Social Network Analysis and Time Varying Graphs

by

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Thesis submitted to the
Faculty of Graduate and Postdoctoral Studies
In partial fulfillment of the requirements
For the Ph.D. degree in
Computer Science

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Abstract

The thesis focuses on the social web and on the analysis of social networks with particular emphasis on their temporal aspects. Social networks are represented here by Time Varying Graphs (TVG), a general model for dynamic graphs borrowed from distributed computing.

In the first part of the thesis we focus on the *temporal aspects* of social networks. We develop various *temporal centrality* measures for TVGs including betweenness, closeness, and eigenvector centralities, which are well known in the context of static graphs. Unfortunately the computational complexity of these temporal centrality metrics are not comparable with their static counterparts. For example, the computation of betweenness becomes intractable in the dynamic setting. For this reason, approximation techniques will also be considered. We apply these temporal measures to two very different datasets, one in the context of knowledge mobilization in a small community of university researchers, the other in the context of Facebook commenting activities among a large number of web users. In both settings, we perform a temporal analysis so to understand the importance of the temporal factors in the dynamics of those networks and to detect nodes that act as "accelerators".

In the second part of the thesis, we focus on a more standard *static* graph representation. We conduct a propagation study on YouTube datasets to understand and compare the *propagation dynamics* of two different types of users: subscribers and friends. Finally, we conclude the thesis with the proposal of a general framework to present, in a comprehensive model, the influence of the social web on e-commerce decision making.

Acknowledgements

First and foremost, my most sincere thanks goes to Prof. Flocchini, who provided me an opportunity to join her team, and provided me with a remarkable scientific and moral support that most students envy. Her ideas, guidance, and recommendations always guided me to the right direction, and her knowledge lightened the dark corners of the route to PhD. As the words come short in expressing my gratitude, there is no doubt that without her precious support it would not be possible to conduct this research. Moreover, I would like to thank Ms. Joanne Gaudet that shared her data, comments, and knowledge with me during the course of this research.

I also thank my fellow lab-mate, and dear friend, Dr. Amir Rahnamai Barghi, who has always been a great support at tough times. He patiently provided me with directions to solve hard mathematical issues that I encountered during the course of this research. I also thank him for the stimulating discussions, for the long days we were working together, and for all the fun we have had in the last two years. Also I thank my friends in the Distributed Computing Lab, especially William Doan, for sharing his ideas with me. I also thank Prof. Morad Benyoucef for his support in the course of this study.

One of my sincerest thanks goes to my family: my parents and to my sisters for supporting me spiritually throughout writing this thesis and throughout my life in general.

Last but not the least, I would like to thank all my friends in Iran and Canada whose company energized me in every step of the way.

In conclusion, I recognize that this research would not have been possible without the financial assistance of MITACS, and IBM Canada Inc., and I express my gratitude to those agencies.

Dedication

I dedicate my dissertation work to my family. A special feeling of gratitude to my loving parents, Abbas and Marziyeh whose words of encouragement and push for tenacity has been the fuel to my enthusiasm for research. And to my sisters Parvaneh and Fatemeh whom I feel never left my side even though being thousands of kilometres away.

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Chapter 1

Introduction

The word social is defined as "liking to be with and talk to people" in Merriam-Webster dictionary. Most people around us are social. In fact, according to Darwin, human is the most developed social animal [32]. Thus, society, and, in general, being social is an inseparable dimension of life. Invention of computers and creation of World Wide Web (WWW), as a new all-purpose tool, threatened the social aspect of life. In 1998, when Internet was booming, Sleek [105] conducted an study that proved the fact that high use of Internet leads to isolation. Certainly, this was not intended as part of this new technology. It has been seen, from early times, that man develops social connection wherever he goes; and Internet was not an exception. Therefore, intentionally, or unintentionally, the social aspect of life gradually entered into the internet world. At the same time, introduction of Web 2.0 and development of Classmates.com[1], which is often cited as the official birth of on-line social networks and social networking, were important media to facilitate importing social aspects to WWW. Thus, social networks started off as a platform to bring the social aspect of life to the Internet world. Nevertheless, they started growing very rapidly and covering activities and applications that has not been thought about; to the extent that on-line social networks are considered as the main rival of traditional Web in terms of applications and usage [107]. The popularity of social networks are such that seven out of ten most visited websites in 2013 were social networking sites[2]. This popularity has driven more and more attention towards on-line social networks.

Social networks have also changed the application of Web. Unlike the traditional Web, which is content-oriented, on-line social networks focused on Web users, and embody them as their first-class entities. On-line social networks have given users the freedom to organize

[1] Classmates.com

[2] According to Alexa.com (http://www.alexa.com/topsites). Accessed: 12/09/2014

the content however they wish. Users can create their own content and links, and join different networks and communities. The linked nature of on-line social networks, gives the ability of sharing knowledge to its users, so the on-line social networks are often denoted as information networks [76].

The extreme popularity of social networks and their wide variety of applications represents a unique opportunity to study, understand, and leverage their potential and properties. Since, on-line social networks offer properties that enhance information and knowledge sharing, one of the mainstream research areas in social network analysis (SNA) was to explore characteristics that leads us to important actors within such networks. Such characteristics have been the target of many studies that revealed interesting facts about centrality measures, information propagation, trust, etc. in social networks.

Evolution of social networks, however, have recently changed the view on social networks. Unlike previous view on social networks that always studied the static snapshots of social networks, the new view sees the social networks as evolving entities that change shapes and structure during their life-cycle. This new view, while attracting popular interest of researchers, is still in infancy, and evaluation of its characteristics is still an open question.

All the above-mentioned characteristics fall under the umbrella of influence study. Influence is fundamental factor of social network analysis that works together with advances in internet technologies, security, and on-line payment mechanisms to highlight the role of the internet as a commercial tool and marketing channel. Emerge of social media tools, and creation of social influence, also impacted business, and boosted e-commerce to a higher level enabling users to actively participate in all stages of e-commerce process. This is clearly reflected in the large amounts of product reviews, news and opinions constantly posted and discussed on sites such as Facebook and Twitter.

This new e-commerce era facilitated by social media is dubbed "social commerce", initially conceptualized by Yahoo Inc. in 2005 as a describing method for a set of on-line collaborative shopping tools such as shared pick lists, user ratings and other user-generated content-sharing of on-line product information and advice. Also, since influence, as part of social commerce, does not happen overnight, and it is a process building up along with other components of social commerce, its analysis from dynamic point of view attracts research communities.

The increase in computational power, increase in the number, availability, and variety of social networks, as well as increase in the social network user base, results in the amount of available information in such networks that can be leveraged for variety of purposes.

This trend shows that the more research is less as new hidden aspects of such networks, whether positively or negatively affecting the society, is being discovered. We hope that our research lightens some dark aspects of combined social network and commerce, and opens new trends for research in this area.

1.1 Motivations and Goals

Social networks have been the centre of attention for the past decade. Many researchers conducted studies to analyse different aspects of social networks including their structure, the formation of communities, the identification of central actors, and so on.

The objectives and the results of this thesis gravitate around three distinct but related topics in the general area of social network analysis.

1. *Temporal Analysis of Social Networks.* Most studies on social network analysis targeted the networks as static entities, and studied their characteristics in a single snapshot encompassing their entire time evolution. Recently, some studies have looked at social networks as systems growing in time [35]. Most of these studies view evolving social networks as a series of snapshots taken from the social graph at different points in time [9]. Even though the study of series of snapshots proved to be helpful in observing how communities change or evolve over time (e.g., [9]), it fails to describe some dynamic features of the network. For example, no measure has been designed and employed to determine how much an actor contributes to fast propagation of information and to understand its temporal centrality in this sense. The same limitation holds for the relationships between actors. Some of the sociometric measures have been adapted to be used in networks that vary over time [101, 110, 112]. However, no algorithm is provided for computing these temporal metrics, consequently, very limited empirical studies have been conducted on time varying social graphs to verify their applicability and correctness. Thus, a large gap exists between the existing social network analysis tools, research and the real nature of social networks.

   One of the goals of this thesis is to target this gap and to propose the study of social networks taking their *temporal dimension* explicitly into account. In fact, the main objective is to describe social networks as temporal entities and to use time as the main parameter in their analysis. Time-varying graphs, where nodes and edges are labeled with their time of existence, are an ideal representation for this purpose; our

intention is to devise network indicators specifically designed for time-varying graphs and to test them in some real scenarios to assess their usefulness.

2. *Propagation in Social Networks.* From the point of view of information propagation, social networks have been under study as the main medium facilitating information propagation. The concept of information propagation is especially interesting for advertisement communities. Such communities have always been interested in increasing the speed and range of information propagation in social networks. Since the introduction of different information propagation models, there have been many studies on the analysis of various models against different real social networks. Those studies have shown that there exists no perfect model that fits all situations, and each social network behaves differently for information dissemination. Thus, the research community's interest has shifted on evaluating each social network setting for its own specific characteristics of information dissemination. To the best of our knowledge, no research has attempted to compare the effects of characteristic and network settings in enabling the spread of information. YouTube provides two of the most common social network settings, namely *followership* and *friendship* and it is a perfect ground to make comparisons between the two settings in the same network.

   A goal of the thesis is to move in this direction studying propagation and its enablers in YouTube with the goal of better understanding the relationship between propagation, friendship and similarity of interests. For example, one fact that we would like to verify is whether it is indeed true, as generally stated, that propagation and friendship occur mostly when actors have similar interests. We also would like to test whether similarity plays a strong role in friendship. Moreover, we aim to explain the effectiveness and efficiency that a friendship network has over followership network (or vice versa) in the same environment. The secondary goal of the propagation analysis is to understand whether similarity of interests is driver of friendship, or propagation, or neither, or both.

3. *Social Commence.* Finally, as mentioned earlier, the diversity of definitions provided for social commerce, a platform that is dependant to social network and social network analysis, is a factor causing a wide range of interpretations of social commerce, and, consequently, assigning different components to this new platform. Therefore, it seems that the design patterns are lost in the design of new social commerce platforms due to the lack of comprehensive framework that comprises design guidelines along with introduction of necessary components for social commerce. This caused a confusion about the nature of social commerce and it is often seen that e-commerce

or mobile commerce is presented as a social commerce solution. Thus, the need for a comprehensive framework is being sensed to fill this vacuity.

With regard to this issue, another goal of this thesis is to introduce a general framework to define *social commerce*, a newly introduced platform whose definition is still not well-defined, and on which there are no guidelines for developing processes.

Note that the three general areas that we consider in the thesis are quite distinct and we treat them using different tools and methods. However, they are deeply interrelated and studies in one can have impact in the other: information propagation, in fact, is a key element in e-commerce, and temporality is a crucial aspect of both.

More specifically, commerce, and e-commerce as one of its sub-categories, has always been interested in increasing revenue and decreasing costs, which can be generally be called as efficiency. Creation of on-line social networks is seen as an opportunity to increase this efficiency. Hence, the e-commerce community incorporated the commercial advantages of social commerce into the concept of e-commerce. The wide use of social commerce for on-line shopping created a demand for an organized and structured definition of social commerce. The most advantage that social commerce provides to on-line shopping is spread of the word of mouth and creation of desire for shopping. Therefore, information propagation aspect of social networks become the boldest feature of social networks for commercial activities. It did not take long before the marketers realize that not everyone makes the same impact on the propagation of information, and the information spreads better through influential users. Therefore, some efforts began to identify the influential actors in social network. These efforts led to realization that nobody stays influential throughout a long time, and the level of influence changes over time, which caused an analysis of social network users over time. In this thesis, we study the full cycle of social commerce, influence propagation, and time effects in influence.

1.2 Contributions

The contributions of the thesis touch the three topics mentioned in the previous Section.

Temporal Analysis of Social Networks.

- *Proposal of Temporal Metrics.* To investigate social networks from a temporal point of view, we propose to represent them as time varying graphs (TVGs). Roughly

speaking, a TVG is a dynamic graph where nodes and edges have a presence function associated to them that specifies when they exist in time. In a TVG, the notion of path changes connotation, in fact, a "temporal" path must not just consists of consecutive edges in the static representation of the TVG (called its footprint), but must also respect the temporal constraints associated to them. Indeed, a TVG with a connected footprint might be disconnected even at every time instant, still having valid temporal paths (or journeys) existing over time. In a TVG, also the notion of shortest paths can be extended in a temporal way to incorporate the concept of "fastest" paths, and the one of paths with earliest arrival time ("foremost" paths). With these temporal extensions, some classical social network parameters based on shortest path (e.g., betweenness, closeness) can be also modified so to account for time in the fastest or foremost way. In the thesis we focus on betweeness and, in particular, on *foremost betweenness*. The classical betweenness measure in static graphs determines the central nodes as indicated by their frequent presence in the shortest paths between the others; foremost betweenness, instead, gives an indication of how frequently a node lies in paths that arrive as early as possible to their destination. We also adapt another classical centrality measure, eigenvector centrality, to the evolving graph setting, where the TVG is divided into a sequence of static graphs that change in time.

- *Computation of Temporal Betweeness.* Unfortunately, the computation of foremost betweenness, being equivalent, in some cases, to the counting of all paths between any two nodes, is a #P complete problem. We design two exponential algorithms to compute it; the first works in any arbitrary TVG, the second is specifically designed for TVGs with a particular temporal structure, which will be treated in the subsequent chapters. Both algorithms have inevitably a high complexity (both in time and space), the advantage of the second over the first is that it can be executed in parallel over portions of the TVGs and thus its computation becomes more manageable. We also design an algorithm to compute an approximate value of temporal betweenness in a TVG, suitable when the social network is too large for an exact calculation to be possible.

- *Eigenvector Centrality.* We also focus on *Eigenvector Centrality*, another classical parameter widely employed in the analysis of social networks, and we design a variant of this parameter suitable to be used in evolving graphs, i.e., sequences of static graphs which change in time. Its computation in this temporal setting requires a mathematical remodelling of the adjacency matrices describing the sequence of

static graphs to ultimately obtain a single matrix representing the whole TVG with the main challenge of preserving the importance and status in time of each edge in the transformation. We propose two adaptations, one incorporating the out-neighbours' degree of each node in time, the other incorporating its own degree.

- *Analysis of Temporal Metrics for Real Data Sets.* To validate the choice of foremost betweenness as a temporal measure, we consider two very different datasets: a small social network describing relations among researchers in a University setting (*KnowledgeNet*), and a very large set of data describing the commenting activities of *Facebook* users.

  KnowledgeNet is a heterogeneous network composed of researchers, publications, laboratories, etc., connected whenever there is knowledge that mobilizes between them. Our Facebook network is composed of users, connected when they write a comment on the same page. Note that while the majority of the pages contains scientific articles, some of them are known to contain hoax information (conspiracy theories); the commenting patterns among legitimate and hoax information is then of particular interest. Both networks have been already studied disregarding any temporal information and representing the relationships as static links. In the thesis, we describe both using the TVG framework and we perform the analysis of foremost betweenness employing our algorithms. We then compare our findings with the results obtained from a static representation of the network. We focus, especially, on betweenness, and we use our algorithms to compute it in both settings: we obtain exact values for KnowledgeNet (which is small enough), while we use just estimates for the Facebook data (which consists of more than 800 thousand nodes).

  Among the observations we can make, we discover that, in both networks, there are actors that were neglected by the static betweenness measure and considered rather marginal, which instead, when observed in a temporal fashion, become quite important because they contribute heavily to the fast relay of information. The reverse observation is also true, some very central element assume much less importance when observed using time as the main metric. In other words, some elements of the network are not often part of shortest path, but they do assume a central role by lying in paths that reach their destination very fast ("accelerator" nodes) and vice-versa.

  In the specific case of the Facebook data, we also notice other general behaviours of users. For example, we identified a very large group of nodes that show importance only in the temporal analysis and, in comparison, a much smaller set of nodes with the opposite behaviour. We also observe that the conspiracy distributors in the

Facebook social network do not gain a huge importance compared to the users who distribute factual information, when analysing the Facebook graph in either static or temporal fashion. Also, one important social observation from Facebook is that users tend to stay in their small community for the first few years of joining the network. It is only after that period that they spread out their activities to other communities.

Propagation in Social Networks.

- *Friends vs. Followers.* To investigate the information propagation in social networks, we propose to evaluate such flow from the point of view friends in the social networks and also from the point of view of followers. While analysing some candidates for the study, we noticed that each social network has special characteristics that may affect information dissemination dramatically, and this eliminates the usefulness of any comparative study done in this regards if it is done on different networks. Among social networks, YouTube provided networks pertaining to friendship and followership in the same environment, enabling a fair comparison between two networks not being concerned about the effects of social networking tool on the result of analysis.

- *Propagation Speed and Range.* We collect ten datasets extracted from *YouTube* using the snowball sampling technique. The collection of datasets started from a random point, pertaining to one user for each dataset collected. Note that to eliminate any inconsistency, for each of the datasets, the starting point for the followership and friendship networks are identical. We, then, measure the speed and the range of propagation in both networks throughout the collected datasets. We observed that the effect of propagation of people who are neither in a friendship network nor in a subscription network is higher than that of friends or subscribers. Meanwhile, we discovered that even though the network of subscribers was denser than the network of friends, the amount of propagation in the subscription network was lower. This might imply that when the relationship is one-way, that is, users are less inclined to contribute to the content.

- *Similarities Among Friends.* In a follow-up study, we measured the relationship between relation and similarities of users involved in the relation; in some cases, this study showed a low correlation. This is important since this is the first time such observation is made in an open social network. We found that the similarity between users increases if they are friends, but this increase does not define similarity as a determining factor in friendship. Considering this, together with the fact that

content propagation in on-line social network is done mostly by non-friends, and knowing that similarity is a driver for content propagation, we can conclude that, within communities, indirect friends are more similar to each other than direct friends (as they participate more in content propagation).

- *Similarity Measures and their Fit in SNA.* Finally, we examined several similarity measures to find the most suitable ones for processing on-line social network data. We found that similarity measures can be categorized into two classes based on their accuracy. We define the accuracy as the amount of friendship ratio over similarity.

Social Commerce.

- *Integration of Social Networks into e-commerce.* Although the integration of social networks into e-commerce is established in most of the on-line commerce applications and tools, there still is not a general, comprehensive, and widely applicable definition for this integration. Every provider of on-line commerce platform defines this integration differently. We provide a comprehensive definition for the integration of social networks and on-line commerce tools.

- *Social networking tools.* Moreover, we explore various social commerce tools with their advantages and projected deficiencies providing a framework that covers the main features of social networking tools that can be used in commercial activities, and defining how these tools should be integrated into on-line commerce for maximum efficiency. Meanwhile, we explain the benefits that using social commerce will bring to the commercial activities in a feature by feature basis.

1.3 Organization of the Thesis

In Chapter 2 we give some background information about on-line social networks reviewing the most common metrics that have been used to analyse their structure. We also review the existing work in the three aspects of social network analysis treated in the thesis: temporality, information propagation, and social commerce.

In Chapter 3 we introduce the notion of time-varying graphs to describe dynamic networks and, in particular, social networks. We also introduce some temporal measures existing in the literature.

In Chapter 4 we discuss the feasibility of implementing some temporal parameters, focusing especially on temporal betweenness noticing that their computability leads to intractable problems. We consider temporal betweenness in general time varying graphs, as well as in some special classes of TVGs that will be relevant in the subsequent Chapters, and we describe exponential algorithms to compute them. Finally, we introduce the notion of temporal Eigenvector Centrality as a generalization of the corresponding static parameter.

In Chapter 5, we focus on an heterogeneous network built over a research community at the University of Ottawa, which we call *Knowledge-Net*. This small network (367 vertices and 719 edges) has been created to study *knowledge mobilization* (see [53, 54]). The network's vertices contain researchers, projects, laboratories, papers, conferences; edges between two vertices represent any form of knowledge mobilizing between the two entities. A study was conducted using classical statistical parameters, to understand how knowledge mobilizes in this environment. The entire study was based on a static representation of a dynamic network and the results did not take the time component into account. In this Chapter we concentrate on this network with the same goal, but employ temporal betweenness so to be able to see the effect of time on the importance of the various actors. In doing so, we identify the elements in the knowledge mobilization community that are important for their temporal role of accelerating the flow of information. Comparing our results with static betweenness measure reveals the presence of "invisible rapids", potential important nodes that are not visibly important in the static analysis (accelerators), and "invisible brooks", elements that act as slow mobilizers, which are considered important in the static analysis. Highlighting these differences, the use of foremost betweenness has proven to be an effective method for measuring knowledge mobilization in a dynamic context. The results of our study is published in:

- Amir Afrasiabi Rad, Paola Flocchini and Joanne Gaudet. "Tempus Fugit: The Impact of Time in Knowledge Mobilization Networks". *1st International Workshop on Dynamics in Networks* (DyNo2015), Workshop of the *2015 IEEE/ACM International Conference on Advances in Social Networks Analysis and Mining* (ASONAM), 2015.

In Chapter 6 we consider *Facebook* data of over 800 thousand users and their commenting activity on 81 pages that have been acquired from Facebook and given to us by a research group in IMT Institute for Advanced Studies [15, 16]. The dataset is particularly interesting as it provides abundant of data on the distribution of legitimate (scientific) and hoax information on Facebook. Bassi et al. [16] have already conducted multiple studies on the dataset from the static point of view of the network. We are, instead, interested in

the analysis of the network from a dynamic point of view and in the observation of the evolution of communities that are formed around the scientific and hoax data. Therefore, to reach our goals, in this Chapter we concentrate on Facebook network employing temporal betweenness and eigenvector centrality measures in order to observe the effect of time on the importance of the various users whether being a science user or a conspiracy distributor. We identify Facebook users who accelerate the flow of information, and become important as the information flows in time. Similar to Chapter 5, we identify these "invisible rapids" and "invisible brooks" by comparing our results with static betweenness measure. By employing the eigenvector analysis and comparing temporal and static results we detect a similar behaviour: nodes that we call "shockers" and "breakers". Shockers denote nodes that are deemed important and influential in time, yet do not appear among static influential nodes. Breakers show the exact opposite characteristics by being statically important and influential, but staying in the unimportant group temporally.

In Chapter 7, we concentrate on data extracted from *YouTube*. By using standard techniques, we analyse rate of propagation of videos among friends and subscribers. We also study the relationship between the popularity of a video and its propagation rate. We, then, conclude by evaluating similarity parameters among users. This study has been performed on ten datasets, each containing the data for around 10,000 users, collected using a snowball sampling method. The analysis is conducted by employing classical statistical metrics, which focus on the static representation of the network without making any temporal assumptions. Our datasets have two separate networks for friendships and followings (subscription), which allow us to analyse both networks in the same settings, and at the same time. The results of this Chapter are published in the following papers:

- Amir Afrasiabi Rad and Benyoucef Morad. "Measuring propagation in online social networks: the case of youtube". Journal of Information Systems Applied Research, (2012). 5(1) pp 26-35.
- Amir Afrasiabi Rad and Benyoucef Morad. "Similarity and Ties in Social Networks: a Study of the YouTube Social Network". Journal of Information Systems Applied Research, (2014). 7(4) pp 14-24.

Finally, in Chapter 8 we introduce social commerce as an emerging platform in software engineering and electronic commerce. Social commerce sparked after the creation of Web 2.0, and, consequently, emerge of social networks and all of their analysis techniques. Therefore, social networks are considered as the backbone and enabler for social commerce.

As social commerce is not yet well-defined, we provide a framework for explaining it, and to understand its ties to social networks, its processes, and its design challenges. Our framework acts as a guideline intended for social commerce platform developers to streamline the features and processes that should be included in their platform. The results of this Chapter are published in the following:

- Amir Afrasiabi Rad and Benyoucef Morad. "A model for understanding social commerce". Journal of Information Systems Applied Research, (2011). 4(2) pp 63-73.

Chapter 2

Background

In this chapter we introduce on-line social networks and we review the most common parameters that have been used to analyse their structure, focusing especially on social influence since it is considered as one of the main motivations for studying social networks and social interactions. We also touch on business motivations for social network analysis at the end of this chapter.

2.1 On-line Social Networks

Social networks, in general, are defined as a social structure containing a set of members and a set of ties between them. The members can be human, animal, and even non-living entities, that have a communication mechanism [116]. On-line social networks are computerized successors of off-line social networks. They were brought to life by the birth of Web 2.0, and can be defined as a system in which users are avatars or representative profiles of their owners (humans or bots), and they may create explicit links to other users or content items. On-line social networks have a huge difference from off-line social network since the on-line versions are easily navigable and processable whereas a huge effort is needed for performing the same operations on the off-line ones [40].

In a comprehensive study of social networks, Boyd and Ellison [40] identify three distinct purposes for the formation of on-line social networks. First, as life becomes more and more hectic, and humans, as well as businesses, need to communicate with disparate geographical locations as part of the global village idea, on-line social networks are useful tools to maintain existing social ties, or make new social connections. Therefore, on-line social networks make it easy for their users to reach their extended networks. Meanwhile, social

networks act as a personal news agency for its members [73]. Being easily navigable, on-line social networks serve as an easy to access medium to find new, interesting content by filtering, recommending, and organizing the content uploaded by users. Later in this thesis, we will see how similarity of on-line social friends makes it easy to access to the content that are interesting to us, and existence of acquaintances help propagating our interests in the on-line social network.

Even though social networks were scientifically defined in 1930s, no mathematical modelling or a formal study was conducted on them until 1950s. It was then that mathematicians represented the social networks, a pure sociological concept at that time, as graphs and started developing theories on their bases. In 1980s, the social network became a mainstream field in mathematics, statistics, psychology, etc. However, it was not until 1990s when social networks were officially introduced to the web by Classmates.com[1] as a by-product of Web 2.0. Classmates.com, although referred as the first on-line social network, did not have full characteristics of social networks as it did not allow direct links between its members, and members only had the choice of forming an affiliation network between the members and the schools they attended. Two years later, SixDegrees.com[2] was created as the first social network allowing the creation of links of between members. On-line social networks, nevertheless, did not gain their popularity until early 2000s, when a number of social networks were created and further developed.

Nowadays, there are multiple social networks, and some of them such as LinkedIn[3], Instagram[4], YouTube[5], etc. are dedicated to a special purposes whereas others such as Facebook[6], MySpace[7], etc. are general purpose social networking sites. It should be noted that all of them, no matter how they are used, contain the basic features of social networks (see Section 2.1.4).

Other than their application, on-line social networks can be categorized into two large classes of open and private networks. The posted content and profiles of members of the open social networks are open to public, or at least to all members of the social network, unless otherwise privatized by the owner. YouTube, and Twitter[8] are examples of open social networks. On the other end of spectrum, exist the private social networks, such

[1] www.classmates.com
[2] www.sixdegrees.com, which is discontinued at present day
[3] www.linkedin.com
[4] www.instagram.com
[5] www.youtube.com
[6] www.facebook.com
[7] www.myspace.com
[8] www.twitter.com

as Facebook, PhotoCircle[9], etc. In these social networks, the default setting preserves complete or at least some privacy for the users, unless otherwise modified by the user, such that the profiles and shared content can only be visible to the friends, and in some cases followers[10].

These categorizations, and also the sociological aspects behind the rapid growth of social networks are still the focus on some areas of social network analysis even though one of the main ideas explaining both concepts are user-centric nature of social networks. Nevertheless, it is not the focus of this thesis, so interested audience are referred to [40, 23].

Before explaining different attributes of social networks, we need to have a short survey on the definition of graph, as the underlying model to study social networks, which will be used extensively in the rest of this thesis.

2.1.1 Graph's Terminology

Graphs are a fundamental construct in complex SNA research, and the use of graph theoretic algorithms and metrics to extract useful information from a social graph is a primary method of analysis in SNA. Formally, a social network is represented as a graph $G = (V, E)$, where $V(G)$, represents the set of vertices, and $E(G)$ refers to the set of edges in the graph (simply V and E when no ambiguity arises) and both consist of a finite number of elements $n = |V|$ and $m = |E|$, respectively ([60]). The edges in the graph between $u \in V$ and $v \in V$ is represented as a pair $(u, v) \in E$.

A graph can be directed or undirected. In a directed graph, an edge e is represented by an ordered pair, and, if an edge (u, v) exists, u is a predecessor of v; we also say that the vertex u dominates node v. A directed graph is called *reflexive* or *digraph* if there is no u such that $(u, u) \in E$. Let $deg(v)$ denote the degree of a vertex v in an undirected graph, let $outd(v)$ and $indeg(v)$ respectively denote the in-degree and the out-degree of vertex v in a directed graph. A digraph is called a *tournament* when there is at least one directed link between any two different vertices. It is also called *transitive* if for any three vertices u, v, z, if both $(u, v), (v, z) \in E$, then $(u, z) \in E$ as well.

Let $d(u, v)$ denote the shortest distance between vertices u and $v \in V$. The eccentricity $\varepsilon(v)$ of $v \in V$ is defined as $max_v\{d(v, u) : u \in V\}$. The *diameter* of G corresponds to the maximum vertex eccentricity $max_v\{\varepsilon(v)\}$.

[9] www.photocircle.com

[10] Friendship represents a mutual tie between users whereas follower-ship is representative of a uni-directional tie between users

A graph is often represented using an adjacency matrix. An adjacency matrix $A(G)$ (simply A when no ambiguity arises) is an $n \times n$ Boolean matrix (with $n = |V(G)|$) where entry $a_{ij} = 1$ if and only if $(i, j) \in E(G)$; it is zero otherwise.

A *walk* is a sequence $v_0, e_1, v_1, ..., v_k$ of vertices v_i and edges e_i such that for any i, edge e_i has endpoints $v_{(i-1)}$ and v_i. A walk that has distinct vertices and edges is called a *path*. In a *cycle* the start and the end points of the path are the same.

Finally, a *hypergraph* is a graph where multiple edges are allowed between pairs of vertices.

Social networks are commonly represented by graphs. In the case of a social network, the set of vertices V of its corresponding graph often represents individuals and set of edges E may represent relationships, friendships, or sometimes communications among them.

2.1.2 Structure of Social Networks

Over the time, the mathematicians, physicists, and computer and social scientists tried to formulate and model the structure of social networks. This evaluation has become easier due to the availability of extensive data on social networks. The structural properties of social networks are determining factors in influence maximization, social network categorization, actor ranking models, and so on. Basically, other then techniques that focus on text mining and Natural Language Processing (NLP), almost all other SNA models are based on the structure of social networks.

As a general classification, we can classify social networks in two big categories: *homogeneous* and *heterogeneous*. Social networks are *homogeneous*, when vertices and edges are all of the same type, or *heterogeneous*, when there exist more than one type of node or edge in the graph ([81]). If we restrict the heterogeneous networks in a way that vertices of the same type cannot have an edge between themselves, we have an *affiliation network* [76]. Affiliation networks, however, can be easily converted to simple networks (homogeneous networks) at the price of information loss. Moreover, social networks might be represented by *hypergraphs* in which hyperedges connect more than two vertices [14]. All the above-mentioned network types can have directed or undirected edges.

In the rest of this section we discuss about structural characteristics of such networks, focusing more on homogeneous social networks. We will see that the Facebook, YouTube, and Knowledge-net networks studied in this thesis, while some being heterogeneous, and other homogeneous, all fall in the categories of small world and scale-free networks.

Random Graphs. Random networks have been heavily studied in the past few decades. Erdos and Reyni [42] were the first researchers who conceptualized the model for random graphs. Their interpretation of random graphs includes a set of edges between pairs of node with equal independent probability. The model is represented by $G(n, p)$, where p refers to the probability that an edge is included in the graph. Thus, all graphs with n nodes and m edges have equal probability $p^m(1-p)^{\binom{n}{2}-m}$.

The parameter p in this model can be thought of as a weighting function. Therefore, when p increases from 0 to 1, the likelihood that the resulting graph become a dense graph increases. Similar to most probabilistic models, the behaviour of random graphs are studied for the cases where n tends to infinity. Random graphs might not have real examples among on-line social networks, but they show very interesting characteristics in aforementioned cases, and are basis for study of some social networks.

Small-world networks. In 1960s, Milgram [82] carried out a famous experiment involving passing letters from one person to another in order to deliver each letter to its designated destination. The experiment showed that the delivery is possible in very small average hops of only six. This result was called *the small world effect* meaning that the vertices of the graph are connected to each other by a very short path. Specifically, a small-world network is a network where the distance d between two randomly chosen vertices grows proportionally to the logarithm of the number of nodes n in the network ($d \propto \log n$) [117]. Others define the same value d proportional to both distance and number of vertices in the graph. For instance, Newmann [87] defines d as $d^{-1} = \frac{1}{\frac{1}{2}n(n+1)} \sum_{i \leq j} {d_{ij}}^{-1}$, where d_{ij} is the shortest distance between i and j. Nevertheless, the logarithmic definition is much more popular and mathematically provable. Figure 2.1 presents the random graph along with small-worlds.

The small-world effect implies that the spread of information in the network is fast. The effect is also tested on real networks and it is proven that some social networks display small world characteristics [6, 85, 86]. Bollabas and Riordan [20] later showed that some social networks posses characteristics resembling to the power law distribution, which led to creation of new class of social network structural models.

Scale-free Networks. in 1965, studies on the academic network of citations showed that the citations that papers receive have a long tailed distribution following Pareto or power law distribution [122]. This was the starting point for mathematically modelling such networks. Barabasi [10] followed the previous studies and mapped them on the study

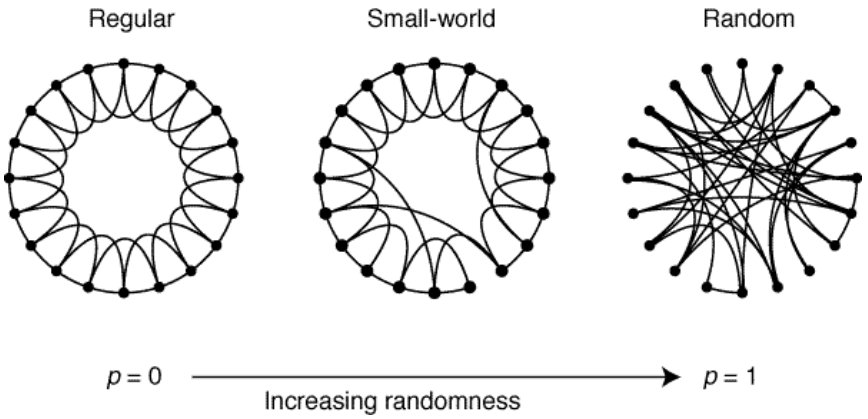


Figure 2.1: Random Graphs Vs. Small-Worlds [117]

of World Wide Web. He discovered that World Wide Web shows indications of power law distribution. He and his colleagues called such networks, which exhibit a power-law degree distribution, Scale-free Networks.

To explain the scale-free behaviour, Barabasi and Albert proposed a mechanism, called preferential attachment, that explained creation process of scale-free networks [10]. Figure 2.2 represents the process of preferential attachment, where a new vertex links to other vertices proportional to their degree. Later, it is discovered that preferential attachment can only explain a subset of real-life scale-free networks [36]. Li et al. [77], recently, offered a more precise model for scale-free networks called scale-free metric. Briefly, let G be a graph with edges E, and the degree of a vertex v by $\deg(v)$. The scale-free metric $SF(G)$ is defined as a value that is calculated directly from the joint degree distribution of the graph. Therefore,

$$SF(G) = \frac{\sum_{(u,v)\in E} \deg(u) \cdot \deg(v)}{\sum_{(u,v)\in E} \deg(u) \cdot \deg_{max}(v)} \tag{2.1}$$

where the denominator is the maximum value in the set of all graphs with degree distribution identical to G. Scale-free measure is always between 0 and 1. If the graph is set in a way that the high degree vertices tend to connect to other high degree vertices, the value tends to be closer to 1, and when the high degree vertices are connected to low degree vertices, the value becomes closer to 0. Therefore, the scale-free networks are often described as self-similar.

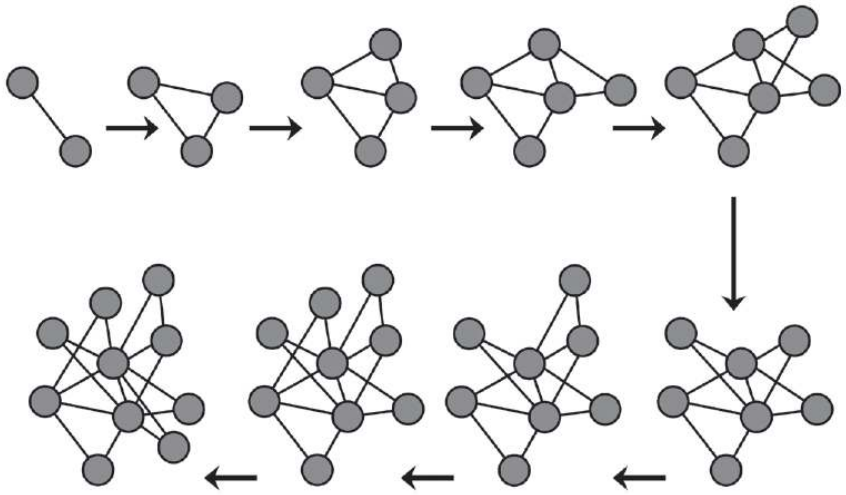


Figure 2.2: The Emergence of a Scale-Free Network as a Result of the Preferential Attachment [21]

2.1.3 Social Network and Communities

Although we do not technically analyse communities in this thesis, this concept is so important topic in SNA that needs introduction. In fact, detecting the communities is one of the main areas of interest in structural SNA. Communities are formed from social network users who are tightly linked together. There is ongoing research on community detection on social networks. Fortunato [47] surveyed almost all important algorithms that lead to detecting communities in the graphs. We, specifically, develop over community detection models based on betweenness and modularity [87]. We will detail such models in the next chapters.

2.1.4 Characteristics of Social Networks

Now that we have defined the most renowned social-network structural models, we provide a brief survey of some important characteristics of social networks.

Diameter

Diameter in graphs is defined as the longest shortest path in the graph. In Social Networks, however, researchers often refer to the diameter with different meanings. For instance, Esley and Kleinberg [38] define the diameter as the average of shortest paths (we call this average diameter) in the graph as opposed to the normal graph-theoretic definition that defines the diameter as the longest shortest path.

As discussed in 2.1.2, social networks, characterized by small-worlds, have small diameters. However, first, as initial models of social networks revolved on the random graph structure, the probability to have social networks with small diameter was very low, but reality proved otherwise. Secondly, not all social networks are small-worlds. The reason why it is said that social networks have small diameters is that a wide range of social networks have small average shortest path length, and consequently are referred to small worlds; World Wide Web being one of the most common examples which is considered a small world due to its small average shortest path (in case of disconnected networks, we have several small worlds).

Why knowing the social network diameter is important? As stated, the diameter is representative of how close (from the geodesic distance point of view) the actors are in the network. Thus, the diameter can be used as a measure of global density of the network, when the definition by Esley and Kleinberg [38] is used.

Navigability

As discussed in 2.1.2, Milgram's experiment showed that most real life social networks are in fact navigable small-worlds meaning that not only do exist short paths connecting most pairs of people, but also each vertex can build (short) paths to any other vertex just by using only local and some structural global knowledge. This characteristics is completely defined, and observable in the small-world model developed by Watts et. al. [118]. Their model is based upon multiple hierarchies defined based on the properties of the vertices and the network structure. The model also incorporates a greedy algorithm that attempts to get closer to the target in various dimensions at every step. Unfortunately, no attempt has yet been made to investigate this model theoretically while many empirical analysis has been conducted on it.

Although we do not directly investigate this model, we will see how navigability affects influence propagation while analysing YouTube social network.

Giant Components

Although we do not refer to components in this thesis, introduction of communities is not complete without introducing connected components in the graph. Components are composed of a set of connected vertices that are disconnected from the rest of the graph. These are different from communities in a sense that vertices in different communities can still be connected to each other, but more densely connected inside the community.

Giant components are defined as connected components containing a large number of vertices, often more than half of the vertices in the graph. Small-world effect implies that that social networks must have a large connected component containing most of the vertices. Giant components are studied in random graphs and small-worlds in different disciplines, mathematics, computer science and physics.

Mixing

Knowing the degree distribution of social networks gives abundant information to understand the network. However, degree distribution is only a local property, and does not guide us to identifying the global structure of the network in a precise manner. Therefore, it is very important to know if the high degree vertices are linked with other high degree vertices, similar to what exists in scale-free networks (see 2.1.2), or they are linked with low degree vertices or any other patterns. These link patterns are called mixing in social networks. The mixing in social networks is studied in various attempts, and the result of the studies exhibited a positive co-relation between the degree of node v and its neighbours [72, 89, 90]. This mixing pattern that is common in social networks, is called assortative mixing.

2.2 Social Influence

One of the important applications of social networks is information dissemination, and this is not possible without social influence. Thus, social influence is an important strategy that is embedded in the concept of social network. Merriam-Webster dictionary defines influence as "the act or power of producing an effect without apparent exertion of force or direct exercise of command". In scholarly articles, social influence is defined as the phenomenon where the actions of a user can induce his/her friends to behave in a similar way [98]. In social networks, influence is created by passing an idea to a networked friend. Social network users pass their ideas by creating new content or reusing pre-generated content (i.e., reposting other people's ideas or quoting other people in interactions). It is apparent that social influence is the result of content that is generated by social network users. In fact, more generated content can trigger more (positive or negative) influence.

Various social factors participate in the influence, and influence occurs for a wide variety of reasons. Flanagin and Metzger [46], for instance, provided the most comprehensive model for user participation in social activities by surveying 684 people from different

demographics. Their survey revealed twelve factors that actively affect how and why people participate in social activities (Table 2.1 shows the top seven of those factors). The survey shows that, in addition to entertainment related reasons, most people engage in social activities to produce ideas and gain or distribute information.

Table 2.1: Top seven reasons for social participation

Top Reasons Ranked by the General Public	Top Reasons Ranked by On-line Users
1. To get information	1. To stay in touch
2. To be entertained	2. To provide others with information
3. To pass the time away when bored	3. To get information
4. To relax	4. To get to know others
5. To generate ideas	5. To be entertained
6. To learn how to do things	6. To have something to do with others
7. To learn about myself and others	7. To pass the time away when bored

In a different study, Dholakia et al. [33], categorized social participation factors into five major categories, namely: Purposive Value, Self-Discovery, Maintaining Interpersonal Interconnectivity, Social Enhancement, and Entertainment Value. All categories have a direct relation with influence in social networks and virtual communities. We summarize all factors affecting influence in Table 2.2.

Although many different models are developed for modelling influence propagation is social networks, most of the aforementioned factors are sometimes ignored in those models, and, specially, in identification and ranking of influential actors in social networks. The challenge causing this issue pertains to difficulties related to collection of such data. Therefore the analysis of influence ranking is usually restricted to graph-theoretical methods called sociometric measures of social networks.

2.2.1 Sociometric Techniques for Ranking

Centrality measures are designed as indicators that identify and rank the most important vertices and edges within a graph. The applications of the centrality measures are very diverse ranging from identifying the influential people in social networks to predicting patterns of disease contamination and propagation, to dividing the graphs into sub-graphs also known as communities. However, it should be noted that centrality indices have three important limitations. First, their application is domain dependant. Therefore, a

Table 2.2: Factors affecting influence

Factor	Description
Connections	It is generally perceived that a higher number of connections is indicative of higher popularity, and popular people are more influential than others [113]. Moreover, when you have more connections, more people hear your voice, and your ideas may be distributed faster and wider. On the other hand, you will also hear more voices; hence your ideas might become blended with other people's ideas.
Networking Purpose	The networking purpose has a significant effect on the choice of content to consume and generate. To identify the networking purpose, the content of communications must be evaluated. To do so, Weng et al. [120], Romero et al. [98], and Huberman et al. [61] introduced the notion of topic-based influence.
Demographics	By evaluating the demographics of social network users, we can determine who shares similar interests with whom [74].
Group Membership	Group membership provides a fast way to identify the interests of users, as users with similar interests in an issue tend to connect together in a group.

measure that applies to a domain, and provides good results is not necessarily useful for other domains as well. Meanwhile, the values for the centrality measures are just relevant to the structure of the graph, and undermine the internal characteristics of the vertices and edges. The centrality values are significantly different for high ranked vertices and edges, but show very little variation in the rest of the graph. Therefore, the ranking of the vertices and edges are not very useful in such cases, except the situations where the goal is to divide the graph into communities. In community detection algorithms that work based on centrality measures all values for centrality are important. In this thesis, we focus on betweenness and eigenvector centrality values, yet we believe that general understanding of centrality measures helps the reader to understand the motivation and results of the chapters along their methodologies and content. Hence, we provide a summery of most renowned centrality measures in Table 2.3, and discuss them in this chapter. However, centrality measures are not limited to the measures discussed here.

Geometric Measures

Geometric measures are those measures in which the importance is a function of distances; more precisely, a geometric centrality depends only on how many nodes exist at every

Table 2.3: Sociometric Techniques for SNA

Metric	Description
Degree	refers to the number of edges that connect the node to other nodes in the network
Closeness	Closeness centrality is defined over the connected graphs, and it is denoted as the average distance of a vertex from all other vertices in the graph
Eigenvector	eigenvector centrality measures the prestige of a node and translates into the phrase: a node is important if it is linked to by other important nodes. Hence, the eigenvector centrality is more meaningful than degree centrality, so that a node receiving many links does not necessarily have a high eigenvector centrality (it might be that all linkers have low or null eigenvector centrality)
Katz	It is very similar to eigenvector centrality with heuristic components
PageRank	PageRank estimates the importance of a vertex by counting the number and quality of links to it.
HITS	very similar to PageRank. It is an iterative algorithm that computes hub score and authority scores both at the same time. A page is authoritative if it is pointed by many good hubs pages which contain good list of authoritative pages , and a hub is good if it points to authoritative pages
SALSA	an extension to HITS in order to assign high scores to hub and authority vertices based on the quantity of edges among them
Betweenness	measures the importance of a vertex based on how often the vertex happens to be located between two or more communities
Clustering Coefficient	Clustering coefficient measures the degree to which graph vertices tend to form a cluster together
Modularity	measure the strength of division of a graph into modules, also known as communities

distance.

— **Degree centralities.** Degree centrality of a node v is the simplest and historically the first centrality measures that has been used in SNA. It is usually defined as the degree $deg(v)$ of node v, divided by $n-1$ to restrict the metric in the $[0,1]$ range (figure 2.3). Degree centrality can be interpreted as the risk of being infected in the graph or the opportunity of infecting others. For that reason, it is one of the most-used measures in the studies about spread of disease in communities (e.g. [22]).

Analogously, one can define the in-degree centrality $indeg(v)$ and the out-degree cen-

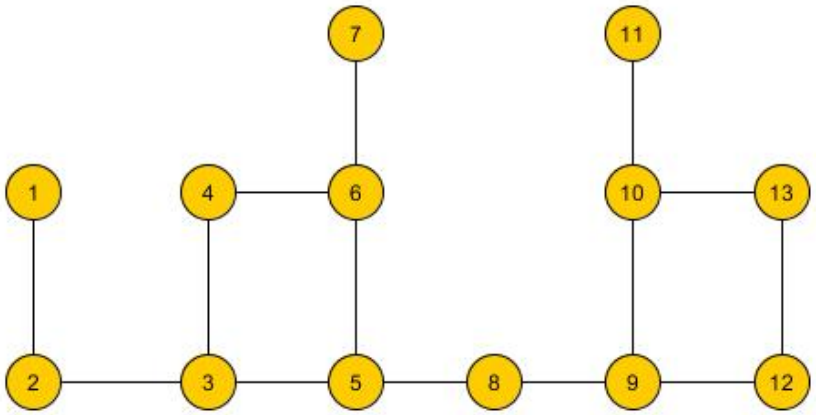


Figure 2.3: Degree centrality of a graph: Vertices 1, 7, 11 have degree 1/12, vertices 3, 5, 6, 9, 10 have degree 0.25, and the rest of vertices have degre 1/6

trality $outdeg(v)$ of a node v in a directed graph. A high in-degree can indicate a tendency towards being a content consumer, or it means a high risk of being infected. If a user with a high in-degree has a low or zero out-degree value it might be an indication of inactivity. A higher number of received messages might also indicate that there are opportunities for the user to be influenced by his/her friends. A vertex with high out-degree has more opportunities to influence others by its behaviour because it has more ways to transfer its characteristics. The combination of in-degree and out-degree centrality measures helps in detecting spammers and inactive users [4]. A user with zero (or close to zero) in-degree and high out-degree might be a spammer.

— **Closeness centrality.** An important node is typically close to others in the network, and can communicate quickly with them. The basis of the closeness centrality is quick communication. Closeness centrality is defined over the connected graphs (its modified version also works for disconnected graphs), and it is the average distance of a vertex from all other vertices in the graph [100]. The metric is usually reversed in order to restrict the closeness value in $[0, 1]$ (figure 2.4):

$$C_C(v) = \sum_{u \in V} \frac{n-1}{d(v,u)} \tag{2.2}$$

Closeness centrality represents the amount of information that can be distributed by a vertex in the graph if it is accepted that information travels through the shortest path between the vertices knowing that the distance between a vertex and itself is zero. Thus, closeness centrality is very popular in analysis that involve travel of information, goods,

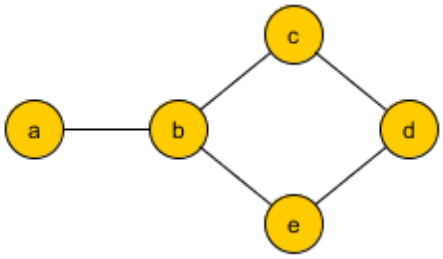


Figure 2.4: Closeness centrality: vertex b has the highest closeness equal to 0.8, and the closeness values are 0.5 for a, and 0.57 for the rest of vertices

vehicles, etc. Conti et al. [28], for instance analyse the disruptions in US flight networks employing closeness centrality extensively.

However, in most cases, the distance that information travel between two vertices in the graph is not necessarily through the shortest path between them. Thus, Newman [91] provided an algorithm that calculates the closeness of a vertex in the graph based on a series of random walks starting from that vertex. This measure of closeness is more realistic to social networks involving people.

In an attempt to extend the applicability of closeness centrality to disconnected graphs, Dangalchev [31] defined closeness as the inverse of 2 to the power of distance:

$$C_{C_D}(v) = \sum_{u \in V} 2^{-d(v,u)} \tag{2.3}$$

The closeness centrality is extended to disconnected directed graphs by Boldi and Vigna [19] as:

$$C_{C_B}(v) = \sum_{u \neq v} d(v,u)^{-1} \tag{2.4}$$

where $1/\infty = 0$.

Prestige Measures

Prestige measures of prominence take into account the differences between the neighbourhood set, considering that their rank would affect the rank of the node. Prestige measures better apply to directed graphs, but are still useful for undirected graphs. We specifically focus on eigenvector centrality in this thesis, but we are compelled that introduction of the

measures that are very close to eigenvector centrality develops better understanding of this metric. Thus, we briefly introduce the background and application a few of eigenvector centrality's siblings.

— **Eigenvector centrality.** Degree centrality awards the same centrality score for every link a vertex receives, but not all vertices are equivalent. Some vertices are more relevant or important than others. For instance, while vertices 2 and 8 in figure 2.3 have the same degree centrality, it is clear that they should be valued differently. Vertex 8 should be considered more important than vertex 2, since vertex 8 is connected to vertices 5 and 9 that are more important than what 2 is connected to (1 and 3). Therefore, being connected to an important vertex, logically, affects your importance more than being connected to a non-popular vertex. Eigenvector centrality measures the prestige of a node giving more importance to nodes that are linked to other important nodes. Hence, the eigenvector centrality is more meaningful than degree centrality, so that a node receiving many links does not necessarily have a high eigenvector centrality (it might be that all linkers have low or null eigenvector centrality). Moreover, a node with high eigenvector centrality is not necessarily highly linked (the node might have few but important linkers) [103]. Eigenvector centrality is tied with popularity. Thus, it is very applicable in the studies analysing and designing campaigns, whether, political [49], advertisement [104], or any other form of campaign.

Eigenvector centrality is computed by:

$$C_E(v) = \lambda^{-1} \sum_u a_{v,u} C_E(u) \tag{2.5}$$

where λ is a constant. However, the recursive dependency of value makes it impossible to calculate the eigenvector centralities using Equation 2.5 in a recursive manner. The reason for this is that the base case for the recursive function does not exist. Hence, eigenvector centrality is redefined as the eigenvector of the adjacency matrix $\lambda \mathbf{x} = A\mathbf{x}$. Therefore, we see that $\mathbf{x}$ is an eigenvector of the adjacency matrix with eigenvalue λ. To make sure that the centralities are non-negative,it can be shown (using the PerronFrobenius theorem) that λ must be the largest eigenvalue of the adjacency matrix A, and $\mathbf{x}$ the corresponding eigenvector ($C_E(v) = x_v$).

Referring back to Figure 2.3, the adjacency matrix for the graph presented in the figure is as following, which shows the eigenvector centralities for vertices 2 and 8 are 0.19 and 0.31 respectively, while 5 has the highest eigenvector centrality of 0.44. This shows the purpose

of eigenvector centrality that differentiates the nodes based on who they are connected to. We discuss about this measure more in this thesis.

$$\mathbf{x} = \begin{pmatrix} 0.08 & 0.19 & 0.39 & 0.32 & 0.44 & 0.37 & 0.15 & 0.31 & 0.31 & 0.24 & 0.10 & 0.20 & 0.18 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \end{pmatrix}$$

— **Katz's index.** Katz centrality was introduced by Leo Katz and is used to measure the degree of influence of an actor in a social network [65]. Katz centrality is normally used in directed networks where measures like eigenvector centrality are rendered useless. Unlike typical centrality measures which consider only the shortest path between a pair of vertices, Katz centrality takes into account the total number of walks between a pair of vertices. Hence, Katz centrality puts a step forward and includes more nodes than the direct neighbours, like eigenvector centrality while penalizing the distant connections by a attenuation factor α in $(0, 1)$. Thus,

$$C_K(v) = \sum_{k=0}^{\infty} \sum_{u} \alpha^k (A^k)_{vu} \tag{2.6}$$

Katz' measure might be used interchangeably with eigenvector centrality, but in directed graphs, it yields extreme usefulness. Due to the fact that this measure can be applied on directed graphs, it is mostly used for direct influence models, such as behavioural modelling of networks, and analysis of influence sources in the network. An example of such studies is Mizruchi's study of behavioural cohesion and similarity in networks [84].

— **PageRank Centrality.** PageRank is one of the most discussed and quoted prestige indices in use today, mainly because of its alleged use in Google's ranking algorithm. PageRank estimates the importance of a vertex by counting the number and quality of links to it. It is generally assumed that the more popular the vertex is, the more links it receives from other vertices [93]. By definition, PageRank of set of web pages is an assignment C_R satisfying:

$$C_R(v) = \beta \sum_{u \in P(v)} \frac{C_R(u)}{outdeg(u)} + \beta \mathbf{r}(v) \tag{2.7}$$

such that β is maximized and L_1-norm of R is equal to one. In the aforementioned equation, $\mathbf{r}$ is some vector over the web pages that corresponds to a source of rank, and $P(v)$ is the set of web pages that v points to.

— **HITS.** Hyperlink-Induced Topic Search (HITS; also known as hubs and authorities) is a graph analysis algorithm, developed by Kleinberg [68]. The idea behind HITS is very similar to that of PageRank's, and is that a page is authoritative if it is pointed by many good hubs - pages which contain good list of authoritative pages -, and a hub is good if it points to authoritative pages. Therefore, HITS algorithm is an iterative algorithm that computes hub score $h(v)$ and authority scores $a(v)$ both at the same time.

$$\begin{aligned} h(i+1) &= a(i)A^T \\ a(i+1) &= h(i+1)A \end{aligned} \tag{2.8}$$

If done for infinite iterations, this process converges to the left dominant eigenvector of the matrix A^TA. This value refers the authority score of vertices of the graph, the hub values can be easily computed based on authority scores (the left dominant eigenvector of AA^T). Therefore, it is obvious that both vectors are left and right singular vectors associated with the dominant singular value in the singular-value decomposition of A [43].

— **SALSA.** Stochastic Approach for Link-Structure Analysis (SALSA) is a graph measure designed by Lempel and Moran [75] as an extension to HITS in order to assign high scores to hub and authority vertices based on the quantity of edges among them. SALSA extends HITS by applying it on L_1-normalized adjacency matrix of A. Therefore,

$$\begin{aligned} \bar{h}(i+1) &= \bar{a}(i)\,\bar{A}^T \\ \bar{a}(i+1) &= \bar{h}(i+1)\,\bar{A} \end{aligned} \tag{2.9}$$

This normalization causes simplification in the process of computation of SALSA, as it does not need the iterative process required for HITS. In this process, we initially compute the connected components of the symmetric graph induced by the matrix A^TA. In this graph, based on the matrix A^TA, two vertices are connected if they had common predecessors in the original graphs. Then, the SALSA scores a vertex by computing the ratio between the size of the component that the vertex belong to it and $|V|$ multiplied by the

ratio between vertex's in-degree and the sum of the in-degrees of all vertices in the same component. Therefore, as opposed to HITS, which is an iterative algorithm, SALSA only needs in-degrees, so only one iteration on the graph would suffice for its computation.

Path-based Measures

Path-based measures work utilizing the graph feature called shortest path(s) that pass through a vertex in the graph. These measures mainly count the paths and manipulate the vertex scores based on the results of counting. A great portion of this thesis is focused on the measures discussed under this category.

— **Betweenness.** Betweenness centrality was first introduced by Freeman [50]. Betweenness measures the importance of a vertex based on how often the vertex happens to be located between two or more communities. The measurement method, however, only counts the number of shortest paths that cross the vertex. If we represent the number of shortest paths that flow between vertices x and y by σ_{xy}, and the number of those paths that cross v by $\sigma_{xy}(v)$, we can define the betweenness of v by:

$$C_B(v) = \sum_{\substack{u,w \neq v, \\ \sigma_{uw} \neq 0}} \frac{\sigma_{uw}(v)}{\sigma_{uw}} \tag{2.10}$$

The intuition behind betweenness is that if a large fraction of shortest paths passes through v, then v is an important vertex that works as a connection point of communities in the graph. Therefore, it is clear that eliminating such vertex from the graph would disrupt the communication between different parts of the graph, and create separate communities. Therefore, betweenness centrality concerns about the information flow, rather than structural positioning of the vertex in the graph, the feature that is the basis for prestige measures. Figure 2.5 expands on this difference in an example.

A few years later, Freeman et al. [51] suggested that not all communications between graph entities go through shortest paths, and in fact messages may choose any path for transmission. In fact, if we take the graph as the set of rivers starting at s and ending at t, each river can only carry a maximum flow of water without flooding. Hence, they developed the flow betweenness concept based on the idea of the maximum flow. The flow betweenness of the vertex v is defined as the maximum flow that can be carried out from s to t passing through v, averaged over all s and t in the graph. Hence, the if g_{st} is the

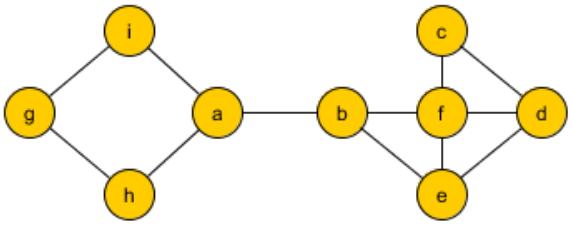


Figure 2.5: Betweenness centrality: vertex b has the highest betweenness as it falls into the path for most interactions between other vertices. However, its structural measures such as degree (3) and eigenvector (0.77) are not very high. The highest degree and eigenvector values correspond to vertex f.

number of paths linking points s and t in a graph, and $g_{st}(v)$ is the number of such paths that contain point v, then:

$$C_B(v) = \sum_{s<t}^{n} \sum_{s<t}^{n} \frac{g_{st}(v)}{g_{st}} \tag{2.11}$$

Newman [91], nevertheless, suggested that not all communications between graph entities go through all paths, and in fact communications choose a random path for transmission. Considering that, and the conditional probability characteristics, the probability of choosing a shorter paths is higher than a longer paths. Newmann denoted these shorter paths as geodesics, a term that we use to refer to this kind of paths in his thesis. Hence, he argued that flow betweenness, like shortest path betweenness, can give counter-intuitive results, and proposed a random walk version of the metric. Random walk betweenness is calculated based on the concept of current flow analogy. Newmann's algorithm provides a more intuitive model for information flow, thus betweenness. Considering the networks in Figure 2.6, the drawbacks of flow betweenness, and the random walk treatment that fixes it are depicted.

Community-based measures

Community-based measures either use communities in the calculation, or are inspired by formation of communities. These metrics heavily depend on the structure of the graph, and make no assumption on the information flow in the graph or geometric characteristics of the graph. Out of these two measures, modularity is used in the computations for Chapter 5.

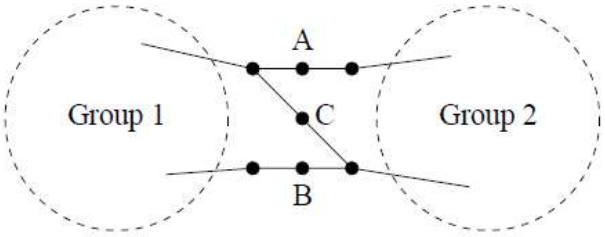


Figure 2.6: Flow betweenness, while claiming that it passes through all the paths, it gives low betweennss value for vertex c. Random walk betweenness solves this issue, by considering a probability for each vertex corresponding to the paths that pass through it. [91]

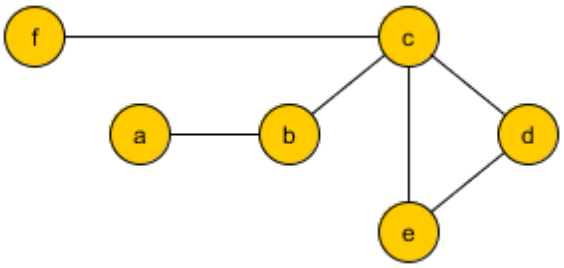


Figure 2.7: Global clustering coefficient is 0.33 as a third of the triplets are closed ($\{c, d, e\}$ is the only closed triplet). From a local point of view, the highest clustering coefficient belongs to d and e, while, for instance, c has the value 1/12.

— **Clustering coefficient.** Clustering coefficient measures the degree to which graph vertices tend to form a cluster together. Evidently, in social network graphs, vertices tend to create tightly knit clusters or cliques. Interestingly, this likelihood is even more than the chance of creation of an edge between two vertices [117]. Clustering coefficient can be measured locally and globally. The global view gives an overall indication of the clustering in the network, whereas local view only gives the results on how embedded a vertex is in the graph.

The clustering coefficient is based on counting the number of triplets (sometimes called strong ties) that are formed in the graph. A triplet consists of three vertices that are linked together, and a closed triplet is denoted to a subgraph of three vertices that form a complete graph K_3 [79] (Figure 2.7). The global view is computed by:

$$C_{Cl_g} = \frac{number\,of\,closed\,triplets}{number\,of\,connected\,triplets\,of\,vertices} \tag{2.12}$$

The local clustering coefficient, however, focuses on one vertex in the graph, and quantifies how close the vertex's neighbours are to forming a clique. The local clustering coef-

ficient for a vertex is computed by taking the ratio of edges between the vertices that fall into the group of its neighbours divided by the number of edges that they could have if the graph was a complete graph. Thus, if we denote neighbours of v (i.e. $|N(v)|$) by n_v, we will have the following equation for computing the local clustering coefficient for the vertex v:

$$C_{Cl_l}(v) = \frac{|\{(u,w) : u,w \in N(v), (u,w) \in E\}|}{n_v(n_v - 1)} \tag{2.13}$$

In case of undirected graphs, the number of edges in the complete graph of neighbours would be divided by 2 resulting the whole equation be multiplied by 2. Watts and Strogatz [117] developed a new interpretation of global clustering coefficient based on local clustering coefficients to explain small-world phenomenon. Their model weights low degree vertices more than high degree vertices:

$$C_{\bar{C}l} = \frac{1}{n}\sum_{v=1}^{n} C_{Cl_l}(v) \tag{2.14}$$

— **Modularity.** Modularity measures how modular the structure of the graph is. It measures the strength of division of a graph into modules, also known as communities. Modularity is mainly used as a quality metric for identifying how good the graph is divided into different communities. High values of modularity shows dense communities and sparse intra community edges, whereas low modularity is representative of non-significant separation between communities. The intuition behind this measure is that vertices inside a community tend to have lots of edges between other vertices in the same community and lower number of edges with other vertices that do not belong to the same community. Thus, modularity is defined as the fraction of edges that fall within a group minus the expected number of edges within that for a random graph with the same degree distribution (some even consider complete graphs) [88]. Thus, supposing that s_v and s_w represent the communities that v and w belong to, and $s_v s_w = 1$ if v and w belong to the same community and -1 otherwise, the modularity for two communities is defined as:

$$Q = \frac{1}{2m}\sum_{vu}\left[A_{vu} - \frac{deg(v)\,deg(u)}{2m}\right]\frac{s_v s_u + 1}{2} \tag{2.15}$$

Modularity is later generalized to cover multiple communities [27].

2.3 Conclusion

In this chapter, we introduced the essential concepts discussed in the thesis, while providing definitions for various terminologies used throughout the thesis. The chapter mainly focused on the static graphs. In the next chapter, we shift our focus to dynamic networks.

Chapter 3

Time-Varying Graphs and Temporal Metrics

In this Chapter we introduce the notion of time-varying graph, a formal model that has been recently proposed to describe dynamic networks encompassing different contexts into a unique framework (e.g., vehicular, ad-hoc, satellite networks, social networks, robotic, military networks, etc.). We also describe several temporal measures, whose corresponding "static" version has been employed to analyse social networks.

3.1 Time Varying Graphs

Similarly to the static graphs that are defined in 2.1, TVGs are also formed from vertices V and edges E. Nevertheless, since they address dynamical systems, the relations between the edges take place over a time span $\mathcal{T} \subseteq \mathbb{T}$ called lifetime of the system. $\mathbb{T}$ is the temporal domain of the system and is equivalent to $\mathbb{N}$ for discrete-time systems and $\mathbb{R}^+$ for continues-time systems. The existence of edges should also be defined in a TVG. The presence function, $\rho : E \times \mathcal{T} \to \{0, 1\}$, indicates whether an edge exists at a given time $t \in \mathcal{T}$. Meanwhile, since the traversal time on every edge might be different from other edges, the latency function, $\zeta : E \times \mathcal{T} \to \mathbb{T}$, depicts the time that takes to traverse the edge from its source to its target at a given time. Therefore, TVGs are described by $\mathcal{G} = (V, E, \mathcal{T}, \rho, \zeta)$. The model may, of course, be extended by defining the vertex presence function ($\psi : V \times \mathcal{T} \to \{0, 1\}$), and vertex latency function ($\phi : V \times \mathcal{T} \to \{0, 1\}$).

Obviously, the definition of TVG imposes no restrictions on the edges or nodes. In particular, if the presence function is always equal to one and latency is equal to zero,

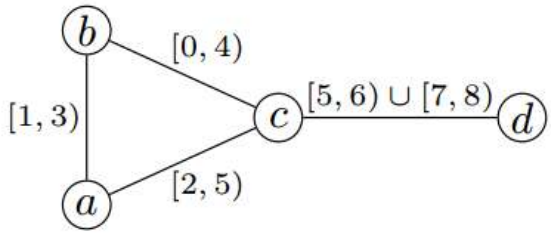


Figure 3.1: TVG visualization by Casteigts et al. [25]

the TVG is equivalent to a static graph. There might be limited restrictions applied, for instance, the latency function can be defined to be constant over time ($\zeta : E \to \mathbb{T}$), over the edges ($\zeta : \mathcal{T} \to \mathbb{T}$), over both ($\zeta \in \mathbb{T}$), or simply ignored. At the same time, there might be two edges connecting two vertices starting and ending at the same time, i.e. $\rho(e_1, t) = \rho(e_2, t)$, but having different latencies, i.e. $\zeta(e_1) \neq \zeta(e_2)$. In the latter case, the TVG contains parallel edges and it describes a multigraph [58]. Therefore, a TVG can be seen as a general case that covers a broad range of graphs, and this illustrates the spectrum of models over which the TVG formalism can stretch.

Several analytical works on dynamic networks ignore ζ, or assume a discrete-time scenario implicitly corresponding every time step to a constant ζ. Also, in some research settings that are delay-free, this factor is automatically equals to zero. Such researches include SNA even though in real-life examples it is impossible to reduce ζ to a value equal to zero, and have null latency. In the rest of this thesis, we assume that all components of TVGs exist unless explicitly indicated.

An example of TVG is shown in Figure 3.1. The labels on the edges represent the time intervals when the edge exists. Note that the same labelling could be used for vertices as well.

3.2 Temporal Concepts

In this section we introduce a number of dynamic network concepts from the TVG framework point of view. We limit our review to the major concepts that frequently appear in various fields and and abundantly referred to in the literature.

3.2.1 The Underlying Graph

The underlying graph $U = (V, E)$ of a TVG $\mathcal{G}$ is a static interpretation of $\mathcal{G}$. U flattens the time dimension of in $\mathcal{G}$ and assumes an edge between two vertices if there exists at least one edge between them in at least one instance of time. Thus, U is, sometimes, referred to as footprint of $\mathcal{G}$. While U is a helpful concept for some applications, it, unfortunately, does not reveal any information about the structure of its corresponding TVG. For instance, from the point of view of the simplest structural concept, connectedness, the footprint graph and TVG do not necessarily have any correlation. Therefore, the connectedness of U does not imply the connectedness of $\mathcal{G}$ over time. In a broader view, the degree distribution of such graphs can be totally different, too.

3.2.2 Points of View

Depending on the problem setting, TVGs can be viewed from different point of views. TVGs can generally be viewed from three different point of views. We can look at the evolution of the system from the point of view of a given edge (edge-centric point of view), or of a given vertex (vertex-centric point of view), or look at the global system (graph-centric point of view) [25].

The edge-centric view revolves around indicating the existence and latency of edges over time. The available times of an edge e is defined as the union of all times when the edge is available, i.e. $\mathcal{I}(e) = \{t \in \mathcal{T} : \rho(e, t) = 1\}$. $\mathcal{I}(e)$ can also be represented by a set of pairs of times t where $t_i < t_{i+1}$ as $\mathcal{I}(e) = \{[t_1, t_2) \cup [t_3, t_4), ...\}$, where the set of the first items in the pairs are appearance sequence $App(e)$, (e.g. $t_1, t_3, ...$), and the set of the second items of the pairs are disappearance sequence $Dis(e)$, (e.g. $t_2, t_4, ...$). Therefore, the notation $\rho_{[t,t')}(e) = 1$ indicates that $\forall t'' \in [t, t'), \rho(e, t'') = 1$. The union of $App(e)$ and $Dis(e)$ are referred to as characteristic times of e, and represented by $\mathcal{S}_{\mathcal{T}}(e)$.

The vertex-centric view, however, has a completely different formalism, and focuses on the successive changes that happen in the neighbourhood of a vertex [92]. The sequence of neighbourhood representation $N_{t_1}(v), N_{t_2}(v), ...$ is useful as it can lead us to define the temporal degree of a node. The temporal degree of v can be defined in a punctual format as $deg_t(v) = |E_t(v)|$, and its integral corresponding degree is defined as $Deg_{\mathcal{T}}(v) = | \cup E_t(u) : t \in \mathcal{T}|$.

In TVGs, each topological event can be viewed as the transformation from one static state to another. Hence, the evolution of the system can also be depicted as a sequence of

snapshots taken at different points of times as static graphs. Indeed, this is the view on which the definition of *evolving graphs* is based [45]. This implies that this point of view can be described based on characteristic times of the edges, and subsequently characteristic times of the graph $\mathcal{S}_{\mathcal{T}}(\mathcal{G}) = \text{sort}(\cup\{\mathcal{S}_{\mathcal{T}}(e) : e \in E\})$. Thus, the sequence of static snapshots of the graph $\mathcal{S}_{\mathcal{G}} = G_1, G_2, ...$, where G_i corresponds to the static snapshot of $\mathcal{G}$ at time $t_i \in \mathcal{S}_{\mathcal{T}}(\mathcal{G})$, describes the graph-centric view of the TVG. Therefore, in a discrete model, a snapshot corresponds to each new appearance of set of edges. When we talk about snapshots in this thesis, we refer to this discrete model unless otherwise stated. It is important to mention that in a continuous time model $G_i \neq G_{i+1}$ always holds, and in a discrete time model, where $t = i$, it is possible to have $G_i = G_{i+1}$.

3.2.3 Journeys

Let us consider TVGs with non zero latency; i.e., $\zeta(e) \neq 0, \forall e \in E$.

In $\mathcal{G}$, a journey $\mathcal{J}$, in its simplest form, is a temporal walk (vertices can appear multiple times in a journey as long as the appearance occur at different times), and defined as a sequence of ordered pairs $\{(e_1, t_1), (e_2, t_2),...,(e_k, t_k)\}$, such that $\{e_1, e_2, ..., e_k\}$, called the journey route and represented by R, is a walk in G, if and only if $\rho(e_i, t_i) = 1$ and $t_{i+1} \geq t_i + \zeta(e_i, t_i)$ for all $i < k$. Of course, one may assume more restrictions and conditions for a journey based on the application. Every journey has a $departure(\mathcal{J})$ and an $arrival(\mathcal{J})$ that refer to journey's starting time t_1 and its last time $t_k + \zeta(e_k, t_k)$.

The set of all journeys in a TVG is denoted by $\mathcal{J}^*_{\mathcal{G}}$, and $\mathcal{J}^*(u, v) \subseteq \mathcal{J}^*_{\mathcal{G}}$ represents journeys starting from u and arriving at v. A journey from u to v can also be defined in terms of its route. Since a journey should have a temporal route, we define σ as the set of points of times indicating when each edge of route from u to v (i.e. $R(u, v)$) is to be traversed. Hence, a journey is defined in terms of R and σ as $\mathcal{J}(u, v, \sigma) = \{R(u, v), \sigma\}$. u can reach v (i.e. $u \rightsquigarrow v$) if and only if $\mathcal{J}^*_{(u,v)} \neq \emptyset$. It is worth mentioning that the existence of journeys are not symmetrical and $u \rightsquigarrow v \not\Leftrightarrow v \rightsquigarrow u$. The horizon of u is also defined as the set $\{v \in V : u \rightsquigarrow v\}$. A journey is direct if there is no delay on any of the vertices that lie on its route from u to v. Otherwise, the journey is indirect and it waits at at least one vertex.

Since journeys are naturally walks over time, their length can be measured from both points of view of time and hop. Before defining the journey lengths, we need to define a few concepts. The hop-count, $|\mathcal{J}(u, v, \sigma)|_h = |R_i| = k$, is the number of edges that the journey traverses. The end-to-end duration of the journey is called the journey time,

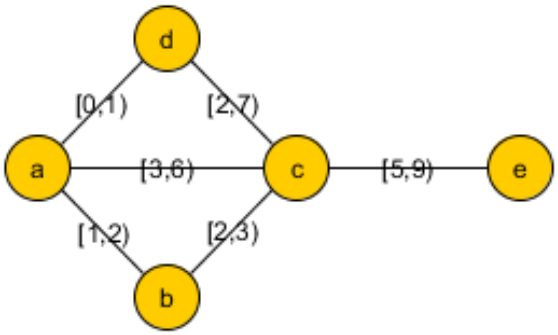


Figure 3.2: If we suppose that $\zeta(a,c) = 3$ and 1 for other edges, we can observe all the journeys in this figure. While $\mathcal{J}_{\hat{d}}(a,c) = \{(a,c)\}$, its foremost journey arriving at time 2 is $\mathcal{J}_{\hat{a}}(a,c) = \{(a,d),(d,c)\}$. Its fastest journey, however, corresponds to $\mathcal{J}_{\hat{l}}(a,c) = \{\{(a,b),(b,c)\},\{(a,d),(d,c)\}\}$. At the same time, if we suppose that the latency of all edges, including $\zeta(a,c)$ are equal to 1, the foremost journey between a and e arriving at time 6 is $\mathcal{J}_{\hat{a}}(a,e) = \{\{(a,d),(d,c),(c,e)\},\{(a,d),(d,c),(c,a),(a,c),(c,e)\},\{(a,c),(c,e)\},\{(a,b),(b,c),(c,e)\},$ $\{(a,b),(b,c),(c,d),(d,c),(c,e)\}\}$. While the fastet and shortest journeys coincide at $\mathcal{J}_{\hat{d}}(a,e) = \mathcal{J}_{\hat{l}}(a,e) = \{\{(a,c),(c,e)\}$.

$|(u,v,\sigma)|_t$ or $t(\mathcal{J})$, and is simply calculated by $arrival(\mathcal{J}) - departure(\mathcal{J})$. The arrival time which is defined as $|\mathcal{J}(u,v,\sigma)|_a = \sigma(e_k) + \zeta(e_k)$ is equal to $arrival(\mathcal{J})$. The latter is simply referred to as $a(\mathcal{J})$. The journey length, therefore, computed based on the number of hops is called the topological length or shortest distance, $\hat{d}(u,v) = \min\{|\mathcal{J}(u,v,\sigma)|_h\}$, whereas the temporal length is two-fold by itself. The end-to-end duration of the journey is called the delay $\hat{l}(u,v)$, and is simply equal to $\min\{|\mathcal{J}(u,v,\sigma)|_t\}$. The second temporal distance is called earliest arrival time, which is defined as $\hat{a}(u,v) = \min|\mathcal{J}(u,v,\sigma)|_a$.

Journeys are divided into three classes based on their variations based on the temporal and topological distance [121]. Journeys with the smallest topological distance are referred as *shortest journeys* $\mathcal{J}_{\hat{d}}(u,v)$, the smallest delay defines the *fastest journeys* $\mathcal{J}_{\hat{l}}(u,v)$, and the journeys that have smallest arrival time are denoted with *foremost journeys* $\mathcal{J}_{\hat{a}}(u,v)$ (Figure 3.2).

A *trail* $\mathcal{L}$ is a journey in which the edges can only appear once. Trails also has variants as shortest $\mathcal{L}_{\hat{d}}(u,v)$, foremost $\mathcal{L}_{\hat{a}}(u,v)$, and fastest $\mathcal{L}_{\hat{l}}(u,v)$.

Finally, a *temporal path* $\mathcal{P}$ is a trail in which the vertices can only appear once. Similar to journeys and trails, temporal paths have variants as shortest $\mathcal{P}_{\hat{d}}(u,v)$, foremost $\mathcal{P}_{\hat{a}}(u,v)$, and fastest $\mathcal{P}_{\hat{l}}(u,v)$.

Let us now consider TVGs where the latency is zero; i.e., $\zeta(e) = 0, \forall e \in E$. In this

case the notion of journey is analogous but has to be redefined by not allowing loops that could give rise to infinite journeys. For instance in Figure 3.2, the journey cannot go back and forth on one edge (e.g. (a, c)) in the same instance of time (e.g. time 4).

3.2.4 Connectivity

In terms of journeys, TVGs show different behaviour depending on their connectedness. Hence, before exploring the temporal metrics of TVGs, we define concept of connectedness in TVGs.

Definition Connectivity over time ($\forall u, v \in V, u \rightsquigarrow v$). A TVG is connected over time if there exist at least a temporal journey from u to v; in other words, every node can reach all other nodes.

Definition Round connectivity ($\forall u, v \in V, \exists \mathcal{J}_1 \in \mathcal{J}^*_{(u,v)}, \exists \mathcal{J}_2 \in \mathcal{J}^*_{(v,u)} : arrival(\mathcal{J}_1) \leq departure(\mathcal{J}_2)$). A TVG is round connected if it is connected over time and for a temporal journey from u to v, there exists a temporal journey back from v to u after the arrival the first journey from u to v.

Definition Recurrent Connectivity ($\forall u, v \in V, \forall t \in \mathcal{T}, \exists \mathcal{J} \in \mathcal{J}^*_{(u,v)} : departure(\mathcal{J}) > t$). A TVG is recurrent connected if at any point t in time, the temporal subgraph $\mathcal{G}_{[t,+\infty)}$ remains connected over time.

Definition Always Connectivity ($\forall t \in \mathcal{T}, \forall u, v \in V$ if $\psi(v, t) = \psi(u, t) = 1, \exists \mathcal{J} \in \mathcal{J}^*_{(u,v)} : departure(\mathcal{J}) = arrival(\mathcal{J}) = t$). A TVG is always connected if it is connected over time,, and at any point t in time, the temporal subgraph $\mathcal{G}_t$ remains connected for the vertices that exist in that subgraph (vertices that are active in that subgraph).

3.3 Temporal Metrics

Most classical metrics used to analyse social networks have a temporal analogous metric when translating the basic concepts of *path*, *walk*, *degree*, *diameter*, etc. into their temporal counterpart. The rest of this section explores such metrics from the temporal point of view. The summary of some of the metrics can be found in Table 3.1.

Table 3.1: Static and Temporal Measures

Concept/Metric	Static	Temporal
Eccentricity	$\varepsilon(v)$: using $d(x,y)$	$\hat{\varepsilon}_{\hat{d}}(v)$: using hops
		$\hat{\varepsilon}_{\hat{l}}(v)$: using delay
		$\hat{\varepsilon}_{\hat{a}}(v)$: using earliest arrival
Degree	$deg(v)$	$\hat{deg}(v,t)$
Closeness	$C_C(v)$: using $d(x,y)$	$C_{C_{\hat{d}}}(v)$: temporal using $\hat{d}(v,u)$
		$C_{C_{\hat{l}}}(v)$: fastness using $\hat{l}(v,u)$
		$C_{C_{\hat{a}}}(v)$: earliness using $\hat{a}(v,u)$
Betweenness	$C_B(v)$: using $d(x,y)$	$C_{B_{\hat{d}}}(v)$: temporal using $\hat{d}(v,u)$
		$C_{B_{\hat{l}}}(v)$: fastness using $\hat{l}(v,u)$
		$C_{B_{\hat{a}}}(v)$: earliness using $\hat{a}(v,u)$
Clustering Coefficient	$C_{Cl_i}(v)$	$C_{\hat{Cl}}(v)$
Modularity	Q	$\hat{Q}$
Eigenvector	$C_E(v)$	ADI*: $C_{\hat{E}_1}(v)$
		SDI*: $C_{\hat{E}_2}(v)$

* These metrics will be defined in Chapter 4.

3.3.1 Degree

In a TVG $\mathcal{G}$ the degree of a node v at time t is indicated by $\hat{deg}(v,t)$. It is easy to see that the definition below is a generalization of the degree in static graphs. If we sum the temporal degrees of all nodes in the TVG, at all time snapshots, the result will be equal to twice the number of edges times the number of snapshots that they appear in, i.e. $2 \times |E(\mathcal{G}) \times \mathcal{T}(\mathcal{G})| = \sum_{v \in V(\mathcal{G})} \sum_{t \in \mathcal{T}} \hat{deg}(v,t)$. Therefore, if $|\mathcal{T}|$ is equal to 1 (i.e. there is only one snapshot), the temporal degree coincides with the static degree of the graph.

We define $max(\hat{deg}(v,t))$ over all t as the *indicator degree* of v, and $\sum_t \hat{deg}(v,t)$ as the *aggregated degree* of v. For instance, in Figure 3.2, the indicator degree of a is 2 as there is no instance of time when a has three incident edges. Its aggregated degree, however, is 3. We will use the temporal degree of a graph specifically its aggregated degree in the definition of temporal eigenvector centrality measure.

3.3.2 Eccentricity and Diameter

Temporal Eccentricity is defined by considering reachability through journeys instead of paths. Let $\hat{\varepsilon}_{\hat{d}}(u)$ gives the maximum number of hops required to get from u to any other node in $\mathcal{G}$. Meanwhile, $\hat{\varepsilon}_{\hat{l}}(u)$ is the maximum delay that takes to get to any node starting u. Considering the arrival times, $\hat{\varepsilon}_{\hat{a}}(u)$ is denoted to the maximum of arrival time to any node starting from u.

The graph diameter, also, has a transformed definition when defined for TVGs. The diameter is defined as the maximum of all eccentricities over the whole graph. Thus, in a TVG, diameter can have three forms, namely, hop diameter, referring to the maximum eccentricity, time diameter (system-lag), denoting the maximum time eccentricity, and rapidity, translated as the maximum of all earliest arrival times in the system. Taking the graph in Figure 3.2 as an example, the hop diameter of the graph is equal to 2, while its system-lag and rapidity are 4 ($a \rightsquigarrow e$) and 8 ($e \rightsquigarrow a$) respectively.

3.3.3 Closeness

Similarly to the case of static graphs, closeness centrality is defined over the graphs connected over time, and it is the inverse of the average temporal distance of a vertex from all other vertices in the graph [100]. In terms of distance, we define three types of closeness in TVGs. The common closeness concerns with the average shortest hop distance between nodes, and defined as [101, 66]:

$$C_{C_{\hat{d}}}(v) = \sum_{u \in V} \frac{n-1}{\hat{d}(v,u)} \tag{3.1}$$

The notion of Equation 3.1 has also been referred to as *efficiency* by Tang et al. [109], who has instead defined closeness as follows:

$$C_{C_{\hat{d}}}(v) = 1 - \left(\frac{1}{W(n-1)} \sum_{u \neq v \in V} \hat{d}(v,u) \right) \tag{3.2}$$

where W is the number of intervals in TVG. As $W = \frac{(t_{max} - t_{min})}{w_s}$, and w_s is the size of each interval. Normalizing the whole model by W, shows that the authors assume that all intervals have the same size w_s. This restricts the application of this method to dynamic graphs of constant interval size. Nevertheless, the factor computed by Equation 3.2 is more similar to computing the farness than closeness.

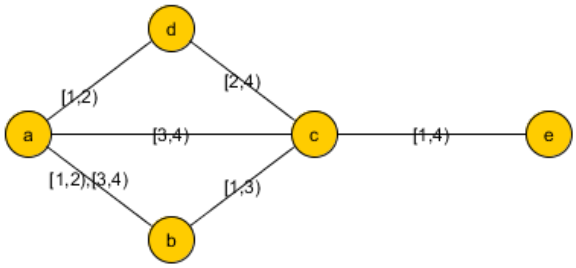


Figure 3.3: Temporal Closeness: in the connected TVG over time $\zeta(c,d) = 2$, and 1 for other edges, hence $\gamma = 1$, we have $C_{C_{\hat{d}}}(a) = 0.87$, $C_{C_{\hat{l}}}(a) = 1.83$, and $C_{C_{\hat{a}}}(a) = 1.11$.

We define the variation of closeness based on the fastest journeys as *fastness*, which refers to the average time that it takes to travel to all vertices. In contrast to the shortest journeys, fastest journeys can have minimum of zero delay. Therefore, as 1/0 is undefined, we add a constant factor γ to the denominator to avoid such situations.Note that γ does not affect the ranking of the nodes as it is applied to closeness values corresponding to all vertices. However, for the networks that do not have zero latency on any of the edges, γ is equal to zero.

$$C_{C_{\hat{l}}}(v) = \sum_{u \in V} \frac{1}{\hat{l}(v,u) + \gamma} \tag{3.3}$$

We define a similar metric based on the average arrival times as *earliness*, as the average earliest arrival times to the vertices in the graph. This metric can also be an indicator for reachability in time for the TVG. The aforementioned measures can be easily normalized in $[0,1]$, which we skip due to simplicity. Figure 3.3 provides an example for temporal closeness.

$$C_{C_{\hat{a}}}(v) = \sum_{u \in V} \frac{1}{\hat{a}(v,u) + \gamma} \tag{3.4}$$

3.3.4 Temporal Katz Score

Calculation of Katz centrality measure in a static setting is defined in Section 2.2.1. Recently, a few efforts have been dedicated to calculate the Katz centrality for the evolving graphs, and the TVGs. Among those, Grindrod et al. [57] developed a model for computing Katz centrality along with their research on communicability of evolving graphs. We explain their model rather in details as Katz score is a similar measure to eigenvector cen-

trality that we have developed in this thesis and will be explained in this chapter and the following chapter. Grindrod et al. [57], first, generalized a conclusion on matrix products from graph theory on evolving graphs, that is the matrix product $A^{[t_1]}A^{[t_2]}...A^{[t_w]}$, where $A^{[t_k]}$ corresponds to the snapshot adjacency matrix $G_k \in \mathcal{S}_\mathcal{G}$, has i,j element that counts the number of dynamic walks of length w from node i to node j on which the mth step of the walk happens at time t_m. Following a common rule in computation of Katz score, they down-weight walks of length w by a factor α^w, where $0 < \alpha < (\lambda^{[t_k]})^{-1}$, for all k, and $\lambda^{[t_k]}$ represents the largest eigenvalue of $A^{[t_k]}$. Letting I be an $N \times N$ identity matrix and noting that the resolvent $(I - \alpha A)^{-1}$ has the expansion $I + \alpha A + \alpha^2 A^2 + ... + \alpha^k A^k = \sum_{k=0}^{\infty} \alpha^k A^k$, the following matrix product is motivated:

$$\mathcal{Q} = (I - \alpha A^{[t_0]})^{-1}(I - \alpha A^{[t_1]})^{-1}...(I - \alpha A^{[t_T]})^{-1} \tag{3.5}$$

where t_T corresponds to the end of system. Therefore, Katz score in evolving graphs can be computed by:

$$C_{\hat{K}}(v) = \sum_{u=1}^{N} \mathcal{Q}_{vu} \tag{3.6}$$

The resolvent sub-graph centrality of node v, $(I - \alpha A)^{-1}{}_v v$, counts the total number of closed walks in the network which are centred at node v, weighing walks of length k by α^k. The defined bounds for α ensures that the matrix $I - \alpha A$ is invertible and that the power series corresponding to $(I - \alpha A)^{-1}$ converges to its inverse. Meanwhile, since $I - \alpha A$ is non-singular, the bounds on α forces $(I - \alpha A)^{-1}$ to be non-negative, which makes it useful for ranking purposes. However, by looking at $(I - \alpha A)^{-1}$, it is apparent that in equations 3.6 and 3.5, it is assumed that a walk can stay in an snapshot for an infinite length. If we disallow the infinite stay in a snapshot, we can, then, define $\mathcal{Q}$ as:

$$\mathcal{Q} = (I - \alpha A^{[t_0]})(I - \alpha A^{[t_1]})...(I - \alpha A^{[t_T]}) \tag{3.7}$$

The matrix $\mathcal{Q}$, now only refers to the situations where only one link can be traversed in any time instance. There is also no need to mention that A can be replaced by its transpose in all above equations depending the type of Katz score that is being computed.

TVGs are a more generalized version of evolving graphs, so computing Katz centrality for them will be different for that reason. The major difference between evolving graphs and TVGs is in the lieu with the existence of latency on the TVG edges. This may cause a

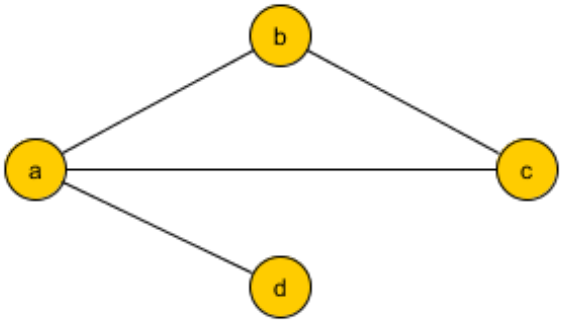


Figure 3.4: A TVG in which the edges and vertices exist all the time with (Case I): $\zeta(a,b) = 2$, and 1 for all other edges; and (case II) $\zeta(a,b) = 2$, $\zeta(a,d) = 0$, and 1 for all other edges.

link to expand in two or more time snapshots. For instance in Figure 3.4, case I, any walk moving on (a,b) will expand in two time intervals. Plus, in TVGs, the maximum length of a walk is equal to the lifetime of the system $|\mathcal{T}|$, so iterating the walk computation until infinite walks makes no sense unless there is an edge with zero latency on the graph (Figure 3.4, case II). We do not rule out such possibilities and design e general model that can be applicable to all the situations. There is till one condition that the granularity of time should be small enough so that all the edges land to their destination at the beginning of an (future) interval, and no edge can arrive to a destination at the middle of an interval. For instance, in Figure 3.4, the granularity of intervals can be equal to $1, 0.5, 0.25, ...$, preferably 1, and any interval length of bigger that 1 is not allowed, as it guarantees that (a,c), for instance, lands to c shorter than the end of interval. Note that edges with zero latency always arrive to their destination at the beginning of an interval.

Therefore, using Equation 3.7 is not possible for a TVG. The reason is that there is no way to construct a normal adjacency matrix A that represents a time snapshot, but its edges start at the beginning of the snapshot and end at the beginning of the next snapshot. Thus, we need to define a new concept for representing such matrices. The adjacency matrix is inspired by the work of Wehmuth et al. [119], especially the TME model. Our model, however, includes the zero latency edges in the TME model. We first explain the notion of TME representation and then generalize it to our purpose.

Wehmuth et al. [119] prove that for any TVG $\mathcal{G}$, there is an isomorphic directed graph H with $N \times |\mathcal{T}|$ vertices for which there is an order preserving bijective function $f : (V \times T)(\mathcal{G}) \to V(H)$, such that any temporal edge $e = (v,u) \in E(\mathcal{G})$, $\rho(e,[t_i,t_j)) = 1$ exists if and only if the edge $(f(v,t_i), f(u,t_j)) \in E(H)$ exists. Therefore, any TVG can be represented by a directed isomorphic graph. Based on that isomorphism, the TVG can

be represented by an adjacency matrix representing graph H. This matrix has $N \times |\mathcal{T}|$ columns and rows, and non-zero entries of the matrix correspond to the temporal edges in the matrix. Wehmuth et al. [119] call this representation as the TME model. For our computational purposes, we fix the granularity of time as explained earlier in this section so each edge lands at the exact starting point of any future or current time interval. We allow landing at the start of the current time interval to make the existence of zero latency edges possible. Let us see an example of TVG adjacency matrix corresponding to Figure 3.4, Case II.

$$A = \begin{array}{c} \left(\begin{array}{cccccccccccc} 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right) \begin{array}{c} a_0 \\ b_0 \\ c_0 \\ d_0 \\ a_1 \\ b_1 \\ c_1 \\ d_1 \\ a_2 \\ b_2 \\ c_2 \\ d_2 \end{array} \\ \begin{array}{cccccccccccc} a_0 & b_0 & c_0 & d_0 & a_1 & b_1 & c_1 & d_1 & a_2 & b_2 & c_2 & d_2 \end{array} \end{array}$$

In A, n_t represents the node n at time t. As it can be seen in Figure 3.4, Case II, the edge starting from a at time zero arrives to d at time zero, so the entry corresponding to row a_0 and column d_0 will be 1. With the same mode, we can fill out A's entries. Note that A represents a directed graph, so the entry corresponding to $A_{a_0 b_1}$ is not necessarily equal to $A_{b_1 a_0}$, even though it happens for the edges with zero latency.

Following simple lemma from graph theory (a proof can be easily found, for example, in [30], Theorem 2.2.1), it can be easily shown that the quantity A^l_{ij} counts the number of different walks ($i \neq j$) or closed walks ($i = j$) of length l between nodes i and j. Therefore, for instance $A^l_{a_0 b_1}$ has the number of walks of length l between a and b that started at time 0 and ended at time 1. Note that if the graph does not have any edge with zero latency, the maximum length of the walk will be equal to $|\mathcal{T}|$. Now that the number of walks can be calculated, the temporal Katz score can be easily computed by its counterpart formula in the static model as:

$$C_K(v) = \sum_{k=0}^{\infty} \sum_{i \in \mathcal{T}} \sum_{u \in V \times (\mathcal{T} \geq i)} \alpha^k (A^k)_{v_i u} \tag{3.8}$$

Equation 3.8 sums all the walks starting from v at all the times arriving at different vertices at times greater than or equal to the start of the walk since the walks cannot go back in time.

3.3.5 Temporal Betweenness

Calculation of betweenness in static graphs is defined in Section 2.2.1. Recently, Santoro et al. [101], have explored a few centrality measures including betweenness in temporal graphs. They extend betweenness to time varying graphs by considering foremost, shortest and fastest journeys instead of paths. The temporal shortest betweenness of node v is then defined as:

$$C^{\mathcal{J}}_{B_{\hat{d}}}(v) = \sum_{u \neq w \neq v \in V} \frac{|\mathcal{J}_{\hat{d}}(u, w, v)|}{|\mathcal{J}_{\hat{d}}(u, w)|} \tag{3.9}$$

where $|\mathcal{J}_{\hat{d}}(u, w)|$ is the number of shortest journeys between u and w in the TVG, and $|\mathcal{J}_{\hat{d}}(u, w, v)|$ is the number of shortest journeys passing through v. The fastest $C^{\mathcal{J}}_{B_{\hat{l}}}(v)$ and foremost $C^{\mathcal{J}}_{B_{\hat{a}}}(v)$ betweenness can easily be calculated by replacing $\mathcal{J}_{\hat{d}}$ in Equation 3.9 by $\mathcal{J}_{\hat{l}}$ and $\mathcal{J}_{\hat{a}}$ respectively.

A different notion, also called temporal betweenness while being essentially a calculation in static graphs, has been introduced by Tang et al. [111] . They define betweenness of a vertex at successive intervals t. To do so, a function $\eta(v, t, u, w)$ is defined so that returns the number of shortest temporal paths from u to w in which vertex v has either received a message at time t or is holding a message from a past time window until the next vertex is met at some time $t' > t$. Similar to all betweenness measures, the betweenness is zero if $|\mathcal{J}_{\hat{d}}(u, w)| = 0$.

$$C^{t}_{B_{\hat{d}}}(v) = \frac{1}{(n-1)(n-2)} \sum_{\substack{u \in V \\ u \neq v}} \sum_{\substack{w \in V \\ w \neq u \\ w \neq v}} \frac{\eta(v, t, u, w)}{|\mathcal{J}_{\hat{d}}(u, w)|} \tag{3.10}$$

Then, the betweenness of v over the whole $\mathcal{T}$ is:

$$C_{B_{\hat{d}}}(v) = \frac{1}{W} \sum_{t=1}^{W} C_{B_{\hat{d}}}^{(t \times w_s)+t_{min}}(v) \tag{3.11}$$

where w_s is the size of each footprint's time window (the size of each interval), and the number of the graphs in the sequence of footprints is denoted by $W = \frac{(t_{max}-t_{min})}{w_s}$.

3.3.6 Clustering Coefficient

The clustering coefficient is used in social network analysis to characterize the network architecture. More formally, by applying to footprints, the clustering coefficient $C_{\hat{Cl}}(v)$ indicates how close to a clique the neighbourhood of v is; in fact, it can also be extended to cover the snapshots by taking the proportion of edges among the neighbourhood of v divided by the maximum number of edges that could potentially exist between them in snapshots.

3.3.7 Modularity

The modularity, measures how the structure of a given network is modular, i.e. how it can be decomposed into subgraphs. Moreover, it can quantify the quality of a division of a network into subgraphs. The higher the modularity is, the denser is the internal connections between nodes within communities compared to the connections between different communities.

Amblard et al. [7] define modularity in TVGs in the footprint of the graph. Therefore, the modularity of a pair of nodes u and v on footprint U is defined as $\frac{deg(u) \times deg(v)}{2m}$. Modularity can easily be generalized into the snapshots of the TVG as following:

$$\hat{Q}_t = \frac{deg_t(u) \times deg_t(v)}{2|E|_t} \tag{3.12}$$

3.4 Conclusion

In this Chapter we introduced the notion of time-varying graph. We also described several temporal measures. In particular, we introduced various betweenness measures corresponding to different types of journeys (foremost, fastest, and shortest) that will be heavily used in the subsequent Chapters.

Chapter 4

Computation of Temporal Measures

In this Chapter we are interested in the computation of some temporal measures and, in particular, temporal betweenness. We study properties of temporal betweenness based on shortest and foremost journeys. While shortest temporal betweenness can be computed in polynomial time in any TVG, we observe that the situation is radically different for foremost betweenness, because its computation is a #P problem and thus intractable. We describe a polynomial algorithm to compute shortest betweenness, an (exponential) algorithm to compute foremost betweenness in general settings, and one to compute it in special TVG classes that will be relevant in the subsequent Chapters. Finally, we conclude the Chapter with the introduction of a new temporal metric: temporal eigenvector centrality that generalizes the well known static metric of eigenvector centrality.

4.1 Temporal betweenness

Temporal betweenness is defined in Chapter 3. We remind that temporal shortest betweenness of v is defined as the number of shortest journeys between u and w passing through v over the total number of shortest journeys between u and w:

$$C_{B_{\hat{d}}}(v) = \sum_{u \neq w \neq v \in V} \frac{|\mathcal{J}_{\hat{d}}(u, w, v)|}{|\mathcal{J}_{\hat{d}}(u, w)|}$$

The foremost betweenness $C_{B_{\hat{a}}}(v)$ can easily be calculated by replacing $\mathcal{J}_{\hat{d}}$ in Equation 3.9 by $\mathcal{J}_{\hat{a}}$ respectively to account for foremost journeys instead of shortest.

In the following we consider TVGs whose lifetime is constituted by a finite interval of time $[t_s, t_e]$. We remind the notion of journeys between two nodes u and w defined in

Chapter 3. A foremost journey is a walk over time that arrives at w on the earliest possible time by leaving u any time on or after t_s; a shortest journey between u and w is a path over time that arrives at w with the minimum number of hops leaving u any time on or after t_s. We also remind the difference between a journey and a temporal path: a temporal path does not traverse the same node more than once, while a journey may pass more than once thought the same node, but the traversal time at those vertices must be different.

4.2 Temporal Shortest Betweenness

As mentioned, temporal shortest betweenness is similar to the non-temporal metric and deals with counting the shortest journeys between all pairs of the vertices in the graph. Temporal shortest journeys differ from shortest paths in a way that a legitimate shortest path might not be temporally feasible (i.e., it might not correspond to a journey) thus requiring a feasibility check. The algorithm described in this section is very general, and it counts all journeys from a node to all the others under any TVG scenario.

The Algorithm

Counting the shortest journeys in TVGs can be done using a BFS-like algorithm on the TVG inspired by the method that is developed in [121] to construct a shortest journey spanning tree from a source to all destinations, and using the same data structure (Figure 4.1).

We need to modify the algorithm to be able to count also the number of shortest journeys through any intermediate node. To do that, instead of computing only one shortest journey, we need to maintain all shortest journeys at different instance of time. Xuan et al. [121] prove that the prefix property exists in shortest journeys such that if the last edge, say (u, v), of a shortest journey between vertex s and vertex v arrives at time t, then the prefix journey (going from s to u) is shorter than all the journeys from s to u ending before t. This property is useful in computing, and consequently counting, shortest journeys. In order to count the shortest journeys, we modify the algorithms defined in [121] to store the number of shortest journeys arriving at each vertex at each step of time and record the number of hops that they have had so far in the journey.

Algorithm 1 receives (G, s) as its input, where G is the TVG and s is the starting node from which the journeys to all other vertices in the TVG are being counted. The results are returned in the combination of $shortestCount[v, k]$ and $shortestIntCount[v, k]$ matrices,

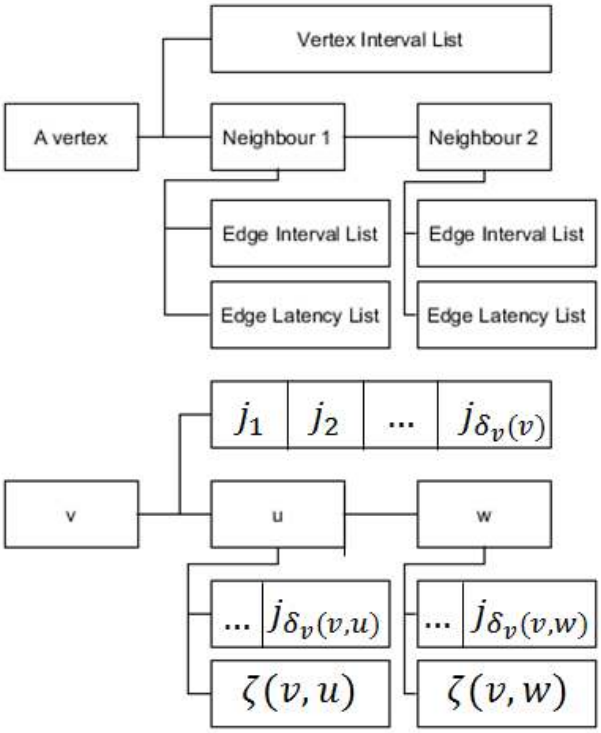


Figure 4.1: The data structure to store TVGs, adopted from [121]

which record the number of shortest journeys from s to v with length k, and the number of such journeys that pass through the nodes that fall on the path of the corresponding journey.

The algorithm starts by adding all the possible predecessors of v into the predecessor list. Matrix $Pred[v,k]$ stores the predecessor of vertex v. Each predecessor falls on a "quasi-shortest path" with length k. We call this path quasi-shortest since it does not always store the shortest path, rather, it carries some longer paths that might contribute to the shortest path at later hops. The arrival time to some vertex v at the current step is stored in variable $arr(v)$, which is recorded for future path feasibility check.

The quasi shortest paths are counted and stored in Matrix $count[v,k]$, while the $intCount[v,k]$ stores the number of journeys that pass through a specific vertex on the quasi shortest path from s to v.

Some quasi-shortest paths are, indeed, the shortest paths that are recorded in the matrix $shortestCount[v,k]$, and the count for their intermediate vertices are stored in the $shortestIntCount[v,k]$. This is determined by checking the local array $t_{LBD}[u]$ that gives for each $u \in V_G$ a lower bound on the departure time, meaning the earliest time that the journey can exit u. $t_{LBD}[u]$ is initialized to infinity and gets updated anytime that a

Algorithm 1: Counting the shortest journeys

input : A TVG G, a vertex $s \in V_G$
output: $shortestCount[v,k]$ that records the number and length of the shortest journeys from s to all $v \in V_G$
begin
  Initialize $t_{LBD}[s] \leftarrow 0, Pred[s,0] \leftarrow (), k \leftarrow 0, arr \leftarrow ()$, $shortestCount[\{.,.\}] \leftarrow 0, count[\{.,.\}] \leftarrow 0, intCount[\{.,.\}] \leftarrow \emptyset$, and define for all $v \neq s, t_{LBD}[v] \leftarrow \infty$
  while *there is* $v \in V_G$ *such that* $t_{LBD}(v) = \infty$ *and* $k < n$ **do**
    $k \leftarrow k + 1$
    $arr \leftarrow ()$
    for $(u,v) \in V_G$ **do**
      Let $t = EarliestTransmit((u,v), t_{LBD}[u])$
      if $(t + \zeta(u,v)) \leq t_{LBD}[v]$ **then**
        add $(u, (t + \zeta(u,v)))$ to $Pred[v,k]$
        $arr[v] \leftarrow min((t + \zeta(u,v)), arr[v])$
    for $(w, k-1) \in Pred[v,k]$ **do**
      $count[\{v,k\}] \leftarrow count[\{v,k\}] + count[\{w,k-1\}]$
      for *each* (x,i) *in* $intCount[\{w,k-1\}]$ **do**
        if x *exists in* $intCount[\{v,k\}]$ *as* (x,j) **then**
          replace (x,j) with $(x,i+j)$ in $intCount[\{v,k\}]$ and update the count for $(x,i+j)$
        else
          add $(x,i+1)$ to the end of $intCount[\{v,k\}]$ and update the count for $(x,i+1)$
    for $v \in V_G$ **do**
      if $t_{LBD}[v] = \infty$ **then**
        $shortestCount[\{v,k\}] = count[\{v,k\}]$
        update $shorthestIntCount[\{v,k\}]$ with $intCount[\{v,k\}]$
        $t_{LBD}[v] = arr[v]$
    $t_{LBD} \leftarrow arr$

shortest journey to a vertex is found. Thus, checking whether the value of $t_{LBD}[u]$ for u is infinity or not, determines if the shortest journey for u is found earlier or not.

It should be noted that function $EarliestTransmit(,)$ gives, for each edge (u,v), and each time instant t, the earliest moment after t when vertex u can transmit a message to v. If such a moment does not exist, $EarliestTransmit(,)$ returns $+\infty$.

–**Time Complexity** Algorithm 1 counts the number of shortest journeys from one node to all the others, and also the number of such journeys that pass through each intermediate vertex. Repeating this procedure for all starting points would provide all the necessary information to compute temporal shortest betweenness for all nodes.

Let δ indicate the maximum number of different time intervals on an edge.

Lemma 4.2.1 *The number of shortest journeys from a single source s to all the vertices in a TVG can be computed in $O(n^3 \log \delta)$.*

Proof Using the data structure proposed in [121], and depicted in Figure 4.1, procedure $EarliestTransmit(,)$ is computed in time $O(\log \delta)$ due to the fact that we can apply binary search to find the earliest transmit time. We call procedure $EarliestTransmit(,)$, m times during the execution of the algorithm for a total of $O(m \log \delta)$. Meanwhile, $t_{LBD}[v], \forall v \in V_G$ becomes a value smaller than infinity if and only if the graph is connected and we have found all the shortest journeys including the longest one, which is equal to the eccentricity. In case of a disconnected TVG, we have to iterate the *while loop* at most n times. The other factor contributing to the complexity appears in the nested loop for that results which amounts to $O(n^2)$ times. ∎

Theorem 4.2.2 *The number of shortest journeys from all vertices to all other vertices in a TVG can be computed in $O(n^4 \log \delta)$.*

Proof The complexity mentioned in Lemma 4.2.1 has to be multiplied by n to repeat the process from every possible starting node. ∎

Since the most time-consuming part of the betweenness algorithm is its path counting, the computation of betweenness is at most in the order of its path counting algorithm.

–**Space complexity.** The algorithm stores five matrices in memory. The predecessor matrix, in the worst case, for each node, needs k times the the sum of degrees in the graph, (i.e., the the eligible predecessor nodes), that is O(km) in the worst case. However, the highest amount of space is allocated to store $intCount$, which is a $m \times (n-1)$ array where each element points to a list of all vertices that fall on the path. In fact $intCount$ stores, for each predecessor (for a maximum of m), the number of at most $n-1$ vertices. If we consider that the from s there is a path to every vertex, the space complexity of the algorithm is then $O(mn^2)$.

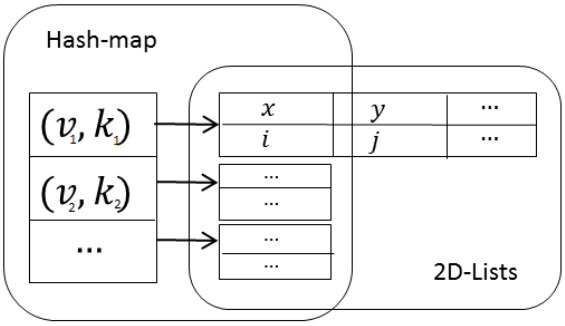


Figure 4.2: Data Structure Used for Storing the Path-counts for Intermediary Vertices *intCount*

–**Practical considerations.** Due to the sparsity of the matrix that stores the path counts, we actually use hash-maps to reduce space and time complexity of the algorithm. In fact, we represent matrices $count[\{.,.\}]$, $intCount[\{.,.\}]$, $shortestCount[\{.,.\}]$, and $shortestIntCount[\{.,.\}]$ as hash-maps, with $count[\{.,.\}]$ and $shortestCount[\{.,.\}]$ receiving an integer as their value, and $intCount[\{.,.\}]$ receiving a list of vertex and path-count (x, i) pair as its value. $intCount[\{v, k\}]$ stores the number of paths (i) starting from s and ending at v with length k that pass through x. Figure 4.2 provides a schematic view for the data structure that we use. While the worst case complexity is still high, hash-maps with a good hash function reduce the access time to the stored items, which in average becomes $O(1)$. In our analysis on real datasets we will make use of this data structure to speed up the computation.

4.3 Temporal Foremost Betweenness

It is easy to see that there exist TVGs where counting all foremost journeys, journey routes, and temporal paths between two vertices is #P-complete. Consider, for example, TVGs where edges always exist (note that a static graph is a particular TVG) and latency is zero. In such a case any journey, journey route, or temporal path between any pair of nodes is a foremost journey, journey route, or temporal path respectively. Counting all of them is then equivalent to counting all paths between them, which is a #P-complete problem (see [115]). In general, it is then unavoidable to have exponential algorithms to compute foremost betweenness.

In this Section we first focus on betweenness based on temporal paths (and journeys) in the general setting (Algorithms 2 and 3). We then focus on betweenness based on journeys

for TVGs with zero latency and instant edges (Algorithm 4). Note that each solution has the same worst case time complexity, proportional to the total number of temporal paths (resp. journey routes) in the TVG. The advantages of the algorithm designed for the special temporal condition of instant edges and zero latency are mainly due to the possibility of reducing the space complexity by being able to use a secondary storage and thus decreasing the main memory requirements.

4.3.1 General Algorithm

In this Section we describe an algorithm for the computation of all foremost temporal paths between a node to all other nodes, which is at the basis of the notion of betweenness, we will then extend it to the case of journeys.

Note that a temporal path $[(x_0, x_1), (x_1, x_2), \ldots, (x_{k-1}, x_k)]$ may contain several journeys, each corresponding to different traversal times. Algorithm CountFormemost, described below, considers only foremost temporal paths; in other words, several occurrences of journeys using the same paths are counted as one.

The input of Algorithm CountFormemost is a pair (G, s) where $G = (V, E)$ is a TVG and s a starting node; the algorithm returns a matrix $Count_s[x, y]$, for all $x, y \in V$ containing the number of foremost temporal paths from s to y passing through x. Note that $Count_s[x, x]$ denotes the number of foremost temporal paths from s to x.

First of all, the foremost arrival times of foremost journeys starting from s to all nodes are computed using the Algorithm from [121]. To each node v is then associated its foremost arrival time $foremost(v)$.

The counting algorithm is very simple and it is based on Depth-First Search (DFS) traversal. It essentially consists of visiting every temporal path of G starting from s, incrementing the appropriate counters every time a newly encountered journey is foremost. A typical DFS traversal visits every node and terminates when they are all visited; in our algorithm, however, we need to repeatedly perform DFS, re-visiting nodes possibly many times, so to traverse all temporal paths.

To do so, the traversal starts as a usual DFS, pushing the incident edges of the source s onto a stack S and visiting one of the adjacent neighbours (say s'), thus discovering a first temporal path $\pi_0 = [(s, s')]$. The current temporal path under visit is kept in a second stack $Path$ (nodes in $Path$ are marked *visited*). At this point the DFS continues pushing on the stack the edges incident to s' that are feasible with π_0 (i.e., the edges whose latest

traversal time is greater than or equal to the earliest arrival time at s'), and updating $Count[s', s']$ if π_0 has a foremost arrival time at s'.

In general, as soon as a temporal path $\pi = [(x_0, x_1), (x_1, x_2), \ldots, , (x_{k-1}, x_k)]$ is encountered in the traversal, $Count[x_i, x_k]$, $i \leq k$ is updated only if π is a foremost temporal path, and, regardless of it being foremost, the traversal continues pushing on the stack the edges incident to x_k that are temporally feasible with π.

Whenever backtracking is performed, however, the already visited nodes on the backtracking path are remarked *unvisited* (and popped from $Path$) in such a way that they can be revisited as part of different temporal paths, not yet explored. The pseudo-code is described in Algorithm 2.

Algorithm 2: CountFormemost.

```
input  : (G, s) : a TVG G = (V, E), s ∈ V
output: Count_s[x, y], ∀x, y ∈ V: number of foremost temporal path from s to
        y ∈ V, passing through x ∈ V
begin
    Path.push(s), Count_s[., .] ← 0
    for all w ∈ Adj(s) do
        S.push(s, w)
    while S ≠ ∅ do
        (x, y) ← S.pop()
        while x ≠ Path.top() do
            Path.pop()
        Let π be the temporal path corresponding to the content of Path
        Let t_{x,y} be the latest possible traversing time of edge (x, y)
        if y ∉ Path and t_{x,y} ≥ arrival(π) then
            Path.push(y)
            for each (y, w) such that w ∉ Path and t_{y,w} ≥ arrival(π) do
                S.push(y, w)
            if arrival(π) = foremost(y) then
                Update Count_s[z, y] for all z ∈ Path
```

– **Complexity.** Let μ_s be the number of temporal paths from s all the other nodes, μ the overall number of temporal paths in $\mathcal{G}$, $n(\mu_s)$ (resp. $n(\mu)$) the number of nodes belonging to those paths, and n the number of nodes of $\mathcal{G}$.

Theorem 4.3.1 *In the worst case, Algorithm 2 computes the number of foremost temporal paths from a single source s to all vertices $y \in V$ passing through any $x \in V$ in $\mathcal{G}$ in $O(n(\mu_s))$ time, and $O(n^2)$ space.*

Proof Correctness of the algorithm is straightforward as it follows multiple DFS traversals to traverse every temporal path from a source. The algorithm traverses every temporal path from s to any other node, and it performs an update for every visited node in each foremost path that it encounters. Thus, in the worst case, it has a $O(n(\mu_s))$ time complexity. As for the space complexity. The stacks S and $Path$ can have at most size $O(m)$, matrix $Count_s$ has size $O(n^2)$. ∎

Note that the size of a temporal path is bounded by n because a node is never revisited in a temporal path.

By repeating the procedure for every starting node, we can then easily compute foremost betweenness.

Theorem 4.3.2 *Employing on Algorithm 2, foremost betweenness based on temporal paths can be computed for every node in $\mathcal{G}$ in $O(n(\mu))$ time, and $O(n^3)$ space.*

Proof For every $s \in V$ we incur in time complexity $O(n(\mu_s))$, repeating the counting procedure for every s we then obtain $O(\sum_s n(\mu_s)) = O(n(\mu))$. Since n matrices of size n^2 are employed, the total space complexity becomes $O(n^3)$. ∎

– **Practical Considerations: reducing space** In Algorithm 2, the data structure that stores the TVG is the same as the one that is explained in Section 4.2, and Figure 4.1. Therefore, the TVG is loaded in memory and the access time to its edges and vertices is as explained in Section 4.2. For large TVGs it is impractical to store in memory a large matrix corresponding to the whole TVG, especially considering that once the number of temporal paths from s to some target y, is determined, it will never be revisited. Thus, it makes sense to store the journey counter in a single dimensional array $count_{sy}[x]$, where x refers to the vertices that are on the route from s to y. The array can be stored on the disk when computation of the journeys for each target y is completed. Note that the theoretical analysis does not change in the worst case, but is much better in general. At each point of time, we consider one path only, which means that the stack $Path$ will have maximum size of order of n. The same applies to the $Count_{sy}$ array, as at each point in time, we only count the number of journeys corresponding to the stored $Path$, so the maximum elements

of the $Count$ linked-hashmap is the same order of n. We used linked-hashmap to represent $Count_{sy}$ because it provides the benefits of linked list, which is easy resizing and keeping the order, and the fast access of hashmap. In practice, we need to keep the sorted order to be able to loop over the $Count_{sy}$ and $Path$ structures to decrease the time complexity, while the random access and fast resizing is useful for cutting and modifying the list as the path changes to save on rehashing time. The space complexity is still $O(n^2)$ in the worst case, much better in a practical setting.

Counting Journey Routes. As explained, Algorithm 2 is applicable to the general TVG, and it counts the number of *temporal paths* in it. The number of *journey routes* in the TVG can be found slightly modifying the same algorithm, and its data structure.

We remind that a node in a temporal path cannot appear more than once, while in a journey it can re-appear but the different occurrences must correspond to different times.

To consider journey routes instead of temporal paths, we need to store the moment when a node is visited in the journey route, so that if it is visited again we can determine whether the second visit happened at a later time. Thus, in both $Path$ and S stacks, we need to store the time stamp, to register the time of the first visit in $Path$ and to register the time for the next visits in the S. Meanwhile, since journey routes allow nodes to be visited more than once, if the visits happens at different times, the condition for pushing the nodes into the stacks has to modified as well. In fact, we push the nodes to the $Path$ or S only if they are not visited yet (i.e., not in the path), or they are visited before (in the path) but at a different time. The function $arriv(x, y, t)$ returns the arrival time to y, leaving x at time t.

Theorem 4.3.3 *Employing on Algorithm 3, foremost betweenness based on journey routes can be computed for every node in G in $O(n(\mu'))$ time, and $O(n^3)$ space, where μ' is the total number of different journey routes in G.*

Proof Following the proof for the Theorem 4.3.2, the only change in the algorithm is the change in the number of possible journeys. Thus, the complexity is computed with the new journey count μ'. ∎

Note that a journey route could contain a node several times, but all in different instants. It can then be bounded by $n\mathcal{T}$, where $\mathcal{T}$ is the lifetime of the system.

Algorithm 3: CountForemostJRoutes.

input : (G, s) : a TVG $G = (V, E)$, $s \in V$
output: $Count_s[x, y]$, $\forall x, y \in V$: number of foremost journey routes from s to $y \in V$, passing through $x \in V$

```
begin
    Path.push(s, 0), Count_s[., .] ← 0
    for all w ∈ Adj(s) do
        S.push(s, w, arriv(s, w, 0))
    while S ≠ ∅ do
        (x, y, t) ← S.pop()
        while x ≠ Path.top() do
            Path.pop()
        Let π be the journey route corresponding to the content of Path
        Let t_{x,y} be the latest possible traversing time of edge (x, y)
        if t_{x,y} ≥ arrival(π) then
            if y ∉ Path or y ∈ Path at time t' < t then
                Path.push(y, arriv(x, y, t))
                for each (y, w) such that t_{y,w} ≥ arrival(π) and either w ∉ Path or
                w ∈ Path at time t' < arriv(y, w, t) do
                    S.push(y, w, arriv(y, w, t))
                if arrival(π) = foremost(y) then
                    Update Count_s[z, y] for all z ∈ Path
```

4.3.2 Algorithm for Zero latency and Instant edges

Algorithm 3 is applicable to the general TVG. In this thesis (Chapter 5), we will deal with a very special type of TVG with very specific temporal restrictions. One such peculiarity is given by *instant edges*, i.e., edges that appear only during a unique time interval, another characteristic is zero latency: an edge can be traversed istantaneously. We now describe a variation of the algorithm specifically designed for those conditions and we compute foremost betweenness based on journey routes in this setting.

More precisely, given a TVG $G = (V, E)$ we assume we can divide time in consecutive intervals $I_1, I_2, \ldots, I_k$ corresponding to k snapshots $G_1, G_2, \ldots, G_k$, where $G_i = (V_i, E_i)$, in such a way that $(x, y) \in E_i$ implies that $(x, y) \notin E_j$, for $j \neq i$. Furthermore, we assume that $\zeta = 0$, that is an edge can be traversed in zero time.

The key idea that can be applied to this very special structure is based on the observation that, given a foremost route $\pi_{x,y}$ from x to y with edges in time intervals I_j, with

$j > i$, and given any journey route $\pi'_{s,x}$ from s to x with edges only in I_i, the concatenation of π' and π is a foremost route from s to y, passing through x.

This observation leads to the design of an algorithm that starts by counting the foremost routes belonging to the last snapshot G_k only, and proceeds backwards using the information already computed. More precisely, when considering snapshot G_i from a source s, the goal is to count all foremost routes involving only edges in $\cup_{j \geq i} E_j$ (i.e., with time intervals in $\cup_{j \geq i} I_j$), and when doing so, all the foremost routes involving only edges strictly in the "future" (i.e., time intervals $\cup_{j > i} I_j$) have been already calculated for any pair of nodes. The already computed information is used when processing snapshot G_i avoid a recalculation in a dynamic programming fashion.

The inputs of Algorithm 4 are a snapshot G_i, and a starting node s. The algorithm returns an array of lists, $Count_s[u, v]$, where each of the list elements refer to vertices falling on the journey. $Count_s[u, v]$, for all $u, v \in V$ contains the number of foremost journeys from s to u passing through v counted so far (i.e., considering only edges in $\cup_{j \geq i} E_j$).

The actual counting algorithm on snapshot G_i is a modified version of Algorithm 3, still based on Depth-First Search (DFS) traversal. However, when a new route is discovered to some node x, if this route is foremost, a normal update is performed like in Algorithm 3: i.e., an increment to $Count_s[v, x]$ is done, v being the node that falls on the journey route from s to x. If instead it is not a foremost route and it is connected to a node that existed in the "future", a special update is performed using the data already calculated for the "future snapshots". In other words, when $s \rightsquigarrow x$ is a prefix of a journey route $x \rightsquigarrow y$ at a later time snapshot, we perform a procedure called special count (procedure 5). The special count involves aggregating the values of $Count_s[v, x]$ with $Count_x[v', y]$, for all node occurring in the journey routes between s and x and between x and y (see Procedure 5).

– **Complexity.** The complexity is the same as the one of the previous algorithm.

Lemma 4.3.4 *Algorithm 4 computes the number of foremost journey routes from all vertices to all other vertices in $O(n(\mu'))$ time, and $O(n^3)$ space.*

Proof For every distinct foremost journey, possibly spanning different snapshots, an update (either normal or special) is ultimately applied for each of its nodes, for a total $O(n(\mu'))$ single updates. Note that the size of a journeys is bounded by nk because a node cannot appear more than k times in a journey.

As for the space complexity: as in the previous algorithm, matrix $Count_s$ has size $O(n^2)$ and the algorithm uses n of these matrices. ∎

Algorithm 4: Counting all foremost journeys in TVGs with zero latency and instant edges.

input : A TVG G, starting node $s \in V$, and snapshot interval i
output: $Count_s[v, u]$ that records the number of the journeys from $s \in V_G$ to all $u \in V_G$ passing through $v \in V_G$ at interval i

```
begin
    Initialize Count_s[.,.] ← 0
    Path.push(s)
    for all w ∈ Adj(s) do
        S.push(s, w)
    while S ≠ ∅ do
        (x, y) ← S.pop()
        while x ≠ Path.top() do
            Path.pop()
        if y ∉ Path then
            Path.push(y)
            if y falls in snapshot i then
                for each (y, w) such that w ∉ Path do
                    S.push(y, w)
                if path is foremost then
                    Count_s[z, y] =Normal Count Count_s[z, y] for all z ∈ Path
            else
                Count_s[z, y] =Special Count Count_s[z, y] for all z ∈ Path
```

We then obtain the same complexity of the previous algorithm to compute foremost betweenness for each node of the network.

Theorem 4.3.5 *Employing on Algorithm 5, foremost betweenness based on journey routes can be computed for every node in G in $O(n(\mu'))$ time, and $O(n^3)$ space, where μ' is the total number of different journey routes in G.*

Proof This theorem is the generalization of Lemma 4.3.4 for G. Note that μ' here refers to the number of journey routes in G, not in an interval. ∎

In this particular case, the size of a journey route can be bounded by nk, where k is the number of snapshots of $\mathcal{G}$.

Algorithm 5: SPECIAL COUNT.

input : $Count_s[.,x]$, in G_i, and $Count_x[.,y]$ in $\cup_{j>i} G_j$
output: $Count_s[v,y]$, $\forall v \in V$: number of foremost journey routes from s to $y \in V$, passing through $v \in V$

begin
  for *each* $v \in U \cup W$ *where* $U =$ *all nodes in* $x \rightsquigarrow y$ *and* $W =$ *all nodes in* $s \rightsquigarrow x$ **do**
    if $v \in s \rightsquigarrow x$ *and* $v \notin x \rightsquigarrow y$ **then**
      $Count_s[v,y] += Count_s[v,x] \times Count_x[y,y]$
    else if $v \notin s \rightsquigarrow x$ *and* $v \in x \rightsquigarrow y$ **then**
      $Count_s[v,y] += Count_s[x,x] \times Count_x[v,y]$
    else if $v \in s \rightsquigarrow x$ *and* $v \in x \rightsquigarrow y$ **then**
      $Count_s[v,y] += Count_s[x,x] \times Count_x[v,y] + Count_s[v,x] \times Count_x[y,y] - Count_s[v,x] \times Count_x[v,y]$

– **Practical Considerations: reducing time.** Algorithm 4 has to be executed in the chronological order of the time corresponding to the different snapshots, starting from the last one, since it uses the previously calculated results in the computation of the new results. Since the graph is divided into independent snapshots, the number of all journeys can be computed separately for each snapshot, and the result of the calculation can be aggregated at the end. This has the advantage of reducing the time complexity of the computation eliminating all the special updates from the first part of the algorithm (while detecting all the journey routes) and deferring it to the second part (when aggregating all the information for the final update). While not being theoretically advantageous, in our case that will be discussed in Chapter 5, this strategy results in a more efficient solution from a practical point of view due to the small number of intervals in our dataset. Thus, instead of performing the special count at each level, we can postpone it to the last step of the algorithm, and loop once through all the collected counts with hard-coded intervals in the loop.

4.4 Temporal Eigenvector Centrality

In this Section we introduce a temporal concept generalizing the common notion of Eigenvector Centrality for static graphs. Eigenvector centrality, as described in Section 2.2.1 is an important ranking measure in SNA. Computing eigenvector centrality depends on the adjacency matrix of the TVG. However, there is not a unique adjacency matrix defined for

TVGs. Nevertheless, in a most simplistic view, the TVG can be seen as a series of static graphs $\mathcal{S}_{\mathcal{G}}$. Suppose that we have an adjacency matrix $A(t)$ for a TVG, which spans over $\mathcal{T}$, defined as:

$$A(t) = (a_{ij}) \tag{4.1}$$

where

$$a_{ij}(t) = \begin{cases} 1, & \text{if } \rho(e(i,j),t) = 1 \\ 0, & \text{if } \rho(e(i,j),t) = 0 \end{cases} \tag{4.2}$$

therefore, the eigenvector centrality corresponding to such matrix will be equal to $\mathbf{x}(t)$ such that

$$A(t)\mathbf{x}(t) = \lambda(t)\mathbf{x}(t) \tag{4.3}$$

However, Equation 4.3 provides a series of eigenvectors each corresponding to the static snapshot G_t. Hence, computation of temporal eigenvector centrality requires a mathematical remodelling of adjacency matrices for TVGs such that we have a single matrix representing the whole TVG. The challenging aspect of this modelling is to preserve the importance and status of each edge in the transformation. We view this problem from two points of view, and each view results in a model. Note that in both models we assume full connectivity over the TVG. This assumption also stands as an assumption for computation of eigenvector centrality in static graphs so it is reasonable to carry it on for the dynamic case.

4.4.1 Adjacent Degree Induced Eigenvector Centrality (ADI)

In the first model, in an attempt to preserve the importance of each vertex in the aggregated adjacency matrix, we include the degree of each vertex v_j that is linked to v_i as an indicator of v_i's importance, and create a matrix $A = (a_{ij}) \geq 0$ corresponding to the TVG such that:

$$a_{ij} = \sum_t a_{ij}(t)\hat{deg}(j,t) \tag{4.4}$$

where $deg_t(j)$ refers to the degree of j at time t. The reasoning behind this is that, based on Katz centrality measure (Section 2.2.1), each vertex receives its importance from what it is connected to. Therefore, if v_i is connected to a node with high importance, it will be important as well.

Since the degree of a vertex cannot be negative, $a_{ij} \geq 0$ is true for all a_{ij}. Due to the assumption about the connectivity of the TVG, the projected adjacency matrix A is connected as well, and thus primitive. The fact that A is primitive confirms that A is irreducible. If A is irreducible, A is known to have largest eigenvalue λ_{max} such that all components of its corresponding eigenvector $\mathbf{x}$ are all positive [64]. Thus, we conclude that solving Equation 4.5 provides the eigenvector centrality values for A and, hence, for the TVG.

$$A\mathbf{x} = \lambda_{max}\mathbf{x} \tag{4.5}$$

such that

$$x_i = \frac{1}{\lambda_{max}} \sum_{j=1}^{n} a_{ij}x_j \tag{4.6}$$

which in fact is

$$x_i = \frac{1}{\lambda_{max}} \sum_{j=1}^{n} \left[\sum_{t} a_{ij}(t)\hat{deg}(j,t) \right] x_j \tag{4.7}$$

Temporal eigenvector centrality of v in ADI eigenvector centrality model is denoted by $C_{\hat{E_1}}(v) = x_v$.

In this model, we need to calculate the degree of each vertex. Depending on the data structure, the computational complexity varies. The best possible data structure is to store edge list for each vertex, so the degree of each vertex can be retrieved at $O(1)$. Then, at each snapshot, we need to extract the degrees corresponding to the target u of each edge that is started at vertex v. This process needs $O(|E|)$ time complexity for all the vertices in an snapshot. Adding up all the results extracted at each snapshot will result in addition of a component of distinct lifetimes of the system $|\mathcal{S}_{\mathcal{T}}(\mathcal{G})|$ to the complexity, and including the eigenvector computation we have $O(|E||\mathcal{S}_{\mathcal{T}}(\mathcal{G})| + |V|)$ as the complexity of algorithm.

Due to the connectivity of the graph, $O(|E|) \geq O(|V|)$ always holds. Therefore, the general complexity of computing the temporal eigenvector centrality for this model is

$O(|E||\mathcal{S}_{\mathcal{T}}(\mathcal{G})|)$.

Although this model represents the ranking of a node in a temporal fashion, it fails to generalize the common static measure with temporarily. Thus, we propose the second model for eigenvector centrality computation in TVGs that is a direct generalization of the static measure.

4.4.2 Self Degree Induced Eigenvector Centrality (SDI)

Also in this case, we make the same connectivity assumption. The SDI model is similar to ADI model in many ways, but the main difference between two is on the method used to preserve the temporal importance. In this model, we preserve the importance of each vertex in the aggregated adjacency matrix, by including its own degree as an indicator of its importance, and create a matrix $A = (a_{ij}) \geq 0$ corresponding to the TVG such that:

$$a_{ij} = \sum_{t} a_{ij}(t) \tag{4.8}$$

By referring to the eigenvector centrality measure in static graphs (Section 2.2.1), we conclude that vertices can perceive the importance of their neighbours and implicitly affect it and get affected by it. Therefore, the importance is automatically transferred over the links in the graph. Since the degree of a vertex cannot be negative, $a_{ij} \geq 0$ is true for all a_{ij}, and, in the same way as for the previous model, we can conclude that A corresponding to the TVG is irreducible. Thus, A has a largest eigenvalue λ_{max} such that all components of its corresponding eigenvector $\mathbf{x}$ are all positive [64], and we can conclude that solving Equation 4.9 provides the eigenvector centrality values for A and, hence, for the TVG.

$$A\mathbf{x} = \lambda_{max}\mathbf{x} \tag{4.9}$$

such that

$$x_i = \frac{1}{\lambda_{max}} \sum_{j=1}^{n} a_{ij} x_j \tag{4.10}$$

which in fact is

$$x_i = \frac{1}{\lambda_{max}} \sum_{j=1}^{n} \left[\sum_{t} a_{ij}(t) \right] x_j \tag{4.11}$$

Temporal eigenvector centrality of v in SDI eigenvector centrality is denoted by $C_{\hat{E}_2}(v) = x_v$. Note that, if the graph is static (i.e. $|\mathcal{S}_\mathcal{T}(\mathcal{G})| = 1$), the equations 4.8 and 4.9 coincide with the definition of eigenvector centrality in static graphs (Section 2.2.1).

This model is very similar to the ADI model, but it does not need the factor $O(|E|)$ for extracting the target degrees. Therefore, the complexity based on the model is $O(|\mathcal{S}_\mathcal{T}(\mathcal{G})| + |V|)$.

4.4.3 Examples

We illustrate the measures with an example (see Figure 4.3). In general, the presented TVG has the following set of adjacency matrices for times t from 1 to 5. This is because, as mentioned earlier, a TVG can be present as a series of snapshots. Therefore, each $A(t = i)$ represents the adjacency matrix for the graph in the snapshot corresponding to $t = i$.

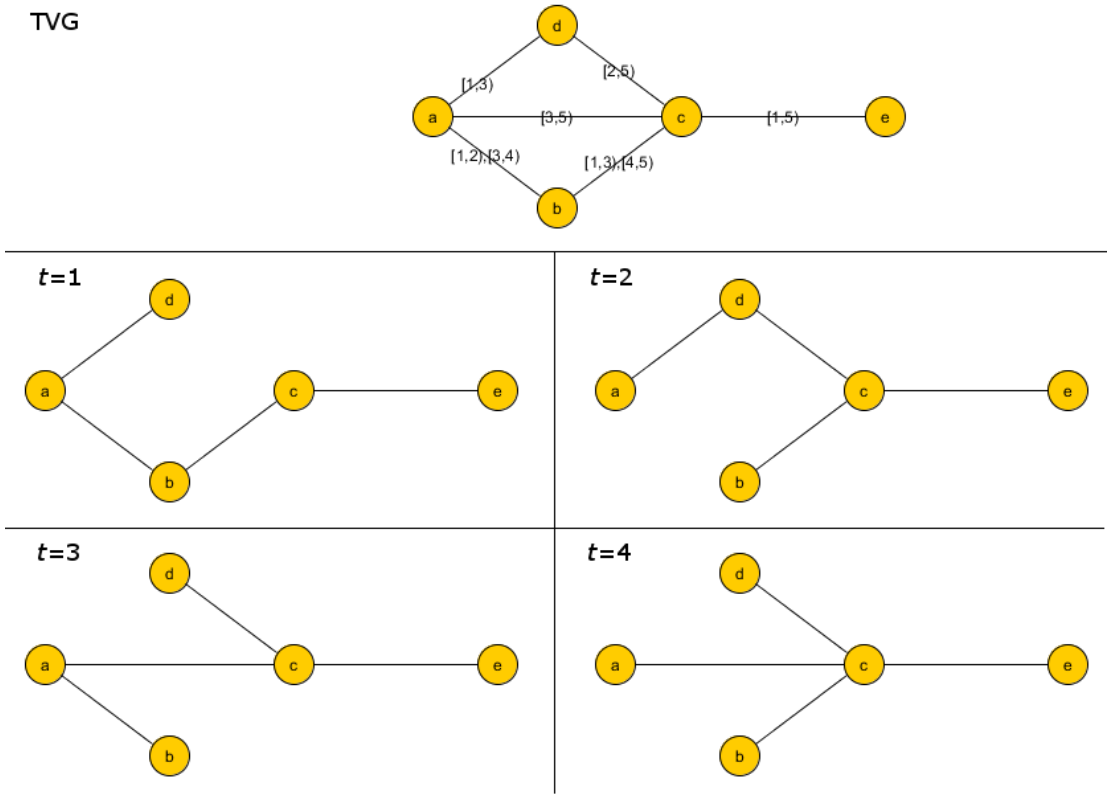


Figure 4.3: Temporal eigenvector centrality: the figure represents a TVG and its snapshots at 4 different timeslots

$$A(t=1) = \begin{array}{c} \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} \\ \begin{matrix} a & b & c & d & e \end{matrix} \end{array} \qquad A(t=2) = \begin{array}{c} \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} \\ \begin{matrix} a & b & c & d & e \end{matrix} \end{array}$$

$$A(t=3) = \begin{array}{c} \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} \\ \begin{matrix} a & b & c & d & e \end{matrix} \end{array} \qquad A(t=4) = \begin{array}{c} \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} \\ \begin{matrix} a & b & c & d & e \end{matrix} \end{array}$$

Based on ADI eigenvector centrality, we generate matrix A in accordance to the procedure described in Equation 4.7.

$$A = \begin{array}{c} \begin{pmatrix} 0 & 3 & 7 & 3 & 0 \\ 3 & 0 & 9 & 0 & 0 \\ 3 & 4 & 0 & 4 & 4 \\ 3 & 0 & 10 & 0 & 0 \\ 0 & 0 & 12 & 0 & 0 \end{pmatrix} \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} \\ \begin{matrix} a & b & c & d & e \end{matrix} \end{array}$$

The eigenvector corresponding to the largest eigenvalue ($\lambda = 14.66$) of matrix A is vector $\mathbf{x}$ as following, and as expected, the importance of c is more than others.

$$\mathbf{x} = \begin{pmatrix} 0.47 \\ 0.50 \\ 0.65 \\ 0.27 \\ 0.18 \end{pmatrix} \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix}$$

For SDI eigenvector centrality, the matrix A is compiled as:

$$A = \begin{array}{c} \left(\begin{array}{ccccc} 0 & 2 & 2 & 2 & 0 \\ 2 & 0 & 3 & 0 & 0 \\ 2 & 3 & 0 & 3 & 4 \\ 2 & 0 & 3 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \end{array} \right) \begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} \\ \begin{array}{ccccc} a & b & c & d & e \end{array} \end{array}$$

The eigenvector corresponding to the largest eigenvalue ($\lambda = 7.08$) of matrix A is vector $\mathbf{x}$ as following, and as expected, the importance of c is more than others.

$$\mathbf{x} = \begin{pmatrix} 0.40 \\ 0.38 \\ 0.64 \\ 0.38 \\ 0.36 \end{pmatrix} \begin{array}{c} a \\ b \\ c \\ d \\ e \end{array}$$

It is important to note that the rankings in the ADI eigenvector centrality is slightly different from rankings in SDI eigenvector centrality due to differences in the models. In our idea, the ADI eigenvector centrality ranks the vertices more rationally as it differentiates between b and d. However, SDI eigenvector centrality, as mentioned earlier, generalizes the static concept more into the TVG, and it ranks most important vertices realistically.

4.5 Conclusion

In this Chapter we discussed the computability and complexity of temporal parameters and, in particular, of temporal betweenness. Our analysis showed that computing foremost betweenness centrality is a #P-complete problem, while shortest temporal betweenness can be computed in polynomial time. We first described a polynomial algorithm to compute shortest temporal betweenness, we then proposed some exponential solutions to compute foremost betweeness, first in the general setting, then in the particular setting that will be treated in the next Chapter. Both solutions have an (inevitable) exponential time worst case complexity; the second solution however, resulted in a much faster execution

in a practical setting. Finally, we introduced a new temporal metric, called *temporal eigenvector centrality*, an adaptation of eigenvector centrality, which is defined for static graphs, to the case of TVGs. We defined two approaches for the calculation of such a measure, one based on the degree of vertices at different times, and the other based on the degree of neighbours at various times.

Both solutions proposed in this Chapter to compute foremost betweenness are unfeasible in any large social network. To be able to use this parameter in most networks, it would be necessary to design good approximation algorithms to compute foremost betweenness in polynomial time. This is the most interesting open problem stemming from this Chapter.

Chapter 5

Temporal Analysis of a Knowledge Mobilization Network

In this Chapter we are interested in understanding knowledge mobilization dynamics on a social network that describes a research community in a seven year period (*Knowledge-Net*). *Knowledge-Net* has been already studied by employing classical parameters on the aggregated static graph that describes its overall structure [54]. The goal of this chapter is to perform a temporal analysis to better understand the temporal dimension of knowledge mobilization in this context. We consider temporal betweenness and we compare the results that we obtain with the static ones. In particular, we observe the emergence of important actors, whose central role was invisible in the static analysis. The results show that this form of temporal betweenness is effective at highlighting the role of nodes whose importance has a temporal nature (e.g., nodes that contribute to mobilization acceleration). The results of this Chapter have been published in [5].

5.1 Introduction

Knowledge Mobilization (KM) refers to the use of knowledge towards the achievement of goals [53]. Scientists, for example, use published papers to produce new knowledge in further publications to reach professional goals. In contrast, patient groups can use scientific knowledge to help foster change in patient practices, and corporations can use scientific knowledge to reach financial goals. Recently, researchers have started to analyse knowledge mobilization networks (KMN) using a social network analysis (SNA) approaches (e.g., see [17, 18, 26, 41, 69]). In particular, [54] proposed a novel approach where a heterogeneous

network composed of a main class of actors subdivided into three sub-types (individual human and non-human actors, organizational actors, and non-human mobilization actors) associated according to one relation, knowledge mobilization (a Mobilization-Network approach). Data covered a seven-year period with static networks for each year. The mobilization network was analysed using classical SNA measures (e.g., node centrality measures, path length, density) to produce understanding for KM using insights from network structure and actor roles [54].

The KM SNA studies mentioned above, however, lack a fundamental component: in fact, their analysis is based on a static representation of KM networks, incapable of sufficiently accounting for the time of appearance and disappearance of relations between actors beyond static longitudinal analysis. Indeed, incorporating the temporal component into analysis is a challenging task, but it is undoubtedly a critical one, because time is an essential feature of these networks. As mentioned in the introduction of the thesis, temporal analysis of dynamic graphs is an important and extensively studied area of research (e.g., see [52, 66, 70, 71, 101, 110, 113]), but there is still much to be discovered. In particular, most temporal studies simply consider network dynamics in successive static snapshots thus capturing only a very partial temporal component by observing how static parameters evolve in time while the network changes.

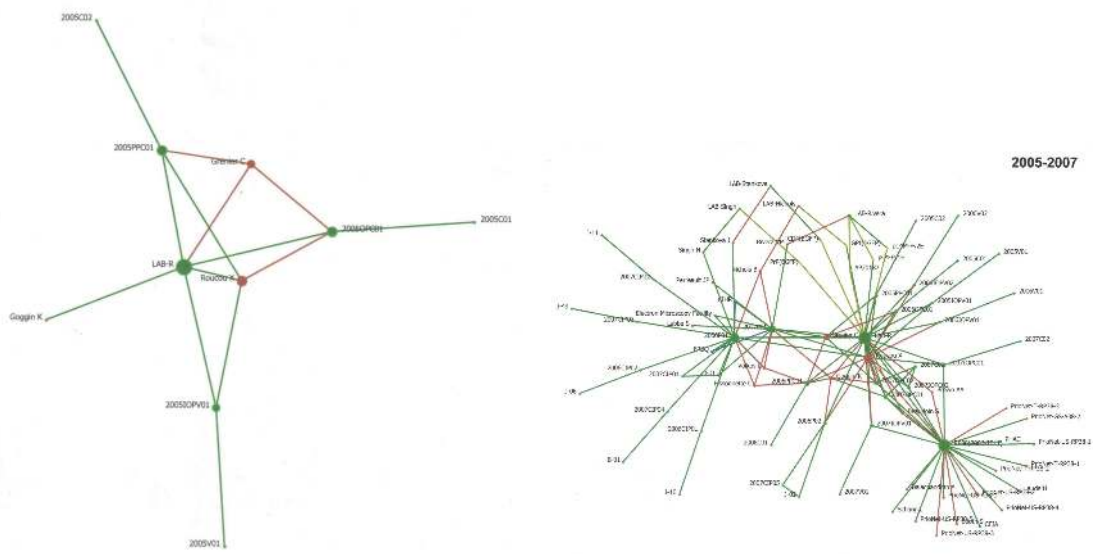
In this chapter, we represent KMN by TVGs and we propose to analyse them in a truly temporal setting. We provide, for the first time on a real data set, an empirical indication of the effectiveness of a temporal betweenness measure specifically designed for TVGs. In particular, we focus on data extracted from [54], here referred to as *Knowledge-Net*.

We first follow the classical approach by considering static snapshots of Knowledge-Net corresponding to the seven years of its existence, and by studying the classical centrality measures in those time intervals, we provide rudimentary indications of the networks' temporal behaviour.

To gain a finer temporal understanding, we then concentrate on *temporal betweenness* following a totally different approach. Instead of simply observing the static network over consecutive time intervals, we focus on the TVG that represent Knowledge-Net and we compute a form of betweenness centrality measure that explicitly and globally takes time into account. We compare the temporal results that we obtain with classical static measures to gain insights into the impact that time has on the network structure and actor roles. We notice that, while many actors maintain the same role in static and dynamic analysis, some display striking differences. In particular, we observe the emergence of important actors that remained invisible in static analysis, and we advance explanations for these.

5.2 Knowledge-Net Data description

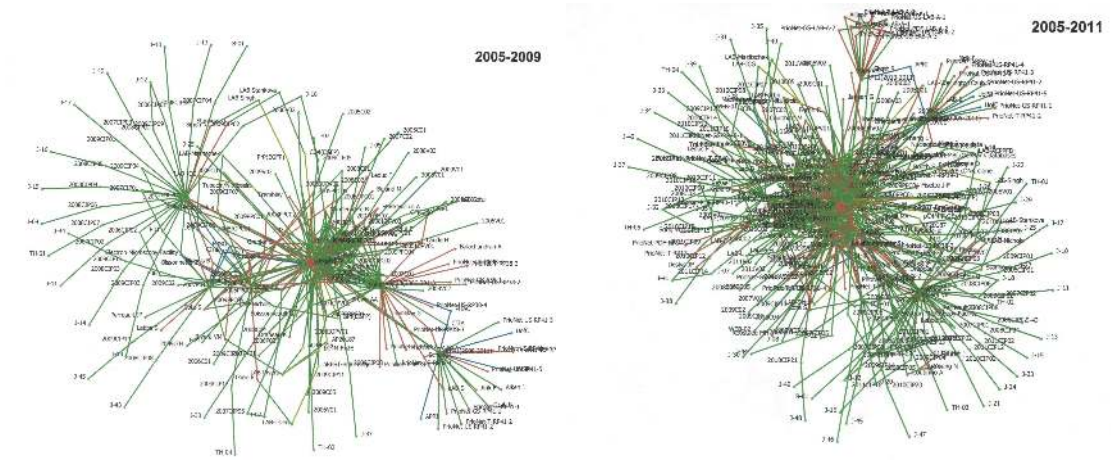
Knowledge-Net is an heterogeneous network where nodes represent human and non-human actors (researchers, projects, conference venues, papers, presentations, laboratories), and edges represent knowledge mobilization between two actors. The network was collected for a period of seven years [54]. Once an entity or a connection is created, it remains in the system for the for entire period of the analysis. The monotonic growth of the network that is apparent in Figure 5.1 and 5.2 is a result of this feature of the network. Meanwhile, the sparse knowledge-net network in 2005, composed of ten vertices transforms into a dense network in 2011 due to the same reason of links staying in the network.



(a) 2005 (b) 2007

Figure 5.1: Growth dynamics of knowledge-net over time.

Table 5.1 provides a description of the *Knowledge-Net* dataset. The dataset consists of 366 vertices and 750 edges in 2011. The number of entities and connections vary over times starting from only 10 vertices and 14 edges in 2005 and accumulating to the final network year in 2011. *Knowledge-Net* is mainly comprised of non-human actors, 272 in total (non-human mobilization actors, NHMA, non-human individual actors, NHIA, and organizational actors, OA), in relation with 94 human actors (HA). Human actors include principle investigators (PI), highly qualified personnel (HQP) and collaborators (CO). It is through non-human mobilization actors (NHMA) that individual, organizational actors and mobilization actors associate and mobilize knowledge to reach goals. For example, scientists mobilize knowledge through articles where not all contributing authors might be in relation with all other authors, yet all relate with the publication [54]. These non-



(a) 2009 (b) 2011

Figure 5.2: Growth dynamics of knowledge-net over time.

human mobilization actors make up the bulk of the network including conference venues, presentations (invited oral, non-invited oral and poster), articles, journals, laboratories, research projects, websites, and theses.

Classical statistical parameters have been calculated for Knowledge-Net, representing it as a static graph where the time of appearance of nodes and edges did not hold any particular meaning. In doing so, several interesting observations were made regarding the centrality of certain nodes as knowledge mobilizers and the presence of communities [54]. In particular, all actor types increased in number over the 7 years indicating a rise in new mobilization relations over time. Although non-human individual actor's absolute numbers remained small (ranging from 3 in 2006 to 15 in 2011), these actors were critical to making visible tacit (non-codified) knowledge mobilization from around the world (mostly laboratory material sharing, including from organizations and universities in the USA, from Norway, and from Canadian universities). Finally, embedded in human individual actor counts were individuals that the laboratory acknowledged in peer-reviewed papers, thus making further tacit and explicit knowledge mobilization visible.

5.3 Design of The Study

The Knowledge-net dataset can be studied for different static and temporal measures. We define the dataset in the sequence of snapshots for easier manipulation, so an edge between two entities exists only for one time unit since its creation - a year in this case. In this model, when knowledge is mobilized between two entities, its actual mobilization is

Table 5.1: *Knowledge-Net* data set with characteristics of actors and their roles at different times

Start	Duration	#Nodes	#Edges	Granularity
2005	7 Years	366	750	1 Year

Actor Type	2005	2006	2007	2008	2009	2010	2011
HIA	3	22	27	46	51	76	94
NHIA	0	3	6	9	9	9	15
NHMA	7	25	43	87	132	194	248
OA	0	5	5	9	9	9	2
Total	10	55	81	151	201	288	366

limited to the period in which the mobilization edge exists (when the mobilization exchange occurred), but its effects continues to be seen in the future.

For this dataset, we implement betweenness centrality analysis. This allows us to understand the effects of each measure in this network.

When representing *Knowledge-Net* as a TVG $\mathcal{G}$ we notice that, due to zero latency and to the fact that edges never disappear once created, any shortest journey route in $\mathcal{G}$ is equivalent to a shortest path on the static graph corresponding to its footprint; moreover, the notion of fastest journey does not have much meaning in this context, because on any route corresponding to a journey, there would be a fastest one. On the other hand, the notion of foremost journey, and in particular of foremost increasing journey, is extremely relevant as it describes timely mobilization flow, i.e., flow that arrives at a node as early as possible.

5.4 Analysis of consecutive snapshots

To provide more clear statistics on the Knowledge-Net dataset and a ground for better understanding of temporal metrics, we first calculated classical statistical measures (e.g., node centrality measures, path length, density) on seven static graphs, corresponding to the seven years of study. The average for each value for the graphs is calculated to represent a benchmark on how the rank for each node is compared to others.

Table 5.2: Some static statistical parameters calculated for successive snapshots

	2005	2006	2007	2008	2009	2010	2011
Ave. Degree	1.40	1.32	1.63	1.84	1.98	2.02	2.04
Diameter	4	5	5	6	6	6	6
Density	0.31	0.04	0.04	0.02	0.02	0.01	0.01
#Communities	4	3	6	8	8	15	12
Modularity	0.17	0.52	0.46	0.47	0.46	0.54	0.54
Ave. Clustering Coefficient	0.41	0.06	0.21	0.22	0.20	0.24	0.23
Ave. Path Length	2.04	3.04	3.06	3.26	3.34	3.46	3.50
Ave. Normalized Closeness	0.51	0.33	0.33	0.31	0.30	0.29	0.29
Ave. Eccentricity	3.10	4.41	4.40	4.70	4.80	4.83	4.83
Ave. Betweenness	4.70	58.36	83.53	169.70	234.89	354.23	456.18
Ave. Normalized Betweenness	0.13	0.03	0.02	0.01	0.01	≈ 0	≈ 0
Ave. Page Rank	0.10	0.01	0.01	≈ 0	≈ 0	≈ 0	≈ 0
Ave. Eigenvector	0.52	0.19	0.15	0.10	0.09	0.07	0.05

The statistical data presented in Table 5.2 provides valuable information about the graph. The steady decrease in the (normalized) centrality values confirms that the net-

work growth is not symmetric, so the centrality values have long tails. The low value of normalized betweenness, along with the low values for density, confirms that the graph is coupled in a way that there are great number of shortest paths between any two arbitrary vertices in the graph. This caused the betweenness for most vertices to be similar and quite low when compared to the ones of nodes with the highest betweenness. Low average path length is a sign that the network presents small world characteristics and the knowledge mobilization to the whole network is expected to be conducted only in a few hops. Meanwhile, the decreasing graph density along with the increasing average degree represent the slow growth in the number of edges compared to the number of nodes. Escalation in the number of communities with increase in graph modularity metrics shows that the knowledge mobilization actors tend to form communities as time progresses. As the normalized average betweenness decreases steadily, it can be concluded that a few vertices at each community play the role of mediators and create the link between communities.

Apart from these general observations, a static analysis of consecutive snapshots, does not provide deep temporal understanding. For example, it does not reflect which entities engage in knowledge mobilization in a timely fashion, e.g. by facilitating fast mobilization, or slowing mobilization flow.

To tackle some of these questions, we represent *Knowledge-Net* as a TVG and we propose to study it by employing a form of temporal betweenness centrality measure that makes use of time in an explicit manner.

5.5 Temporal Growing Betweenness Centrality

Calculation of betweenness in static and temporal graphs is extensively defined in Section 3.3.5, and analysed in Chapter 4. TVGs are attributed by vertices and edges that appear and disappear at different times. There are, however, situations where those graph elements do not disappear once they are created. A very common example could be academic networks in which the papers and citations stay in the network once they appeared in it. Once authors publish a paper, and cites a colleagues paper, they cannot un-publish or not cite it in a later time. Thus, the link exists in the TVG from its birth until the end of system. This notion is different from generic TVG in which, for instance, a cellular device connects to an access point and disconnects from it at a later time. In terms of information travelling and influence propagation, the aforementioned networks behave significantly differently. In the latter case, the existence of the edge, including its every reappearance, is an opportunity for information dissemination over the edge, but

the same is not true for the described academic network. The existence of the edge after its first appearance, provided that there is no reappearances, does not convey any tool for information dissemination, and therefore should not be included in the possible journeys, and, thus, be removed from betweenness computation.

Considering these factors, we develop a betweenness measure that we call *growing betweenness*. A growing journey ($\mathcal{J}_{\mathcal{G}}$) is a journey route with non decreasing time associated to its edges' first appearance. In other words, a growing journey route $[(e_1, t_1), \ldots, (e_i, t_i), \ldots, (e_k, t_k)]$ is such that $\rho(e_i, t_i) = 1$, $t_i = birth(e_i)$, and $t_{i+1} \geq t_i + \zeta(e_i, t_i)$ for all $i < k$. The growing journey can have variations as foremost growing journey, fastest growing journey, and shortest growing journey. A growing betweenness measure is a temporal measure that computes the betweenness based on growing journeys. The computation of growing betweenness is the same as the one explained in Chapter 3. Equation 5.1 is provided as a reminder in which $|d(x, y)|$ and $|d'(x, y, v)|$ refer to growing journeys.

$$C_{B_{\hat{d}}}^{\mathcal{J}}(v) = \sum_{u \neq w \neq v \in V} \frac{|\mathcal{J}_{\hat{d}}(u, w, v)|}{|\mathcal{J}_{\hat{d}}(u, w)|} \tag{5.1}$$

where $|\mathcal{J}_{\hat{d}}(u, w)|$ is the number of shortest journeys between u and w in the TVG, and $|\mathcal{J}_{\hat{d}}(u, w, v)|$ is the number of shortest journeys passing through v. The fastest $C_{B_{\hat{l}}}^{\mathcal{J}}(v)$ and foremost $C_{B_{\hat{a}}}^{\mathcal{J}}(v)$ betweenness can easily be calculated by replacing $\mathcal{J}_{\hat{d}}$ in Equation 3.9 by $\mathcal{J}_{\hat{l}}$ and $\mathcal{J}_{\hat{a}}$ respectively.

In the following we will refer to growing betweenness simply as betweenness.

5.6 Foremost Betweenness of Knowledge-Net

In this Section we focus on *Knowledge-Net*, and we study $C_{B_{\hat{a}}}^{\mathcal{J}}(v)$ for all v. Nodes are ranked according to their betweenness values and their ranks are compared with the ones obtained calculating their static betweenness $C_B(v)$ in the same time frame. Given the different meaning of those two measures, we expect to see the emergence of different behaviours, and, in particular, we hope to be able to detect nodes with important temporal roles that were left undetected in the static analysis.

5.6.1 Foremost Betweenness during the lifetime of the system

Table 5.3 shows the temporally ranked actors accompanied by their static ranks, and the high ranked static actors with their temporal ranks, both with lifetime $\mathcal{T}$ = [2005-2011]. In

our naming convention, an actor named $Xi(yy)$ is of type X, birth date yy and it is indexed by i; types are abbreviated as follows: H (human), L (Lab), A (article), C (conference), J (journal), P (project), C (paper citing a publication), I (invited oral presentation), O (oral presentation). Note that only the nodes whose betweenness has a significant value are considered, in fact betweenness values tend to lose their importance, especially when the differences in the values of two consecutive ranks are very small [50].

Interestingly, the four highest ranked nodes are the same under both measures; in particular, the highest ranked node (L1(05)) corresponds to the main laboratory where the data is collected and it is clearly the most important actor in the network whether considered in a temporal or in a static way. On the other hand, the table reveals several differences worth exploring. From a first look we see that, while the vertices highest ranked statically appear also among the highest ranked temporal ones, there are some nodes with insignificant static betweenness, whose temporal betweenness is extremely high. This is the case, for example, of nodes S1(10) and J1(06).

The case of node S1(10)

To provide some interpretation for this behaviour we observe vertex S1(10) in more details. This vertex corresponds to a poster presentation at a conference in 2010. We explore two insights. First, although S1(10) has a relatively low degree, it has a great variety of temporal connections. Only three out of ten incident edges of S1(10) are connected to actors that are born on and after 2010, and the rest of the neighbours appear in different times, accounting for at least one neighbour appearing each year for which the data is collected. This helps the node to operate as a temporal bridge between different time instances and to perhaps act as a knowledge mobilization accelerator.

Second, S1(10) is close to the centre of the only static community present in [2010-2011] and it is connected to the two most important vertices in the network. The existence of a single dense community, and the proximity to two most productive vertices can explain its negligible static centrality value: while still connecting various vertices S1(10) is not the shortest connector and its betweenness value is thus low. However, a closer temporal look reveals that it plays an important role as an interaction bridge between all the actors that appear in 2010 and later, and the ones that appear earlier than 2010. This role remained invisible in static analysis, and only emerges when we pay attention to the time of appearance of vertices and edges. On the basis of these observations, we can interpret S1(10)'s high temporal betweenness value as providing a fast bridge from vertices created earlier and those appearing later in time. This lends support to the importance of poster

Table 5.3: List of highest ranked actors according to temporal (resp. static) betweenness, accompanied by the corresponding static (resp. temporal) rank in lifetime [2005-2011].

Temporal to Static				Static to Temporal		
Actor	Temporal Rank	Static Rank		Actor	Static Rank	Temporal Rank
L1(05)	1	1		L1(05)	1	1
H1(05)	2	2		H1(05)	2	2
A1(06)	3	3		A1(06)	3	3
A2(08)	4	4		A2(08)	4	4
P1(06)	5	8		A5(08)	5	12
A3(07)	6	9		A4(09)	6	7
A4(09)	7	6		P2(08)	7	9
S1(10)	8	115		P1(06)	8	5
P2(08)	9	7		A3(07)	9	6
J1(06)	10	160		P3(10)	10	17
C1(07)	11	223		A6(11)	11	18
A5(08)	12	5		A8(09)	12	36
I1(09)	13	28		P4(10)	13	22
O1(05)	14	45		P5(11)	14	27
S2(05)	15	46		H2(05)	15	44
I2(05)	16	47		A7(09)	16	21
P3(10)	17	10		A9(10)	17	31
A6(11)	18	11		P5(11)	18	69
C2(10)	19	133		P6(10)	19	23
J2(09)	20	182				
A7(09)	21	16				

presentations that can blend tacit and explicit knowledge mobilization in human - poster presentation - human relations during conferences and continue into future mobilization with new non-human actors as was the case for S1(10) [12].

The case of node J1(06)

J1(06), the *Journal of Neurochemistry*, behaves similarly to S1(10) with its high temporal and low static rank. As opposed to S1(10), this node is introduced very early in the network (2006); however, it is only active (i.e. has new incident edges) in 2006 and 2007. It has only three neighbours, A1(06), A3(07), and C1(07), all highly ranked vertices statically (A1(06), A3(07)), or temporally (C1(07)). Since its neighbouring vertices are directly connected to each other or in close proximity of two hops, J1(06) fails to act as a static short bridge among graph entities. However, its early introduction and proximity to the most prominent knowledge mobilizers helps it become an important temporal player in the network. This is because temporal journeys overlook geodesic distances and are instead concerned with temporal distances for vertices. These observations might explain the high temporal rank of J1(06) in the knowledge mobilization network.

5.6.2 A Finer look at foremost betweenness

A key question is whether the birth-date of a node is an important factor influencing its temporal betweenness. To gain insights, we conducted a finer temporal analysis by considering $C^{\mathcal{J}}_{B_{\hat{a}}}(v)$ for all possible birth-dates, i.e, for $\mathcal{T}$ = [x, 2011], $\forall x \in \{2005, 2006, 2007, 2008, 2009, 2010, 2011\}$. This allowed us to observe how temporal betweenness varies depending on the considered birth-date.

Before concentrating on selected vertices (statically or temporally important with at least one interval), and analysing them in more detail, we briefly describe a temporal community detection mechanism that we employ in analysis.

Detection of temporal communities

We approximately detect communities existing in temporal networks. To detect communities involving x, we first determine the temporal foremost journeys arriving at or leaving from x. We then replace each journey with a single edge, creating a static graph with an edge between x and all the vertices that are reachable from or can reach x in a foremost manner. For instance, Figure 5.3 shows the transformation of a graph into a directed

weighted graph that is used for community detection. We finally apply existing directed weighted community detection algorithms to compute communities around x [56]. The model is an approximation since it overlooks the role that is played in communities by vertices that fall along journeys while not being their start or end-points; however, it is sufficient for our purposes to give an indication of the community formation around a node.

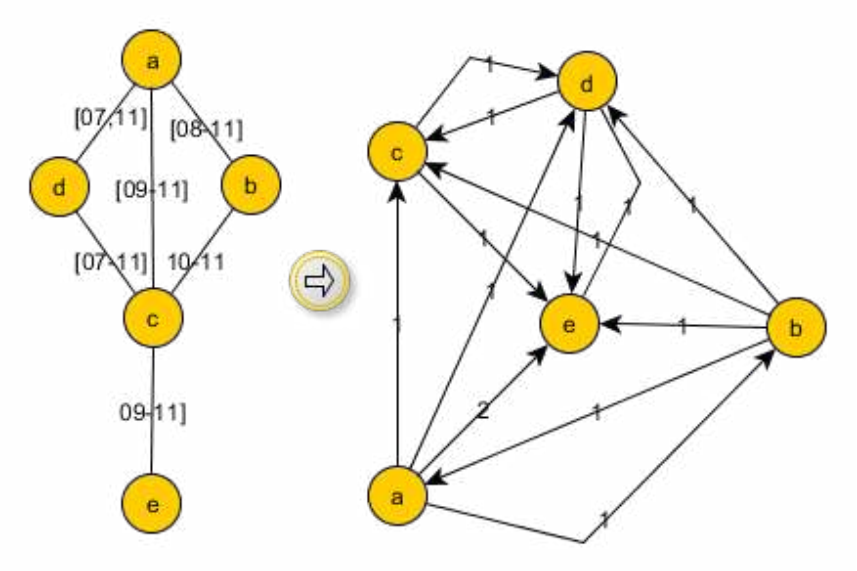


Figure 5.3: Transformation of a temporal graph into a weighted graph used for community detection.

The case of node P1(06)

This is a research project led by the principle investigator at L1(05). The project was launched in 2006 and its official institutional and funded elements wrapped-up in 2011. Data in Table 5.3 support that P1(06) has similar temporal and static ranks with regards to its betweenness in lifetime [2005-2011]. One could conclude that the temporal element does not provide additional information on its importance and that the edges that are incident to P(06)-1 convey the same temporal and static flow. However, there is still an unanswered question on whether or not edges act similarly if we start observing the system at different times. Will a vertex keep its importance throughout the system's lifetime?

The result of such analysis is provided in Figure 5.4, where $C^{\mathcal{J}}_{B_{\hat{a}}}(P1(06))$ is calculated for each birth-date (indicated in the horizontal axis), with all intervals ending in 2011.

While both equally important during the entire lifetime [2005-2011] of the study, this project seems to assume a rather more relevant temporal role when observing the system

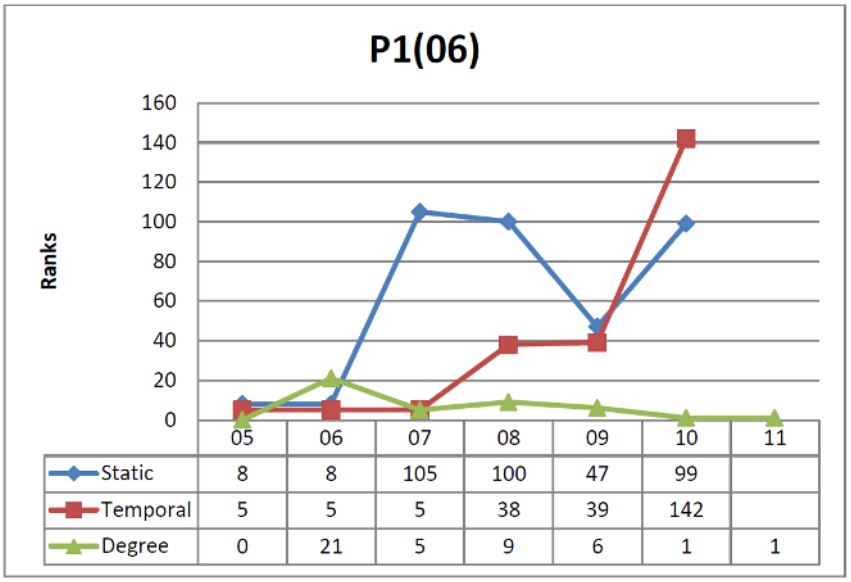


Figure 5.4: Comparison between different values for vertex P1(06). Ranks of the vertex in the last interval are not provided as both betweenness values are zero.

in a lifetime starting in year 2007 (i.e., $\mathcal{T}$ =[2007-2011]), when its static betweenness is instead negligible. This seems to indicate that the temporal flow of edges incident to P1(06) appearing from 2007 on is more significant than the flow of the edges that appeared previously.

With further analysis of P1(06)'s neighbourhood in [2007-2011], we can formulate technical explanations for this behaviour. First, its direct neighbours also have better temporal betweenness than static betweenness. Moreover, its neighbours belong to various communities, both temporally and statically. However, looking at the graph statically, we see several additional shortest paths that do not pass through P1(06) (thus making it less important in connecting those communities). In contrast, looking at the graph temporally P1(06) acts as a mediator and accelerator between communities. More specifically, we observe that the connections P1(06) creates in 2006 contribute to the merge of different communities that appear only in 2007 and later. When observing within interval [2006-2011], we then see that P1(06) is quite central from a static point of view, because the appearance of time of edges does not matter but, when observing it in lifetime [2007-2011] node P1(06) loses this role and becomes statically peripheral because the newer connections relay information in an efficient temporal manner.

In other words, it seems that P1(06) has an important role for knowledge acceleration in the period 2007-2011, a role that was hidden in the static analysis and that does not emerge even from an analysis of consecutive static snapshots. For research funders, revealing a

research project's potentially invisible mobilization capacity is relevant. Research projects can thus be understood beyond mobilization outputs and more in terms of networked temporal bridges to broader impact.

The case of node A3(07)

The conditions for A3(07), a paper published in 2007, illustrate a different temporal phenomenon. Node A3(07) has several incident edges in 2007 (similarly to node P1(06)) when both betweenness measures are high. Peering deeper into the temporal communities formed around A3(07) is revealing: up to 2007, this vertex is two degrees from vertices that connect two different communities in the static graph. The situation radically changes however with the arrival of edges in 2008 that modify the structure of those communities and push A3(07) to the periphery. The shift is dramatic from a temporal perspective because A3(07) loses it accelerator role where its temporal betweenness becomes negligible, while statically there is only a slight decrease in betweenness. The reason for a dampened decrease in static betweenness is that this vertex is close to the centre of the static community, connecting peripheral vertices to the most central nodes of the network (such as L1(05) and H1(05)). It is mainly proximity to these important vertices that sustains A3(07)'s static centrality.

Such temporal insights lend further support to understanding mobilization through a network lens coupled with sensitivity to time. A temporal shift to the periphery for an actor translates into decreased potential for sustained mobilization.

5.7 Invisible Rapids and Brooks

On the basis of our observations, we define two concepts to differentiate the static and temporal flow of vertices in Knowledge Mobilization networks. We call *rapids* the nodes with high foremost betweenness, meaning that they can potentially mobilize knowledge in a timelier manner; and *brooks* the ones with low foremost betweenness. Moreover, we call *invisible rapids* vertices whose temporal betweenness rank is considerably more significant than their static rank and were thus undetected by the static betweenness centrality measure, and *invisible brooks* the ones whose static betweenness is considerably higher than their temporal betweenness, meaning that these vertices can potentially be effective knowledge mobilizers, yet they are not acting as effectively as others due to slow or non-timely relations.

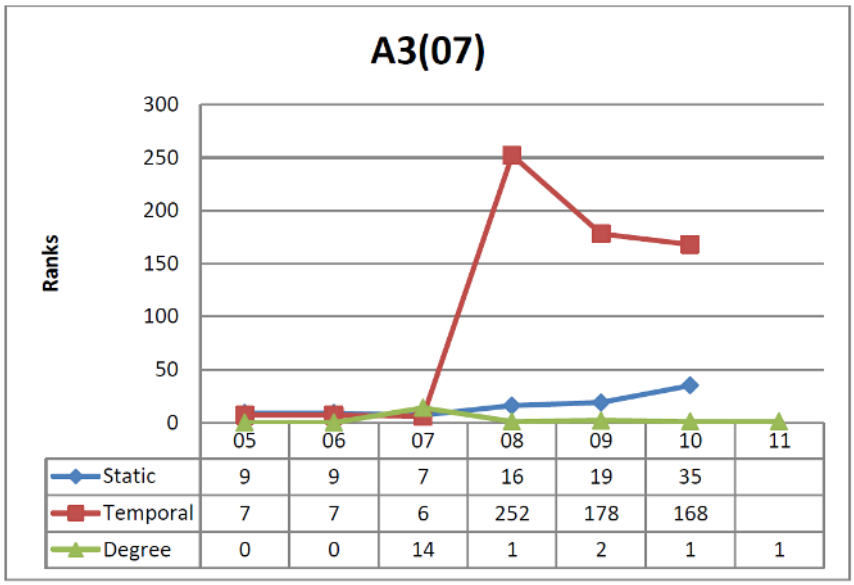


Figure 5.5: Comparison between different values for vertex A3(07). Ranks of the vertex in the last interval are not provided as both betweenness values are zero.

Invisible rapids and brooks can be detected in different lifetimes as their temporal role might be restricted to some time intervals only; for example, as we have seen in the previous Section, S1(10) and J1(06) are invisible rapids in $\mathcal{T}$ = [2005-2011], P1(06) is an invisible rapid in $\mathcal{T}$ = [2007-2011], A3(07) is an invisible brook in $\mathcal{T}$ = [2008-2011]. Tables 5.4 and 5.5 indicate the major invisible rapids and brooks observed in *Knowledge-Net*.

The presence of a poster presentation, a research project, two journals and a conference publication among the invisible rapids supports that different types of mobilization actors can impact timely mobilization while not being as effective at creating short paths among entities for knowledge mobilization. In other words, they can play a role of accelerating knowledge mobilization, but to a concentrated group of actors.

In comparison with invisible rapids, for invisible brooks there is a wider variety in the type of mobilization actors that act as brooks which does not readily lend itself to generalization.

Interestingly, we see the presence of journals among invisible rapids and brooks. From our analysis, it seems that journals can hold strikingly opposite roles: on the one hand they can contribute considerably to more timely mobilization of knowledge while not being very strong bridges between communities; while on the other hand, they can play critical roles in bridging network communities, but at a slow pace. A brook, the journal *Biochemica et Biophysica Acta-Molecular Cell Research* (J3(08)), for example, helped mobilize knowledge in two papers for L1(05) (in 2008 and 2009) and is a journal in which a paper (in 2011)

Table 5.4: Major invisible rapids

Actor	Time Interval	Temp. Rank	Stat. Rank	Type
S1(10)	[05-11]	8	115	poster
	[06-11]	8	113	
	[07-11]	7	115	
	[08-11]	5	104	
J1(06)	[05-11]	10	160	journal
	[06-11]	10	154	
	[07-11]	10	223	
C1(07)	[05-11]	11	223	citing publication
	[06-11]	11	220	
P1(06)	[07-11]	5	105	project
J2(09)	[06-11]	17	179	journal
	[07-11]	16	182	

citing a L1(05) publication was also published. Given expected variability in potential mobilization for a journal, it is not surprising to see these mobilization actors at both ends of the spectrum.

In contrast, the presence of a research project as an invisible rapid is meaningful. It is meaningful in two ways. First, because when public funders invest in research projects as mobilization actor, an implicit if not explicit measure of success is timely mobilization with potential impact inside and outside of academia [54]. Ranking as a rapid (for a mobilization actor) is one measure that could therefore help funding agencies monitor and detect temporal change in mobilization networks. Second, a research project as rapid is meaningful because by its very nature a research project can help accelerate mobilization for the full range of mobilization actors, including other research projects. As such, it is not surprising that they can become temporal conduits to knowledge mobilization in all of its forms.

Table 5.5: Major invisible brooks

Actor	Time Interval	Temp. Rank	Stat. Rank	Type
J3(08)	[08-11]	9	117	journal
	[09-11]	12	84	
C3(11)	[08-11]	10	191	citing publication
	[09-11]	15	153	
C4(11)	[08-11]	15	105	citing publication
H2(05)	[06-11]	16	118	person
	[07-11]	15	134	
A3(07)	[08-11]	16	187	publication
C5(07)	[08-11]	18	158	citing publication

5.8 Conclusion

In this Chapter, we proposed the use of a temporal betweenness measure to analyse a knowledge mobilization network that had been already studied using classical "static" parameters. Our goal was to see the impact on the perceived static central nodes when employing a measure that explicitly takes time into account. We observed interesting differences. In particular, we witnessed the emergence of rapids: nodes whose static centrality was considered negligible, but whose temporal centrality seems relevant to analysis. Our interpretation is that rapids contribute to accelerate mobilization flow in the network and, as such, they can remain undetected when analysis is performed statically. The combination of static and temporal betweenness appears complementary to provide insights into the importance and role of nodes in a network.

Temporal network analysis as performed here is especially pertinent for KM research that must take time into account to understand academic research impact beyond the narrow short-term context of academia. Measures of temporal betweenness, as studied in this chapter, can provide researchers and funders with critical tools to more confidently investigate the role of specific mobilization actors for short and long-term impact within and beyond academia.

In conclusion, we focused here on a form of temporal betweenness designed to detect

accelerators. This is only a first step towards understanding temporal dimensions of social networks; other measures are already under investigation.

Chapter 6

Temporal Analysis of a Facebook Network

In this Chapter, we study the user interaction dynamics that occur around scientific and hoax data in Facebook pages. We base our study on data already collected and partially analyzed from a static point of view [16]. Similarly to the previous Chapter, our goal is to perform a temporal analysis to identify users that act as accelerators in this very different setting. We focus mainly on betweenness centrality and we compute it both on a static representation of the graph, and on a temporal one, highlighting the differences between the two analysis.

Since the data is extremely large (over 800 thousand nodes), we cannot compute exact temporal betweenness values. We then employ an "hybrid" method that uses the same technique already employed in the previous Chapter for exact calculation among key nodes of the graph, with an estimation module that only provides approximated values in less important (but very large) portions of the graph.

6.1 Introduction

Facebook, the second most popular social network after MySpace, has been the target of many researchers' study due to its popularity, availability of APIs, and abundance of features. However, the popularity of a social networking site attracts hoax broadcasters and spammers too. Recently, Bassi et al. [16] started collecting Facebook data from both conspiracy and legitimate Facebook pages in an attempt to study the activity of Facebook users on false versus correct information. They also analysed the formation

of communities around the information and user interest observing that users who are in communities formed around conspiracy information are less reluctant to participate in debates that take place in the communities formed around the scientific data. The opposite, nevertheless, is not true, and members of scientific communities are more eager to get involved in discussions happening around conspiracy data.

The aforementioned analysis is conducted on aggregated static social network that is collected over a timespan of four years. The analysis, therefore, does not consider the change of behaviour over time. It is very important to distinguish how the users evolve during this period of time. It is also interesting to know whether the users in the centre of science communities who appear to participate in conspiracy discussions have always been a central science fan, or were hoax followers gradually moving from conspiracy communities to scientific ones. If the latter happened, the participations in conspiracy debates that appear to be from a science community member v might just be an old record that is preserved in the aggregated graph from the times when v was a member of conspiracy community. In this Chapter, we consider these types of questions by evaluating the Facebook network over time.

6.2 Data Description

Our dataset is acquired from the research group in Italy who collected Facebook post and commenting activities on 83 Facebook pages, [16, 15].

The Data. The data is composed of public Facebook pages and all the comments on those pages along with all the user interactions on those posts in the time span of four years. The comments are tagged with the time of post.

Table 6.1 provides the statistical description of our data, when considered in an atemporal setting (i.e., without indication on when the commenting activities are performed). The Facebook pages that are monitored in this study fall into four categories. Two major categories are science and conspiracy: 34 pages disseminate factual and scientific information, 39 pages propagate conspiracy. Among the rest of the pages, 6 pages actively try to catch and inform about hoax messages (hoaxbusters), while 2 pages intentionally distribute hoax information, the latter is managed by the data collectors for the research purposes. With the focus on interactions, a "like" corresponds to a positive feedback, while "share" stands for the willingness to increase the visibility of the content. "Commenting", however, is the only medium for debating about the content, which may correspond to a

Table 6.1: Facebook data description [15]

	Total	Science	Conspiracy	Hoaxbusters	Troll
Pages	83	34	39	6	3
Posts	310, 858	62, 075	208, 591	4,502	35,690
Likes	9, 232, 105	2, 505, 399	6, 659, 382	67,324	0
Comments	5,373981	180,918	836,591	17,883	4,338,589
Unique Commenter	841,275	53,438	226,534	5,115	648,825
Unique Likers	1,121,699	332,357	864,047	12,427	0

positive or negative feedback. Since we are interested in the dissemination of information as part of debating the factual or hoax information presented in the context of messages, we chose to focus only on commenting activity of the users.

The data can be then seen as a tripartite graph (Figure 6.1) composed of vertices corresponding to Facebook users, Facebook pages, Facebook posts/comments. It can also be described as an affiliation network of any type of the aforementioned vertices. In particular, in this Chapter we consider three affiliation networks: network of Facebook users affiliated by commenting on the same page (Figure 6.2), network of Facebook users affiliated by commenting on the same post, and network of Facebook pages affiliated by being commented on by the same user. Nevertheless, the networks that can be extracted from the dataset are not limited to those that are explored in this chapter.

Following the notion of flattening affiliation graphs, we consider that the users who comment on the same posts (affiliated by the post) are linked to each other. This also makes the users commenting on the same page to be affiliated to each other. Both of the

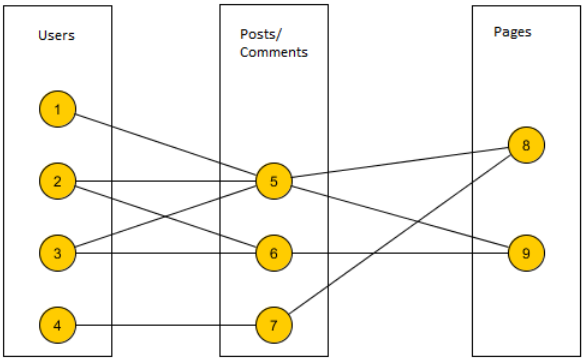


Figure 6.1: Facebook network dataset composition

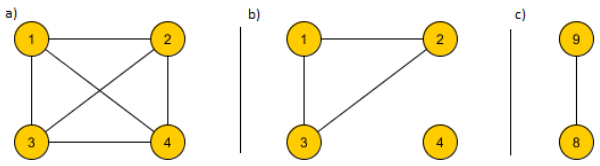


Figure 6.2: Simplified affiliation graphs extracted from the facebook network: a) Simplification of affiliation graph of users commenting on the same post in Figure 6.1, b) Simplification of affiliation graph of users commenting on the same page in Figure 6.1, and c) Simplification of affiliation graph of Facebook pages being commented on by the same users in Figure 6.1

above notions create cliques on the level of pages and/or posts, meaning that a clique of users are always formed around pages or posts.

The Graph Representation. Let $P_1, \ldots P_k$ indicate the Facebook pages, and let $U^t(P_i)$ be the set of users commenting on page P_i at time t, and $P^t(u)$ the set of pages user u comments on during the same time. When referring to the entire life time of the system we shall omit t. Lifetime of the system $\mathcal{T}$ refers to the time frame in which the system as a whole exist. Lifetime, starts with the creation of first node and ends when the last temporal event in the graph takes places.

We represent the *Facebook graph* as a TVG $\mathcal{G} = (V, E)$, where $v \in V$ is a Facebook user, and $(u, v) \in E$ if $u, v \in P_i$, for some P_i (i.e., u and v write a comment on the same page). The presence function indicates the time intervals when a connection exists. The connections may span a long period of time depending on their latency. The latency of a connection (ζ) is the time that it takes for a connection to convey a message from the starting node to the target.

Note that the footprint of the TVG $\mathcal{G}$ is a collection of interconnected cliques corresponding to the Facebook pages: let V_i denote the clique corresponding to $U(P_i)$.

A *bridge* is a node v such that $v \in V_i \cap V_j$ for some i, j, $i \neq j$. Note that a node could obviously act as a bridge for several different pages, and it could do so at different times. Let $V^t_{\mathcal{B}}(G) = \{v \in V : |P^t(v)| > 1\}$ be the set of bridges during time interval t. In Figure 6.3, for instance, node A acts as a bridge that connects three different pages together. It also, in collaboration with B, act as a parallel bridge between P_2, P_3, and P_4, which are identified in different colour in Figure 6.3. Parallel bridges dramatically affect the value of betweenness for the vertices that play a part in the existence of such bridges. Thus, the value of betweenness is equally distributed among these bridges. It is evident that fewer parallel bridges mean higher betweenness value for the vertices involved in them.

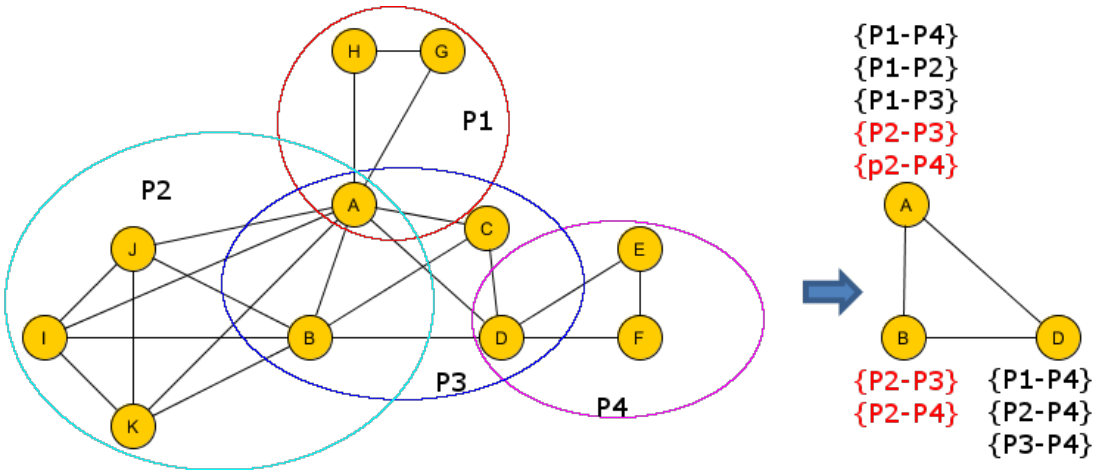


Figure 6.3: The footprint of a Facebook graph and the corresponding PU graph.

A *Page-User graph* (or *PU* graph) is composed by the cliques formed around pages of Facebook that are connected to each other when sharing a vertex. More precisely: in a Page-User TVG $G_{PU} = (V_{\mathcal{B}}, E_{\mathcal{B}})$ vertices are bridges, and two bridges are connected if they belong to the same page: $(u, v) \in E_{\mathcal{B}}$ if $u, v \in V_i$ for some V_i. The presence function indicates the time intervals when such a connection exists.

Figure 6.3 (left) shows an example of a small portion of the footprint of a Facebook graph, composed by four pages and eleven users. As it can be seen in the figure, users A, B, D are bridges. More precisely, A is a bridge between P_1, P_2 and P_3, B is a bridge between P_2 and P_3, and D is a bridge between P_3 and P_4 The corresponding PU graph is shown in Figure 6.3 (right). Figure 6.4 presents the same transformation for TVGs.

The non-bridge nodes (i.e., the ones that belong to one page only) are called *neutral* vertices, $V_{\mathcal{N}} = \{V - V_{\mathcal{B}}\}$. Neutral vertices, as their name indicates, play a peripheral role in the Facebook graph as they do not actively participate in defining or affecting the centrality values of the graph.

Some Characteristics of the Data. Table 6.2 describes some interesting characteristics of the data. As it can be seen, except users contributing to Hoaxbuster pages, most users tend to contribute to only one group of pages. It is also interesting to note that almost 740,000 users contributed only to one page throughout the data collection. A quick comparison with the number of users who contributed to only one category of pages (753,945) shows that most users who contribute to one category, tend to contribute to only one page within the category in a month to month basis.

However, the bridges are very active and once a vertex becomes a bridge, it actively

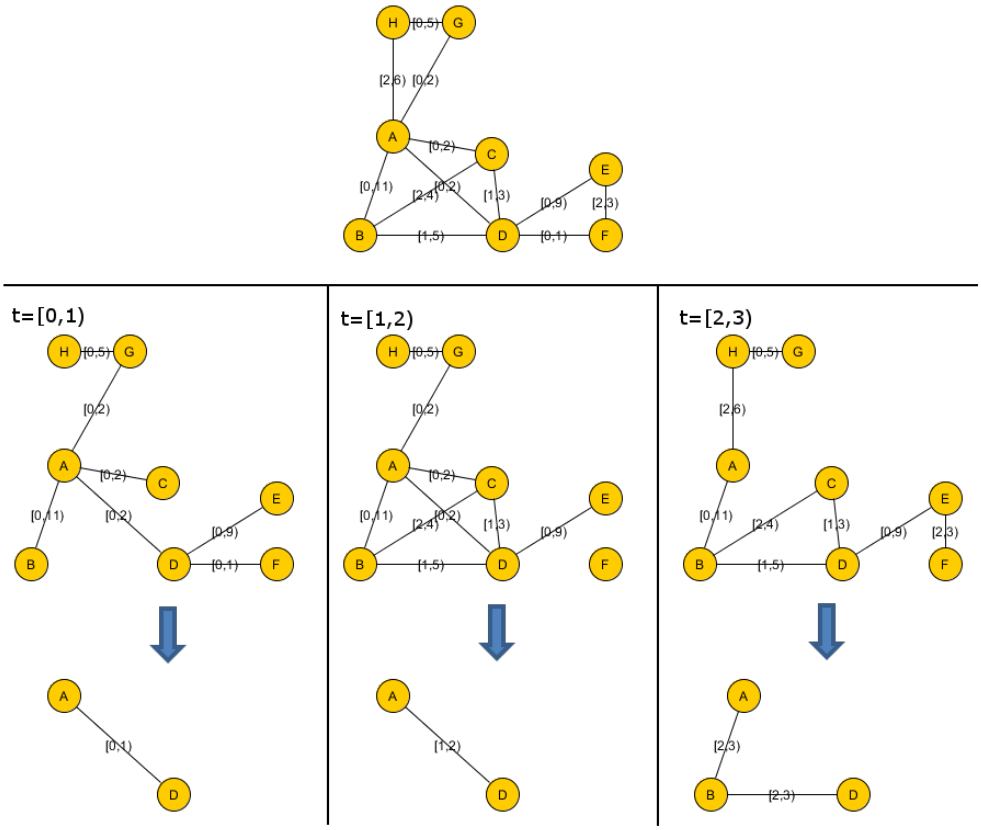


Figure 6.4: The footprint of a Facebook TVG and the corresponding PU TVG. B in the graph for $t = [2, 3)]$ is not a bridge but since the bridges are not connected directly, it works as an intermediary.

participates in many pages. Table 6.3 shows a level of participation of bridges in different number of pages.

As we can see, users tend to participate in fewer pages rather than many pages at the same time. This is understandable as participation in discussions is time consuming, and not many users tend to spend that amount of time on Facebook pages. Note that the number of participations do not add up to 125000 bridges, as one node participating in 19 pages is counted 19 times as a bridge. We plan to discover what factors affect the betweenness value of the nodes. Would participation in 19 pages has a boosting effect, or the structure of the network, the time of participation, or the number of neutral vertices in the pages are deterministic factors in this regard.

These observations motivates the focus on bridges for centrality calculation, since they represent a much smaller portion of the graph while being clearly more "central" than the rest.

Table 6.2: The description of PU graph

	Total	Science	Conspiracy	Hoaxbusters	Troll
Pages (cliques)	82	33	39	6	3
Users (being only part of one page)	753, 945	36, 519	148, 294	2,004	567,128
Bridges	125, 047	16, 919	78, 240	3,111	81,696

Table 6.3: The yearly description of PU graph

#Pages	2	3	4	5	6	7	8	9	10	11	12	19
#Users	47534	5588	1084	300	98	31	11	9	2	2	1	1

6.3 Design of the study

Our study is divided into two parts, we first perform a static analysis, and then a temporal one. For the static analysis, we focus on betweenness and eigenvector centrality measures of bridges in the static representation of the whole Facebook graph. For the temporal analysis we consider the corresponding TVG and we focus on two measures: foremost betweeness of bridges, and eigenvector centrality.

6.3.1 Static Betweenness Centrality of Bridges

Betweenness centrality is well-defined and explained in Section 2.2.1. Because of the special structure of the Facebook graph, we can focus on the betweenness of bridges, disregarding the betweenness of the neutral nodes, which are clearly not central. The betweenness value of bridges, on the other hand, are closely tied with the number of neutral vertices that they connect in different pages. Thus, betweenness centrality of a bridge v can be reformulated in terms of pages as follows:

$$C_B(v) = \sum_{\substack{P_s,P_e \in P \\ P_s \neq P_e \\ \Psi_{P_s,P_e} \neq 0}} \frac{\Psi_{P_s,P_e}(v)}{\Psi_{P_s,P_e}} \tag{6.1}$$

where P_s, P_e refer to the starting and target pages, P refers to the set of all pages. Ψ_{P_s,P_e} represents the number of paths from P_s to P_e, and $\Psi_{P_s,P_e}(v)$ is the number of those paths that pass through v and can be calculated as follows:

$$\Psi_{P_s,P_e}(v) = \begin{cases} (|V_{P_s}| - (|V_{P_s} \cap V_{P_e}|)) \times (|V_{P_e}| - (|V_{P_s} \cap V_{P_e}|)), & \text{if } v \text{ is bridging } P_s \text{ and } P_e \\ 0, & \text{otherwise} \end{cases} \tag{6.2}$$

where $|V_{P_s}|$ refers to the number of vertices in page (clique) P_s. Meanwhile,

$$\Psi_{P_s,P_e} = \sum_{v \in V} \Psi_{P_s,P_e}(v) \tag{6.3}$$

Equation 6.1 is also general for the cliques that are distant from each other. Even in such cases, the only vertex count is necessary to be done in P_s and P_e, as the paths never take longer route by going through neutral vertices in intermediary cliques.

6.3.2 Temporal Betweenness Centrality of Bridges

Calculation of betweenness in temporal graphs is extensively defined in Section 3.3.5, and analysed in Chapter 4. TVGs are attributed by vertices and edges that appear and disappear at different times. In the case of the Facebook graph, the edges appear with the first comment being posted on a certain Facebook page appear, and disappear after the last comment is posted. We assume that it takes one second for each message to travel through the network, so the latency that is assigned to each edge in the graph is equal to one second.

In information propagation applications, the early transfer of information is extremely important, for this reason we focus our analysis on foremost betweenness, defined in Section 4.3.

Algorithm 2 described in Chapter 4 computes the foremost betweenness of all nodes in a TVG in exponential time. As we have seen, the problem is inherently complex and we cannot avoid having an exact algorithm with such a high time complexity. Since the Facebook graph we are dealing with is extremely large, it would be unfeasible to perform an exact analysis. In the following we propose a method that combines exact computations of the number of journeys through bridges, with the estimate of number of journeys between them. More precisely, instead of using Algorithm 2 on the whole Facebook graph, we apply it to the PU graph only. In other words, we traverse only the subgraph of the Facebook graph composed by bridges. To count the overall number of foremost journeys

between nodes, however, we need also to know the number of such journeys between any pair of adjacent bridges passing through neutral nodes that are not traversed. Instead of performing an exact count of those journeys, we compute an estimate. The estimation algorithm is based on a variant of an existing algorithm [96] that provides an estimate of all paths between two vertices in static graphs. Following this idea, our algorithm traverses the PU graph in DFS (as in Algorithm 2) and when the traversal reaches a bridge y from bridge x, the estimation module estimates the number of journeys between x and y and updates the corresponding counters before proceeding with the traversal.

Estimation Module. As mentioned earlier, the algorithm developed by Roberts and Kroese (RK-Algorithm) [96] provides an estimate of the count of all paths between two vertices in static graphs. We would like to use the same algorithm to obtain an estimate of the number of foremost journeys between two connected bridges x and y in a certain portion of the Facebook graph. To do so, however, we need to perform a pre-processing phase which transforms the portion of the TVG under scrutiny into a static graph that represent *only journeys*. In other words, given a TVG G we would like to obtain a sub-graph G' such that any path between x and y in the static representation of G' is also a journey in the original TVG G. The result of this process will be the graph (here called *feasible graph*) to be given in input to the RK-Algorithm. Note that since a bridge is connecting two or more cliques, the construction of feasible graphs will always concern two consecutive bridges x and y, and the goal will be to transform (portion of) the clique into a feasible graph between x and y. To extract the feasible graph from a graph G from x to y with earliest entering time t from x and latest exit time t' from y, we simply prune G to only contain edges and nodes that exist in time frame $[t, t']$ (we call it procedure ExtractFeasible(G, x, y, t, t')).

Example of graph extraction. *Figure 6.5 shows an example of a sub-graph generation for time frame* $[2, 5)$*. In Figure 6.5-b, parts of the clique, such as the edge between d and a, and also between d and b are removed, as they cannot be traversed in the time frame* $[2, 5)$*, the arrival and exit times to the clique.*

The feasible graph extraction and the estimation are applied every time we traverse an edge (x, y) between two bridges during the execution of Algorithm 2 on the PU graph, appropriately computing the earliest entering time $t_{x,y}$ (which can be obtained by employing directly the algorithm described in [121]) and the latest possible exit time $t_{y,x}$ (by running the algorithm in reverse starting from y at its foremost arrival time, back to x recording the latest possible time that each clique can have to arrive at destination in a feasible way).

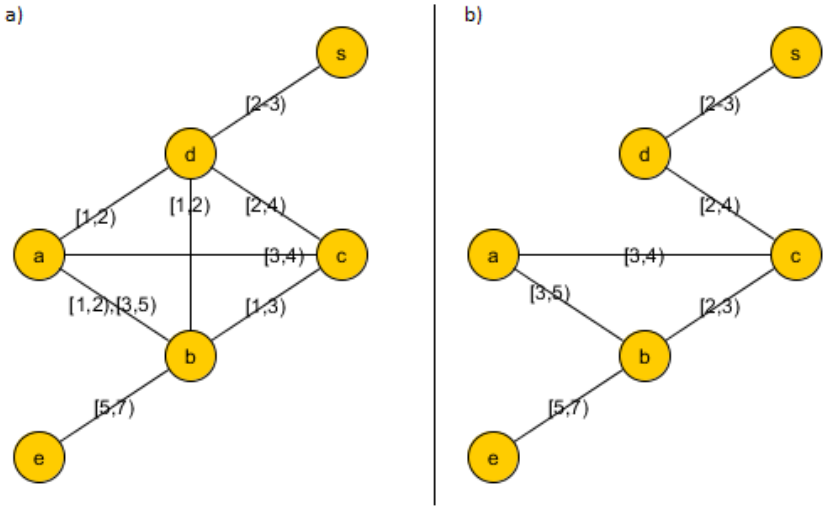


Figure 6.5: Sub-graph creation for foremost path count estimation, starting at s and ending at e

Example of earliest and latest traversal times. *For instance, in Figure 6.6, we travel from s to e, starting at $t = 0$ which leads to foremost arrival to e at $t = 7$, supposing that the latency on all edges, including the clique, is 1. Thus we know that the earliest time that we arrive to the clique is $t = 2$. However, hen travelling from s to e, we can exit the clique at times $t = 3, 4, 5$ and still arrive at e at its foremost time. Thus, simply assuming that the clique is only reachable at times $t[2, 3)$ because we exit the clique at $t = 3$ is a wrong assumption. Clearly, the clique is reachable beyond that and in time frame equal to $t = [2, 5)$. Forward calculation of foremost times provides us with the arrival time to the clique, yet it does not provide the right exit time. Therefore, we run foremost time calculation in the reverse order starting from e at its foremost arrival time ($t = 7$) to s. The reverse foremost time calculation assumes travelling backwards in time, so the arrival time to b would be the latest possible time that we exit the clique and still arrive at e at its foremost time.*

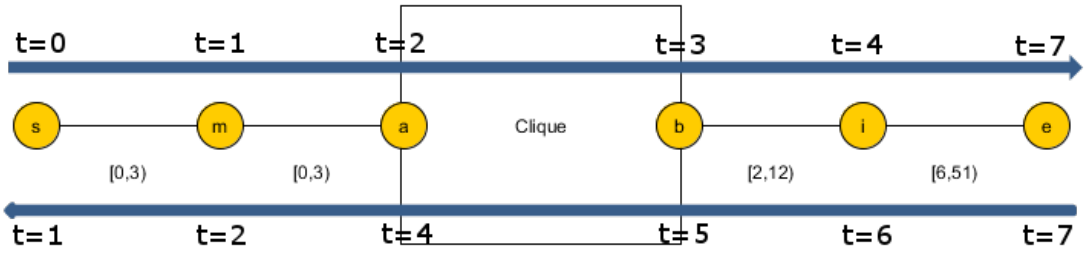


Figure 6.6: Process of forward and reverse foremost time calculation

In conclusion, Algorithm 8 has exactly the same structure and complexity of Algorithm

Algorithm 6: ExtractFeasible.

input : TVG $G_m = (V_m, E_m)$, $x, y \in V$, $V_{visited}$, x's arrival time t, and y's exit time t'
output: TVG $G'_m = (V'_m, E'_m)$
begin
  $G'_m = G_m$
  for *all* $e \in E'_m$ *in* G'_m **do**
    if e *does not exists in time* $[t, t')$ **then**
      remove e from E'_m
      for *all endpoints* v *of* e **do**
        if v *is isolate* **then**
          remove v from V'_m

Algorithm 7: EstimateForemost.

input : TVG $G = (V, E)$, $x, y \in V$, $V_{visited}$, x's arrival time t, and y's exit time t'
output: $Count_{x,y}$, $\forall x, y \in V$: number of estimated foremost temporal path from x to y
begin
  Create $G_m \subseteq G$ with $V_m = V \setminus V_{visited}$
  $G'_m = ExtractFeasible(G_m, x, y, t, t')$
  Add $V(G_m)$ to $V_{visited}$
  $estimation$ = RK-Algorithm(G'_m, x, y)
  Update $Count_{x,y}$

2 and it traverses the PU graph in DFS, as before. As the traversal reaches a bridge y from bridge x, it estimates the number of journeys between x and y in the visited part of the clique between them and updates the estimated count ($ECount$) list, and consequently the path counter $Counter$. The estimated count of journeys are added to the path when the target is visited. The set $V_{visited}$ makes sure that we do not take into consideration a neutral node twice in the course of a journey.

6.4 Static Analysis of the Facebook Dataset

To provide more clear statistics on the dataset, and to set a ground for better understanding of temporal metrics, we first calculated classical statistical measures (i.e., node centrality measures) on the aggregated static Facebook graph. The static Facebook graph structure resembles the structure shown in Figure 6.3, yet its user cliques are more intertwined,

Algorithm 8: CountForemostEstimated.

input : a TVG $G = (V, E)$, $s \in V$, and the graph of bridges G_{PU}
output: $Count_s[x, y]$, $\forall x, y \in V$: number of foremost temporal path from s to $y \in V$, passing through $x \in V$

begin
  $Path.push(s), Count_s[., .] \leftarrow 0, ECount.push(1), Counter = 1, V_{visited} \leftarrow \emptyset$
  for *all* $w \in Adj(s)$ *in* G_{PU} **do**
    $S.push(s, w)$
  while $S \neq \emptyset$ **do**
    $(x, y) \leftarrow S.pop()$
    while $x \neq Path.top()$ **do**
      $Path.pop()$
      $Counter = \frac{Counter}{ECount.pop()}$
      $V_{visited}.pop()$
    Let π be the temporal path corresponding to the content of $Path$
    Let $t_{x,y}$ be the latest possible traversing time of edge (x, y)
    Let $t_{y,x}$ be the earliest possible traversing time of edge (y, x)
    if $y \notin Path$ *and* $t_{x,y} \geq arrival(\pi)$ **then**
      $Path.push(y)$
      $count_{x,y} =$ EstimateForemost $(x, y, G, V_{visited}, arrival(\pi), t_{y,x})$
      $ECount.push(count_{x,y})$
      $Counter = Counter \times ECount.top()$
      for *each* (y, w) *such that* $w \notin Path$ *and* $t_{y,w} \geq arrival(\pi)$ **do**
        $S.push(y, w)$
      if $arrival(\pi) = foremost(y)$ **then**
        Update $Count_s[z, y]$ with $Counter$ for all $z \in Path$

meaning that more users appear in various pages and hence more bridges exist in the graph.

We perform the analysis on the full graph (containing both bridges and neutral vertices) but we record only the result for bridges. This creates a graph of more than 125 thousand users acting as bridges in various pages. Note that the numbers in Table 6.2 do not add up for bridges as the users acting as bridge in one category might act as a bridge in a different category as well.

In the resulting graph, we computed betweenness and eigenvector centralities in an annual and monthly basis to be able to provide grounds for comparison between the temporal results and static measures.

As we will discuss in details later, we take two approaches for annual analysis: a) snap-

Table 6.4: Static statistical parameters referring to *bridges only*, calculated for successive snapshots of the Facebook graph

	2009	2010	2011	2012	2013	2014
Nodes ($betweenness > \alpha$)	0	1	248	15832	37385	15941
Edges	0	0	1201	24820	111583	22012
Ave. Yearly Degree	0	0	2.11	2.74	3.01	2.03
Ave. Normalized Betweenness	0	0	8.5×10^{-3}	1.0×10^{-2}	1.2×10^{-2}	8.1×10^{-3}

shot approach, which is generating the graph for each year, and b) aggregated approach, which is generating the graph for the initial time (2009) to the end of every year (e.g. [2009-2010], [2009-2011], ... , [2009-2014]). However, since, in monthly analysis, the snapshot approach sometimes results in disconnected graphs, we only considered the aggregated approach in that case.

6.4.1 Facebook Static Analysis: snapshot approach

In the first step, we process the Facebook graph for its static betweenness values in various snapshots. The average for each snapshot's betweenness value for the graphs is calculated to represent a benchmark on how the rank for each vertex is compared to the others. Note that Table 6.4 only presents annual computation of betweenness that is calculated as described in Section 6.3.1.

The statistical data presented in Table 6.4 provides valuable information about the graph. The graph initially does not contain many links and vertices in 2009 and 2010, but the it grows steadily until 2013. The nodes join the PU graph as soon as they start acting as bridges. We omit years 2009 and 2010 for the rest of this analysis, as they do not give us enough insight about the graph and the characteristics of the bridges. The steady growth of the graph stops in 2014 as it was closer to the end of data collection. This slow growth in the graph led to an increase in the density in 2014. The initial increase in the eigenvector centrality of the graph is an indicator that the links create a structure where

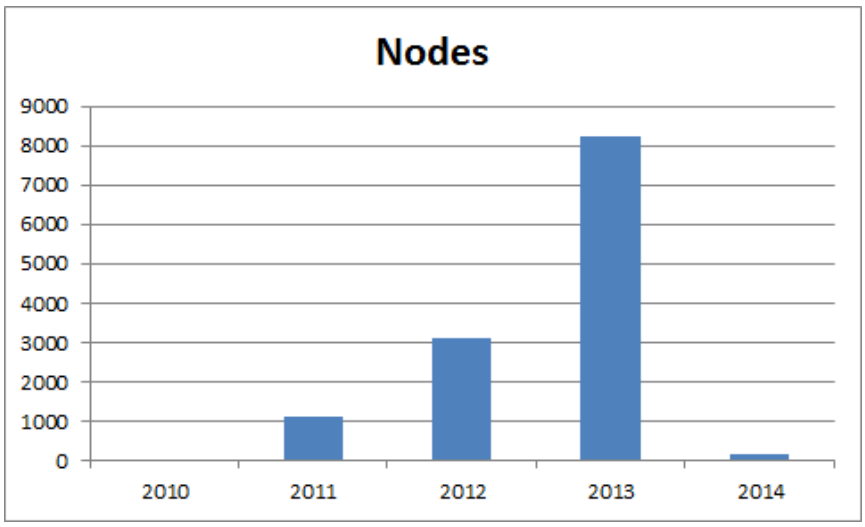


Figure 6.7: Distribution of the top ranked nodes by joining time (averaged over snapshots)

the nodes boosted each other's values.

The table also shows that betweenness values gradually grow as the time progresses, except in 2014. The distribution of the nodes with this betweenness shows that most of the high rank nodes (top 10%) are introduced to the graph in 2013 (Figure 6.7). The high concentration of high ranked nodes from 2011 to 2013 is the main reason for gradual increase in betweenness values.

As most users in the PU graph are conspiracy users, it was expected to see more conspiracy users in the top ranked nodes. However, this is not the case, as highest ranked nodes are mostly science users (Figure 6.8). The high number of science users, nevertheless, demonstrates their strategic positioning in the graph meaning that they connect more sub-networks avoiding the creation of (or tending to create fewer) parallel bridges.

6.4.2 Facebook Static Analysis: aggregated approach

The aggregated approach does not divide the graph into the small sub-graphs. Quite conversely, the graph always grows as time advances because we are considering larger time windows at each step.

Processing the graph in successive growing time windows, we obtain a series of graphs where each of them is an aggregation of all nodes and edges that existed from the system's birth until the time that graph is generated. The average for each graph's betweenness value is calculated and presented in Table 6.5.

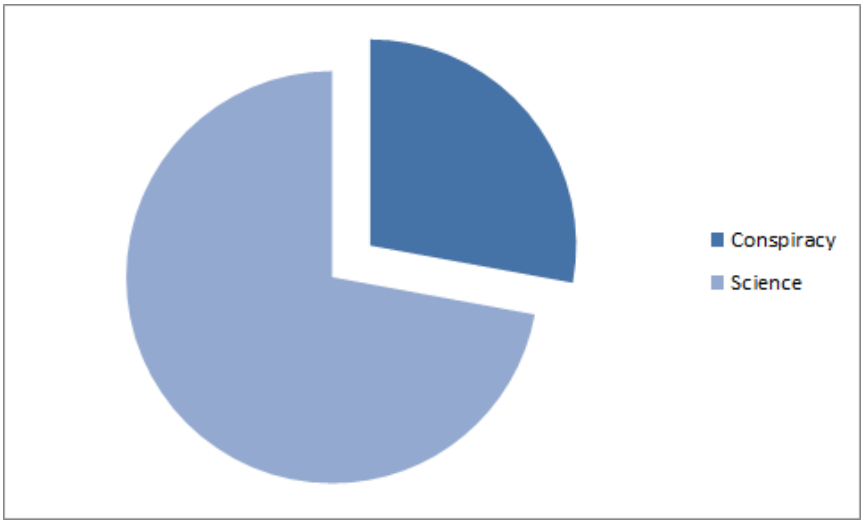


Figure 6.8: Distribution of top ranked nodes based on their activities (averaged over snapshots)

The table shows that the betweenness value gradually grows as time progresses and the graph becomes larger, with an exception in [2009,2014]. Compared to Table 6.4, the values are generally higher. This shows that the graph grew in sub-graphs rather than in a large component. Similarly to the snapshot analysis, the distribution of betweenness values shows that most of the high rank nodes (top 10%) are introduced to the graph in 2013, or more precisely in the [2009,2013] period (Figure 6.7).

Science nodes, again, have more presence among these high ranked nodes (Figure 6.8). This is again a more robust indication that the science users contributed to more communities with less parallelism rather than focusing on their own community.

Analysing the eigenvector centrality, however, provides quite different observations. Although we observe the same jump in the high ranked nodes in [2009,2013], the pattern of eigenvector centrality measures are different. The eigenvector centrality of the single node that joined in [2009,2010] is higher than the average of the eigenvector values of the nodes that joined in [2009,2011], and [2009,2012]. We also observe a similar pattern difference in the number of nodes with high eigenvector centrality values (Figure 6.11). As eigenvector centrality represents the links to highly connected nodes, and betweenness represents the links to highly isolated communities, we can conclude that even bridges prefer to be connected in their communities rather than reaching out to more communities. This would also confirms the structure of the Facebook graph as disparate cliques around various pages with connections between them.

Table 6.5: Static statistical parameters referring to *bridges only*, calculated for aggregated sub-graphs of the Facebook graph

	[2009,2009]	[2009,2010]	[2009,2011]	[2009,2012]	[2009,2013]	[2009,2014]
Nodes ($betweenness > \alpha$)	0	1	248	15963	46368	54661
Edges	0	0	1207	29051	262532	411801
Ave. Yearly Degree	0	0	1.88	2.48	3.10	3.20
Ave. Normalized Betweenness	0	0	8.5×10^{-3}	4.7×10^{-2}	8.6×10^{-2}	3.7×10^{-2}

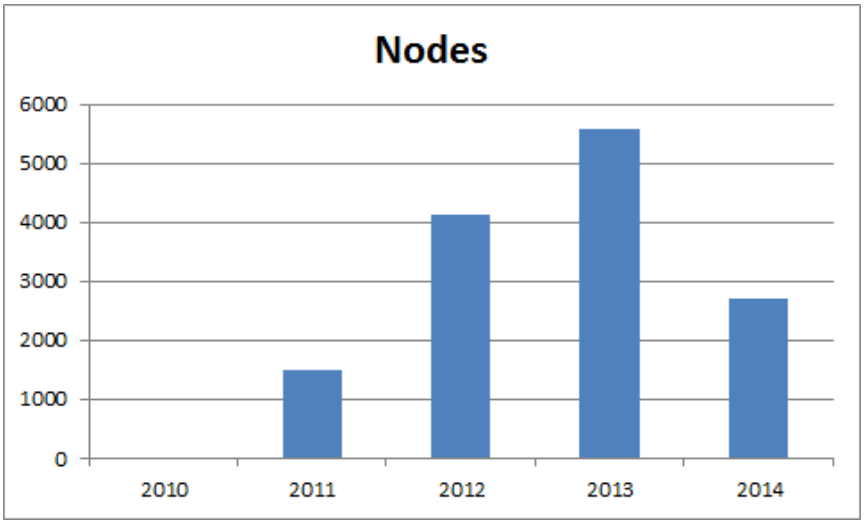


Figure 6.9: Distribution of the top ranked nodes by joining time (full graph [2009,2014])

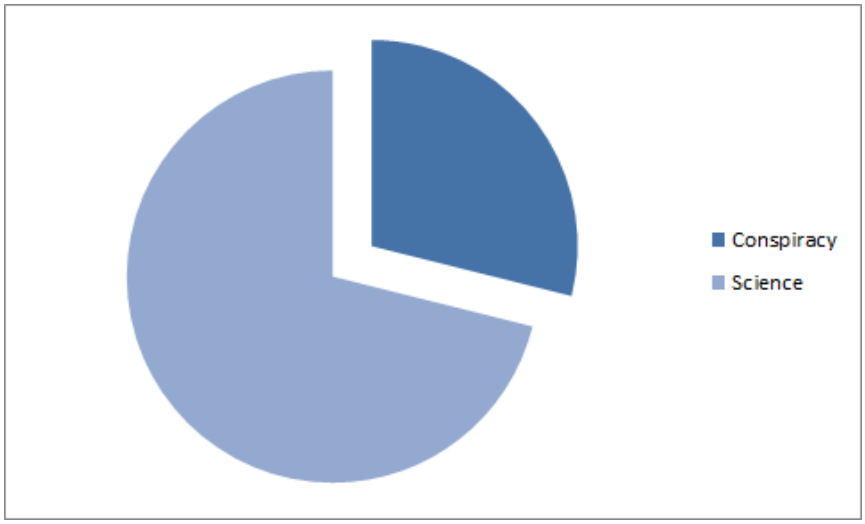


Figure 6.10: Distribution of top ranked nodes (eigenvector) based on their activities

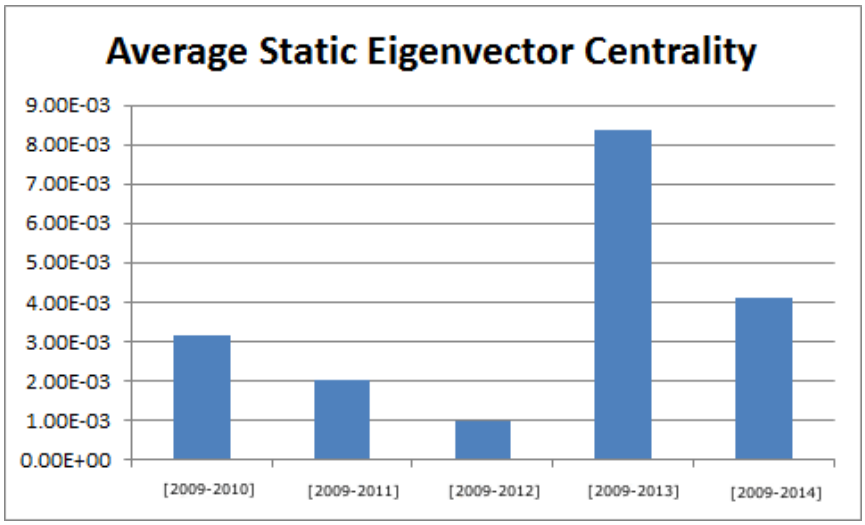


Figure 6.11: Static Eigenvector Centrality of PU Graph in its Lifetime [2009,2014]

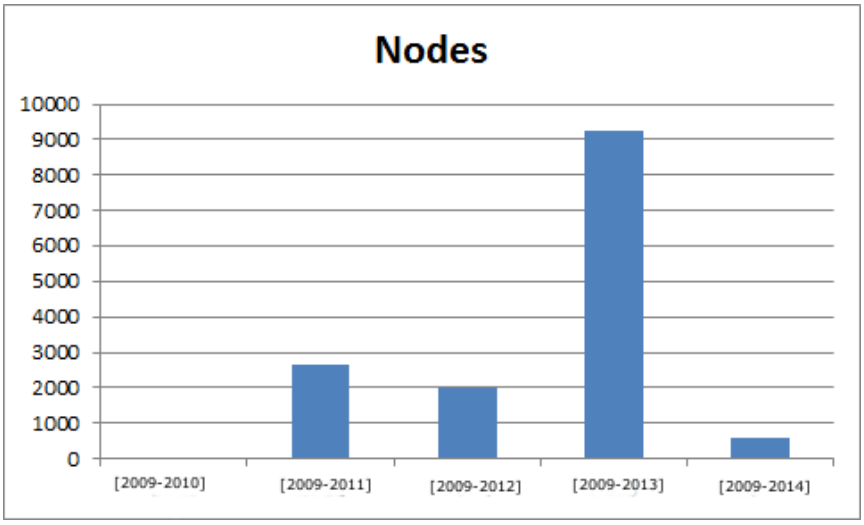


Figure 6.12: Distribution of Top Ranked Static Eigenvector Centrality Nodes Based on Joining Times [2009,2014]

6.5 Foremost Betweenness of Bridges

In this Section we focus on foremost betweenness of bridges using Algorithm 8 described in Section 6.3.2. Bridges are ranked according to their foremost betweenness values and their ranks are compared with the ones obtained calculating their static betweenness using the aggregated and snapshot approaches in the same time-frame (annual, and monthly analysis). Given the different meaning of static and foremost measures, we expect to see the emergence of different behaviours, and, in particular, we hope to be able to detect nodes with important temporal roles whose centrality was left undetected in the static analysis.

6.5.1 Foremost Betweenness during the lifetime of the system

Table 6.6 shows the temporally high ranked Facebook users in Facebook graph accompanied by their static ranks, and the high ranked static users with their temporal ranks, both with lifetime $\mathcal{T} = [2009\text{-}2014]$.

To preserve the privacy of the users, the users are assigned an ID accompanied with their birth date. In our naming convention, a user is named $ID(y)(a)$ where its birth date is y, a value between 0 and 4 corresponding to 2010 to 2014, a represent the ratio percentage of scientific activities and hoax activities, and ID represents its assigned ID. Any negative value of a shows that the user has been more active in the hoax community and the ratio shows the hoax activity over scientific ones. Note that only the nodes whose betweenness

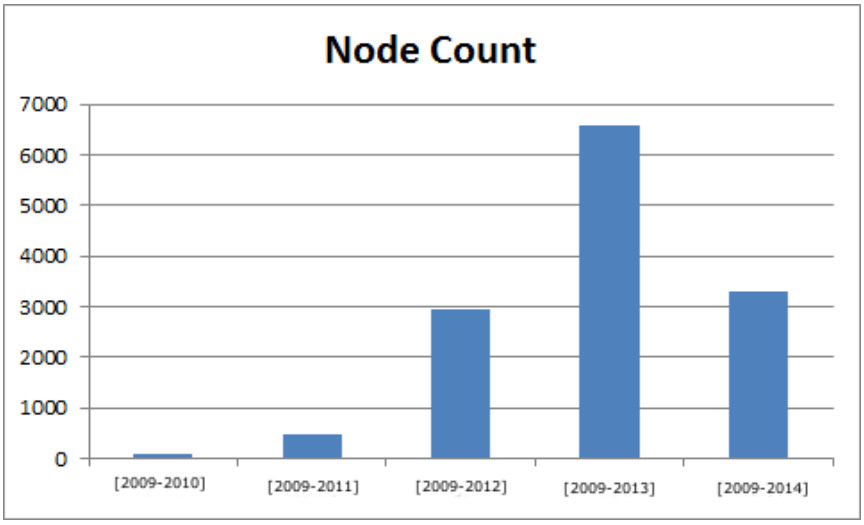


Figure 6.13: Distribution of the top ranked nodes by joining time during the lifetime of the system [2009,2014]

has a significant value are represented in the table. While there are significant values that do not appear in the table, it should be noted that betweenness values tend to lose their importance, especially when the differences in the values of two consecutive ranks are very small [50]. Note that, as it can be seen in the table, for simplicity of representation, and to be able to deal with smaller ID values, we regenerated IDs for the vertices based on their temporal rank.

Interestingly, all of the vertices that fall in the top 20 significant betweenness values have joined PU graph on or after 2012. However, not all high ranked Facebook graph members are in the same situation. Figure 6.13 represents the distribution of the top 10% nodes based on the time that they joined PU graph. It can be observed that the nodes that joined in [2009,2013] had better bridging affect, thus appeared more among the top ranked nodes. However, the nodes that joined in [2009,2014], although fewer than those that joined in [2009,2013], have higher betweenness values, meaning that they connect more communities both in the number and in isolation (Figure 6.14). This is in compliance with what that is observed in Section 6.4. Thus, static and temporal betweenness follow similar patterns considering the joining time of the nodes to the graph.

Even though the number of scientific pages is much smaller than the number of the hoax pages, the contribution of science users seems to be more critical than the conspiracy distributors. Comparing the results obtained from tables 6.2, 6.3, and Figure 6.15 we see that the high number of bridges observed in conspiracy pages (Table 6.2) represents the highly isolated activity of bridges to distribute hoax information among other conspiracy

Table 6.6: List of highest ranked users according to temporal (resp. static) betweenness, accompanied by the corresponding static (resp. temporal) rank in lifetime [2010-2014].

Temporal to Static			Static to Temporal		
Actor	Temporal Rank	Static Rank	Actor	Static Rank	Temporal Rank
1(1)(99.99)	1	7	7(3)(100)	1	7
2(2)(100)	2	11	1021(2)(100)	2	1021
3(2)(-100)	3	170	4833(3)(100)	3	4833
4(2)(100)	4	39309	3969(2)(-100)	4	3969
5(2)(-100)	5	208	4117(2)(100)	5	4117
6(3)(100)	6	23	15(3)(99.99)	6	15
7(3)(100)	7	1	1(2)(99.99)	7	1
8(2)(100)	8	22	5009(2)(-100)	8	5009
9(3)(99.96)	9	13297	307(2)(99.77)	9	307
10(2)(100)	10	221	31(4)(99.57)	10	31
11(3)(100)	11	15014	2(2)(100)	11	2
12(4)(99.57)	12	1001	24(2)(99.96)	12	24
13(2)(-100)	13	1655	9497(3)(99.96)	13	9497
14(2)(100)	14	34944	58(3)(99.35)	14	58
15(3)(99.99)	15	6	11014(3)(100)	15	11014
16(3)(99.95)	16	37234	1355(2)(-100)	16	1355
17(3)(-100)	17	17	17(3)(-100)	17	17
18(3)(100)	18	447	515(3)(100)	18	515
19(4)(100)	19	21133	34(2)(100)	19	34
20(3)(100)	20	82	63(2)(99.98)	20	63

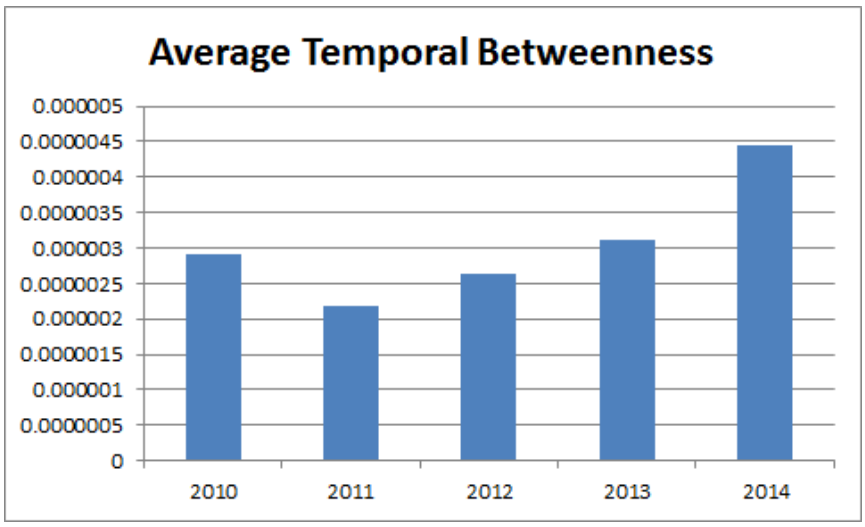


Figure 6.14: The variation in temporal betweenness of top ranked nodes

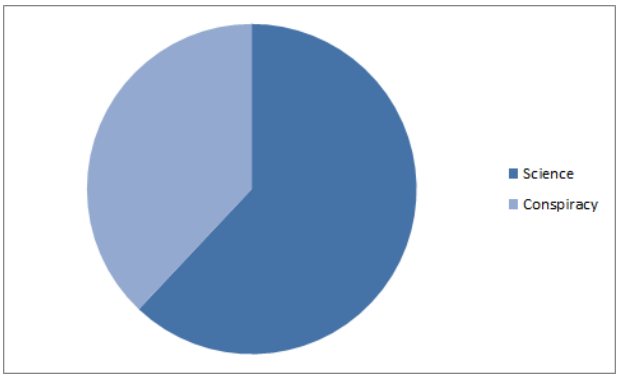


Figure 6.15: Distribution of Mainly Science vs. Mostly Conspiracy Users Among Top Nodes

pages. On the other hand, the science activists are more inclined to create a bridge between science and conspiracy, which ultimately, creates a bridge between two diverse and large communities. The low number of science fans in the PU graph is another contributor to the high betweenness of the scientific bridges in the Facebook graph.

6.6 Foremost Betweenness of Bridges in time intervals

In this section we consider the TVG corresponding to the Facebook graph is successive time window, all starting from the beginning and terminating in consecutive years. We then compare the results with the one obtained by the aggregated yearly static analysis.

Table 6.7: Statistical parameters calculated for the aggregated PU graph

	[2009,2011]	[2009,2012]	[2009,2013]	[2009,2014]
Ave. Normalized Betweenness	8.4×10^{-3}	1.0×10^{-3}	8.0×10^{-2}	8.0×10^{-2}
Foremost-Static Betweenness Correlation	0.38	0.40	0.50	0.46

Table 6.8: Statistical parameters calculated for top nodes in aggregated PU graph in time

	[2009,2011]	[2009,2012]	[2009,2013]	[2009,2014]
Foremost-Static Betweenness Correlation - Top Static	0.32	0.38	0.42	0.46
Foremost-Static Betweenness Correlation - Top Foremost	0.36	0.49	0.49	0.41

Table 6.7 provides a statistical view on the graph metrics, both static and temporal in an aggregated yearly basis. Thus, for instance, the values for year [2009,2011] considers the PU graph from 2009 to 2011. In the case of the temporal analysis, this graph is a TVG with links labeled with their time of existence, while in the case of the static analysis it is just a static graph with no time indication.

As it can be observed, the static and foremost betweenness are similar in the case of yearly analysis, yet no conclusion can be deduced from the static analysis regarding the foremost value and vice versa. The correlation is smaller in the graph of interval [2009,2011] where the number of nodes and edges are smaller. As the graph grows, the correlation increases. Part of the reason might be the very large number of negligible and close to zero values of betweenness. We analysed the correlation of the top nodes in both static and temporal measures (Table 6.8).

The correlation increases between the top nodes for both measurements as the graph grows in time. However, the elimination of less significant nodes has lowered the value of correlation. Observing the correlation values proves that the static analysis of the graph cannot provide predictive results about the temporal betweenness of the nodes.

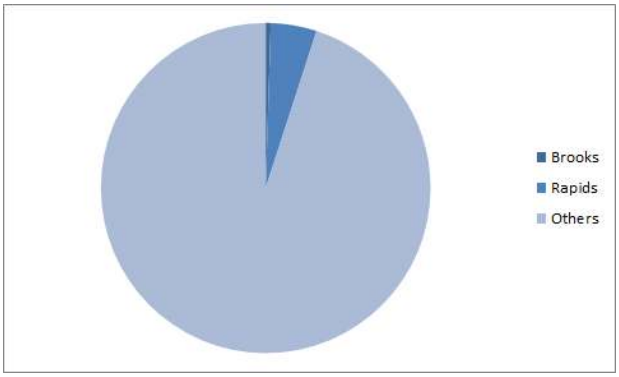


Figure 6.16: Composition of Top 10% with regards to rapids and brooks

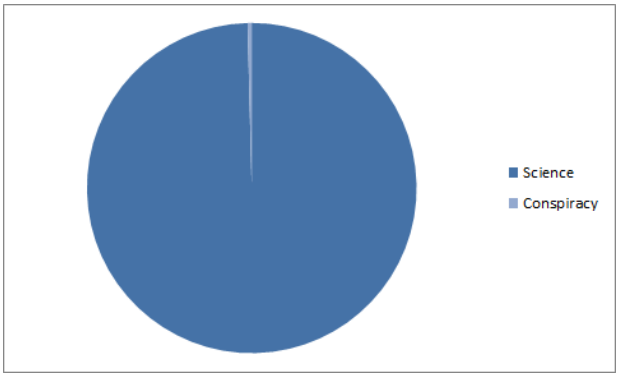


Figure 6.17: Distribution of Rapids Among Science and Conspiracy Users

6.7 Rapids and Brooks in Facebook Dataset

In the Facebook graph we consider a node to be a rapid if it falls in the top 10% high ranked temporal nodes, but its static betweenness is less than the static betweenness value of half of the other nodes. Table 6.6 provides a small sample of rapids and brooks that exist in Facebook graph. Similarly to the case of knowledge-Net, the number of rapids is much higher than the number of brooks, even though they do not form the majority of population (Figure 6.16).

Concentrating on the rapids, since most of the high ranked temporal nodes are science users, it is expected that most of the rapids, if any, be science users as well. Figure 6.17 shows the distribution of rapids between science and conspiracy communities. More than 99% of the rapids correspond to users who belong to science communities. This shows that science users distribute scientific data faster and to a wider community than conspiracy users.

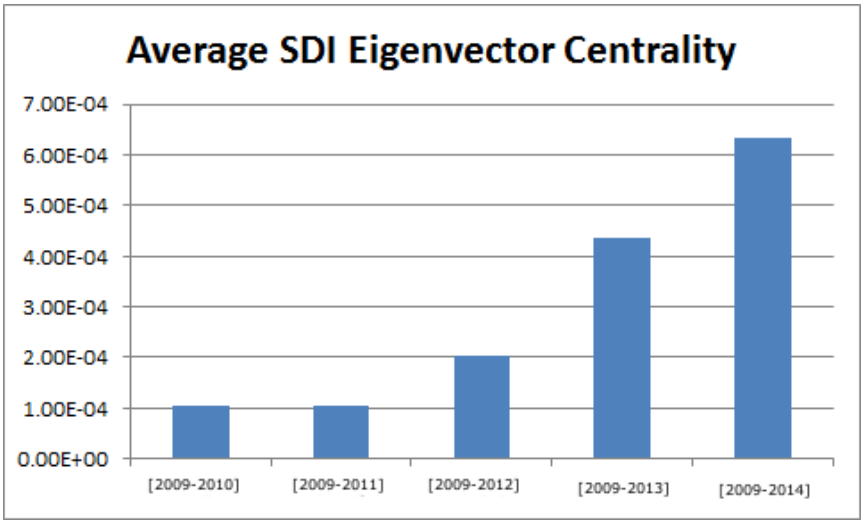


Figure 6.18: SDI eigenvector centrality of PU Graph in [209,2014]

6.8 Temporal Eigenvector Centrality of Facebook Graph

In this Section we focus on *PU graph*, and we study temporal eigenvector centrality for all nodes in that graph. We ranked the vertices according to their eigenvector centrality values and their ranks are compared with the ones obtained calculating their static counterparts in the aggregated system's lifetime and also in the same time-frame snapshot. Given that the measures are different in their nature, we expect to see the emergence of different behaviours, moreover, it is interesting to observe whether eigenvector and betweenness centralities are correlated or not.

6.8.1 Temporal Eigenvector Centrality in The System Lifetime

We first compute the eigenvector centrality of the system during its whole lifetime. Temporal eigenvector centrality can be measured in two ways: SDI and ADI. In SDI, mostly the node affects its importance while in ADI, the neighbours of a node are more indicative of its importance than the node itself. Figures 6.18 and 6.19 show the temporal eigenvector centrality of the PU graph based on the times when nodes joined the graph.

Comparing Figures 6.14 and 6.18 we see a high correlation between the SDI eigenvector centrality and temporal betweenness. This correlation might confirm the hypothesis that the SDI model is closer to static eigenvector centrality, and also to betweenness as it relies on the impact of the node on its vicinity.

Meanwhile, SDI and ADI show similar patterns except in [2009,2013]. Investigating

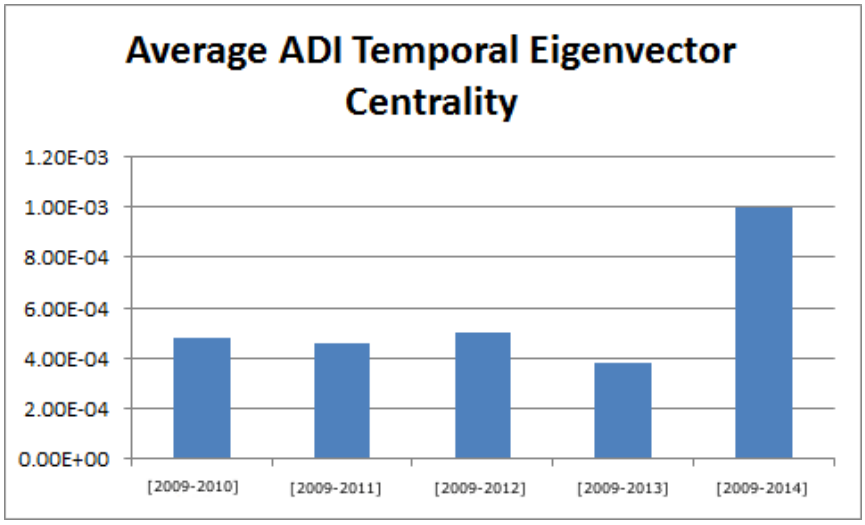


Figure 6.19: ADI Eigenvector centrality of PU graph in [2009,2014]

the graph, we see that the majority of nodes joined the PU graph in [2009,2013]. At the same time, the number of edges that are created in that year is almost three times the number of nodes (Table 6.4). This causes the nodes that joined in that year, to have high probability of being adjacent to more neighbouring nodes. This makes most of the nodes equally important and equally between, which resulted in higher betweenness and lower eigenvector centrality. This also explains the very close value of ADI and SDI eigenvectors for such nodes joining in [2009,2013].

However, referring to Table 6.4, one might ask why such characteristics are not observed for the nodes in [2009,2011], while the ratio of the edges to nodes are even higher than [2009,2013]. In [2009,2011] the edges, hence the degree, are not equally distributed among all the nodes, and some nodes have extremely high degrees while most others have only degree of one or two, or three. This causes an uneven distribution of importance values no matter what the graph's structural characteristics are. Nevertheless, in 2013, the degree is almost evenly distributed among all the nodes. Hence, the graph structure is similar in different sub-graphs of PU. Therefore, the importance values corresponding to different nodes are closer to each other.

6.8.2 Shockers and Breakers

Comparing SDI and ADI values of PU graph with the static eigenvector centrality of PU graph in its lifetime (Figure 6.11) shows no correlation between these values, except an abnormal behaviour in [2009,2013] which is also seen in SDI and ADI models. No

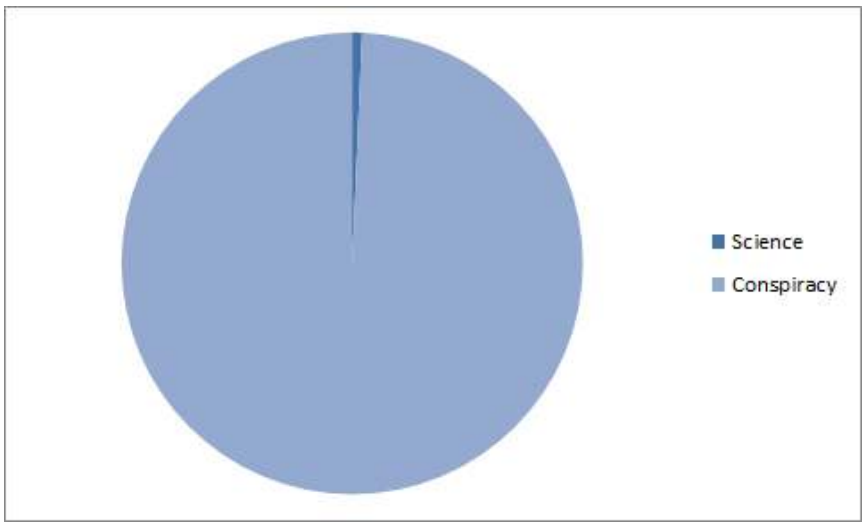


Figure 6.20: Distribution of Shockers in Science and Conspiracy

correlation between static and temporal values ensures that temporal analysis provides more informative facts about node activities in time that is hidden by static analysis.

Among the top 10% high ranked nodes in SDI model, 4.1% appear only in very low static ranks. The similar value for ADI is 8%. These nodes are not very good overall influencers, yet they spark at different moments and influence their neighbours. We call such nodes *shockers* as they sock their influencee when such high monumental influence is not expected from them. Most shockers are among the conspiracy actors, which explains why they might have the shock effect. The conspiracy actors usually create a buzz around a hoax news, so that even the science users get involved in it in order to educate the public on that matter (Figure 6.20). This increases their hubbiness factor (eigenvector centrality) briefly. As this action appears more frequently, the created momentum results in high temporal eigenvector centrality.

On the other end of the spectrum, there are users who have high static eigenvector centralities and very low temporal eigenvector centralities. We call such nodes *breakers*, who break the news slowly to their neighbours. Contrary to shockers, the number of breakers are not high in the PU Graph, corresponding to 1.19% and 1.72% of top 10% static nodes, respectively for SDI and ADI. It is interesting to mention that most breakers are science users (Figure 6.21). The involvement of science users is consistent in different topics, and their followers consistently follow them in different discussions as they share interest in scientific topics.

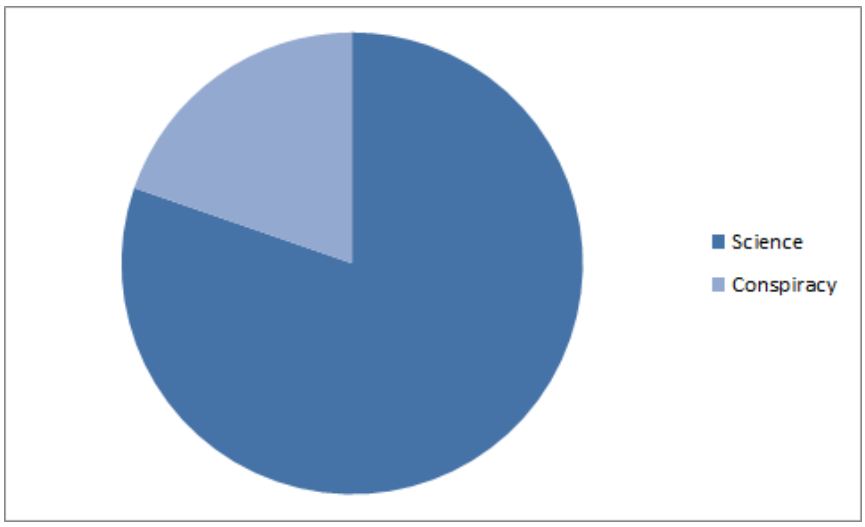


Figure 6.21: Distribution of Breakers Among Science and Conspiracy Users

6.9 Conclusions

In this Chapter we performed a temporal analysis of Facebook data relative to the commenting activities of a large number of users on several Facebook pages.

Facebook users create cliques around the Facebook pages, and these cliques are interconnected. Given the very large number of users and thus the impossibility of computing exact betweenness values, we took a novel approach that focuses on those users who interconnect these cliques (bridges) giving a more passive role to the less important users who belong only to single cliques. Following this approach we calculated an estimate of foremost betweenness.

Foremost betweenness indicated the users who contribute most to the fast convey of information through comments, showing that these users belong, for the most part, to scientific community. This seems to indicate that, even if there is a larger number of conspiracy users, these users are not highly important from a temporal point of view and that science users distribute scientific data faster and to a wider community than conspiracy users. The analysis also shows that, as expected, static betweenness is unable to predict this type of importance and that the results of static and temporal betweenness values are not highly correlated.

Finally, we measured the eigenvector centrality of the PU Graph in both ADI and SDI approaches and, also in this case, we identify two groups of users like we did in the case of foremost betweenness.

In conclusion, both betweenness and eigenvector centrality indicate that science users

are more temporally important than conspiracy users. They bridge more groups to each other, hence they have a high betweenness. Moreover, they have friends who follow their conversation persistently over the time. Thus, since persistence has a great effect in temporal eigenvector measure, their eigenvector centrality value is higher than conspiracy users who do not have persistent followers.

Chapter 7

Propagation Study in YouTube

In this Chapter we study various aspects of a social networking website (YouTube) whose data has been collected in a snowball sampling method. More precisely, we study the friendship and the subscription networks created around a high number of users starting from a randomly selected video. We analyse the propagation of videos among friends and subscribers noticing that the average number of hops a video traverses in this period of time is quite small in both cases. We then study the relationship between the popularity of a video and its propagation rate discovering that there is no direct link between the two parameters. Finally, we analyse users similarities discovering that the users that are friends are not necessarily similar in interests, while the interest similarity of subscribers are relatively higher. In contrast with the previous two Chapters, the analysis carried out inhere is based on "static" parameters employed on the aggregation of the network over time. The results of this Chapter have been published in [2, 3].

7.1 YouTube Social Network

YouTube, a subsidiary of Google, is the largest video sharing website containing about 43% of all videos found on the Internet [1]. Since its launch in 2005, the popularity of YouTube has consistently increased, and more web users, from various demographics, registered on this video sharing website to benefit from its contents and features. Statistics from 2010 state that more than 35 hours of video are uploaded to YouTube every minute[2]. But

[1] ComScore - Accessed: July 8, 2011. `http://www.comscore.com/Insights/Press-Releases/2010/6/comScore-Releases-May-2010-US-Online-Video-Rankings`

[2] YouTube - Press Statistics. Retrieved July 9, 2011, from `http://www.youtube.com/t/press_statistics`

YouTube is not just a video sharing website. It also accounts for being a social network since it has a large number of registered users (i.e., channels) who can upload videos, follow other channels (i.e., subscribe), and be friends with other channels. Thus, there are many channels in YouTube with millions of friends and subscribers[2]. Most importantly, in order to fully qualify as a social network, YouTube has to enable users to communicate with each other. YouTube satisfies this requirement by implementing a broad infrastructure that allows users to communicate with each other in many different ways which resulted in users commenting on nearly 50% of YouTube videos[2]. YouTube's communication infrastructure includes the following features:

- Private messaging: channels can send private messages to each other
- Commenting on channels: channels can comment on other channels
- Commenting on videos: channels can comment on videos posted by themselves or other channels
- Marking a video as favourite (favourite-marking): channels can favorite uploaded videos
- Publishing video descriptions: the uploader channel can write a video description for its uploads
- Liking or disliking a video description or a comment (rating): channels can like or dislike video descriptions or comments that are posted by other channels
- Replying to a comment: every channel can reply to a comment. This is simply the act of commenting on comments.

YouTube provides the advantage of allowing two types of relationships between channels: friendship, which creates a two-way relationship for channels, and subscription, which allows channels to get updates on any other channel while having a one-way relationship with those channels. This feature allows us to evaluate our model on friendship and subscription on the same social network with the same communication features. Note that since private messages are not extractable, from an external observer's view point, the communication features are the same for both friends and subscribers. The existence of this feature is very important as it gives the opportunity to analyse the behaviour and communication patterns of friends and subscribers, as well as their influence on content propagation.

7.2 Data Collection

Google (YouTube's owner) published a library of APIs and tools that enable developers to connect their applications with Google products. APIs are a set of message formats that facilitate communication between different applications. In order to collect data we used YouTube APIs[3], and crawled a subset of the YouTube network. We randomly selected a YouTube video and chose its uploader as our starting point. In addition to recording all publicly available communications, uploads, and their information, we located the uploader's friends and subscribers. We continued crawling by performing the same tasks for the friends and subscribers. Note that we conducted this operation separately for friends and subscribers, as each has its own network hierarchy. In this way, only for the friendship network, we collected 10 different subsets of YouTube social network in a snowball sampling method with different starting points.

The extracted friendship network can be described as an undirected graph $G_f(V, E, E')$, where V is the set of users, $(x, y) \in E$ is a link between two users such that x and y are friends, and E' is a collection of directed edges between x and y such that represent commenting activity of x on a video posted by y, or on an activity that y conducted on user z's video (z is linked to y). Edges in E' are labelled with the video corresponding to the activity. Similarly, the subscription network is a directed graph $G_s(V, E, E')$, with V representing the collection of users and $(x, y) \in E$ shows a subscription link, such that x is subscribed to y. In this case, E' represents the same concept defined for G_f.

We should mention that we collected the interactions as signs of content propagation because YouTube has (had) a system that reveals (revealed) recent activities of friends and subscribers, so every comment is (was) visible to all neighbouring vertices. We did not evaluate the content of comments, so spam might be among our collected data. However, considering that we are mainly interested in comments made by friends or subscribers or their networks, the amount of spam can be small compared to meaningful comments, the small error created by spam can be ignored. Table 7.1 contains the statistics of our collected data.

Table 7.1: The Statistics of Collected Data

Dataset#	Data Description	Statistics
	#Users	8,984

[3]YouTube APIs - `http://code.google.com/apis/youtube/overview.html`

Dataset 1

Dataset#	Data Description	Statistics	
	#Videos	113,562	
	Friendship	#Link	8,986
		Max Degree	57
		Average Degree	2.66
	Subscription	#Link	16,830
		Max Degree	456
		Average Degree	19.77
Dataset 2	#Users	9,633	
	#Videos	332,296	
	Friendship	#Link	13,863
		Max Degree	63
		Average Degree	2.68
	Subscription	#Link	40,358
		Max Degree	457
		Average Degree	25.17
Dataset 3	#Users	15,193	
	#Videos	88,670	
	Friendship	#Link	20,230
		Max Degree	60
		Average Degree	2.55
	Subscription	#Link	29,986
		Max Degree	350
		Average Degree	21.68
Dataset 4	#Users	12,069	
	#Videos	48,500	
	Friendship	#Link	15,193

Dataset#	Data Description	Statistics	
		Max Degree	73
		Average Degree	2.58
	Subscription	#Link	19,620
		Max Degree	500
		Average Degree	23.34
Dataset 5	#Users	17,180	
	#Videos	10,089	
	Friendship	#Link	22,076
		Max Degree	106
		Average Degree	2.78
	Subscription	#Link	29,630
		Max Degree	875
		Average Degree	19.94
Dataset 6	#Users	19,888	
	#Videos	86,920	
	Friendship	#Link	27,170
		Max Degree	26
		Average Degree	2.00
	Subscription	#Link	32,157
		Max Degree	25
		Average Degree	16.21
Dataset 7	#Users	7,111	
	#Videos	52,972	
	Friendship	#Link	7,515
		Max Degree	24
		Average Degree	2.72

Dataset#	Data Description	Statistics	
	Subscription	#Link	24,446
		Max Degree	25
		Average Degree	16.50
Dataset 8	#Users	3,267	
	#Videos	21,785	
	Friendship	#Link	3,461
		Max Degree	26
		Average Degree	2.01
	Subscription	# Link	31,725
		Max Degree	25
		Average Degree	16.59
Dataset 9	#Users	17,055	
	#Videos	92,080	
	Friendship	#Link	17,358
		Max Degree	26
		Average Degree	2.17
	Subscription	#Link	179,138
		Max Degree	25
		Average Degree	16.95
Dataset 10	#Users	6,650	
	#Videos	47,778	
	Friendship	#Link	6,651
		Max Degree	66
		Average Degree	2.53
	Subscription	#Link	60,530
		Max Degree	25

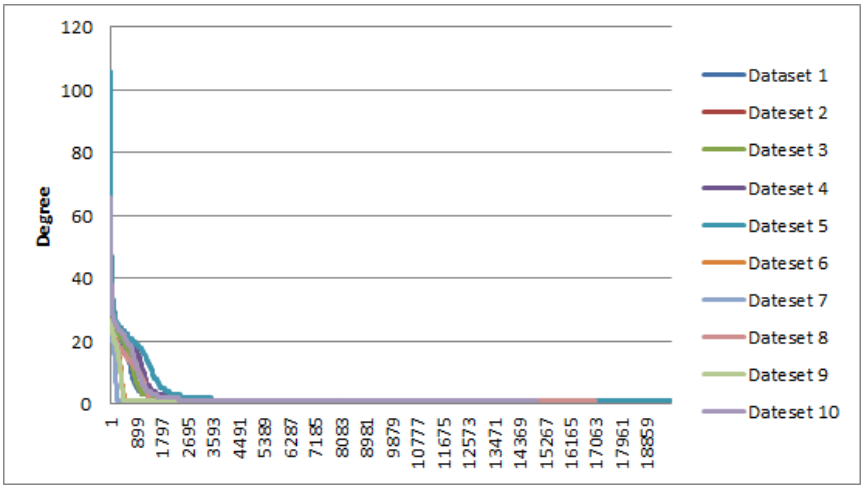


Figure 7.1: The Degree Distribution of YouTube Friendship Social Network

Dataset#	Data Description	Statistics	
		Average Degree	15.95

7.2.1 YouTube Statistics

Since analysis of 10 datasets showed very similar results, we focus on providing figures and analysis for just two datasets (datasets 1, for friendship, and 9, for subscription) in the rest of this chapter. Analysis of the extracted network of YouTube (from this point on, we refer to the extracted subset of YouTube as simply YouTube network) users shows that with the extraction of about 9000 friends using snowball sampling, we reached a maximum of five hops from the seed user. This gives an estimate about the connectedness rate in the friendship network in the YouTube social network.

To explore the statistics of our extracted networks, we focus on the degree distribution which can show how the network behaves. The degree in G_f is defined as the number of friends that a user have (Table 7.1). These statistics mean that users tend to have a small number of friends on YouTube (Figure 7.1). To better visualize the graph, we removed high degree nodes as well as those that have zero degree in Figure 7.2. Figure 7.2 represents all the friendship network samples considering that the sampling for datasets 1 through 5 was based on friendship, and it was based on subscription on the rest of datasets.

On the other hand, statistics for the subscription network are different. The subscription degree (out-degree) in G_s is defined as the number of channels that a user is subscribed to (Figure 7.3; for better clarification on the visual chart, we removed the few very high

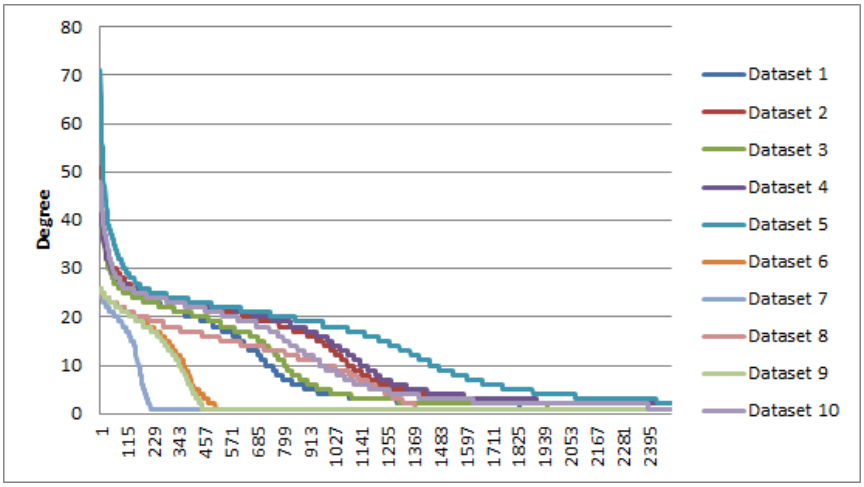


Figure 7.2: The Degree Distribution of YouTube Friendship Social Network excluding very high degree nodes as well as nodes with degree equal to zero

degree nodes in Figure 7.4). However, the number of users with zero subscription is still high. Interestingly enough, the number of users with high number of subscription is high, too. Meanwhile, it can be seen from the graph that the mode of subscription is 25, meaning that many users tend to subscribe to 25 channels, which is interesting by itself. This means that the ease of subscription and lack of necessity to be approved by the other user are factors that encourage users to subscribe to other channels rather than create a friendship link. These statistics help us understand the underlying network structure of the crawled data.

Figures 7.1, 7.2, 7.3, and 7.4 reveal an interesting fact about the networks of friendship and subscription. On the charts, the two networks seem to have dissimilar distributions. However, both networks experience a peak at around degree value in 20s.

7.2.2 Limitations in Data Collection

Unfortunately, YouTube does not keep track of more than 7500 comments for each video, so we could not evaluate the speed of propagation. However, the most popular video was uploaded in 2006, and still receives comments. All the first thousand popular videos received their last comment on the day of data collection in 2011.

Moreover, this limitation may affect our results if friends and subscribers were among the people who commented first on the videos. To measure this effect, we selected a smaller dataset of videos with less than 7500 comments and ran the analysis on them. Our analysis, nevertheless, showed similar results on propagation magnitude, and its correlation

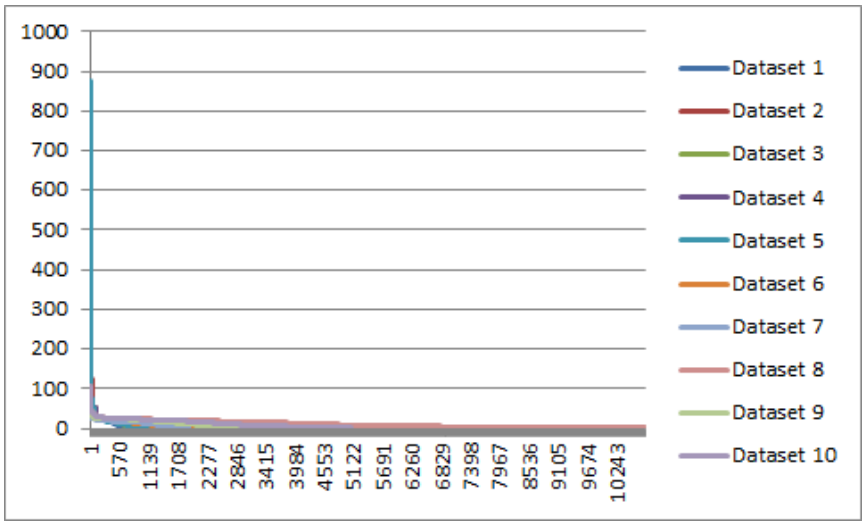


Figure 7.3: The Degree Distribution of YouTube Subscription Network

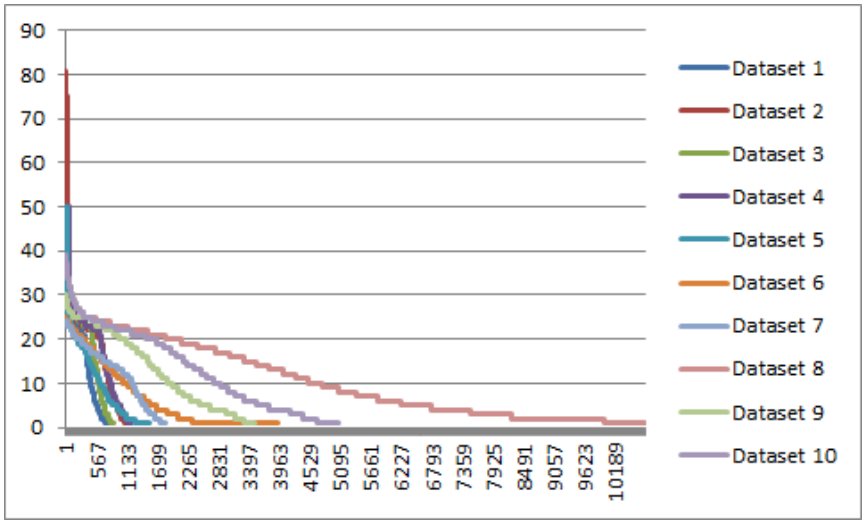


Figure 7.4: The Degree Distribution of YouTube Subscription Network (few very high degree nodes are removed)

with popularity.

7.3 Propagation in YouTube

YouTube data can be propagated by different means, and is not restricted to commenting inside the YouTube network. These methods range from inside network propagation to exporting the video on a personal blog or website. Table 7.2 provides a set of methods that contribute to content propagation in YouTube.

Table 7.2: Video propagation methods in YouTube

Propagation Method	Description
Sharing	Users can share YouTube videos by email, posting on blog, etc.
Recommended Videos	Videos that are recommended by YouTube based on user's previous visits.
Featuring	YouTube features some videos on its first page.
Suggested Videos	Videos that are similar to the video that the user is watching.
Search Results	Videos that appear in search results.
Recent Activities	Videos that were involved in recent activities of user's subscribers or friends.

Since we are interested in content propagation on YouTube that is generated by friends or subscribers, we are interested in the users' recent activities (i.e., five most recent uploads, commenting, rating, etc. that appear on every user's profile page) that are visible to friends and subscribers. Rating, favourite-marking, Commenting on a video, and uploading a new video are the commonly observed recent activities, with rating being the most common one. As YouTube does not allow access to ratings or favourite-markings per user, we only extracted the networks of users who commented on each others' videos. These networks include data on comments that are made on videos by users who have a path through friendship or subscription to the uploader. In other words, we eliminated from our analysis comments that were not made by friends, subscribers, and their networks.

In the context of YouTube networks G_f and G_s, we define propagation as the existence of activity edge $e \in E'$ from a vertex v to a vertex u corresponding to video vid.

Propagation Magnitude in YouTube

The first step in analysing the propagation is to analyse the magnitude, or the longest hop, by which data propagates. Formally, we define propagation magnitude as the eccentricity of the uploader v of video vid in the edge induced subgraph of G_f and G_s limiting the edges to the subset of E' that have label vid. Our dataset of five hops shows interesting results. We discuss them in the friendship and subscription datasets.

Propagation Magnitude in Friendship Network. As mentioned earlier, we focus the section on the results on dataset 1. We recorded a total 16.4 million interactions on videos that are posted in our friendship dataset. Since we are only interested in interactions between friends, we pre-processed our data to extract the underlying network of interactions between friends. This resulted in a huge reduction in our sample graph. This illustrates our first finding: in an open social network, the amount of interactions between strangers accounts for a high percentage of the total interactions.

This finding is verified by a reduction of our captured interactions to 133 thousand interactions, a reduction rate of 98.76%, when we filtered out the interactions between channels that do not have a friendship path to the uploader node.

Analysis of propagation in the friendship network revealed that videos are propagated at most to three hops of friends (a hop denotes a link between two levels of friendship). Meanwhile, the distribution of propagation reveals that only a small fraction of the videos is propagated to the second and third levels of friends (Table 7.3).

Table 7.3: Propagation of videos in friendship network

Propagation Magnitude	#Videos	%Propagated Videos	%Total Videos
1 hop	1,289	96.84%	1.14%
2 hop	40	3.00%	0.04%
3 hop	2	0.16%	0.01%

The propagation of videos through friendship is not significant. However, looking at the users involved in propagating the videos suggests that a huge part of propagation is carried out by a small number of users. We observed that the commenting pattern in the friendship network follows a power law distribution with the exponent of 2.02, meaning that the contents are highly propagated through a small number of highly active users (Figure 7.5).

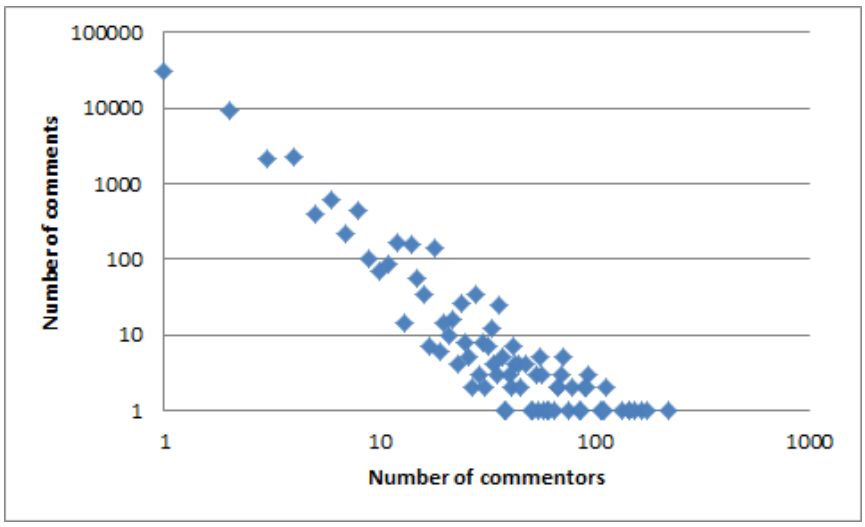


Figure 7.5: Log-Log chart of YouTube commenting, pertaining to friends in dataset 1

Propagation Magnitude in Subscription Network. In the same way, we recorded a total 44.7 million interactions on videos, in dataset 9, that are posted in our subscription network. Since we are only interested in interactions between subscribers, we pre-processed our data to extract only the interactions between subscribers. Similar to the friendship network, this resulted in a huge reduction in our sample graph. The captured interactions were reduced to 27 thousand, much less than the interactions in the friendship network. This reduction has a rate of 99.93%, which means that almost all interactions happen between users who do not have a path through subscription. This was a surprise because since the connectedness of the subscription network is far higher than the friendship network, it was expected that subscribers have more effect on propagation than friends. The low effect on propagation may be due to lower personal connection between subscribers, hence subscribers are less inclined to leave comments.

Meanwhile, our analysis of propagation in the subscription network revealed that videos are propagated at most to two hops of subscribers. Moreover, the distribution of propagation suggests that only a small fraction of the videos are propagated to the second level of subscribers (Table 7.4).

Table 7.4: Propagation of videos in subscription network

Propagation Magnitude	#Videos	%Propagated Videos	%Total Videos
1 hop	269	96.76%	0.88%
2 hop	9	3.24%	0.03%

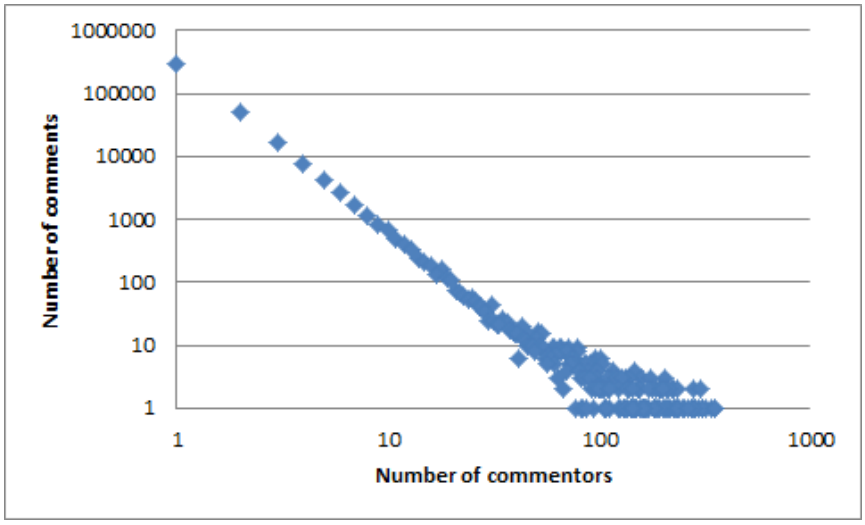


Figure 7.6: Log-Log chart of YouTube commenting, pertaining to subscribers in dataset 6

Similar to the friendship network, the propagation of videos through subscription is not significant. However, looking at the users who are involved in propagating the videos still suggests that a huge part of propagation is carried out by a small number of subscribers. We observed that the commenting pattern in the subscription network follows a power law distribution with the exponent of 2.01, meaning that the content is highly propagated through a small number of highly active users (Figure 7.6).

7.4 Propagation and Popularity in YouTube

In the next step, we investigated the popularity of videos in relation to their propagation, in order to understand whether the popularity of videos drives or is driven by propagation, or if friends and subscribers choose the videos to comment on based on other considerations. To do so, we selected a set of ten highly propagated videos in addition to ten highly popular videos from each dataset, and evaluated the correlation of popularity and propagation of videos. Popularity is defined by the view count of the video in the YouTube website (Figure 7.7). We measure the popularity of a video by its view count and ratings. Table 7.5 shows statistics of the five most popular videos in our datasets. These videos may or may not be propagated by network members, and these statistics show general popularities of videos without considering their propagation. Note that three of five popular videos are common in both networks. This infers the similarity of growth patterns in both networks.

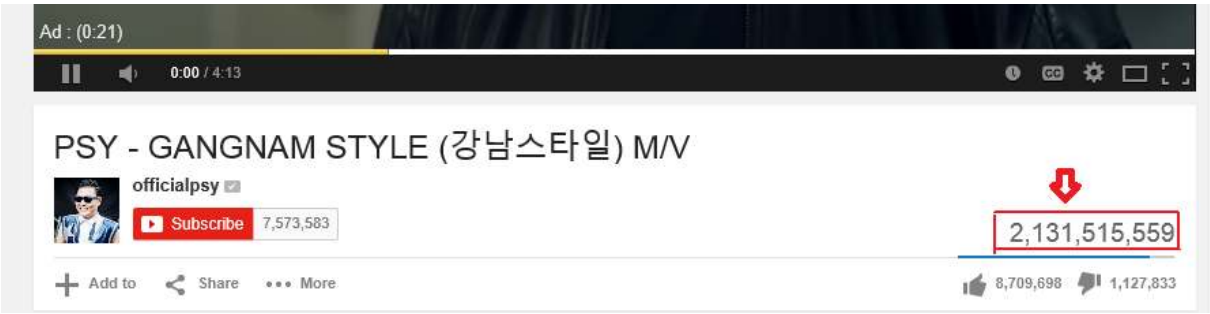


Figure 7.7: YouTube Viewcount

Table 7.5: Statistics of popular videos in datasets

#Dataset	#Views	Rating
Dataset 1 (Friendship)	1.8×10^8	4.68
	8.6×10^7	4.91
	4.8×10^7	4.83
	4.6×10^7	4.54
	3.8×10^7	4.93
Dataset 9 (Subscription)	1.6×10^7	4.91
	4.8×10^7	4.83
	3.8×10^7	4.93
	3.4×10^7	4.91
	3.6×10^6	4.50

7.4.1 Propagation and popularity in friendship network

To measure the correlation between popularity and propagation in the friendship network, we extracted the five most popular and the five longest propagated nodes from the network of friendship interactions, i.e., the friends who commented on each other's posts (Table 7.6). In our first observation, none of the videos that appeared in the network's most popular videos (Table 7.5) appeared in the most popular and deepest propagated set in the friendship interaction network, and the most popular video in the friendship interaction network was, in fact, ranked 1570 out of 113 thousand videos in the total friendship network. Meanwhile, the longest propagated videos had average popularities in the friendship network. These figures mean that the propagation of videos by friends does not affect the popularity of videos, and vice versa.

Table 7.6: The deepest propagated, and the most popular videos in friendship network

Type	Propagation Depth (#hops)	#Views	Rating
Longest Propagated	3	575	5
	3	231	3.67
	2	71953	3.78
	2	61429	3.95
	2	30914	4.75
Most Popular	1	562261	4.94
	1	558523	4.89
	1	78220	4.94
	1	78074	4.93
	1	76163	4.39

7.4.2 Propagation and popularity in subscription network

We applied the methodology that we used for the friendship network on the subscription network. The analysis of the subscription network shows that the most popular video (Table 7.7) ranked 747 out of 332 thousand videos in the total subscription network (Table 7.5). On the other hand, videos that are propagated the most in the subscription network are also subscription network's most popular videos. Therefore, there is a correlation between the popularity and the level of propagation by subscribers, meaning that more propagated videos by subscribers become popular at least among the subscribers and their network or vice versa.

7.5 Discussion on YouTube Propagation

Advertisement is a costly process for businesses, and in some cases, it takes a considerable amount of the business budget. Businesses have always looked into ways to advertise their products and services at a lower cost. Viral marketing and advertisement on social networks provided a solution for this requirement. However, there is still a considerable cost associated with viral marketing even though it is lower than, say, banner ads. This cost is mainly associated with influencing the first person, and encouraging him/her to spread the word, in addition to making sure that the word will spread to the next levels in the

Table 7.7: The deepest propagated, and the most popular videos in subscription network

Type	Propagation Depth (#hops)	#Views	Rating
Longest Propagated	2	72001	3.78
	2	61429	3.95
	2	30935	4.75
	2	7262	4.43
	2	5072	3.96
Most Popular	1	562261	4.94
	2	72001	3.98
	2	61429	3.95
	1	37203	4.82
	2	30935	4.75

network. Therefore, businesses may be interested in finding the most appropriate person and the most appropriate network to do the advertisement. The low propagation rate among friends and followers in an open social network suggests that open social networks are not generally well suited for businesses that need to spread the word in communities. Meanwhile, the better propagation rate among friends (compared to followers) suggests that the focus of businesses should be on friendship networks.

At the same time, our research suggests that in friendship networks, the popularity of the message does not affect its propagation, while in follower networks it does. Therefore, businesses may need to focus on making the message itself interesting (popular) within follower networks more than they do within friendship networks. Therefore, our study reveals that content propagation in on-line open social networks follows different patterns compared to what has been observed in off-line social networks (i.e., pre-internet social networks) [94]. Although the actions of individuals are usually open to a wide range of other users in both off-line and on-line open social networks, interestingly, propagation in off-line social networks is mostly affected by the number of ties (i.e., friends, co-workers, and family) and their networks, while our study revealed that in an on-line open social network, propagation is far more affected by individuals who are neither in the network of friends nor the network of followers of the content generator.

Other studies also revealed contradictory results. For instance, Crandall et al. [29] studied multiple on-line and off-line social networks and discovered that an increase in

similarity between on-line social network users boosts both the magnitude and speed of content propagation. On the other hand, and focusing merely on off-line social networks, Feld [44] discovered that similarity is one of the major factors that define the strength of ties between members of a social network. Note that from this point on, we only focus on friendship. Thus, a tie means the existence of a direct path between two social network users. It can be argued that since friends of a user u have stronger ties with u (assuming that friendship in on-line social networks has the same meaning as friendship in the off-line world), and consequently a greater similarity, they should participate more in propagating the user's content, and consequently affect its propagation more than non-friends.

According to the literature, similarity is a boosting agent for content propagation, while our study interestingly showed that strangers (non-friends, and non-followers) affected YouTube content propagation more than friends. Our objective, here, is to analyse communities (communities are formed by ties between users of a social network, and detected using random walks [95]) within the YouTube social network to measure the similarity between members of those communities. For that we compute and analyse similarity metrics within the entire social network, and within its communities. This gives us a comparative tool for investigating similarity values. We also evaluate the ratio of friendship over similarity with the goal of understanding if similar community members are in fact friends.

We focus on interest similarity since it is one of the most effective similarity measures contributing to the propagation of content or influence [108]. Although on-line social networks differ in their settings and content types, and probably follow different similarity patterns, a look at the work of Mislove et al. [83] leads us to conclude that social networks that fall into the same category based on their privacy settings, user demographics, and applications, display similar information dissemination and similarity patterns. Considering that, we selected YouTube for our analysis as a good representative of on-line open social networks. We measure interest similarity between YouTube users based on the common topics they share with their friends, followers, and strangers in communities. We measure the similarity of connected and unconnected users in each community, and analyse the ratio of links between similar users versus dissimilar users. This will lead us to answer the question: do similar users in communities befriend each other, and to what extent?

Researchers in sociology, mathematics, and physics have proposed different similarity measures, and Social Network Analysis has adopted them to study similarity in social networks. In this Chapter we evaluate some of these similarity measures in a real social network setting and evaluate them based on the ratio of friendship between similar users.

7.6 Interest Similarity and Ties in YouTube

According to Crandall et al. [29], friends and followers in social networks are either similar to each other at the time the friendship (or follower) tie is made (aka selection process) or they grow in similarity over time after they become friends or followers through social influence. Also, rising similarity between two individuals is an indicator of current, and more specifically future, interactions between them [29, 44]. Therefore, we argue that current activities of friends and followers of a user, who are presumed to have a certain degree of similarity, can be a predicator of that user's next activity. Hence, friends, also recognized as the most similar people by Crandall et al. [29], should have the greatest effect on content propagation. But the question is: are friends the most similar people in their community? This section attempts to answer this question by analysing data extracted from YouTube for similarity friendship ratios (the ratio yielding that what percentage of similar users in communities are friends). To do so, we utilize the similarity measures defined in Section 3. Note that we cleaned the YouTube dataset to only keep friends in our evaluation and ignored all follower links in order to comply with the findings of Crandall et al. [29] who only consider reciprocated links (here, YouTube friends).

Before we proceed, it is important to comprehend that communities are different from groups, where communities are concepts that are generated based on existing links between social network members, and groups are a feature introduced on social networks to gather users with similar profiles into a single place. Since the access to the group data is not provided by YouTube APIs, we used a dataset collected by Mislove et al. [83], and certainly is different from the datasets that we saw earlier in this chapter.

7.6.1 Similarity Measures and Functions

This section is devoted to a review of popular similarity measures used in social networks analysis. According to Lin [78], similarity is a function of commonality and difference, in a way that if two objects are not exactly the same, their similarity depends positively on the amount of their common features, and will have negative relations with their differences.

Many similarity measures have been developed; each tied to an application or requiring a specific domain and design. Therefore, not all similarity measures are suitable to be applied on social networks to compute interest similarity. To measure the similarity of YouTube users, first, we selected a set of similarity measures that can be applied to interest similarity, and then we applied each measure (all of them discussed in this section) as a

function of common group memberships of YouTube users. According to Baatarjav et al. [8], a group in a social network has specific characteristics that match the profiles of most of its members. Therefore, users who share a set of group memberships should have a similar profile. Note that analysing similarity based only on group membership may not provide results as accurate as those that can be obtained by semantically analysing, for instance, the content of users' postings, and considering the demographic information of users.

Jaccard and Dice's Similarity Coefficient

Jaccard and Dice's similarity coefficient measures are specific to measuring set similarity [34, 63]. These measures were first developed to measure similarities in ecological studies, but their nature of set operations made them applicable for measuring social similarity. They are computed by dividing the intersection of sets over their union. Jaccard and Dice's index can easily be converted to each other and provide monotonic asymmetric results. Therefore, in this chapter, we only use Jaccard similarity coefficient for simplicity. Jaccard index is calculated using the following equation:

$$J(U_1, U_2) = \frac{|H_1 \cap H_2|}{|H_1 \cup H_2|} \tag{7.1}$$

where H_1 and H_2 are the group membership sets of user sets U_1 and U_2, respectively.

Russel and Rao Similarity

Russell and Rao similarity measure [99] is close to Jaccard's similarity coefficient. Russell and Rao measure the similarity of the common items compared to the whole vector including the attributes, here groups, that are absent from both vectors. In other words, the Russell and Rao similarity measure computes the common group memberships versus the whole set of unique groups in the system, and is calculated by:

$$R(U_1, U_2) = \frac{|H_1 \cap H_2|}{|H|} \tag{7.2}$$

where $|H|$ represents the total number of group memberships.

Roger and Tanimoto Similarity

Roger and Tanimoto [97] devised a measure that is suitable for comparing the similarity of Boolean vectors. Their model gives double weight to disagreements. The Roger and Tanimoto index is calculated by:

$$T(U_1, U_2) = \frac{|H_1 \cap H_2| + |H_1^c \cap H_2^c|}{-3|H_1 \cap H_2| + 2(|H_1| + |H_2|) + |H_1^c \cap H_2^c|} \tag{7.3}$$

where H_i^c represents the groups that do not have user U_i as their member.

Sokal and Sneath Similarity

Sokal and Sneath similarity measure [106] is comparable to Dice's measure and to Roger and Tanimoto measure. The only difference between Sokal and Sneath and Roger and Tanimoto similarity measures is in the heuristic constant components of the formulas, which produce almost similar results. Sokal and Sneath give double weight to matches instead of differences. Sokal and Sneath, however, founded their model on the Jaccard and Dice similarity measure by extending it to integrate dissimilarity of items into the calculation of similarity. It is calculated by:

$$S(U_1, U_2) = \frac{|H_1 \cap H_2| + |H_1^c \cap H_2^c|}{|H_1 \cap H_2| + |H_1| + |H_2| + 2|H_1^c \cap H_2^c|} \tag{7.4}$$

L^1 and L^2 -Norms

With regard to sets, L^1-Norm, and L^2-Norm [64] evaluate similarity to be the overlap between two groups divided by their sizes. L^2-Norm compared to L^1-Norm decreases the level of effect that the sizes of individual sets have on the similarity measure. L^1 and L^2 -Norms are measured by:

$$L^1(U_1, U_2) = \frac{|H_1 \cap H_2|}{|H_1||H_2|} \tag{7.5}$$

$$L^2(U_1, U_2) = \frac{|H_1 \cap H_2|}{\sqrt{|H_1||H_2|}} \tag{7.6}$$

7.6.2 Data Description

Before developing our analysis, the data must be cleaned and made ready for analysis. We have access to a large dataset of over 1.15 million YouTube users and their group memberships along with information about ties between them. This dataset was collected and formerly used in an analysis by Mislove et al. [83]. The dataset covers more than 30 thousand groups and contains over 290 thousands recorded group memberships, so on average, every user in the dataset is a member of roughly four groups. Every user, on average, has more than four reciprocatory and non-reciprocatory ties with other users. The most connected user has over 28 thousand links, while the majority of users only have one link. Figure 7.8 shows the distribution of tie frequency per user in the YouTube social network.

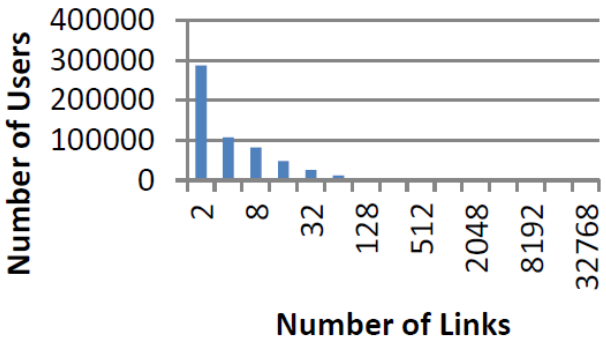


Figure 7.8: Frequency of ties per user

The highest number of ties in the network belongs to a user with 28,644 connections while the second most connected user only has 11,239 connections. Interestingly, about 183 thousand users only have only one connection, and more than 500 thousand are not connected at all. This shows the level of uneven distribution of inactivity and activity in the YouTube social network. As it is apparent in Figure 7.8, most users have less than 128 ties. The full statistics of the YouTube dataset used in this study can be found in Table 7.8.

A more detailed look at the statistics shows that about 8% of the users are members of groups, which accounts for about 10 memberships per group. From this point on, our analysis only considers users who are group members, and we simply discard from our analysis the users who did not use YouTube's group feature. The statistical data also illustrates that, on average, users have three common group memberships, which shows a great potential for similarity between users.

Table 7.8: YouTube dataset statistics

Type of Data	Statistics
Users	$1,157,827$
Groups	$30,087$
Users that are member of at least one group	$94,238$
Users that are not members of any group	$1,063,589$
Links	$4,945,382$
Group memberships	$293,360$
Groups that a user with highest number of membership is subscribed in	$1,035$
Memberships for a group that has highest number of memberships	$7,591$
Communities	$139,142$

As planned, we then extracted communities from the YouTube dataset. To do so, we relied on the random walk community detection technique described in [95]. The Random Walk community detection method discovers communities based on their structural similarity. It first estimates the distance of vertices, as a metric for estimating structural similarity, and assigns it to them as a weight. The next step is applying a hierarchical clustering model in order to identify clusters (communities). The algorithm works at the time complexity of $O(n^2 \log n)$, which is suitable for analysing large graphs. We identified over 139 thousand communities with an average of 11 members per community, the largest community having 73 members.

7.6.3 Analysis of Similarities

As detailed earlier in this Chapter, we use common group memberships of users in the YouTube social network to measure the similarities between them. We argue that users who are members of the same set of groups are more likely to have similar interests, and that the similarity of interests increases as the number of common group memberships increases.

In order to perform this analysis, we implemented six applications, each of them responsible for performing one similarity measurement operation. The programs performed their analysis on a cleaned database of YouTube users that were previously clustered for communities using our RandomWalk clustering program developed using C++ and the

iGraph[4] library.

To measure similarities, we selected six well-defined and generally accepted similarity measures as detailed in Section three of this Chapter. Table 7.9 describes the result of applying each technique on YouTube social network, and its extracted communities. It, also, shows that for every similarity measure, the similarity of users within the communities is greater than the similarity within the entire social network. Being connected increases similarity, and therefore community members are more similar to each other than the rest of the network.

Table 7.9: Similarity Measures and the Result of Applying Them on the YouTube Social Network and its Communities

Metric	Social Network Average	Average Over Communities
Jaccard	0.14	0.31
Russel and Rao	0.90	0.91
L1	0.12	0.17
L2	0.26	0.34
Sokal and Sneath Similarity	0.50	0.54
Roger and Tanimoto Similarity	0.40	0.47

However, being a member of a community does not necessarily indicate friendship. A community is a collection of users who have transitive connections to each other. Therefore, there is a path between most community members. This also results in a high clustering coefficient for every node in the community. This means that a community is created from the collection of friends, friends of friends and so on. Based on our analysis, it is still not clear how much similarity induces friendship. To be able to answer this question, we selected users who have a more than average similarity with each other in their community, and examined if they are friends or not.

The result of our analysis shows that there is not a high correlation between similarity and friendship in communities (Table 7.10). In other words, most similar users are not necessarily friends even in small communities within the social network. Note that being in the same community means either a direct friendship or the existence of a short path with many mutual friends between two users. The friendship similarity ratio in small communities of connected people is not large (a range of 11% to maximum 38%), which is an indicator of our observation. In the beginning of this chapter, we observed that

[4]iGraph - `www.igraph.sourceforge.net`

content propagation in social network communities is done mostly by non-friends or non-followers. Also, as argued in the literature, content propagation happens where there is a high similarity between the propagator and propagatee. Therefore, it can be deduced that it is possible for indirect friends to be more similar that direct friends. Thus, a comparison of the results presented in tables 7.9 and 7.10 suggests that the higher average of similarity in communities might be the result of high similarity between indirect friends rather than similarity between friends.

Table 7.10: Similarity Measures and the Result of Applying Them on the YouTube Social Network and its Communities

Metric	Similarity Friendship Ratio in Communities
Jaccard	0.12
Russel and Rao	0.38
L1	0.32
L2	0.11
Sokal and Sneath Similarity	0.12
Roger and Tanimoto Similarity	0.21

7.7 Discussion

Our analysis shows that every similarity measurement method consistently yielded some degree of similarity between users in communities. Based on the proposition in [44], the higher similarity within communities was expected to be higher than the average similarity in the whole social network. This was confirmed by our results. However, the subsequent analysis that resulted in relatively low friendship similarity ratios in the communities was unexpected. Feld [44] proposes similarity as a determining factor in social ties in off-line social network. Nevertheless, the situation can be different in on-line social networks. Off-line social networks are known to be free of fake friends and spammers which is certainly not the case for on-line social networks [80]. The problem starts to grow when we realize that fake friends have on average six times more friends than legitimate users (i.e., users whose friends are real) [80]. Therefore, unless we have a mechanism to separate fake friends from real friends, the results cannot show the true ratio. Nonetheless, the friendship similarity ratio is so low that the general finding of low similarity between friends stands even if fake friends are removed from the network. The only difference would be a slight increase in

the ratio.

Based on the research done by Feld [44], it is expected that, in off-line social networks, similar people be friends with each other. Our study on YouTube found that this is not necessarily the case for on-line social networks. However, considering Feld's study, we expect that friends should have higher similarity. Therefore, similarity measures that result in a higher ratio between friendship and similarity provide more accurate results in the case of on-line social network.

By looking at the results presented in Table 7.10, the similarity measures that resulted in higher values of friendship similarity ratios in communities are Russel and Rao and L1 similarities. We have a second category including Jaccard, L_2, and Sokal and Sneath Similarity, with relatively similar results. Comparing these results with the values presented in Table 7.9, we see that even though the similarity values resulting from different techniques vary, the techniques can be categorized into two major categories with regards to their approximate accuracy. A conclusion about which category provides better results will depend on more research to be conducted on the correlation between friendship and similarity in on-line social networks. In which case, a higher correlation will play in favour of the first category of measurement techniques, and a lower correlation will favour the second category.

7.8 Conclusion

In this chapter we analysed the YouTube social network with regards to the propagation of videos to understand the characteristics of propagation. We crawled two subsets of the YouTube user network for friendship and subscription and analysed the propagation, and the role of friends and subscribers in content dissemination. We observed that the effect on propagation of people who are not either in a friendship network or a subscription network is higher than that of friends or subscribers. Meanwhile, we discovered that even though the network of subscribers was denser than the network of friends, the propagation in the subscription network was lower. This might imply that when the relationship is one-way, users are less inclined to contribute to the content.

Although our extracted data did not initially include user relations to the level of more than five hops, this limitation did not affect our study of the magnitude of propagation, and the correlation of propagation and popularity as even the most popular videos did not propagate more than three hops in their networks. Our result shows a low correlation between popularity and propagation in general. However, the correlation of popularity and

propagation in the friendship network is more than what exists in the subscription network. This may be due to the fact that friends feel more obliged than subscribers to contribute comments about the contents posted by their peers. On the other hand, subscribers may, most of the time, only comment on what interests them.

We also analysed the ties that exist between users and their common group memberships (which we used as an indicator of similarity of interests), to assess the relation between friendship and the similarity of interest inside communities of users within a social network. We found that the similarity between users increases if they are friends, but this increase does not define similarity as a determining factor in friendship. Considering that, and also the fact that content propagation in on-line social network communities is done mostly by non-friends, and knowing that similarity is a driver for content propagation, we can conclude that, within communities, indirect friends are more similar to each other than direct friends (as they participate more in content propagation). The second possibility is that the YouTube communities are formed from users that have little similarity whether friends or non-friends. The deterministic conclusion on the findings discussed above needs more exploration on the similarities between indirect friends, which is one the paths for our future study.

Furthermore, we examined several similarity measures to find the most suitable ones for processing on-line social network data. We found that similarity measures can be categorized into two categories based on their accuracy, which is measured by the friendship ratio. The results yielded by the Russel and Rao as well as L_1 similarity measures led to higher friendship similarity ratio, and Jaccard, L_2, and Sokal and Sneath Similarity fell in the second category. More research is needed to determine which category provides better results for on-line social networks.

Our analysis can be developed further to extract larger and better facts from a social network like YouTube. One of the limitations of this research is the lack of comprehensive data on the YouTube network. We only used a sample of YouTube, where users are group members, and we ignored users who are not members of a group. This resulted in a large YouTube user base. Therefore, a higher group membership rate would have improved the results. Time permitting, we also intend to extend our analysis to a temporal YouTube network including the timestamps for each connection and comment.

Chapter 8

Social Commerce: a Platform Founded on SNA

Social commerce is an emerging platform in software engineering and electronic commerce era that is sparked after creation of Web 2.0, and emerge of social networks. Social networks and all of their analysis techniques are the backbone and enabler of social commerce. Social commerce, as a newly introduced platform, is not yet well-defined. This includes vague ideas on how social networks should effectively be used in this new platform. In this chapter, we provide a framework for explaining social commerce, its ties to social network, its processes, and its challenges. Our framework works as a guideline for social commerce platform developers for streamlining the features and processes that should be included in their platform. The results of this Chapter have been published in [1].

8.1 Understanding Social Commerce

The concept of consumer buying behaviour is not new. It refers to the decision making process which evolves in multiple steps including the act of buying and using products and services. Studying consumer buying behaviour helps in understanding the influential factors on purchase decisions, and answers the question of why customers buy what they buy. It also enables firms to comprehend the reaction of customers to their marketing strategies. Understanding why, where, what, and how customers buy improves marketing campaigns and gives a better prediction of customers' response.

Consumer Buying behaviour model more or less points to six prevalent stages pertaining to customer behaviour, namely Need Recognition, Product Brokerage, Merchant

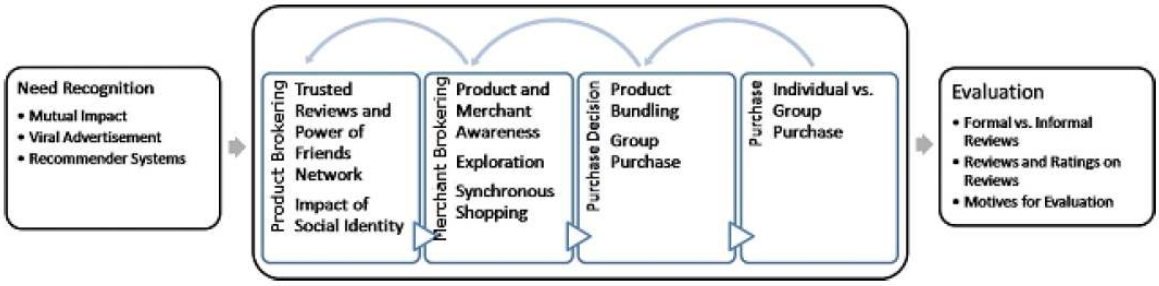


Figure 8.1: Model for understanding social commerce

Brokerage, Purchase Decision, Purchase, and Evaluation. As the basis of our proposed model, we will detail these stages in the next sections. Note that although each stage represents a decision making step in the purchase process, not all customers follow them in the specified order. For instance, in traditional marketplaces most low cost purchases are made without previous intention or research as customers see products on the shelf and make the decision to buy or not to buy. Even for more expensive products, the order of the stages can change. For instance, to buy a laptop, a customer might be determined to buy a Mac, so he immediately starts browsing through Apple products, placing the Merchant Brokerage stage before Product Brokerage. Nevertheless, in most cases customers follow the stages sequentially.

The adoption of social networks introduced a new set of components to the e-commerce environment. Fisher[1] divides these components into six categories: Social Shopping, Rating and Reviews, Recommendation and Referrals, Forums and Communities, Social Media, and Social Advertising. Each component has brought new challenges and advantages to the on-line shopping experience, urging for the analysis of consumer buying behaviour in the context of social networks. In our proposed model, we evaluate the effects of the above-mentioned components on social shopping behaviour from the viewpoints of consumers and businesses. Including businesses in the model should improve the analysis since businesses are usually part of consumer networks and they affect consumer decisions. In the following subsections we detail the stages of our model (Figure 8.1).

8.1.1 Need Recognition

The first stage in a customer's purchase decision making process is identifying the need for a specific product or service. Although this is considered the first stage in the process,

[1]Fisher - `http://goo.gl/SA37C2`

the role played by businesses in creating brand and product awareness begins long before customers become aware of a need.

Need recognition is associated with many issues that must be addressed for a clear understanding of the entire social shopping process. One of these issues has to do with customer needs and wants. Campbell [24] defines need as the requirement, necessity, or the feeling of deficiency; and associates want with phrases such as "desire", "fancy", "love", "attracted to", and "fond of". The contrast between need and want rests on the difference between deprivation and desire. Need refers to a state of deprivation, and it occurs when there is a lack of necessary items to maintain an existing condition, whereas want refers to a motivational disposition to experience the pleasure of owning a product or service.

Customer needs and wants can be motivated by social networks. For instance, two kinds of social influence correlated to the generation and recognition of customer wants and needs are observed [11]. Normative social influence (aka subjective norm) creates a social and psychological pressure (i.e., want) on people to purchase a product (or service) - regardless of an individual's interest in the product - since not adopting that product may paint them as old fashioned in their society or network of friends. Therefore, some purchases have a positive correlation with prestige and competition. However, informational social influence is a learning process achieved through observing early adopters' experiences with a special product (or service) aiming to understand the motives for acquiring it. The product can then be modified to address those needs more effectively, and the product profile should address the issue of attracting customers with similar needs. For instance, if your friend brags about his new phone that checks emails, then the need for checking emails on the go may be awakened in you.

Businesses, on the other hand, are interested in awakening the need or generating the want in customers. The key to make their products known to potential customers is effective advertisement. Note that CRM systems can assist businesses in predicting their potential customers and their potential needs.

How can the social web improve the need recognition process? Within social networks, nodes are the individual actors and links are the relationships between these actors. A social network is simply a map of relevant links between nodes. Links usually represent common interests or needs between actors on which they establish their relationships [102], and thus they often form a subgroup. We believe that social networks can improve the need recognition process using the following three methodologies.

Mutual Impact

A customer's decision to buy a product or service is often influenced by family, friends, colleagues, business partners, etc. Due to mutual influences, it is more likely to observe similar purchase behaviours among customers with strong ties in a social network. Adopting a product by a network of people connected to an individual may awaken the need for the product in that individual or create a desire (want) for acquiring that product or in some cases a similar product.

Back in 1996, Hotmail employed the effect of mutual impact to increase its user base. Hotmail increased its users from 0.5 million to 12 million by adding a simple message to the end of each sent email.

Viral Advertisement

While popular social networks base their business model on advertising [114], identifying the effective target for advertisement has always been challenging[2]. Indeed, only 40% of customers are source of positive social influence, while 12% create negative influence. Almost half of social network users have no social influence at all [62]. A positively influential customer offers the opportunity for targeting an effective, but maybe small, portion of customers, resulting in a decrease in advertisement cost. Observing similar purchasing behaviour helps identify subgroups of customers with strong ties and likely common interests. Businesses can create profiles of their products within an on-line community to increase their interaction within that community. For instance, Kiva[3], a charity loan organization, created a profile on Facebook so people can become friends with Kiva and promote its service. This resulted in the formation of support groups among Facebook members, some even launching campaigns and competing to show support for various causes.

A different methodology consists of advertising a product to an on-line community member who has strong ties to other members or is positioned between sub-communities. The community member may, then, intentionally or unintentionally mention the product in his/her posts which creates a special form of viral advertising called "blogvertising" (i.e., advertising a product indirectly by talking about it in blog posts). Seth Godin, a renowned business author, provided an electronic version of his new book for free to his blog readers, who are also bloggers and social network users, and asked them to post it on their blogs,

[2]Heather Green (2008), Bloomberg BusinessWeek – Accessed July 2010, `http://www.businessweek.com/stories/2008-09-24/making-social-networks-profitable`

[3]Kiva Org. – `www.kiva.org`

twitter, etc. if they found it interesting. Also, several e-commerce websites provide the functionality of posting purchases on Facebook immediately after the purchase, so more people become aware of the purchased product.

Recommender Systems

Recommender systems use various techniques to make accurate recommendations, social recommendations being among those techniques. After detecting the sub-communities and analysing the behaviour of individuals and their community-wide connections, recommender systems can be employed to better predict the current and future needs of the community. "Customers who purchased this also purchased " uses community behaviour to identify similarities in the interests of people in products. The accuracy of recommendations increases by incorporating the different facts about users such as social ties and demographics.

8.1.2 Product Brokerage

Product Brokerage (aka Information Search) is the stage where consumers determine what to buy after a need or want has been recognized. This is achieved through a comprehensive search on products, followed by a critical evaluation of candidate products information. The search procedure is normally conducted through "Internal" or "External" search or both. Internal search focuses on personal knowledge and past experiences, whereas external search utilizes marketers dominated sources, comparison shopping, public sources, and friends and relatives who can affect the decision through word-of-mouth. Social networks have the potential of improving the product brokering process by providing a resourceful environment of individuals with different experiences and specialties who spread the word-of-mouth and potentially lower the cost of search for different products [59]. Social networks can assist in achieving this lower cost search medium by providing the following:

Trusted Reviews and Power of Friends Network

Trusted reviews may appear in two forms, formal and informal. When customers visit a merchants website, they provide formal reviews on the products there and then. In contrast, informal reviews are provided whenever customers informally share some opinions on products among their social network of friends. Informal reviews can have more credibility

since they originate from members of the same on-line community who supposedly share the same values.

A friend who uses Twitter to comment on his recent purchase and describes the product with passion or disappointment affects his friends more than a formal review. Plus, friends may re-tweet (i.e., repost) the comment if they trust the original author. The re-tweet may be re-tweeted again to reach larger communities. In open social networks such as Twitter, users can search for products and reach thousands of informal, and sometimes formal, reviews about these products.

Impact of Social Identity

Purchases and memberships can signal customers' social identity [13]; therefore a customer's social identity may hinder the purchase of specific products. People may converge or diverge in their choice of products based on how much their choice will signal their social identity. A colour, cloth, or hairstyle is socially accepted to represent a group, but if other people start to adopt the same style, then the meaning of adopting that specific style may become diffuse. For instance, Berger and Heath discuss the example of Harley motorcycles which are a symbol of toughness, so many buy a Harley to signal their tough social identity, and the social identity that is associated with Harley motorcycles may stop many people from buying them. However, if different groups, e.g., accountants, start to adopt Harleys, their tough social identity may disappear over time.

Synchronous Shopping

Social networks give users in different locations the opportunity to shop together simultaneously. With Web 2.0, web pages can be embedded into chat tools, and a group of people is able to browse the web together while they communicate regarding product profiles[1]. This synchronous shopping method preserves the fun of shopping together while benefiting from each other's ideas. Actually, this method mirrors the off-line shopping experience where a group of shoppers visit a mall and help the potential buyer by discussing products and brands. Mattel, producer of Barbie dolls, provides synchronous shopping on its website, so kids in different locations can play together and design their own Barbie doll.

8.1.3 Merchant Brokerage

The Merchant Brokerage stage compares merchant alternatives. The result of the comparison may lead to the next stage of the social commerce process or a return back to the previous stage to conduct more searches (Appendix 2). In this stage, the buyer establishes criteria for evaluating merchant related product specifications, along with promotions and accessories that a merchant provides. Plus, the merchant-customer relationship plays a role in the buyers decision to select a merchant. Scanzoni (Scanzoni, 1979) identified five phases in the development of merchant-customer relationships in a conventional marketplace, namely awareness, exploration, expansion, commitment, and dissolution. We believe the same phases apply to an on-line marketplace, the first two having a direct impact on merchant brokerage.

Awareness

Awareness refers to one party recognizing another party as a feasible exchange partner. That means customers will understand that a merchant provides their needed product or service in the desired condition. The presence of the merchant in social networks, whether formally or informally, amplifies the customers awareness of the merchant. Amazon developed a method to amplify its recognition by providing affiliated links to its users, so whenever users talk about a book on their blog they can use the affiliated link to direct others to the book description hosted on Amazon. In this win-win situation, book descriptions are readily available to customers, while Amazon benefits from recognition and increased sales.

Exploration

Customers evaluate the benefits, burdens, commitments, and conditions of the deal associated with the seller. Trial purchases are suggested as an enabler for the evaluation of benefits and drawbacks while increasing trust (Dwyer et al., 1987). But social networks help in skipping the trial purchase step and going straight to the exploration phase. The quality of the reviews and ratings associated with the merchant, especially those coming from trusted parties, speed up this stage. Customers usually rely on other peoples recommendations. For instance, a Twitter account named "AskAroundOttawa" gives the opportunity to Ottawa residents to get fast feedback regarding Ottawa related issues. One user may receive hundreds of feedbacks for inquiring about a restaurant serving a specific

cuisine. Moreover, merchants can provide promotions and discounts on their social profile which updates users more frequently than a website.

Techniques and applications discussed during the product brokerage stage are also useful for merchant brokerage if they are focused on merchants. For instance, if a merchant provides a synchronous shopping functionality on its website, users will be attracted and the fact that they are using the service means that they have already chosen the merchant to do their purchase.

8.1.4 Purchase Decision

This stage (aka negotiation) is where the price and other terms of the transaction are determined. Similar to the previous two, this stage does not always lead to the next stage. There is a possibility that the customer returns to the previous stages to do more analysis (Figure 8.1). As social networks rely on members and communities, two types of purchases exist: individual purchases and group purchases (aka group buying). The value of social networks is more apparent in group purchases.

Once a customer decides on the merchant and proceeds to the purchase stage, the merchant will try to extract maximum benefit from the purchase, for instance using recommender systems to suggest accessories or related products. Recommender systems leverage customers activities within social networks to identify their interests and habits then recommend the right product to them. Bundled products which usually translate into better prices for the customer may start a new social shopping trend. If there is a choice in the suggested accessories, customers may go back to the product and merchant brokerage stages to revisit the decision on the choice of accessories.

The purchase process can involve multiple customers, especially when the merchandise is a subscription to a digital product (e.g., Safari Books). Although wholesale and group prices were always available for different products, most products are sold one at a time because customers usually need one item. However, social communities have the potential to change that. Communities within a social network can be formed to adopt a product, so sales increase and price decreases. CommunityShopper[4] has recently launched a service that enables customers to purchase products in groups. Customers can join the service and form groups by showing interest in different products, leading to a group purchase. CommunityShopper also leverages the power of other social networks, so any purchase or show of interest can be posted on the user's Twitter account.

[4]CommunityShopper – Accessed July 2010, `www.communityshopper.com`

In general, social networks potentially empower customers and merchants in the following ways: (1) Product Bundling: recommender systems recommend accessories or related products to customers based on their social relations. (2) Group Purchase: enabling customers to use their collective buying power to obtain lower prices.

8.1.5 Purchase

Although purchase is an important stage in social commerce, social networks do not affect it dramatically if the purchase is done off-line. Based on what we described previously, the purchase can be done individually or in a group. In case of an individual purchase through a social network, the customer can leverage feedback from his network. For instance, the status of a member of Movie Fans[5] is updated when he purchases a movie ticket. If friends view his status and dislike his choice of theatre, they may suggest better venues. He may then consider their suggestion for his next movie outing. In case of a group purchase, merchants, customers and their social network benefit from the purchase. Customers acquire the product for a lower cost, while social networks multiply sales for the merchants. Moreover, merchants can promote the product by enabling customers to post their purchases on their social profiles (perhaps to gain social acceptance). Also, the merchant may ask the customer to recommend a product to friends or recommend people who are interested in a product to the merchant.

Nevertheless, in some types of purchases where the purchase has "a duration" associated with it, the effect of social networks on this stage may increase. For instance, when a customer orders food in a restaurant, he is committed to pay even though the payment will be completed in the near future. The purchase action begins when the order is received. If the user posts his location and his intention to dine on a social networking site such as Foursquare[6], friends (i.e., members of his social network) can join him. Foursquare encourages users to be frequent buyers and to post their status on the website, rewarding them with social recognition and promotions.

8.1.6 Evaluation

The post-purchase stage is the final and probably the most influential stage in the social commerce model. It affects all previous stages, involves customer service, and more importantly the evaluation of the satisfaction with the buying experience. It acts as a transition

[5]NetFlix Inc. – Accessed July 2010. `www.community.netflix.com`
[6]Foresquare Labs, inc. – `www.foursquare.com`

stage for customers to go from being influenced to becoming potential influencers. The rationality of the decision made by the customer is evaluated, leading to satisfaction or cognitive dissonance. On-line reviews are important if we accept that on-line customer review systems are one of the most powerful channels to generate on-line word-of-mouth [48, 55]. However, not all researchers agree on the impact of on-line reviews on sales. The disagreement results from the fact that some researchers focus on the persuasive aspect of on-line reviews and on assessing the quality of products in the reviews, while others focus on user awareness and spreading the word without paying attention to the quality of the products [37]. Nevertheless reviews have a positive relationship with the quality of the shopping experience. If a product sells well, then the number of reviews will grow and will eventually cause more recognition [39]. The number of positive reviews during a certain amount of time is also indicative of more future sales, so merchants can predict sales and assign resources for more production.

Reviews can be divided into three categories: Customer Reviews, Expert Reviews, and Sponsored Reviews. Although it is expected that expert reviews have the most effect on customer decision making, in reality, informal and user generated reviews affect customers the most [39]. Businesses should therefore focus on encouraging customer generated reviews.

In social networks, customers are encouraged to leave reviews for several reasons. An important one is that social network members seek recognition and try to show that they are always first in line, which is more verifiable in social networks where members know each other, hence they expect social satisfaction. Foursquare, for example, provides badges to grant social recognition to its users when they post reviews. Another incentive for leaving reviews is to help friends with decision making by providing personal experiences and history of products or services. While the number and quality of reviews change based on products, more attention is directed towards the comments of a critic [39]. Trusting a critic's reviews in a network of friends is easier since the users are aware of the background of the critic [67].

In light of the above, social networks are better for review generation than merchants' websites.

8.2 Conclusion

Web 2.0 generated a new e-commerce stream named social commerce, enabling customers to harness the power of the social web to make more accurate decisions. Although social

networks have an impact on customers' purchase decisions, few studies have focused on such influences because until recently data about the effects of social interaction on sales has not been adequately captured. With more customers using the social web, businesses developed tools to reach more of them to create product and brand awareness.

This Chapter reviewed and leveraged existing frameworks to present the influence of the social web on e-commerce decision making in a comprehensive model. The model guides all actors involved in the social commerce (businesses, developers, and customers) in leveraging the power of social networks. This includes enabling businesses to improve their marketing campaigns and increase sales. On the other end, customers are empowered through more informed purchases. All of this is possible when the developers build more focused tools to target the communities.

By using the right tools in the right way, e-commerce companies can ultimately increase sales while lowering marketing cost. We believe that e-commerce companies can benefit from the analysis of customer behaviour in the social shopping experience. They should also recognize and apply the right strategies at the right purchase decision making stage. The model guides business through the process of selecting the right strategies for different products and different target groups, as the model provides a comprehensive overview of possible techniques for employing social networks in business and their positive and negative effects. The result makes the social web an additional tool to be used by businesses in influencing customer purchases.

The model explores various social commerce tools with their advantages and projected deficiencies. Developers of social commerce systems can use the model improve current technologies.

Customers who may not have complete information about a product or service are eager to learn from other customers. Furthermore, human psychology suggests that people are interested to own what their friends have, whether they need it or not. Viewing products or hearing about them may awaken needs in customers. High quality reviews and functionalities on e-commerce websites that connect merchants to customer networks may encourage or discourage purchases of specific products from specific merchants. Customers are the ultimate beneficiaries from the model since it improves the services provided to them by business and developers.

In conclusion, our findings show that the main driver for social commerce is user interaction and involvement. Companies should encourage users to engage more in providing product and merchant related comments on their social networks and a comprehensive understanding of social commerce strategies is required for managers.

Chapter 9

Conclusions

9.1 Summary

The thesis focuses on both static and temporal aspects of social networks and it studies some of their structural and dynamical properties, especially taking time into consideration.

We proposed some temporal measures (temporal shortest/ foremost/fastest betweenness, and temporal eigenvector centrality), and we designed algorithms to compute some of these parameters. We focused in particular on *foremost betweenness* and we designed a novel solution to compute exact betweenness of all nodes in a graph. Since the problem is intractable and our solution exact, the algorithm runs, inevitably, in exponential time. Thus, we proposed a variant that, while still having an exponential time complexity, can be implemented on small enough networks running some parts of the algorithms in parallel. Finally, we also proposed a temporal version of the classical *Eigenvector centrality* measure by augmenting time as a factor of weight to the adjacency matrix of the graph. Our method has two variants focusing on the degree over time of the node being analysed as adjacency matrix weight (SDI), and the degree over time of the neighbours as the weight in the adjacency matrix of the graph (ADI). Eigenvector centrality can be easily computed in polynomial time in both models.

We then investigated three very different datasets: a knowledge mobilization network, a network of users commenting on Facebook pages, a YouTube sharing video network. The first two networks have been studied to test our proposed temporal measures in a real setting, so to gain some understanding of the temporal centrality of their nodes; the third has been considered with the different goal of measuring propagation of influence, and analysing user similarities.

In the context of *knowledge mobilization*, we unveiled the presence of accelerator nodes: actors that contribute to the fast flow of knowledge in the network. This type of temporal importance was not detectable in the static analysis, where centrality of a node is only related to its connections to the other nodes regardless of their time of existence. Indeed, the whole concept of network *accelerator* is new and nodes of this type in a real social network setting have been detected for the first time in this thesis. The results of this Chapter have been published in [5].

In the context of Facebook Social network we could not compute exact foremost betweenness because the graph created over the commenting activities of uses on Facebook pages is too large. Hence, we proposed a new idea to combine the exact algorithm employed for the (much smaller) knowledge mobilization network, with an approximate component to obtain just an estimate of foremost betweenness. While not being exact, the analysis gives an indication of Facebook users whose commenting pattern is particularly effective for the fast convey of information through different pages. This type of behaviour by Facebook users has never been observed before and it could be a very useful measure to detect relevant users in other temporal settings; for example, in situations where timely connections assume a particular importance that needs to be reflected by centrality measures.

In the context of YouTube social network, our focus is diverted from temporal analysis, and we mainly focus on the static graph representation to measure the propagation dynamics on a YouTube network. YouTube provides one of the most appropriate test-beds for our purpose as it accommodates two of the most common social network links: followership and friendship. This kind of analysis is being conducted for the first time in this thesis. We measure the speed and depth of propagation by following a YouTube post and its comments throughout the network in a snowball-like model. Speed analysis might deem temporal, as we measure to see how fast the post propagates in the graph, by collecting the time-tags on the communications. We also measure the similarity of users to understand how similar are friends in an open social network, like YouTube. Our results show the importance of link structure in social networks for information propagation. The results of this Chapter have been published in [2, 3].

We conclude the thesis with a discussion on social commerce. The concept of social commerce is very crucial from the SNA point of view, as it embodies the most important application of SNA. In fact, the financial implications of exploring important nodes was one of the main reasons for the birth of SNA. Nevertheless, our investigations show that no proper definition and design framework existed for the important concept of Social Commerce. We propose a framework that describes the influence of the social web on e-

commerce decision making. While defining social commerce, the framework guides through the design choices through all the steps of social commerce design and decision making process, giving designers a better understanding about the values of different characteristics of social networks that add value to social commerce process. The results of this Chapter have been published in [1].

9.2 Open Problems

The thesis is only a first step towards the temporal analysis of social networks. In fact, many challenging problems are open. Some interesting directions are indicated below.

- Foremost and fastest betweenness are among the very few measures proposed so far to be employed in time-varying graphs, and their computation is inherently complex. A useful and interesting direction would be the design of novel temporal measures that are computable in polynomial time and that provide significant information about the temporal aspects of social networks;

- In the thesis we concentrated mostly on exact computations, thus leading to very time consuming algorithms, only suitable for very small networks (like *KnowledgeNet* in Chapter 5). We also introduced an hybrid method where foremost betweenness is computed combining an exact component with an approximate one, for the Facebook graph of Chapter 6. In that chapter, in fact, we computed an approximate foremost betweenness based on the estimate of the number of foremost journeys between pair of nodes. It is not known, however, how close to the exact value this approximation is, and this is left as an open problem. In general, the study of approximation algorithms to compute intractable temporal measures (like foremost betweenness) that can provide reasonable approximated values is a very interesting research direction that should be pursued;

- In static networks, topological structures have been often the object of investigation as they reveal interesting aspects of social networks. The investigation of different dynamical topological structures and their effects on the centrality measures over time is still an open area of research.

- There has been some research on detection of dynamic communities on temporal social networks, such as temporal modularity measure. However, the identification

of temporal communities is a complex task and is far from being accurate. The identification of dynamic communities is still open problem.

- Although propagation in social networks has been the focus of studies for years, study of probabilistic measures over the real social networks has not been studied comprehensively yet. Thus, it would be a very interesting issue to analyse probabilistic measures in the context of various real life networks so that we identify which probabilistic measure applies to what type of social network.

- Since the beginning of this thesis, there has been a good progress on the various aspects of social commerce, in terms of definitions and of concepts development. At the same time, technological development in big data analysis and web service technologies requires redesign and redevelopment of frameworks that accommodate new advances in the technology. Furthermore, the introduction of mashups and internet of things can provide added advantages to the use of social networks in commerce. To explore how social networks can be combined with mashups to be embedded as a solution for social commerce is still an open problem.

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