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Social Conflict and Growth

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Despite the predictions of the neoclassical theory of economic growth, we observe that poor countries have invested at lower rates and have not grown faster than rich countries. To explain these empirical regularities we provide a game-theoretic model of conflict between social groups over the distribution of income. Among all possible equilibria, we concentrate on those that are on the constrained Pareto frontier. We study how the level of wealth and the degree of inequality affects growth. We show how lower wealth can lead to lower growth and even to stagnation when the incentives to domestic accumulation are weakened by redistributive considerations.

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1. Introduction

Neoclassical growth theory predicts that poor countries, because of the law of diminishing returns, grow at faster rates than rich countries. This inverse relation between wealth levels and growth rates should further be strengthened by the diffusion of technology and the opportunities for catching up. Yet, despite concerted efforts at faster development, we observe that poor countries have invested at lower rates, exhibited more intense social conflict and political instability, and consequently have not grown faster than rich countries. The empirical relationship between income levels and growth rates is flat and possibly hump-shaped, not downward sloping.[1] One possible explanation for these results has been given by theories of endogenous growth that appeal to human capital to eliminate labor as a fixed factor, or by introducing elements of increasing returns (see Lucas, 1988; Romer, 1986, 1990). An alternative approach known as the *conditional convergence* hypothesis (see Barro, 1991; Mankiw, Romer, and Weil, 1992; Levine and Renelt, 1992) suggests that the lower growth rates observed in poorer countries are essentially due to lower rates of accumulation in physical and human capital. When factor accumulations or savings rates are taken into account, the predicted negative relation between growth rates and initial income levels is reestablished. Indeed, it has been observed that investment rates in physical and human capital (primary schooling) are positively correlated with income levels (see Fisher, 1991). Furthermore, investment rates show a robust negative correlation with various measures of political instability (see Barro, 1991; Levine and Renelt, 1992; Venieris and

Gupta, 1986), and there is a negative relationship between measures of political instability and levels of income (see Alesina and Perotti, 1993; Londregan and Poole, 1990). The cross-country evidence suggests then that poorer countries are more prone to political instability, have lower investment rates, and consequently may not have realized their growth potential to catch up with rich countries.

In this paper we offer a model that can explain why poor countries have tended to invest and accumulate at rates lower than the rich countries. We pursue an alternative game-theoretic course that emphasizes the relationships between levels of wealth, social and political conflict, and incentives for accumulation. As such our work is indirectly related to papers by Persson and Tabellini (1994), Alesina and Rodrik (1994), Grossman (1991), and Tornell and Velasco (1992).

We have in mind a situation where organized social groups can capture, or attempt to capture, a larger share of the output either by means of direct appropriation or by manipulating the political system to implement favorable transfers, regulations, and other redistributive policies.[2] Depending on the country, these groups may represent, among others, organized labor, industrial and business associations, the military, the bureaucracy, or racial, ethnic, and tribal groups.[3] Such redistributive and expropriative activities undertaken by social groups can create significant disincentives to accumulate, which as we show below, can be stronger at lower levels of wealth than at higher ones, so that poorer countries grow more slowly or even stagnate at lower levels of wealth. We obtain these results in our model without having to rely on the alternative and probably complementary framework that requires nonconvexities or threshold effects in the production technology (see, for example, Lucas, 1988; Romer, 1986, 1990; Azariadis and Drazen, 1990).

To capture the empirical relationship between wealth and growth discussed above, we use a simple dynamic game framework. Social groups are modeled as players who independently choose a consumption level; the residual output, if any, becomes the capital or the productive resource in the following period. We consider equilibrium paths of accumulation in which players receive utilities that are at least as high as those that they could obtain by defecting: that is, by appropriating higher immediate consumption levels and suffering some retaliation later on. (For a related framework of analysis, see Marcet and Marimon, 1990; Chari and Kehoe, 1990; Tornell, 1992.) We focus, however, on those subgame-perfect equilibria that are second best—that is, on the subset of subgame-perfect equilibria that lie on the constrained Pareto frontier. There are three main reasons for this choice. First, it is conceivable that those are, in fact, the equilibria we observe because institutional arrangements and social norms that select an efficient outcome among equilibria may develop.[4] Second, as we discuss later, these are equilibria in which the economy grows at the fastest rate; so they provide an upper bound to the growth rate. Finally, in this way we obtain an indirect measure of the effects of the strategic behavior of groups: it is at least as large as the difference between first and second best. Within the set of second best, we analyze the effects of wealth on growth and on steady-state income levels.

Whether high or low levels of wealth depress investment and growth rates critically depends on the curvatures of technology and preferences. To illustrate these possibilities we study two opposite cases. At first we consider the case in which lower wealth leads to lower growth and sometimes, though not necessarily always, to a "growth trap." This case is likely when there are sufficiently high diminishing returns in utility: when wealth levels are high, appropriating a large amount of consumption, at the expense of future retaliation, becomes less attractive. At lower wealth levels however, when consumption is low and marginal utility is high, the opposite may be true, especially if the marginal product of capital does not become high enough to deter defection with a high level of consumption. In the extreme, we may observe "growth traps." Even though first-best policies lead to growth, along second-best equilibria growth may have to be slower or may not even be possible at low wealth levels because of incentive constraints: the accumulation of wealth by one player can lead to appropriation and to high consumption by other players and therefore may not be sustainable as an equilibrium.

Another possibility is for incentive constraints to bind at high wealth levels and not at low ones. This type of case occurs when marginal utility does not diminish too strongly while the marginal product of capital does. Defecting with a large amount of consumption at high levels of wealth may now become attractive. Because of diminishing returns in production however, capital may be too precious at low levels of wealth, and players may initially follow first-best policies of accumulation. Inefficiency sets in at higher levels of wealth and first-best policies may have to be abandoned as the incentives for appropriation grow and redistributive pressures increase. The possibility that inefficiencies are associated with stable and wealthy economies in which organized groups have had the time to mature and exert redistributive pressures has been suggested by Mancur Olson (1982).

The discussion above and the theoretical analysis in the subsequent sections suggest that structural and specific aspects of the economy, as modeled by the preferences and technology, determine whether the strategic behavior of interest groups is likely to diminish growth at high levels or at low levels of wealth. Whether group conflict and strategic behavior can lead to slow growth at high or low wealth levels, or maybe even at both high and low wealth levels, remains therefore an empirical matter. We illustrate the theoretical possibilities in Sections 5 and 6 below.

In Section 2 we show that when incentive constraints are binding, the fastest-growing subgame perfect equilibrium is the symmetric (egalitarian) second best. For instance, if incentive constraints are binding at low levels of wealth, then the growth rate of the symmetric (egalitarian) second-best equilibrium sets an upper bound to the growth rates at low wealth levels. Growth rates on all other equilibria, including the nonsymmetric or inegalitarian second best, must be even lower. Our model therefore also indicates that for any given level of wealth, there is a tradeoff between growth and inequality, where inequality is measured by the disparities of consumption rates and welfare levels. (For a similar view based on credit market imperfections, see Galor and Zeira, 1993.) High rates of accumulation in economies with pronounced and persistent inequalities may not be sustainable because the disadvantaged groups can undertake redistributive actions or exert redistributive pressures that discourage domestic investment. The political consensus necessary for efficient growth may not be attainable if income inequality is too severe. Recent empirical work has confirmed the inverse relationship of income inequality with investment and growth (see, for example, Venieris and Gupta, 1988; Alesina and Rodrik, 1994; Persson and Tabellini, 1994).

Our paper is organized as follows. The next section sets up the problem in a general framework. Section 3 works out a simple and illustrative example of a second-best problem where incentive constraints retard growth but accumulation rates do not depend on wealth. Section 4 provides some general conditions under which a political "growth trap" occurs without having to explicitly compute the "second best." Section 5 computes an explicit example of a case where growth is slow relative to first-best levels only when wealth levels are low. It also provides an example of a discontinuous value function. Section 6 illustrates the "Olson" case where first-best policies are optimal at low stock levels but cannot be sustained at high stock levels. Section 7 concludes with a brief overview.

2. The Second-Best Problem

We consider two players characterized by two concave and strictly increasing utility functions $U_i, i = 1, 2$, and a common discount factor $\beta \in (0, 1)$. k_t represents the capital stock at time t. The production function f is concave, increasing, and $f(0) \geq 0$. The feasible paths of the consumption sequences must satisfy $f(k_t) - c_t^1 - c_t^2 \leq k_{t+1}$, and $c_t^i \geq 0$, $t = 0, 1, \ldots; i = 1, 2$. In our game, histories at time t are sequences of consumption pairs $h_t = (c_1^1, c_1^2, \ldots, c_t^1, c_t^2)$, and strategies are maps from histories to consumptions. For a given initial stock k, the *second-best value* is defined by

$$v_{sb}(k) \equiv \sup \sum_{t=0}^{\infty} \beta^t [\alpha_1 U_1(c_t^1) + \alpha_2 U_2(c_t^2)], \quad (1)$$

where the supremum is taken over the sequences $(c_t^1, c_t^2)_{t \geq 0}$ of subgame perfect equilibrium (SPE) outcomes and $\alpha_1, \alpha_2 \geq 0$.

The purpose of this section is to prove that the second best is achieved over a smaller set of SPE. To avoid ambiguities, we describe in detail how the *allocation of consumption* is regulated. It will be useful to distinguish between attempted consumption and actual consumption (the first is the consumption a player is trying to get, the second is what he actually gets under the allocation rule). For a given capital stock k and attempted consumptions c_1 and c_2, the actual allocated consumption is

$$A_1(c_1, c_2, k) = \begin{cases} c_1 & \text{if } c_1 + c_2 \leq f(k) \text{ or } c_1 \leq f(k)/2 \\ f(k) - c_2 & \text{if } c_1 + c_2 \geq f(k) \text{ and } c_1 \geq f(k)/2 \geq c_2 \\ f(k)/2 & \text{if } c_1, c_2 \geq f(k)/2 \end{cases}$$

and similarly for A_2. Note that if $c_2 \leq f(k)/2$, then $A_1(c_1, c_2, k) = \min\{c_1, f(k) - c_2\}$.

Since the utility function of both players is strictly increasing, the following pair of strategies, which we call *fast consumption strategies*, constitutes an equilibrium for all values of the capital stock k:

$$\bar{c}_1(k) = \bar{c}_2(k) = f(k).$$

Note in fact that in this case the allocation rule gives $A_1(\bar{c}_1, \bar{c}_2, k) = A_2(\bar{c}_2, \bar{c}_1, k) = f(k)/2$ to both players. Also note that if the second player attempts to consume $f(k)$, for any choice of c_1 the capital stock in the next period is zero. So by reducing c_1 the first player can only reduce his payoff.[5]

Under the allocation rule described above, the worst SPE is easily described. This allows us to utilize the worst equilibrium in order to sustain any other SPE with trigger strategies. Note that one may also arbitrarily choose a simple SPE—for example, the interior Markovian one under which the sum of attempted consumptions never exceeds output—and then study the set of SPE that are enforcable with trigger strategies using that Markovian SPE as a threat.

As noted above, it is clear that the pair $(\bar{c}_1, \bar{c}_2)$ is a SPE, since the utility functions of the players are strictly increasing. The value of this equilibrium to player i is given by

$$\bar{v}_i(k) = \sum_{t=0}^{\infty} \beta^t U_i(A_i(\bar{c}_1(k_t), \bar{c}_2(k_t), k_t)), \qquad i = 1, 2,$$

where $k_0 = k$, $k_{t+1} = f(k_t) - A_1(\bar{c}_1(k_t), \bar{c}_2(k_t), k_t) - A_2(\bar{c}_1(k_t), \bar{c}_2(k_t), k_t)$, $t \geq 0$. Of course, if $f(0) = 0$, the above summation reduces to $U_i(f(k)/2)$. A *trigger strategy pair* is described by an agreed consumption path $(c_t^1, c_t^2)_{t\geq 0}$ and the threat of a shift to a fast-consumption equilibrium after the first defection is detected. The *individual rationality constraint* for player i on an outcome path is the condition

$$\sum_t \beta^t U(c_t^i) \geq \bar{v}_i(k).$$

Clearly, in a SPE, the outcome of the equilibrium of any subgame satisfies this inequality.

Consider now a trigger strategy equilibrium. For any capital stock k and equilibrium consumption c of the other player, the value of defection is the value for a player of deviating optimally—that is,

$$v_i^D(k, c) = \text{Max}\{w^D(k, c), \bar{v}_i(k)\},$$

$$w^D(k, c) = \max_{0 \leq c' \leq f(k) - c} U(c') + \beta U\left(\frac{1}{2} f(f(k) - c - c')\right) + \frac{\beta^2}{1 - \beta} U(f(0)/2), \tag{2}$$

and

$$\begin{aligned} k_1 &= f(k) - c - A_i(k, c, c'), \\ k_{t+1} &= f(k_t) - A_1(\bar{c}_1(k_t), \bar{c}_2(k_t), k_t) - A_2(\bar{c}_1(k_t), \bar{c}_2(k_t), k_t), \qquad t \geq 1. \end{aligned} \tag{3}$$

Note that this optimization problem can be expressed without the maximization operator in defining v_i^D by simply adding the constraint $v_i^D(k, c) \geq \bar{v}_i(k)$. We denote by $c_i^D(k, c)$ the consumption giving the optimal deviation: in the games we consider such optimal consumption exists and is unique.

The following lemma is clear. We state and prove it for completeness.

Lemma 2.1: *Let $(c_t^1, c_t^2)_{t\geq 0}$ be the outcome of a SPE—ξ, say. Then the trigger strategy pair with this agreed consumption path, ξ' is an SPE.*

Proof. For any history h_t, we denote $v_i(h_t)$ the value to the ith player of the equilibrium in ξ starting with h_t. We only need to consider equilibrium histories $h_{t-1} = (c_1^1, c_2^1, \ldots, c_{t-1}^1, c_{t-1}^2)$. Let k_t be the capital stock. We claim that c_t^2 is an optimal choice for player 2 next period, in ξ'. The best alternative choice is $c_2^D(k_t, c_t^1) \equiv c^D$. In the equilibrium ξ such a choice would give him a payoff of $U_2(c^D)$ plus the equilibrium value of the subgame starting at (h_t, c_t^1, c^D). In the equilibrium of this subgame, the individual rationality constraint is satisfied, so our claim follows from the expression below:

$$\begin{aligned} U_2(c_t^2) + \beta v_2(h_t, c_t^1, c_t^2) &\geq U_2(c^D) + \beta v_2(h_t, c_t^1, c^D) \\ &\geq U_2(c^D) + \beta \bar{v}_2(f(k_t) - c_t^1 - c^D) \\ &= v_2^D(k_t, c_t^1). \end{aligned}$$

■

It follows that the supremum in the definition of a second best is the same as the supremum over trigger strategy equilibria. As a first implication of this reduction we prove that the second-best value is in fact achieved. We turn to this now. Let $\alpha_1, \alpha_2 \geq 0$ be weights attached to the players. From what we have seen, the second best is the solution of the problem:

$$v_{sb}(k) \equiv \sup_{\{(c_t^1, c_t^2)_{t\geq 0}\}} \sum_t \beta^t [\alpha_1 U_1(c_t^1) + \alpha_2 U_2(c_t^2)], \tag{4}$$

subject to $f(k_t) - c_t^1 - c_t^2 \geq k_{t+1}$, and

$$\sum_t {}^{\infty}_{\circ}\, \beta^{t+i} U_j(c_{t+i}^j) \geq v^D(k_i, c_i^{j'}) \qquad i = 1, 2, \ldots; \qquad j = 1, 2.$$

Below we refer to this as *the second-best problem.* It is easy to show that under mild assumptions a second-best solution exists and that the value function v_{sb} is uppersemicontinuous[6] (see Benhabib and Rustichini, 1991).

It is clear from equation (4) that there will be a second-best equilibrium for each set of weights (α_1, α_2) and that the equilibrium will be symmetric when the weights are equal. The proposition below establishes that when constraints are binding, a symmetric or egalitarian second-best equilibrium is also the fastest growing subgame perfect equilibrium. Since in the examples below we study the symmetric second best this proposition will be useful to illustrate the extent to which growth is curtailed by incentive constraints across different equilibria.

Proposition 2.1: *For a given k, let $(c_1^t, c_2^t)_{t\geq 0}$ be a second-best equilibrium outcome, starting from k, which is symmetric and such that $v_i(k) = v^D(k, c_{i'})$, for $i = 1, 2$. If $k_1 = f(k) - c_1 - c_2$, and k_1' is the next period capital stock for any other subgame perfect equilibrium, then $k_1' \leq k_1$.*

For a proof see Benhabib and Rustichini (1991).

3. A Simple Example of Second-Best Equilibrium with No Wealth Dependence

We will start by exploring a simple case of a second-best equilibrium to illustrate how growth rates may differ drastically between first-best and incentive constrained second-best equilibria. This first example is simple because growth rates on equilibrium paths will turn out to be independent of the levels of wealth—that is, of the capital stock. The more interesting and complex cases will be studied later.

Let each of the two identical players in this example have an instantaneous utility function given by $U(c) = (1-\epsilon)^{-1}c^{1-\epsilon}$ with $0 < \epsilon < 1$, with discount rate $0 < \beta < 1$. Let the production function be $y = ak$ with $a > 0$. The total utility of each player among a first-best equilibrium is described by the dynamic program

$$\hat{v}(k) = \text{Max}_{0 \leq c \leq \frac{y}{2}} (1-\epsilon)^{-1}c^{1-\epsilon} + \beta\hat{v}(y-2c). \tag{5}$$

The consumption function that solves this program is given by $\hat{c} = \lambda y$, where

$$\hat{\lambda} = \frac{1}{2}\left(1 - \beta^{\frac{1}{\epsilon}} a^{\frac{(1-\epsilon)}{\epsilon}}\right) \geq 0 \tag{6}$$

and where we have imposed the restrictions $a > 1$, $\beta^{1/\epsilon}a^{(1-\epsilon)/\epsilon} < 1$ to avoid negative consumption levels and to ensure a well-defined value function. For any $\lambda \geq 0$, the value function is given by $v(k) = s(\lambda)y^{(1-\epsilon)}$, where

$$s(\lambda) = \frac{(1-\epsilon)^{-1}\lambda^{1-\epsilon}}{1 - \beta(a(1-2\lambda))^{1-\epsilon}}. \tag{7}$$

We note for further use below that s is derived here for arbitrary $\lambda \geq 0$, not only for the first-best λ.

When a player defects against first-best play by his opponent, he must choose his consumption in the current period taking into account that trigger strategies will be enacted subsequently. Optimal defection value is therefore

$$\begin{aligned} v^D(k, \hat{c}(k)) &= \text{Max}_{0 \leq c_D \leq (1-\hat{\lambda})y} (1-\epsilon)^{-1} c_D^{1-\epsilon} \\ &\quad + \beta(1-\epsilon)^{-1}\left(a((1-\hat{\lambda})y - c_D)/2\right)^{1-\epsilon}, \end{aligned} \tag{8}$$

where $\hat{c}(k) = \hat{\lambda}y$. This value reflects the trigger strategy equilibrium in which, following a defection, all output is consumed in equal shares by the two players. The optimal defection policy for consumption against another player consuming λy with $\lambda \leq 1/2$ is given by

$$c_D(k, \lambda y) = M(1-\lambda)y = \lambda_D y,$$

where $M = ((\frac{\beta a}{2})^{1/\epsilon}(\frac{2}{a})+1)^{-1} < 1$. The value of optimal defection is given by $v^D(k, \lambda y) = s_D y^{(1-\epsilon)}$, where

$$\begin{aligned} s_D &= (1-\lambda)^{1-\epsilon}(1-\epsilon)^{-1}[M^{1-\epsilon} + \beta((1-M)a/2)^{1-\epsilon}] \\ &= (1-\lambda)^{1-\epsilon}(1-\epsilon)^{-1}M^{-\epsilon}. \end{aligned} \tag{9}$$

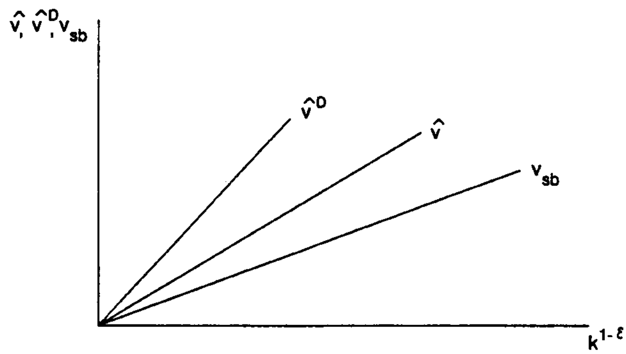

Figure 1.

For first-best policies to constitute an equilibrium, the value that they generate for each player must dominate the values of defection at each point on the equilibrium path—that is, $v(k) \geq v^D(k, \hat{c}(k))$ for all k on the equilibrium path. As we illustrate later in examples, however, $v(k)$ and $v^D(k)$ can intersect so that first-best outcomes can be enforced from some values of k but not from others, as discussed in Benhabib and Radner (1992) or Benhabib and Ferri (1987).

A symmetric second-best equilibrium with incentive compatibility constraints will be given by the solution to the following problem:

$$v_{sb}(k) = \text{Max}_{0 \leq c \leq \frac{y}{2}} (1 - \epsilon)^{-1} c^{1-\epsilon} + v_{sb}(ak - 2c),$$

subject to $v_{sb}(k) \geq v^D(k, c)$.

Clearly, if $s(\hat{\lambda}) \geq s_d(\hat{\lambda})$, then the first-best equilibrium is enforced. If, however, $s(\hat{\lambda}) \leq s_d(\hat{\lambda})$, then the second best must differ from the first best. The consumption function is then given by $c = \lambda_s y$, where $\lambda_s \leq (1/2)$ is defined by $s(\lambda_s) = s_d(\lambda_s)$ (for a formal proof see Benhabib and Rustichini, 1991).

Figure 1 illustrates the second-best solution when incentive constraints are binding. The solution is to find λ_s, which equates the value for each player of following the consumption policy $c_{sb} = \lambda_s y$ with the value of defecting from it. In other words, consumption must be increased and accumulation slowed down to the point where defection is no longer attractive. $\hat{v}^D$ in Figure 1 is the value of defecting against a player following first-best strategies.

The following numerical values illustrate the effects of incentive compatibility constraints on economic growth along the symmetric equilibrium. We set $a = 3.3$, $\beta = 0.325$ (implausibly high discounting, of course), and $\epsilon = .5$. For these values $v^D(k, \hat{c}(k)) > \hat{v}(k)$ for all $k > 0$, where $\hat{v}(k)$ corresponds to the first-best values with policies $c = \lambda y$. We compute $\lambda = 0.326$, $\lambda_s = 0.349$. These magnitudes imply that if the first best could be sustained, the capital stock would perpetually grow at 15 percent. On the second-best path, however, the economy grows at -0.0015 percent—that is, it contracts. Of course, parameters were chosen to make this stark point. Slightly different parameters would allow positive growth along the second-best equilibrium but at a slower rate than the first best.

Of course, in some cases the first best may be enforcable as an equilibrium from all stocks so that incentive constraints do not bind. We note that for the parameters above it is easy to check that $y - 2c_{sb}(k) \geq 0$ and $y - \hat{c}(k) - c_D(k) \geq 0$ for all $k \geq 0$.

4. Wealth Dependent Growth

In general, it is not at all easy to compute second-best equilibria even for simple problems. For example, if we slightly alter the linear production function of the previous section to $y = ak + b$, $b > 0$, it is no longer possible to find a constant λ to equate $v(k)$ and $v^D(k, c)$. In particular $\hat{v}(k)$ and $v^D(k, \hat{c})$ may intersect at some k. If $\hat{v}(k) \geq v^D(k, \hat{c})$ and $k \geq \underline{k}$, first-best policies will be sustainable as equilibria for $k \geq \underline{k}$. From initial conditions below $\underline{k}$ where $\hat{v}(k) < v^D(k, \hat{c}(k))$, it may be possible to construct "switching" equilibria (which are not necessarily second best), along which growth occurs at a rate slower than first-best rates until $\underline{k}$ is reached, and first-best policies are followed once $\underline{k}$ is attained. In this section we will derive conditions under which the second-best growth rates will be wealth dependent: in particular we will find general conditions under which first-best growth rates are sustainable from high stocks, while growth is not at all possible from low stocks because of incentive compatibility constraints. The intuition for the result is simple: relative to first-best levels, consumption rates must be increased and accumulation slowed to prevent defection. When stocks are low, consumption must be increased so much to prevent defection that growth is no longer possible. Examples will follow.

Proposition 4.1: *Assume that for some k (1) $\hat{v}(k) < v^D(k, \hat{c}(k))$ and (2) $f(k) - 2\tilde{c} \leq k$, where $\tilde{c}$ is the least c such that $U(c) + \beta\hat{v}(f(k) - 2c) = v^D(k, c)$. Then $f(k) - 2c \leq k$ for any c which is the first period consumption rate of a symmetric SPE (that is growth is impossible). In particular this is true for second best equilibria.*

Proof: Assume that $f(k) - 2c > k$; then clearly $c < \tilde{c}$. But then, if $v(k)$ is the value of the equilibrium,

$$v^D(k, c) \leq v(k) \tag{10}$$

$$\leq U(c) + \beta\hat{v}(f(k) - 2c) \tag{11}$$

$$< v^D(k, c), \tag{12}$$

where (10) holds by definition of SPE, (11) holds because $\hat{v} \geq v$, and (12) holds by the fact that $U(c') + \beta\hat{v}(f(k) - 2c') < v^D(k, c')$ for every $c \in (\hat{c}(k), \tilde{c})$; this interval is nonempty because of the assumptions 1 and 2. We have derived a contradiction. We note that if a $\tilde{c}$ as defined above does not exist, condition 2 can be taken to be trivially satisfied. Finally, since the fastest growing SPE is the symmetric second best by Proposition 2.1, the result applies to all SPE and all second best equilibria as well. ■

We can now construct a simple example that applies Proposition 4.1 above without explicitly computing the second-best solution. We slightly alter the example of the previous section by adding a constant to the production function: $y = ak + b$. In this case the first-best consumption policy is

$$\hat{c} = \text{Min}(\hat{\lambda} y + \hat{\eta}, y/2), \tag{13}$$

where[7]

$$\hat{\eta} = \hat{\lambda} b/(a-1) \geq 0. \tag{14}$$

For any λ and $\eta = \lambda b/(a-1)$, the first-best value function is

$$v(k) = s\left(ak + \frac{b}{a-1}\right)^{1-\epsilon}, \tag{15}$$

where s is given by (7) as before.

The optimal defection policy against a player consuming $c = \lambda y + \eta$ is

$$c_D(k, \lambda y + \eta) = \min\left(M\left((1-\lambda)y + \left(\frac{a-1}{a} - \lambda\right)\left(\frac{b}{a-1}\right)\right), (1-\lambda)y - \eta\right), \tag{16}$$

where $c \leq y/2$ and $\eta = \lambda b/(a-1)$. The defection value is then given by

$$\begin{aligned} v^D(k, (1-\lambda)y + \eta) &= s_D\left[y + \left(\frac{((a-1)/a) - \lambda}{1-\lambda}\right)(b/(a-1))\right]^{1-\epsilon} \\ &\quad + \left(\frac{\beta^2}{1-\beta}\right)\left(\frac{1}{1-\epsilon}\right)\left(\left(\frac{1}{2}\right)b\right)^{1-\epsilon}, \end{aligned} \tag{17}$$

where s_D is defined in (9).

We now assign parameter values to the example above as follows: $a = 1.058, b = 0.025$, $\beta = .95$, $e = .2$. With these values, for $k \geq 1.2$ we have $\hat{v}(k) > v^D(k, \hat{c}(k))$, while for $k \leq 1.1$, $\hat{v}(k) < v^D(k, \hat{c}(k))$. It is easily shown that for $k > 0.02$ the first-best strategies lead to growth at the rate of about 2.5 percent. Thus for $k \geq 1.2$, this growth rate can be sustained as a first-best equilibrium. However, for k in [0.4, 0.9], conditions of the above proposition apply. For $k = 0.4$ ($= 0.9$) $\tilde{c}$ defined in the proposition is given by 0.10015 (0.05055) and $y - 2\tilde{c} - k < 0$. Therefore even the second-best equilibrium cannot generate growth for $k \in [0.4, 0.9]$. We can check that for $k \geq 0.4$ we have $y - 2\hat{c}(k) > 0$ and $y - \hat{c}(k) - c_D(k) > 0$.

The above example and proposition allow us to starkly establish how growth rates can depend on wealth because incentive constraints can be strongly binding at some wealth levels and weakly binding at others. In fact, in our example incentive constraints are not binding at all for $k \geq 1.2$, but they are binding and prevent positive growth for $k \in [0.4, 0.9]$. The empirically more relevant issue however arises in the region $k \in [0.9, 1.2]$ where growth is possible along the second best but at rates lower than the first-best constraints. In the next section we provide an explicit example to illustrate this.

5. Slow-Growth Equilibria and Growth Traps

In the next example we derive explicitly a second-best policy for which growth toward a high steady state occurs from large stocks but not from low stocks. First-best policies that are not incentive compatible always lead to the unique steady state with positive stocks, but second-best policies may not. The value function for this example is discontinuous even though technology and preferences are convex and continuous. As in the previous section, the players have identical preferences and are equally weighted.

As mentioned in the introduction, the case of slow growth at low wealth levels is more likely to obtain when marginal utility is low at high wealth levels and when the marginal product of capital is not too high at low levels of capital—as, for example, in the case of technologies that are linear in capital. Under diminishing marginal utility, deviation at high wealth levels with a large chunk of consumption is less appealing. At the same time, if the marginal product of capital is not too high at low wealth levels, the benefits of fast accumulation can be more easily sacrificed, especially if consumption is low and marginal utility is sufficiently high to make defection more attractive. The example below illustrates these points.

We consider the production function

$$f(k) = \begin{cases} Ak & k \leq 1 \\ A + B(k-1) & k \geq 1, \quad B/2 < 1^{8} \end{cases} \tag{18}$$

with $A = 5/2$ and $\beta = 1/2$. The utility function is

$$U(c) = \begin{cases} c & \text{if } c \leq 1 \\ 1 + b(c-1) & \text{if } c \geq 1. \end{cases}$$

Since $A\beta > 1 > B\beta$, $k = 1$ is a steady-state stock for the optimal growth problem with $c = 3/4$ as the steady-state consumption. We assume b is small:

$$B\beta < b < \frac{A\beta}{2} < 1.$$

We spare the reader a detailed discussion of the details of the example. For a complete treatment we refer to Benhabib and Rustichini (1991, sec. 4.2 and app. A.2.1). The conclusions of the example are clear from Figures 2a and 2b.

There are five values of the capital stock that mark the extreme points of intervals over which the second-best value and the consumption policy have different behavior.[9] $k_0 = 1$ is the first-best and second-best steady state with positive stocks. The value function is piecewise linear. Two points are noteworthy. First, the value is convex on the interval $[0, k_3)$ and concave in $[k_3, +\infty)$. Second, the value function is discontinuous (but upper-semicontinuous) at k_3. We emphasize again that both the production and utility functions are concave (so that the first-best value function is necessarily concave and continuous). For $k \geq k_1$ the second-best value (and the consumption policy) are the same as the first best. For lower values, the consumption level for second best has to be higher to satisfy

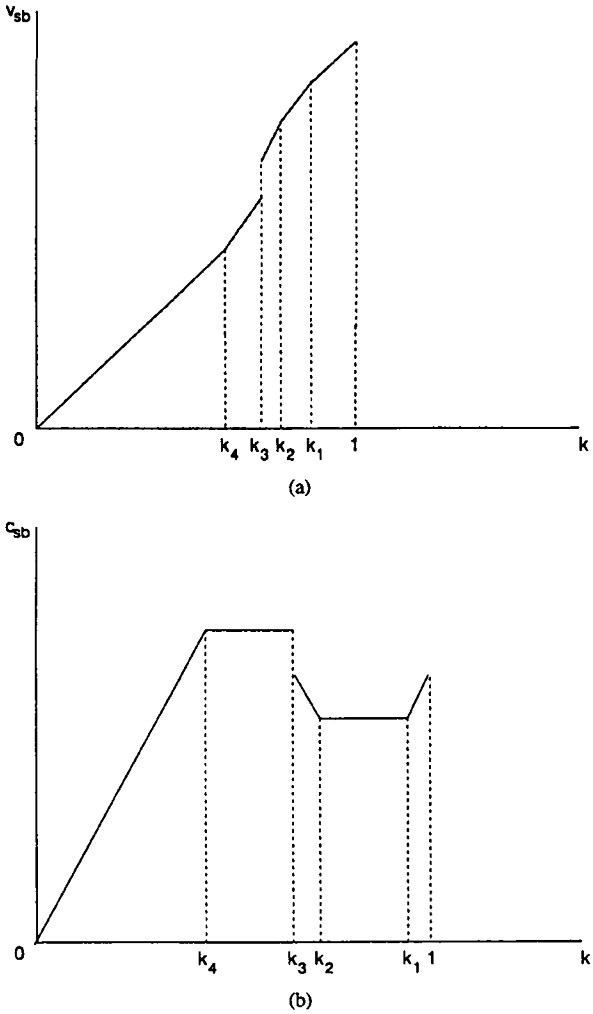

Figure 2.

the equilibrium conditions; growth is slower, and the second-best value is strictly below the first best. More precisely, for $k \in [k_3, k_1]$ the second-best policy prescribes the minimum consumption that makes the second-best value equal to the value of defection. Consumption is decreasing over the range where the incentive compatibility constraint is binding and then increasing when the second-best solution is the first best. Overall, the second-best consumption is nonmonotonic, even in the region where we have steady growth. Note that over $[k_3, k_1)$ the first-best consumption is lower than the second best. As k increases, the incentive constraint becomes less binding, and second-best consumption decreases with k along the equilibrium. The intermediate region $([k_2, k_1])$ has the lowest consumption above k_4. Finally, the consumption policy is continuous except at k_3. The reason for the discontinuity of the consumption policy and the value function may be understood as follows. As k decreases, progressively higher levels of consumption are needed in order to make the value of second best, and the value of defecting from it, equal to each other. To higher levels of consumption corresponds a reduction in both the continuation value and the postdefection value. The rate at which these two second-period values change is different. The rate of the defection value is constant while the rate of the second-best continuation value is changing with k because the second-best value is concave. When the difference between these two rates changes sign and becomes negative, no increase in consumption can equate the second best and the value of defection and at the same time allow the capital stock not to decline. (Computations underlying this example are in Benhabib and Rustichini, 1991.)

The dynamics of the capital stock in the second-best solution gives even more dramatic evidence of the discontinuity at k_3. For $k \in [k_3, k_1]$, the capital stock grows toward the same steady state as the first best but more slowly: we denote this region as one of "slow growth." For $k < k_3$, we have a "growth trap": the capital stock contracts to zero, which is a *stable* steady state for the dynamics of the second best. The region of wealth for which we have "slow growth," the interval (k_3, k_1), is probably of greater empirical interest. Here the growth rate is lower than the first best. Only after the threshold wealth level of k_1 has been reached does growth resume its higher first-best level.[10]

6. An "Olson" Case

In the previous sections we showed how incentive constraints could result in equilibria for which growth occurs from high stocks but not from low stocks. In the following example the opposite is true. In contrast with the previous examples, the utility function is linear, but the production function is Cobb-Douglas. The linear utility makes it attractive to defect from high stocks with a large consumption. On the other hand, with a Cobb-Douglas technology, when stocks are low, their marginal product is high and defection is unattractive. Therefore at low stocks players follow first-best policies to accumulate the precious capital. As stocks get larger, defection becomes more attractive, and accumulation has to slow down. First-best policies are abandoned, and the economy stops short of the first-best steady state. In the spirit of the work of Mancur Olson (1982) inefficiency emerges at high rather than low levels of wealth.

Let the production function be $f(k) = k^\alpha, \alpha \in (0, 1)$, and the utility function be $U(c) = c$. The optimal solution has a steady state given by

$$k^* = (\alpha\beta)^{\frac{1}{1-\alpha}}. \tag{19}$$

The optimal policy is, as usual,

$$\hat{c}(k) = \begin{cases} 0 & \text{if } k \leq k^{*\frac{1}{\alpha}} \\ \frac{k^\alpha - k^*}{2} & \text{if } k \geq k^{*\frac{1}{\alpha}}, \end{cases} \tag{20}$$

and the first-best value of the steady state is

$$\hat{v}(k^*) = \frac{(\alpha\beta)^{\frac{1}{1-\alpha}}}{1-\beta}.$$

Consider now a given level of capital stock k and consumption c for one of the players. Then the value of defection for the other player is

$$v^D(k,c) \equiv \max_{c' \geq 0} c' + \frac{\beta}{2}(k^\alpha - c - c')^\alpha. \tag{21}$$

The optimal defection consumption is clearly in the interval $[0, k^\alpha - c)$:

$$c_D(k,c) = \begin{cases} 0 & \text{if } (k^\alpha - c)^{\alpha-1} > \frac{2}{\alpha\beta}, \\ k^\alpha - c - \gamma & \text{otherwise} \end{cases} \tag{22}$$

with $\gamma = (\alpha\beta/2)^{1/1-\alpha}$. The associated value function for defection is

$$v^D(k,c) = \begin{cases} \frac{\beta}{2}(k^\alpha - c)^\alpha & \text{if } (k^\alpha - c)^{\alpha-1} > \frac{2}{\alpha\beta}, \\ k^\alpha - c + \zeta & \text{otherwise} \end{cases} \tag{23}$$

where $\zeta = \gamma^\alpha(\beta/2) - \gamma$. Note that if the net stock left by the other player, $k^\alpha - c$, is too low, then the optimal defection policy is to consume nothing.

Before we proceed, we define the set of incentive compatible steady states; in this example these are values of k such that the following inequality holds:

$$\frac{f(k)-k}{2(1-\beta)} \geq v^D\left(k, \frac{f(k)-k}{2}\right). \tag{24}$$

These are therefore the values of k such that the value for each player of keeping k as a steady state is larger than the value of defecting from this pair of capital stock and consumption. This set will be useful in determining the second-best value and policy. For any value of α, β the inequality above is equivalent to $\beta k^\alpha - (2-\beta)k - 2\zeta(1-\beta) \geq 0$, so the set of values of k that satisfy the above inequality is a (possibly empty) interval, $\underline{k} \leq k \leq \bar{k}$. Note that for the proposition below, the lowest k from which $\bar{k}$ may be reached in one step by consuming nothing is given by $\bar{k}^{1/\alpha}$. A proof is in Benhabib and Rustichini (1991).

Proposition 6.1: *On the interval* $[\max\{\underline{k}, \bar{k}^{1/\alpha}\}, \bar{k}]$ *we have (1) if* $k^* < \bar{k}$, *the first-best and second-best value functions and consumption policies coincide:* $v_{sb}(k) = \hat{v}(k)$, $c_{sb}(k) = \hat{c}(k)$; *(2) if* $\bar{k} \leq k^*$, *that is the first-best steady state is above the range of incentive compatible sustainable steady states, then the second-best values are* $v_{sb}(k) = (k^\alpha + \bar{k})/2 + \zeta$, *and* $c_{sb}(k) = (k^\alpha - \bar{k}/2$; *(3) if* $\underline{k} < \bar{k}^{1/\alpha}$, *the second-best value strictly dominates defection—that is,* $v_{sb}(\bar{k}^{1/\alpha}) > v^D(\bar{k}^{1/\alpha}, c_{sb}(\bar{k}^{1/\alpha}))$. *Furthermore for an interval* $[k_1, \bar{k}^{1/\alpha}]$ *we have*

$$v_{sb}(k) = \beta\frac{k^{\alpha^2} + \bar{k}}{2} + \beta\zeta, \qquad \hat{c}(k) = c_{sb}(k) = 0,$$

that is, the first- and second-best consumption policies coincide, but the value functions do not.

It is easy to show that both the case $\bar{k} < k^*$ and the case $k^* < \bar{k}$ are possible. For example, for $(\alpha, \beta) = (.975, .97)$, case 2 in Proposition 7.1 occurs with $k^* = .1074$, $\bar{k} = .0906$. However, when $(\alpha, \beta) = (.9142, .92)$, then $k^* = .1329$, $\bar{k} = .1542$. It is also easy to see that, for different values of α, both cases may occur for any given value of the discount factor. In the case where $\bar{k} < k^*$, we know that the second-best policy over $[\bar{k}^{1/\alpha}, \bar{k}]$ is to consume as much as needed to go to $\bar{k}$ in one step. However, on $[k_1, \bar{k}^{1/\alpha}]$ the second-best consumption is the same as the first-best consumption, which is zero. The second-best accumulation path then stops at $\bar{k}$, while the first best grows to k^*. For higher k, second-best consumption is higher than the first best. Therefore on $[k_1, \bar{k}^{1/\alpha}]$, when stocks are low, players follow first-best strategies but stop doing so above $\bar{k}^{1/\alpha}$. (Note also that $v_{sb}(\bar{k}^{1/\alpha}) - v^D(\bar{k}^{1/\alpha}, c_{sb}(\bar{k}^{1/\alpha})) > 0$ for the (α, β) pairs given above; this implies $\underline{k} < \bar{k}^{1/\alpha}$.)

7. Final Remarks

Some empirical evidence suggests that poor countries fail to grow at rates suggested by standard economic models because their saving and investment rates are low. We provide a political economy model of interest groups in which returns to investment are appropriable by other groups. This framework transforms the accumulation problem into a commons problem that may lead to underinvestment equilibria. Our contribution is to focus on second-best subgame perfect equilibria to show that growth rates can indeed be wealth dependent. Poor countries may indeed accumulate at lower rates because even for the best sustainable equilibria, the incentives for appropriation can be much stronger at low levels of wealth than at high ones, and therefore the momentary advantages of defection can be overcome only with high consumption and low investment rates. This, of course, is not the only explanation for why poor countries save little, but it is a plausible one. We also show through examples that second-best equilibria can result in low growth at high rather than low wealth levels, and we refer to such situations as "Olson" cases, after the work of Mancur Olson. With some work it should be possible to come up with examples of second-best equilibria that combine the two polar cases and generate a u-shaped relation between growth rates and the level of wealth.

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Notes

1. On this empirical regularity there is wide agreement, see De Long (1988), Baumol and Wolff (1988, fig. 2), and Easterly (1991).
2. Some of the nonviolent redistributive mechanisms that are used in developing countries include nationalization; bursts of inflationary finance to sustain the incomes of government bureaucracies and the military; the squeezing of the agricultural sectors in favor of politically powerful urban coalitions through exchange rate policies, price controls, and monopolistic marketing boards; legislation and other measures that alter the bargaining power of labor (either positively or negatively); the allocation of highly desirable government and civil service jobs and university admissions to favored ethnic and tribal groups; and large-scale bureaucratic corruption tolerated and condoned by the government.
3. For specific and detailed discussions in particular cases, see O'Donnell (1988) or Olson (1982).
4. As pointed out by a referee, such institutional arrangements may be easier to implement at higher income and wealth levels and account for lower growth rates of poor countries.
5. While we adopted a symmetric specification for the allocation rule, this can easily be modified. To assign asymmetric appropriation power to the players we could have assumed that one of the players can obtain up to, say, three-fourths of the output under fast consumption strategies $(\bar{c}_1, \bar{c}_2)$, and confine the consumption of the opposing player to one-fourth of the output. What sustains fast consumption as an equilibrium is that any attempt to save by a player is defeated because the opposing player then exhausts the residual. Positive savings may be possible if the fast consumption rates of one or both players are bounded—that is, if $f(k) - \bar{c}_j > 0$ where $\bar{c}_j$ is the bound for the consumption rate of the jth player. Conditions under which fast consumption rates are still equilibrium strategies when positive savings rates are possible have been studied (in a continuous time framework) by Benhabib and Radner (1992) for the case of linear utility and by Rustichini (1992) with nonlinear utility.
6. Note that a standard contraction mapping argument to ensure that the continuity of the value function cannot be used. Section 5 provides an example of a discontinuous value function. In general, standard conditions to apply dynamic programming methods and the contraction mapping, which ensure the continuity of the value function, may not hold for our problem. Blackwell's discounting condition $T(v+\alpha) \leq T(v) + \beta\alpha$ can be violated because the constraint set for $v+\alpha$ becomes larger, allowing consumption levels that would be ruled out under v.
7. A sufficient interiority condition for $\hat{c} = \lambda y + \hat{\eta} \leq y/2$ for all $k \geq 0$ is computed to be $\beta a \geq 1$. This condition will be satisfied in all examples below.
8. Note that the utility and production functions are made piecewise linear for computational convenience. They may be approximated with nonlinear functions that smooth the corners to generate an example of switching with nonlinear preferences and technology. It would be interesting to resolve the question of whether given an arbitrary concave production function, it is possible to construct a utility function that generates a switching equilibrium.
9. The k_i's in Figures 2a and 2b are $k_0 = 1, k_1 = 14/15, k_2 = 68/75, k_3 = 1018/1125, k_4 = 4/5, k_5 = 0$. Let the value function be $v_{sb} = a_i k + b_i$ and the consumptions be given by $c_{sb} = m_i k + n_i$. The values a_i, b_i, m_i, η_i hold over the interval $[k_{i+1}, k_i]$, and are given by

$$
\begin{aligned}
a_0 &= \frac{5}{4}, a_1 = \frac{25}{16}, a_2 = \frac{175}{32}, a_3 = \frac{25}{16}, a_4 = \frac{5}{4}; \\
b_0 &= \frac{1}{4}, b_1 = -\frac{1}{24}, b_2 = -\frac{43}{12}, b_3 = -\frac{1}{4}, b_4 = 0; \\
m_0 &= \frac{5}{4}, m_1 = 0; m_2 = -\frac{25}{4}; m_3 = 0; m_4 = \frac{5}{4}; \\
n_0 &= -\frac{1}{2}; n_1 = \frac{2}{3}; n_2 = \frac{19}{3}; n_3 = 1; n_4 = 0.
\end{aligned}
$$

10. Countries like Korea and Chile, among others, provide possible examples of a switch from low to high growth rates in the recent past. In both cases suppression of interest groups under authoritarian regimes may have increased the cost of defection and appropriation, making way for first-best growth. Later, once the original switching threshold was crossed, first-best growth may have become self-sustaining and the authoritarian regimes toppled, as they no longer were necessary to sustain growth. A more recent example, following the path of Chile, may be Peru. Also, for an exposition of the role of group conflict in retarding growth in Argentina prior to the recent acceleration in growth rates, see Mallon and Sourrille (1975) and O'Donnell (1988).

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