1}$$

Note that  $g = g'$  is allowed and this reduces these two measures to *self-support*, and to the standard clustering coefficient respectively. We refer to  $g'$  as the *base network* and  $g$  as the *context network*.

The reason for considering variations on the support measure is that it is quite possible that exchange of one type of favor is supported via relationships involving exchange of some other sort of favor or some other valuable interaction. The corresponding variations on the standard clustering coefficients are supplied in order to have appropriate benchmarks.

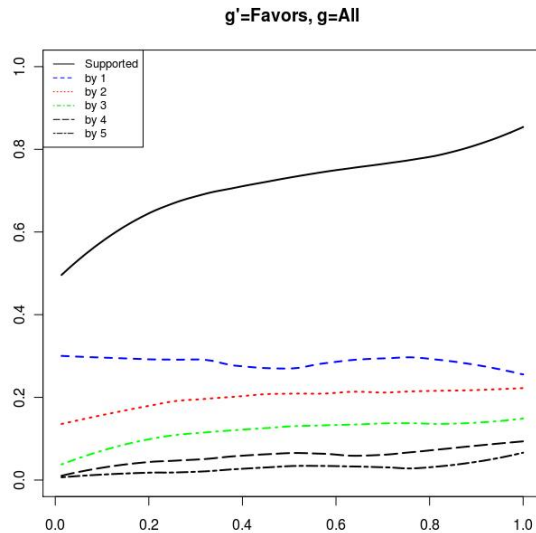


Figure 7: The inverse cumulative distribution function of support levels in the villages: The horizontal axis is the fraction of villages having support no more than the amount listed on the vertical axis. The upper-most curve is the inverse CDF of the fraction of supported *favor* relationships in the *All* network. The five curves below list the breakdown of the fraction for the marginal village by various levels of support: “by  $k$ ” indicates the fraction of links in that village that are supported by exactly  $k$  other nodes (so that  $i$  and  $j$  have  $k$  friends in common), and so the five lines below sum to the curve above.

### 6.3 Support in the Data

Among the relationships described in Table 1, we identified those that can be accurately described as favor relationships. In what follows we focus on the average support and clustering measures setting  $g' = Favors$  and  $g = All$  where *Favors* and *All* are defined as in Table 2.

We provide a similar analysis for all sorts of combinations of relationships in the Supplementary Appendix.

Figure 7 shows the inverse cumulative distribution function of support in the favor networks in our sample of 75 villages along with the plots of the fraction of links supported by exactly  $k$  other links in the marginal village.

The support measure is generally quite high (especially given that it is likely to be biased downwards by the measurement error of missing nodes). It ranges from more than 50% to over 80% depending on the village. Moreover, when we look at certain kinds of favor relationships, the support measure exceeds 85% on average across the villages (see the

supplementary appendix).

Our theory suggests that in robust favor exchange networks, the support measure should be 100 percent, presuming that each relationship requires support in order to sustain favor exchange. In the data we see support that is less than 100 percent. Of course, this could be due to some relationships having frequent enough interaction to be self-sustaining. It could also be due to various forms of measurement error, such as missing nodes and also potentially missing links even within the observed networks. We discuss the measurement error possibility in more detail in the Supporting Appendix.

## 6.4 Comparing Support to Clustering

As mentioned before, our measure of support provides a new network characteristic. We now compare it to the clustering coefficient in the villages.

We calculate the clustering in each village and compare it to the corresponding support measure. In both cases we work with the base of favor relationships and the context of all relationships. Similar comparisons and conclusions hold for other variations of comparisons as shown in the Supplementary Appendix.

Figure 8 shows that support levels are not only higher than clustering, but by an order of magnitude.

There are various ways in which we can see that support is significantly higher than clustering in these villages. The comparison of having it be higher in 75/75 villages has a p-value that is effectively 0. We can also see that the average ratio of support over clustering across the villages is 2.94 with a standard error of 0.38 and this has a p-value of effectively 0 of being significantly higher than 1.

We can also see how the two compare for various types of relationships, as seen in Table 3.

<b>G</b>	<b>Favors</b>	<b>Physical Favors</b>	<b>Intangible Favors</b>	<b>Hedonic Relationships</b>	<b>All</b>
$Clus(G, All)$	0.234	0.257	0.249	0.236	0.222
$S(G, All)$	0.717	0.72	0.733	0.661	0.696

Table 3: Clustering and Support Measures for various types of relationships. In every case the *All* network serves as context.

Thus, we see that support significantly exceeds clustering. This suggests that support should be a very useful measure of network cohesion more generally, and especially in situations where social pressures may be needed to provide incentives.

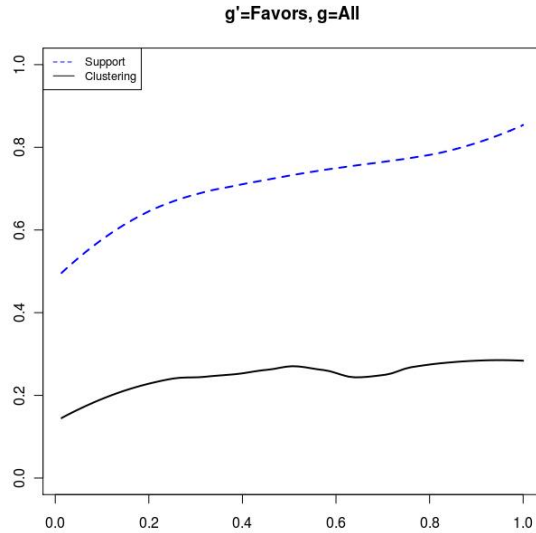


Figure 8: The inverse cumulative distribution function of support and clustering levels in the villages: The horizontal axis is the fraction of villages having support/clustering no more than the amount listed on the vertical axis. The upper-most curve is support and the lower-most is the clustering coefficient of the marginal village.

Moreover, the comparison of support over clustering could also be a useful statistic in more general social network analysis. If the ratio is close to 1, then effectively a typical agent would just have one group of friends, while a higher ratio would suggest that a typical agent has several disjoint groups of friends.

## 6.5 Comparing Support in Different Sorts of Relationships

The data also allow us to compare the support measures of favor networks with those of hedonic networks (HR).

We can test whether there are statistically significant differences in support among networks of different types. To do this, we compare the values of any given support measure village by village. If there were no difference in support between relationships of the types being compared, then in any given village each one would have a 50% chance to be larger than the other. Thus, under the null hypothesis that there is no difference in support, the number of villages where one has a higher support than the other should have a binomial distribution.

When we examine the data we see significant differences between the support of different types of relationships. For example, in 72 out of 75 villages, the support of intangible favors



$S(IFavors, All)$  is higher than that of social relationships  $S(Hedonic, All)$  (both relative to the all network). Of course the probability of a binomial random variable realizing a one in 72 out of 75 trials is effectively 0.

Table 4 shows the comparison of support measures of various relationships (based on the context of *All* relationships). The entry in the table is how many times out of 75, the support measure in the row is higher than that in the column. The Supporting Appendix has comparisons for other contexts with similar patterns.

<i>Network g'</i>	Favors	Physical Favors	Intangible Favors	Hedonic	All
Favors	–	30**	24***	72***	60***
Physical Favors	45**	–	38	69***	56***
Intangible Favors	51***	37	–	72***	57***
Hedonic	3***	6***	3***	–	6***
All	15***	19***	18***	69***	–

\*\*\* significant difference at 1% level      \*\* significant difference at 5% level

Table 4: Comparison of Support Measures.

Entry  $i, j$  is the number of villages for which  $S(\mathbf{g}'_i, All) > S(\mathbf{g}'_j, All)$

## 6.6 How Observed Support Compares to that Expected in a Random Network

We close with a look at whether observed support is statistically significant, even when one corrects for geography.

This answers whether support patterns are something that arose simply because of geography, or whether the patterns really reflect social structure. For example, underlying social processes are likely to be influenced by geography as costs of interacting will often decrease with proximity. It is plausible that the likelihood of relationships increases with the physical proximity of the agents.<sup>30</sup> And, since physical proximity has some transitive features, relationships that are geographically correlated may display completed triads and therefore high support levels. To address this, we estimate exponential random graph models (“ergms”) using the observed networks. In particular, we estimate ergms with the likelihood of a link being present depend on (i) whether the link is supported, (ii) the observed density of the network, and (iii) the physical distance of the agents involved (measured using the

<sup>30</sup>This could be for two reasons, both having links emerge because of proximity, and having people likely to be linked locate close to each other.

GPS coordinates of their respective households)<sup>31</sup> As should be expected, in every network (i.e., every village) the coefficient associated to physical distance was negative and significant at well less than the 1% level. Nonetheless, the coefficients on support are still large and statistically significant.<sup>32</sup> Figure 9 shows the support coefficient estimates with 99% confidence bars when analyzing  $g' = \textit{Favors}$  with  $g = \textit{All}$  as a context. The Supplementary Appendix contains the full set of ergm estimates for many other base-context pairs. The ergm estimates suggest that the observed support levels are not merely an artifact of geographic proximity. More specifically support of a link in the *All* network is a significant predictor (in the statistical sense) of the presence of that link in the favors network, when controlling for density and for geographic proximity of the agents involved.

## 7 Conclusion

Our analysis of favor exchange provides various insights. We have shown that renegotiation results in specific critical structures and that robustness involves social quilts and more generally in supported links. Support provides a new local characteristic of networks and insight into closure and an operationalization of a sort of social capital which emphasizes social structure.

Our empirical analysis finds high levels of support in favor networks in rural Indian villages. We also find that support levels are much higher than clustering, and that support for favor networks is higher than that of more “hedonic” networks.

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<sup>31</sup>The exponential random graph model that we estimate is:

$$\log(\Pr(G = \mathbf{g})) = \beta_0 + \beta_1 \sum_{i < j} \mathbf{g}_{ij} + \beta_2 \sum_{i < j} \mathbf{g}_{ij} \mathbf{s}(\mathbf{g}, \mathbf{g}')_{ij} + \beta_3 \sum_{i < j} d(i, j) \mathbf{g}_{ij}$$

where,  $G$  is a random variable taking values in the space of all possible graphs on  $N$  nodes,  $\mathbf{g}$  is a particular network,  $\mathbf{s}(\mathbf{g})$  is the associated indication of whether the link  $ij$  is supported or not (which may be calculated relative to a related “all network”  $g'$ ), and  $d(i, j)$  denotes the physical distance between  $i$  and  $j$  as measured by the GPS coordinates of their households, and normalized by the mean distance between surveyed households in the village. Given that such a model has  $g$  on both sides of the equation and in a non-linear manner, estimating such models is complicated along several dimensions. The standard method uses an MCMC (Markov chain Monte Carlo) method due to Snijders (2002).

<sup>32</sup>To make sense of the size of the coefficient on support being roughly 2, from the ergm specification some simple algebra shows that

$$\log[\Pr(g_{ij} = 1|g_{-ij}) / \Pr(g_{ij} = 0|g_{-ij})] = \beta_1 + \beta_2 \mathbf{s}(\mathbf{g}, \mathbf{g}')_{ij} + \beta_3 d(i, j),$$

so that having a link be supported roughly increases by 2 or more the log odds ratio that the link will be present in the network.

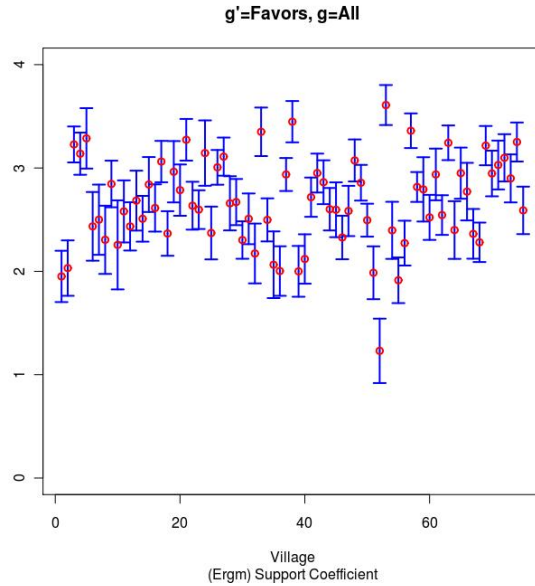


Figure 9: The coefficients associated to the support terms in exponential random graph models of the “favors” networks in the 75 villages. (The value of a link’s support binary variable is 1 if and only if the link is supported in the *All* network.)

## 7.1 Information and Robustness

In closing, we discuss some issues regarding the information observed by the agents in the society.

We have deliberately looked at a complete information setting for two reasons. First, in many applications, including the Indian villages we look at empirically, word of mouth communication travels much faster than actions and so if someone behaves badly other people hear about it quickly. Gossip serves a strong purpose. Second, much of the previous literature has focused on the information as the driver of network structure in providing incentives and so our analysis is completely complementary.

Nonetheless, there is an important observation that comes out. Our analysis ends up yielding social quilts which end up having strong informational robustness properties in addition to the properties that we have investigated. In particular, agents only need to know what the agents whom they are linked to directly are doing, and for all of those agents they also have common friends - so they are both directly, and indirectly connected at a short distance through an independent channel to all of the agents whose behavior they have to be aware of in order to best respond. Thus, even with very limited communication, the robust social networks that we have uncovered can be sustained. Our analysis ends up yielding networks that might otherwise be justified for their informational properties from a

completely different perspective.

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## Appendix: Proofs of Results

**Proof of Proposition 2:** Let  $g'$  be a smallest nonempty network (in the sense of set inclusion) that is a subset of  $g$  and lies in  $G(m)$ . Such a network exists (possibly  $g$  itself) and is thus critical by definition. The second statement is easily verified by construction, but also follows as a corollary to Theorem 1 which is proven below. ■

### Proof of Theorem 1:

We first show that if  $g \in RPN_k$  then  $g \in TC_k$ . Given a network  $g \in RPN_k$ , by the definition it follows that  $g$  is sustainable on the equilibrium path. So for any  $i$  and  $ij \in g$ , if  $i$  is called upon to do a favor for  $j$  and does not, then at least one possible continuation<sup>33</sup> must lead to a network  $g' \subseteq g - ij$  such that  $g' \in RPN_{k'} = TC_{k'}$ ,  $d_i(g') \leq d_i(g) - m$ , and there is no  $g'' \subset g - ij$  such that  $g'' \in RPN_{k''}$  and  $D(g'') > D(g')$ . If this were not the case, then if  $i$  did not perform the favor, he or she would save the cost  $c$  and lose at most  $m - 1$  links in any continuation. Thus,  $i$  would benefit from deviating and not performing the favor since by the definition of  $m$ , (1) holds and so the cost of the favor outweighs the loss in future payoffs from losing no more than  $m - 1$  links, which contradicts the fact that  $g$  is sustained as an equilibrium. Thus, for every  $i$  and  $ij$ , there exists  $g' \subset g - ij$  such that  $g' \in TC_{k'}$  for some  $k'$ ,  $d_i(g') \leq d_i(g) - m$ , and there is no  $g'' \in TC_{k''}$  such that  $g'' \subset g - ij$  and  $D(g'') > D(g')$ . Therefore  $g \in TC_k$ .

Next, we show that if  $g \in TC_k$  then  $g \in RPN_k$ . We do this by induction on the number of links in a network. In order to establish the result, we also need to be careful about what happens starting at subgames that are off the equilibrium path. As such, we work with a stronger induction hypothesis, with the induction indexed by  $k$ .

The induction hypothesis is that starting from any node and any  $g_0 \in G_k$ , there exists a pure strategy subgame perfect equilibrium continuation such that

- (i) there is a unique network  $g_1 \in RPN_{k_1}$  for some  $k_1 \leq k$  that is reached in the continuation, with  $g_1 = g_0$  if  $g_0 \in TC_k$ ,
- (ii) on the equilibrium continuation path a favor is performed if and only if it corresponds to a link in  $g_1$ , and
- (iii) in any subgame starting with some network  $g' \in G_{k'}$  with  $k' \leq k$  if  $g'$  is played in perpetuity with some probability in the continuation then  $g'' \in RPN_{k''}$  for some  $k''$  and there does not exist any  $g''' \subset g'$  such that  $g''' \in RPN_{k'''}$  and  $u_i(g''') \geq u_i(g'')$  for all  $i$  with strict inequality for some  $i$ .

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<sup>33</sup>Even though agents use pure strategies, nature picks which favors are asked in the future, so there are potentially many continuations from any given node.

As a first step in the induction note that it follows directly from the definitions that  $RPN_0 = \{\emptyset\} = TC_0$ . Note also that starting from  $g_0 = \emptyset$  there is a unique subgame perfect equilibrium continuation (no favors can be supplied and no links can be maintained) and so it follows directly that conditions (i)-(iii) are satisfied. So, let us presume that the induction hypothesis holds for all  $k' < k$ . We show that the same is true for  $k$ .

Begin with the case such that  $g_0 = g \in TC_k$ . On the equilibrium path, have all agents maintain all links (so  $L_i(g_t) = N_i(g_t)$  whenever  $g_t = g_0 = g$ ) and perform all favors. The off the equilibrium path strategies are described as follows. If an agent  $i$  is called upon to provide a favor for an agent  $j$  such that  $ij \in g$  and does not do the favor, then the continuation is as follows. Given that  $g \in TC_k$ , by the definition of transitive criticality, there exists  $g' \subseteq g - ij$  such that  $g' \in TC_{k'} = RPN_{k'}$ ,  $d_i(g') \leq d_i(g) - m$  and there is no  $g'' \in TC_{k''} = RPE_{k''}$  for any  $k''$  such that  $g'' \subset g - ij$  and  $D(g'') > D(g')$ . Denote this network by  $g(i, j) = g'$ . Following  $i$ 's failure to provide a favor to  $j$ , have the continuation be such that  $L_\ell(g - ij) = N_\ell(g(i, j))$  for all  $\ell$ . This results in the network  $g(i, j) \in RPN_{k'}$  following the link announcement phase, and so from then on there is a pure strategy subgame perfect equilibrium sustaining  $g(i, j)$  and satisfying (i) - (iii) by the induction step, and so have agents play the strategies corresponding to such an equilibrium in that continuation. At all other nodes off the equilibrium path for which strategies are not already specified we are necessarily at a network with fewer links, and so pick a pure strategy equilibrium continuation that satisfies (i) - (iii), which is possible by the induction hypothesis.

This satisfies (i)-(iii) by construction. To check that this a subgame perfect equilibrium, by the specification of the strategies above, we only need to check that no agent wants to deviate from the equilibrium path, and also that following some  $i$ 's failure to provide a favor to  $j$ , no agent  $\ell$  wants to deviate from  $L_\ell(g - ij) = N_\ell(g(i, j))$ . We only need to check these sorts of deviations since all other continuations were specified to be pure strategy subgame perfect equilibrium continuations. By construction, an agent  $i$  who is called upon to do a favor for an agent  $j$  who deviates will end up losing at least  $m$  links, and so by (1) this cannot be an improving deviation. Next, consider, some agent  $\ell$ 's incentive to deviate from  $L_\ell = N_\ell(g_0)$  if  $g_0$  is still in play, or else from  $L_\ell(g - ij) = N_\ell(g(i, j))$  following some  $i$ 's failure to provide a favor to  $j$ . By not deviating the agent gets the payoff from  $g_0$  or  $g(i, j)$  in perpetuity. By deviating, the agent  $\ell$  will end up with a continuation starting from a network  $g'' \subset g_0$  or  $g'' \subset g(i, j)$ , respectively, where the agent has not gained any links and may have lost some links. Since each link has a positive future expected value, this cannot be an improving deviation.

Next, let us show that from any node in the continuation from some initial  $g_0 \notin TC_k$  there exists a pure strategy subgame perfect equilibrium continuation satisfying (i)-(iii). There are two types of nodes to consider. One is a node at which some agent  $i$  is called upon to provide



a favor for an agent  $j$  such that  $ij \in g_0$ , and another is a node where agents announce the links they wish to sustain.

First, consider starting at  $g_0$  and a node where agents announce the links that they wish to sustain. Find some  $g'$  that has the maximal  $k' < k$  of links such that  $g' \in RPN_{k'}$  and  $g' \subset g_0$ . For each  $\ell$  set  $L'_\ell = N_\ell(g')$  and then from  $g'$  play a continuation satisfying (i) - (iii) (by the induction step). If any agent deviates, to  $L$  such that  $L'_\ell \subset L$ , then play the same continuation as this will not affect the network formed. Otherwise, the continuation will lead to some  $g''$  with strictly fewer links for  $\ell$  and the continuation will necessarily result in a lower expected continuation payoff. This establishes the claim for this sort of node.

Next, let us consider a node at which some agent  $i$  is called upon to provide a favor for an agent  $j$  such that  $ij \in g_0$ . There are two cases that can follow: one where  $i$  performs the favor and so the resulting network is then  $g_0$ . In that case, we have just shown that there is a pure strategy subgame perfect equilibrium continuation satisfying (i) to (iii). Let  $g'$  be the network sustained on the equilibrium path in one of these that has the most links for  $i$ . If  $i$  does not perform the favor, then  $g - ij \in G_{k-1}$  is reached. By the induction hypothesis again there is a pure strategy subgame perfect equilibrium continuations satisfying (i)-(iii), and let  $g''$  be a network sustained by one of these that has the most links for  $i$ . Now, based on those two continuations, have  $i$  choose a pure strategy best response. The claim follows. ■

### Proof of Theorem 2:

We first prove that  $m$ -quilts, or tree unions of  $m$ -cliques,<sup>34</sup> are robust against social contagion. The proof proceeds by induction on the size  $k$  of the tree union.

When  $k = 1$ , it is a single  $m$ -clique, and so it is renegotiation proof since it is critical. Note that also that the only subnetwork of a clique that is in  $G(m)$  is the empty network. Thus, it follows that any equilibrium continuation in (any) equilibrium supporting the clique is the empty network. Thus, a clique is robust against social contagion. Suppose it is true for all  $k' < k$ . We show that a tree union of  $k$   $m$ -cliques is robust against social contagion. To do this, we show that tree unions of  $m$ -cliques and some nonempty strict subnetworks of  $m$ -cliques cannot be renegotiation-proof. This is enough to establish that tree unions of  $m$ -cliques are robust against social contagion, since it shows that link deletions can be punished by deleting all links in the particular  $m$ -clique which the deleted link belonged to and that will not be Pareto dominated by any continuation renegotiation-proof equilibrium.

Begin with a tree union of  $k$   $m$ -cliques,  $g_1, \dots, g_k$ .

Let  $g^0 = \left( \bigcup_{h=1 \dots m_0-1} g_h \right) \cup \left( \bigcup_{h=m_0 \dots k} g_h^0 \right)$ , with  $m_0 \leq k$ ,  $g_h^0 \subset g_h$ ,  $g_h^0 \neq g_h \forall h \geq m_0$  where at least one  $g_h^0$  in the union is nonempty. So this is the tree union of  $m$ -cliques and some

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<sup>34</sup>It is straightforward to verify that a union of  $m$ -cliques is an  $m$ -quilt (has no simple cycles of more than  $m + 1$  nodes) if and only if it is a tree union of  $m$ -cliques.

nonempty strict subnetworks of  $m$ -cliques. Suppose to the contrary that it is renegotiation-proof.

Note that  $\bigcup_{h=m_0 \dots k} g_h^0$  is a tree union of networks, and it must therefore have some leafs. Pick one such leaf and denote it  $g_{h^*}^0$ . Since  $g_{h^*}^0$  is a strict subset of the  $m$ -clique  $g_{h^*}$  and a leaf of the subtree, there is some agent  $i_0$  who has a positive number of links, less than  $m$ , in the subtree. Suppose this agent were to fail to provide a favor on a link  $i_0 j_0$  in  $g_{h^*}^0$ . Since by assumption  $g^0 \in RPN$ , agent  $i_0$  would have to lose at least  $m$  links if he or she failed to provide a favor on any link  $i_0 j_0$  in the subtree. Since the agent does not have enough links to lose in the subtree, he or she would have to lose links in  $\bigcup_{h=1 \dots m_0-1} g_h$ . Denote the continuation by  $g^1$  which must be renegotiation-proof. Note that  $g^1$  cannot be a strict subset of  $\bigcup_{h=1 \dots m_0-1} g_h$ , since by the inductive hypothesis  $\bigcup_{h=1 \dots m_0} g_h \in RPN$ . Therefore  $g^1$  must have some links from  $\bigcup_{h=m_0 \dots k} g_h^0$ . In particular  $g^1 = \left( \bigcup_{h=1 \dots m_1-1} g_h \right) \cup \left( \bigcup_{h=m_1 \dots k} g_h^1 \right)$ , where  $g_h^1 \subset g_h$ ,  $g_h^1 \neq g_h \forall h \geq m_1$  and  $m_1 < m_0$ . This last inequality results from the fact that  $i_0$  lost links in  $\bigcup_{h=1 \dots m_0} g_h$ . Again, any agent who has fewer than  $m$  links in  $\bigcup_{h=m_1 \dots k} g_h^1$  must have links in  $\bigcup_{h=1 \dots m_1-1} g_h$ . We then derive a subnetwork  $g^2$  from  $g^1$  analogously to the way we derived  $g^1$  from  $g^0$ . Proceeding in this fashion we produce a finite sequence of renegotiation proof networks  $g^0, g^1, \dots, g^\ell$ , with  $m_x < m_{x-1}$  at each iteration and there is always at least one link in  $\bigcup_{h=m_x \dots k} g_h^x$ . Continue until  $m_\ell = 0$ . Using the same argument with which we found  $i_0$ , it can be seen that we would find some node with less than  $m$  links in total, contradicting  $g^\ell \in RPN$ .

We now prove that robustness against social contagion implies that a network must be a social quilt. Suppose that  $g$  is robust against social contagion. If there is an  $m$ -clique,  $g_c \subset g$ , that has at most one node  $i$  connected with nodes outside of the clique then  $g - g_c$  is also robust against social contagion. This follows since if any agent  $j \neq i$  who is in  $g_c$  does not perform a favor, then in order for  $g$  to have been sustainable  $j$  must expect to lose all of his or her links in the continuation (as the continuation must be in  $G(m)$  to be sustainable and  $j$  will have fewer than  $m$  links). Then since all agents other than  $i$  in  $g_c$  will have fewer than  $m$  links in the continuation, they must all lose all links in order for the continuation to be sustainable. Thus, the clique must disappear in the continuation, but by robustness against social contagion no other links can be deleted. So, eliminate  $g_c$  and continue with the network  $g - g_c$ . If repeating this process leads to an empty network, then  $g$  must have been a social quilt. Suppose instead, that this elimination process leads to some nonempty  $g'$  (and note that  $g'$  is robust by the definition of robustness) such that  $g'$  contains no  $m$ -clique where at most one agent has links outside of the clique.

By the above process, any remaining  $m$ -cliques that are subnetworks of  $g'$  must satisfy the condition that the clique has at least two agents who have links outside of the clique. So, identify some remaining  $m$ -clique that is a subnetwork of  $g'$  and delete a link between a pair of agents, say  $i$  and  $j$ , who have links outside of the clique. The remaining network is still in  $G(m)$  since  $i$  and  $j$  necessarily had more than  $m$  links in  $g'$ . Note also that this removal of the link  $ij$  does not change the fact that each other clique in  $g'$  has two agents with links outside of the clique (as if either of those agents is  $i$  or  $j$  then they still maintain  $m - 1$  links each in the original clique). Repeat this process once for each clique in the original network such that at least two agents have links outside of the clique until there are no  $m$ -cliques left in the resulting network, and let this nonempty resulting network be  $g'' \in G(m)$ .

Thus, a direct extension (literally word for word) of the proof of Proposition 2 implies that there is a subnetwork of  $g''$  that is “minimally” critical<sup>35</sup> and nonempty. Let that network be  $g'''$ . It follows from our derivation of  $g''$  that  $g'''$  cannot be a clique. However, since  $g'''$  is minimally critical, then by robustness and the renegotiation-proofness of  $g$ , there is an equilibrium where in any subgame starting from  $g'''$ , it is sustained. However, starting from  $g'''$  if any link is deleted then all links must be deleted in any equilibrium continuation by the definition of minimal criticality. This contradicts the robustness of  $g$ , since  $g'''$  is not a clique and hence there is some link  $ij$  and some  $k$  who is not linked to both  $i$  and  $j$  who loses a link as a result of the deletion of the link  $ij$ . ■

**Proof of Proposition 3:** First, we analyze the fraction of subgame perfect equilibria. Denote by  $SPE_n$  the set of subgame perfect networks on  $n$  nodes. A network is a subgame perfect network if and only if no node in the network has at least 1 and no more than  $m - 1$  links. We set a very loose upper bound on the number of networks in which at least one node has at least 1 and no more than  $m - 1$  links by first picking any node and its links and then allowing the remaining  $n - 1$  nodes to link among themselves however they want, and then multiplying by  $n$  to allow for any starting node. A lower bound on the cardinality of

$$SPE_n \text{ is } 2^{\frac{n(n-1)}{2}} \text{ minus this bound. Therefore: } |SPE_n| \geq 2^{\frac{n(n-1)}{2}} - n \sum_{k=1}^{m-1} \binom{n-1}{k} 2^{\frac{(n-1)(n-2)}{2}} \geq 2^{\frac{n(n-1)}{2}} - (m-1)n \binom{n-1}{m-1} 2^{\frac{(n-1)(n-2)}{2}},$$

where the inequality on the right holds for any  $m$  such that  $n - 1 > 2(m - 1)$ . This implies that

$$\frac{|SPE_n|}{|G_n|} \geq 1 - \frac{(m-1)n \binom{n-1}{m-1} 2^{\frac{(n-1)(n-2)}{2}}}{2^{\frac{n(n-1)}{2}}}$$

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<sup>35</sup>A critical network is called minimally critical if any deletion of a link leads the collapse of the whole network, that is if any subnetwork of the original network that lies in  $G(m)$  is the empty network.

$$\begin{aligned}
&= 1 - \frac{(m-1)n \binom{n-1}{m-1} 2^{\frac{(n-1)(n-2)}{2}}}{\sum_{k=0}^{n-1} \binom{n-1}{k} 2^{\frac{(n-1)(n-2)}{2}}} \\
&= 1 - \frac{(m-1)n \binom{n-1}{m-1}}{\sum_{k=0}^{n-1} \binom{n-1}{k}} \\
&= 1 - \frac{(m-1)^2 \binom{n}{m-1}}{\sum_{k=0}^{n-1} \binom{n-1}{k}} \rightarrow 1 \text{ as } n \text{ goes to infinity}
\end{aligned}$$

Next, we find an upper bound on the fraction of social quilts that goes to 0 as  $n$  grows. There are two possible sets of degrees that an agent can have in a network:  $A = \{km < n | k \text{ is a nonnegative integer}\}$  and  $B = \{k < n | k \text{ is an integer}\} - A$ . In a social quilt  $g$ , every agent  $i$  has a degree  $d_i(g) \in A$ . We show that the number of networks such that all agents have degrees in  $A$  is a fraction of no more than  $\frac{1}{2}^{n-1}$  of all possible networks.

To do this we catalog networks  $g$  by first forming a network among the agents 1 to  $n-1$ , and then considering links between those agents and agent  $n$ . We show that regardless of the starting network  $g_0$ , there is at most one configuration of links for agent  $n$  (out of  $2^{n-1}$  possible) that will allow all agents to have degrees in  $A$ . The result then follows directly. Beginning with the network  $g_0$  among agents 1 to  $n-1$ , if  $d_i(g_0) \in A$  then it must be that there is no link between  $i$  and  $n$  in  $g$ . In contrast, if  $d_i(g_0) \notin A$  then in order for  $d_i(g)$  to be in  $A$  it would have to be that  $i$  and  $n$  are linked in  $g$ . Thus, if there is some configuration of links between  $n$  and the other agents that results in the correct configuration of degrees, there is at most one such configuration out of the  $2^{n-1}$  possible configurations. ■

**Proof of Theorem 3:** Suppose to the contrary that  $g$  is robust against social contagion and  $ij$  is not supported. Consider  $h \notin \{i, j\}$  and delete a link of  $h$  (and there is at least one such  $h$  who has a link, as the single link  $ij$  is not sustainable independently as part of a subgame perfect equilibrium). This leads to a continuation  $g'$  that is robust against social contagion and such that  $ij \in g'$ , as otherwise by robustness both  $i$  and  $j$  would have to be neighbors of  $h$  which would contradict the fact that  $ij$  is not supported. Iterate on this argument. Eventually, we reach the empty network which is a contradiction since  $ij$  cannot be deleted as the result of deletion of a link of just a neighbor of one of the two nodes (by the definition of robustness against social contagion). ■