

## Soft branes in supersymmetry-breaking backgrounds

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**ABSTRACT:** We revisit the analysis of effective field theories resulting from non-supersymmetric perturbations to supersymmetric flux compactifications of the type-IIB superstring with an eye towards those resulting from the backreaction of a small number of  $\overline{D3}$ -branes. Independently of the background, we show that the low-energy Lagrangian describing the fluctuations of a stack of probe D3-branes exhibits soft supersymmetry breaking, despite perturbations to marginal operators that were not fully considered in some previous treatments. We take this as an indication that the breaking of supersymmetry by  $\overline{D3}$ -branes or other sources may be spontaneous rather than explicit. In support of this, we consider the action of an  $\overline{D3}$ -brane probing an otherwise supersymmetric configuration and identify a candidate for the corresponding goldstino.

**KEYWORDS:** Flux compactifications, Supersymmetry Breaking, D-branes

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**1 Introduction**

A persistent problem in the development of realistic string compactifications is the implementation of supersymmetry breaking in a genuinely stringy and controllable manner. The tension comes from the fact that string theory is usually defined in 10 (or 11) dimensions with a large number of supercharges, while realistic phenomenology requires such compactifications to reduce at low energies to non-supersymmetric 4d theories. Added to this tension is the issue of moduli stabilization whose details can significantly affect the vacuum structure and supersymmetry-breaking terms in the dimensionally reduced theories. One can contemplate bypassing the intermediate stage of realizing an effective 4d supergravity at low energies by constructing stabilized vacua with supersymmetry broken at or above the compactification scale. The construction of such vacua has proven to be a difficult task as one often encounters (perturbative) instabilities. Thus far, explicit tachyon-free examples of this kind with the broad features of the Standard Model have not yet been found, and there are generic statistical arguments suggesting that such vacua are rare [1] (see also [2]). Furthermore, the scale of supersymmetry breaking in such constructions is typically much larger than the electroweak scale  $m_Z \sim 100$  GeV, with no apparent relation between them. In light of these issues, studies of supersymmetry breaking in string theory often takes a different route. Most work on the subject begins with an effective 4d supergravity, as there are several potential phenomenological benefits for supersymmetry (at least the reduced version, e.g.,  $\mathcal{N}_4 = 1$ ) to persist at intermediate scales. Other than protecting

certain operators from large quantum corrections, subsequent breaking of supersymmetry in the effective low-energy supergravity provides a nice mechanism to trigger spontaneous electroweak symmetry breaking thus tying the electroweak scale to the supersymmetry-breaking scale. Furthermore, such a framework of intermediate or low-scale breaking has the advantage that a myriad of  $\mathcal{N}_4 = 1$  string constructions with semi-realistic spectra are readily available, while there are fewer examples for those exhibiting high-scale supersymmetry breaking. Traditionally, the source of supersymmetry breaking in this context is assumed to be the result of some hidden sector dynamics. Recent developments have extended the possibilities to include other supersymmetry-breaking sources such as fluxes and anti-branes. It is useful to note that while we make a distinction for those constructions that admit a 4d supergravity description at an intermediate scale, such vacua, when lifted to 10d, should correspond to a non-supersymmetric background when the backreaction of the supersymmetry-breaking effects is taken into account.<sup>1</sup> Thus, from a 10d point of view, the nature of the problem is not that different from some of those constructions whose supersymmetry-breaking scale is at or above the compactification scale.<sup>2</sup> Studying the effects of such supersymmetry-breaking backgrounds on the gauge sector, irrespective of the origin of such breaking, will be one goal of the present work

A particularly well-explored corner of  $\mathcal{N}_4 = 1$  constructions are the class of flux compactifications of type-IIB string theory [3–8] commonly known as GKP compactifications.<sup>3</sup> This class of constructions invokes closed-string flux which, in addition to stabilizing many moduli, allows for constructing strongly warped regions. Such strongly warped geometries provide a mechanism to generate a hierarchy of scales via gravitational redshift, realizing the bottom-up idea of [9]. This fact was exploited in the KKLT construction [10]. By combining this strong warping with the quantum effects required to stabilize the Kähler structure of the internal space, it was argued in [10] that supersymmetry can be broken at a hierarchically suppressed scale by the addition of a small number of anti-branes which naturally inhabit points of strongest warping. The KKLT framework has been widely explored in the context of string inflation (for reviews see [11–16]) and in phenomenological scenarios such as mirage mediation [17, 18] and variations thereof [19, 20]. Furthermore, the gauge/gravity correspondence is often realized with strongly warped geometries [21–24]. The addition of a relatively small number of anti-branes to an otherwise supersymmetric construction can, under certain circumstances, be described as a metastable non-supersymmetric state in a dual supersymmetric gauge theory [25, 26]. This has been used to construct gravity duals of gauge mediation scenarios [27–32] (see also [33–35])

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<sup>1</sup>The supersymmetry-breaking effects here are not restricted to localized sources, but include also fluxes as well as sources of dynamical supersymmetry breaking in the hidden sector realized as instantons on branes.

<sup>2</sup>We are cautious in using the qualification “some” here. If the scale of supersymmetry is above the string scale, one would expect in addition to the supergravity fields that a tower of string states to come into play.

<sup>3</sup>Strictly speaking, the compactifications of [8] are not necessarily supersymmetric. However, since we are primarily interested in supersymmetric GKP compactifications, we will use the term “GKP” to indicate the  $\mathcal{N}_4 = 1$  setups of [8]. Furthermore, GKP compactifications alone does not provide a mechanism for compactification, but our analysis will not depend on whether or not the Kähler structure moduli are stabilized.

for related ideas) which are otherwise difficult to analyze using conventional (perturbative) field theoretical techniques. Finally, anti-branes also play an important role in the large-volume scenario [36], where non-perturbative effects are played against  $\ell_s$ -corrections to produce intermediate-scale supersymmetry breaking without relying on strong warping.

Despite the wide applications of this framework, the nature of supersymmetry breaking by  $\overline{D3}$ -branes remains somewhat mysterious, even setting aside the subtleties involved in the backreaction of the  $\overline{D3}$ s [26, 37–42]. In particular, it is not clear whether or not the breaking should be considered explicit breaking or spontaneous breaking from the 4d point of view, although the common folklore holds that it is the former. An argument often given for  $\overline{D3}$  branes providing an explicit source of breaking is that the  $\overline{D3}$ s preserve the “wrong” supersymmetry, meaning the supersymmetry that is broken by the D3 charge carried by the fluxes in a GKP compactification. Indeed, such explicit breaking seems to be reflected in the effective potential used in [10] in which the so-called uplift potential, corresponding to the tension of the  $\overline{D3}$ s, is not included with the  $F$ -term potential.<sup>4</sup>

On the other hand, the  $\overline{D3}$ s can be thought of as a soliton of closed strings, especially when the number of anti-branes is large, in which case it is simply a non-supersymmetric configuration in a supersymmetric theory; such a state of affairs is, by definition, spontaneous breaking of supersymmetry. In this sense, the case of an anti-brane is quite similar to the case of branes intersecting at angles. Although for special angles, two intersecting branes preserve some of the same supercharges, for generic angles they will not. One might be tempted to call this explicit breaking for precisely the same reason as in the  $\overline{D3}$  case: at generic angles the branes do not preserve the same supersymmetry. Yet since such angles are controlled by geometric and brane moduli, the breaking by a non-trivial angle can be controlled by 4d fields and therefore seems to be manifestly spontaneous breaking (see, e.g. [44] for related discussions). Indeed, this was considered in, for example, [45] where the theory for the corresponding goldstino, which indicates the spontaneous breaking of supersymmetry, was discussed. Since  $\overline{Dp}$ -branes differ from  $Dp$ -branes precisely in their orientation, the case of an  $Dp$ - $\overline{Dp}$  pair is in some sense an extreme version of branes intersecting at angles.<sup>5</sup> Finally, the case of  $\overline{D3}$ s in a GKP compactification is not intrinsically distinct from the case of  $Dp$ -branes in flat space as both involve supercharges of 10d background being projected out by the localized sources. In the latter case the massless scalars on the worldvolume are the goldstones associated with the spontaneous breaking of translational symmetry. The massless fermions should be viewed in the same light, as resulting from the spontaneous breaking of maximal supersymmetry. Indeed, the supersymmetric generalization of the Dirac-Born-Infeld (DBI) action contains in it a Akulov-Volkov-like (AV) action for goldstini [46, 47]. Although this fact was understood long ago (see e.g. [48]) it seems, in our opinion, to be under-appreciated.

In this work, we explore this question of explicit and spontaneous breaking by considering non-supersymmetric perturbations to supersymmetric GKP compactifications.

<sup>4</sup>This of course leaves open the possibility that the  $\overline{D3}$  is a source of  $D$ -term breaking as suggested in, for example, [43]. We will provide some evidence for this possibility as well.

<sup>5</sup>This is admittedly a bit of a cheat; for example, for spacetime filling 3-branes transverse to a compact space, there is no finite-energy way to rotate the branes.

Although much of our analysis is agnostic with respect to the source of these non-supersymmetric perturbations, we have in mind those resulting from the addition of  $p$   $\overline{\text{D3}}$ s such that  $p$  is much less than the number of flux quanta that builds a warped region. In generic cases, even though only a single combination of fields is “directly” sourced by the  $\overline{\text{D3}}$ s, all other closed string fields are perturbed, including non-Hermitian components of the internal metric. As a diagnostic of such breaking, we probe the resulting background with a stack of D3 branes and consider the resulting effective field theory. Such a situation has been considered previously in the literature from both the D-brane [43, 49–51] and worldsheet points of view [52, 53], but none to our knowledge takes fully into account the non-Hermitian perturbations to the internal metric (though see [27] for a related case) or explicitly analyze Yukawa couplings. Although this may seem like a slight distinction, the internal metric is the matter-field metric for position moduli of the D3 and so it modifies the marginal operators (as well as operators of other dimensions) of the D3-brane effective field theory. Since the soft terms that result from the spontaneous breaking of supersymmetry are all relevant operators (at least in the  $m_p \rightarrow \infty$  limit), this would seem to hint at explicit breaking. Nevertheless, we find that a simple non-holomorphic field redefinition puts the effective field theory into a form that manifestly exhibits only soft breaking. As generic explicit breaking should lead to hard terms, even in the limit as  $m_p \rightarrow \infty$ , we take this as an indication that the breaking of supersymmetry may be spontaneous.

Let us stress that since D3s are local objects, the analysis of the D3 action is, through marginal order, fairly insensitive to the form that the internal metric takes (so long as it is not singular) and previous analyses of the D3-action are straightforwardly adopted to the case of a general metric. Indeed though the analyses of [43, 50] take the ansatz where the internal metric remains Calabi-Yau, their results are largely valid in more general cases<sup>6</sup> except for the fact that they rely on the underlying Calabi-Yau to give a complex structure to the open-string effective field theory. In this light, our goal is to not to greatly extend the technical advances of these works, but instead to make steps towards a conceptual understanding of supersymmetry breaking.

We also emphasize that even though we primarily work in the context of a non-supersymmetric perturbation to GKP, the D3-brane Lagrangian seems to be soft independently of the background or even if the closed-string equations of motion are applied. However, in the case in which the background is a result of the backreaction of  $\overline{\text{D3}}$ s in GKP, we are also able to identify the gaugino living on the  $\overline{\text{D3}}$ -brane as a candidate for the goldstino that is expected to be present if supersymmetry is spontaneously broken. For this reason, much of our discussion is framed within the context of supersymmetry breaking by the addition of anti-branes.

This paper is organized as follows. In section 2, we review GKP compactifications and argue that the addition of an  $\overline{\text{D3}}$  brane will generically perturb all closed string fields including the internal metric. In section 3, we discuss the effective field theory of a stack of D3-branes or  $\overline{\text{D3}}$ -branes probing such a geometry. In section 4, we review the nature of soft

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<sup>6</sup>Notable exceptions are the non-renormalizable couplings between open and closed strings considered in [50] which depend on an understanding of the light closed-string spectrum that is not available in general.

breaking of supersymmetry and show how the action presented in the previous section falls into this class, though the supersymmetry that is “least” broken is not quite that preserved by GKP. In section 5, we discuss a candidate for a goldstino field on an  $\overline{\text{D3}}$  probing a GKP compactification. Some concluding remarks are given in section 6 and our conventions are summarized in appendix A.

## 2 Non-supersymmetric perturbations to GKP compactifications

In this section, we discuss non-supersymmetric perturbations to  $\mathcal{N}_4 = 1$  GKP compactifications [8] of the type-IIB superstring, with an emphasis on those resulting from the addition of a number of  $\overline{\text{D3}}$ -branes that is small compared to the amount of flux in the supersymmetric case. GKP compactifications are of the form  $R^{3,1} \times_w X^6$  where  $\times_w$  indicates a non-trivial fibration of  $R^{3,1}$  over the compact internal space  $X^6$ . The metric takes the familiar warped ansatz

$$ds_{10}^2 = \hat{g}_{MN} dx^M dx^N = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} g_{mn} dy^m dy^n. \tag{2.1a}$$

The geometry is supported by a 3-form flux  $G_{(3)} = F_{(3)} + ie^{-\phi} H_{(3)}$  without legs on the external space  $R^{3,1}$  and a 5-form flux

$$F_{(5)} = (1 + \hat{*}) \mathcal{F}_{(5)}, \quad \mathcal{F}_{(5)} = d\alpha \wedge \text{dvol}_{R^{3,1}}, \tag{2.1b}$$

in which  $\text{dvol}_{R^{3,1}}$  is the volume form for  $R^{3,1}$  and  $\hat{*}$  is the 10d Hodge- $*$  built from the metric  $\hat{g}_{MN}$ . Our interest is in the regime where dimensional reduction on the  $X^6$  produces an effective 4d theory. Such a theory will exhibit  $\mathcal{N}_4 \geq 1$  if [8, 54]

1.  $X^6$  is a Kähler manifold and  $g_{mn}$  is the associated Kähler metric,
2. the 3-form flux is primitive and has Hodge type (2, 1) and is therefore imaginary self-dual (ISD),  $iG_{(3)} = *G_{(3)}$ , where  $*$  (without the hat) denotes the 6d Hodge- $*$  built from  $g_{mn}$ ,
3. the 5-form flux and the warp factor are related by  $e^{4A} = \alpha$ ,
4. the axiodilaton  $\tau = C_{(0)} + ie^{-\phi}$  varies holomorphically over  $X^6$ .

A construction satisfying these requirements is called a GKP compactification (though see footnote 3). These compactifications must in addition contain certain sources (D3-branes, O3-branes, or 7-branes) to ensure the cancellation of tadpoles; however, these sources will not play a significant role in our analysis.

As reviewed in the introduction, GKP compactifications are a particularly interesting region of the landscape since, while they are based upon the comparatively well-understood Kähler and Calabi-Yau geometries, the presence of non-trivial 3-form flux can stabilize the complex structure of  $X^6$ , the axiodilaton, and the deformation moduli for 7-branes. Additionally, these constructions can accommodate low-scale supersymmetry breaking as large amounts of flux can produce strongly warped regions. Since ISD flux carries D3

charge,  $\overline{D3}$ s, which carry the opposite-sign charge and hence break the supersymmetry preserved by GKP, are naturally attracted to the regions of strongest warping and so the corresponding scale of supersymmetry breaking can be highly redshifted. In some cases, when D3-branes are absent, the  $\overline{D3}$ s will be perturbatively stable, only decaying into flux and D3-branes after undergoing a Myers-like effect [55] followed by a quantum tunneling process [25].

In order to perform a detailed study of such constructions, the influence of such  $\overline{D3}$ -branes on the background must be considered. The most studied example is the Klebanov-Strassler (KS) geometry [56] which results from ISD 3-form flux threading the deformed conifold. The backreaction of a small number of  $\overline{D3}$ s on the KS geometry has been a topic of recent interest [26, 37–39, 41, 42]. Due to the presence of the background 3-form flux of KS, the addition of the  $\overline{D3}$ s produces a non-ISD flux and in fact, near the anti-branes, all Hodge types of 3-form flux are present [37]. Furthermore, it was pointed out in [27] that the  $\overline{D3}$ s perturb the metric in such a way that the internal metric  $g_{mn}$  is no longer Hermitian with respect to the original complex structure: the backreaction of the  $\overline{D3}$ s includes non-vanishing metric components  $g_{zz}$  and  $g_{z\bar{z}}$  when expressed in terms of the complex coordinates of the original deformed conifold.

The fact that such non-Hermitian components will generically appear after the addition of  $\overline{D3}$ s can be easily seen from the type-IIB equations of motion. Let us again consider the ansatz (2.1) but relax the conditions for supersymmetry. It is useful to construct the combinations

$$\Phi_{\pm} = e^{4A} \pm \alpha, \quad G_{\pm} = (*_6 \pm i)G_{(3)}, \quad \Lambda = \Phi_+ G_- + \Phi_- G_+. \quad (2.2)$$

The equations of motion and Bianchi identities (A.4) can be expressed in these fields as [8, 57]

$$0 = \nabla^2 \Phi_{\pm} - \frac{(\Phi_+ + \Phi_-)^2}{16 \operatorname{Im} \tau} |G_{\pm}|^2 - \frac{2}{\Phi_+ + \Phi_-} |\partial \Phi_{\pm}|^2, \quad (2.3a)$$

$$0 = d\Lambda + \frac{i}{2 \operatorname{Im} \tau} d\tau \wedge (\Lambda + \bar{\Lambda}), \quad (2.3b)$$

$$0 = d(G_{(3)} - \tau H_{(3)}), \quad (2.3c)$$

$$0 = \nabla^2 \tau + \frac{i}{\operatorname{Im} \tau} (\partial \tau)^2 + \frac{i}{8} (\Phi_+ + \Phi_-) G_+ \cdot G_-, \quad (2.3d)$$

$$0 = R_{mn} - \frac{1}{2 (\operatorname{Im} \tau)^2} \partial_{(m} \tau \partial_{n)} \bar{\tau} - \frac{2}{(\Phi_+ + \Phi_-)^2} \partial_{(m} \Phi_+ \partial_{n)} \Phi_- + \frac{\Phi_+ + \Phi_-}{16 \cdot 2! \operatorname{Im} \tau} \left[ G_{+(m}{}^{pq} \bar{G}_{-n) pq} + G_{-(m}{}^{pq} \bar{G}_{+n) pq} \right], \quad (2.3e)$$

in which, for simplicity of presentation, we have omitted terms resulting from localized sources. For  $p$ -forms we use the notation

$$X_{(p)} \cdot Y_{(p)} = \frac{1}{p!} X_{m_1 \dots m_p} Y^{m_1 \dots m_p}, \quad |X_{(p)}|^2 = X_{(p)} \cdot \bar{X}_{(p)}. \quad (2.4)$$

Note that we have defined  $\bar{G}_\pm = (G_\pm)^*$  so that, for example,  $\bar{G}_+$  is imaginary anti-self-dual (IASD). Here and throughout we perform contractions and construct connections with the unwarped metric  $g_{mn}$  unless otherwise noted.  $\mathcal{N}_4 \geq 0$  GKP compactifications are characterized by the conditions  $\Phi_- = 0$  and  $G_- = 0$  with  $\mathcal{N}_4 \geq 1$  having the additional requirement that  $G_{(3)}$  is a primitive  $(2, 1)$ -form. For non-vanishing  $\Phi_+$ , we can recast the equation of motion for  $\Phi_+$  as [58]

$$0 = \nabla^2 \Phi_+^{-1} + \frac{1}{16 \operatorname{Im} \tau} \frac{(\Phi_+ + \Phi_-)^2}{\Phi_+^2} |G_+|^2 + \frac{2}{\Phi_+} \left[ \frac{1}{(\Phi_+ + \Phi_-)} - \frac{1}{\Phi_+} \right] (\partial \Phi_+)^2. \quad (2.5)$$

For the moment, we will specialize to the case in which we start with  $G_- = 0$ ,  $\Phi_- = 0$  and  $\tau$  is a constant so that  $X^6$  is a Calabi-Yau. We can then consider a perturbation such as, for example, the addition of a small number of  $\overline{\text{D3}}$ -branes. Then remarkably the linearized equations of motion for the perturbations take a nearly triangular form [58]

$$\nabla^2 \delta \Phi_- = 0, \quad (2.6a)$$

$$d(\Phi_+ \delta G_-) = -d(\delta \Phi_- G_+), \quad (2.6b)$$

$$(* + i) \delta G_- = 0, \quad (2.6c)$$

$$\nabla^2 \delta \tau = -\frac{i}{8} \Phi_+ (G_+ \cdot \delta G_-), \quad (2.6d)$$

$$-\frac{1}{2} \Delta \delta g_{mn} = \frac{2}{\Phi_+^2} \partial_{(m} \Phi_+ \partial_{n)} \delta \Phi_- - \frac{\Phi_+}{16 \cdot 2!} \left[ G_{+(m}{}^{pq} \delta \bar{G}_{-n)pq} + \delta G_{-(m}{}^{pq} \bar{G}_{+n)pq} \right], \quad (2.6e)$$

$$d\delta G_+ = d(\delta G_- + 2i\delta\tau H_{(3)}), \quad (2.6f)$$

$$(* - i) \delta G_+ = 0, \quad (2.6g)$$

$$\begin{aligned} -\nabla^2 \delta \Phi_+^{-1} &= (\delta \nabla)^2 \Phi_+^{-1} - \frac{1}{16} \operatorname{Im} \delta \tau |G_+|^2 \\ &+ \frac{1}{16} \left[ G_+ \cdot \delta \bar{G}_+ + \delta G_+ \cdot \bar{G}_+ + \frac{1}{2!} G_{+m_1 n_1 p_1} \bar{G}_{+m_2 n_2 p_2} g^{m_1 m_2} g^{n_1 n_2} \delta g^{p_1 p_2} \right] \\ &+ \left[ \frac{1}{8} \Phi_+^{-1} |G_+|^2 - 2\Phi_+^{-4} (\partial \Phi_+)^2 \right] \delta \Phi_-. \end{aligned} \quad (2.6h)$$

Here  $\delta \Psi$  denotes a perturbation to a field,  $\Psi \rightarrow \Psi + \delta \Psi$  and

$$\Delta \delta g_{mn} := \nabla^2 \delta g_{mn} + \nabla_m \nabla_n (g^{pq} \delta g_{pq}) - 2\nabla^p \nabla_{(m} \delta g_{n)p}. \quad (2.7)$$

We have additionally set the unperturbed constant axiodilaton to  $\tau = i$  and again omitted the explicit appearances of source terms. Although we will not make use of it, this pattern of triangularity continues order-by-order in perturbation theory.<sup>7</sup>

<sup>7</sup>We note that the equations of motion may not always be truly triangular. For example, in general the metric is characterized by many functions and (2.6e) will generically not have any special structure for those functions. This is the case for perturbations to the KS geometry [38] except in the nearly-conformal region [58].



This form of the equations of motion is useful since it is precisely the mode  $\Phi_-$  that is “directly” sourced by a  $\overline{\text{D3}}$ -brane in the sense that only the equation of motion for  $\Phi_-$  has a  $\delta$ -function term in the presence of  $\overline{\text{D3}}$ s. That an  $\overline{\text{D3}}$  sources  $\Phi_-$  can be most easily seen by placing an  $\overline{\text{D3}}$  in flat space where the only field that becomes non-trivial is  $\Phi_-$ . From (2.6), we see that in the presence of  $G_+ \neq 0$ , once  $\delta\Phi_-$  is non-zero,  $\delta G_-$  is non-zero as well, and indeed  $\delta G_-$  generically possess all Hodge types.<sup>8</sup> The presence of  $\delta G_- \neq 0$  gives a source for  $\delta\tau$  and generically both the real and imaginary components are non-vanishing.<sup>9</sup> Inserting the directly sourced  $\delta\Phi_-$  and the indirectly sourced  $\delta G_-$  into the equation for  $\delta g_{mn}$  generically forces all components to be non-vanishing. For example, a (2, 1)  $G_+$  and (3, 0)  $\delta G_-$  act as a source  $\delta g_{z\bar{z}}$  component.<sup>10</sup> Similarly,  $\Phi_+$  and  $\Phi_-$  are real functions and so  $\delta\Phi_- \neq 0$  should also generically source all components of the metric.<sup>11</sup> Following the remainder of the equations as above also leads us to conclude that  $G_+$  and  $\Phi_+$  are perturbed from the original background values. We note also that this argument implies that an initial singularity in  $\delta\Phi_-$ , such as that appearing in [26, 37, 38], is felt by all perturbed fields, even if  $\Phi_-$  is the only field directly sourced.

The presence of the singularities in the fields not directly sourced by the  $\overline{\text{D3}}$ s is perhaps surprising and has been a topic of recent discussion [38–41, 61–64]. The  $\overline{\text{D3}}$ s directly source  $\delta\Phi_-$  and so the corresponding divergence is as physically acceptable as the divergence in the electric field at the position of a point-charge in classical Maxwell theory. In contrast, the 3-form flux and other fields are not directly sourced by these fields and so the corresponding singularities might be seen as suspect. Here we take the point of view that, due to the non-linearity of the supergravity equations of motion and the fact that all of the fields couple to each other, once one sort of singularity is accepted, divergences in all other fields must be accepted as well. Indeed, presumably there exists some stringy mechanism that resolves the singularity in  $\Phi_-$  (for example, an  $\overline{\text{D3}}$ , even in flat space, should have some finite width comparable to the string length) and once  $\Phi_-$  is rendered finite, there is no reason to expect that any of the other singularities will be present (however, the linearized analysis of the supergravity equations of motion is expected to be inapplicable). We therefore view it as plausible that the divergences will be resolved in a full treatment and so accept the apparent singularities as being a consequence of an incomplete treatment (see also [42] for responses to the objections related to these divergences). Nevertheless, because supergravity may break down near the position of  $\overline{\text{D3}}$ s, we will assume in what follows that we are evaluating our fields sufficiently far away from any such sources.

To summarize, we have argued that in a generic  $\mathcal{N}_4 = 1$  GKP compactification, the addition of an  $\overline{\text{D3}}$ -brane will cause the configuration to move away from all of the super-

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<sup>8</sup>This genericity is violated in, for example, [26] where the imposed R-symmetry requires  $G_{(3,0)} = 0$  (where the Hodge-type is given in terms of the original complex structure).

<sup>9</sup>For the example of KS,  $\tau$  is pure imaginary after the addition of the  $\overline{\text{D3}}$  brane since  $H_{(3)}$  and  $F_{(3)}$  thread dual cycles and so  $F_{(3)} \cdot H_{(3)} \propto \text{Im}(G_+ \cdot G_-) = 0$  automatically, even after the perturbation.

<sup>10</sup>In [26], there was a non-vanishing  $\delta g_{z\bar{z}}$  even though no (3, 0) flux was sourced. This is because the left-hand side of (2.6e) involves all components of the perturbed metric and so even sourcing the  $\bar{z}z$  component of  $\Delta\delta g_{mn}$  will generically result in non-vanishing  $\delta g_{z\bar{z}}$ .

<sup>11</sup>Note that at least in some simple fluxless cases such as a D3- $\overline{\text{D3}}$  pair in flat space, we can choose a coordinate system such that the internal space is still Hermitian with respect to the original complex structure, but at the expense of having a different scaling factor for the transverse metric [26, 59, 60].

symmetry conditions, perturbing all Hodge-types of flux, causing the axiodilaton to be non-vanishing, and forcing the internal metric to be no-longer Hermitian, even though the  $\overline{D3}$  itself directly sources only  $\Phi_-$ . For simplicity and since the equations of motion are almost triangular, we have worked in the special case in which the axiodilaton is constant in the unperturbed geometry. However, it would be rather surprising if in the more generic case of varying axiodilaton that these perturbations were not produced. Hence, it what follows we will drop the assumption of constant axiodilaton. Further, although we have emphasized in this section perturbations due to the presence of  $\overline{D3}$ -branes, the analysis of the D3-action will be independent of the source of these perturbations, though we will assume that supergravity is still applicable.

Note that in the above discussion, we have neglected the influence of the non-perturbative effects that are required to stabilize the Kähler structure [10]. Such non-perturbative reactions will backreact on the geometry in such a way that it will be better described as a generalized complex geometry [57, 65, 66]. Although such effects might naively seem to be negligible, they may spoil important properties such as sequestering [67]. In principle, we could try to fold the backreaction of the non-perturbative effects into the perturbations of GKP that in the above we attributed to the supersymmetry-breaking sources. However, the points that we wish to make are independent of whether or not the Kähler structure is in fact stabilized and so we will leave the incorporation of such effects for future work.

### 3 Effective action for D3s

In this section, we consider the effective action for a stack of coincident D3-branes probing a perturbation to an  $\mathcal{N}_4 \geq 1$  GKP compactification. Our analysis is similar to that performed in [43, 50, 51] (see also [52, 53]) and indeed we recover many of the same results, except that we take into account the fact that non-supersymmetric fluxes will generically cause the internal metric to no longer be Hermitian with respect to the unperturbed complex structure. We perform the analysis for both probe D3-branes and  $\overline{D3}$ -branes but will frequently, in this section, use “D3” to denote a 3-brane of either charge. In section 4, we will re-express the resulting action for a D3 in the language of softly-broken supersymmetry and comment on how our results relate to those appearing elsewhere in the literature.

#### 3.1 Bosonic action

The effective action for the light open-string bosonic fluctuations of a single  $Dp$ -brane in either type-II string theory consists of the familiar DBI and Chern-Simons (CS) terms which in the 10d Einstein frame take the form

$$S_{Dp} = S_{Dp}^{\text{DBI}} + S_{Dp}^{\text{CS}}, \tag{3.1a}$$

$$S_{Dp}^{\text{DBI}} = -\tau_{Dp} \int d^{p+1}\xi e^{\frac{p-3}{4}\phi} \sqrt{-\det(\hat{M}_{\alpha\beta})}, \tag{3.1b}$$

$$S_{Dp}^{\text{CS}} = \pm \tau_{Dp} \int P \left[ \sum_n C_{(n)} \wedge e^{B^{(2)}} \right] \wedge e^{\ell_s^2 f^{(2)}}, \tag{3.1c}$$

where the upper (lower) sign applies for a  $Dp$ -brane ( $\overline{Dp}$ -brane). The integral is over the worldvolume of the brane and the tension and charge of a  $Dp$  brane are given by  $\tau_{Dp}^{-1} = \frac{1}{2\pi} \ell_s^{p+1} g_s$ . Away from orientifold planes, the bosonic fields consist of a  $U(1)$  gauge-field  $A_{(1)}$  with field strength  $f_{(2)} = dA_{(1)}$  and the transverse deformations which enter through the pullback of bulk fields to the worldvolume denoted by  $P$ .  $\xi^\alpha$  are the worldvolume coordinates and choosing the static gauge we have

$$P[v_\alpha] = v_\alpha + \ell_s^2 v_i \partial_\alpha \varphi^i, \tag{3.2}$$

where we have defined the worldvolume scalars  $\varphi^i = \ell_s^{-2} X^i$  in which  $X^i$  are coordinates transverse to the worldvolume. Here we have defined

$$\hat{M}_{\alpha\beta} = P[\hat{E}_{\alpha\beta}] + e^{-\phi/2} \ell_s^2 f_{\alpha\beta}, \quad \hat{E}_{MN} = \hat{g}_{MN} + e^{-\phi/2} B_{MN}. \tag{3.3}$$

For a stack of  $N$   $Dp$ -branes the gauge symmetry on the common worldvolume is promoted to a  $U(N)$  gauge symmetry and the transverse deformations  $\varphi^i$  are promoted to adjoint-valued fields. The DBI and CS actions then become modified to [55]

$$S_{Dp}^{\text{DBI}} = -\tau_{Dp} \int d^{p+1} \xi \text{Str} \left\{ e^{\frac{p-3}{4}\phi} \sqrt{-\det(\hat{M}_{\alpha\beta}) \det(Q_j^i)} \right\}, \tag{3.4a}$$

$$S_{Dp}^{\text{CS}} = \pm \tau_{Dp} \int \text{Str} \left\{ P \left[ e^{i\ell_s^2 \iota_\varphi^2} \left( \sum_n C_{(n)} \wedge e^{B_{(2)}} \right) \right] \wedge e^{\ell_s^2 f_{(2)}} \right\}. \tag{3.4b}$$

In the static gauge in which we work, we redefine

$$\hat{M}_{\alpha\beta} = P \left[ \hat{E}_{\alpha\beta} + e^{\phi/2} \hat{E}_{\alpha i} (Q^{-1} - \delta)^{ij} \hat{E}_{j\beta} \right] + e^{-\phi/2} \ell_s^2 f_{\alpha\beta}, \tag{3.5}$$

in which

$$Q_j^i = \delta_j^i + i\ell_s^2 [\varphi^i, \varphi^k] e^{\phi/2} \hat{E}_{kj}. \tag{3.6}$$

The field strength is modified to  $f_{(2)} = dA_{(1)} - iA_{(1)} \wedge A_{(1)}$  and the pullback to a non-Abelian pullback

$$P[v_\alpha] = v_\alpha + \ell_s^2 v_i D_\alpha \varphi^i, \tag{3.7}$$

where

$$D_\alpha = \partial_\alpha - i[A_\alpha, \cdot], \tag{3.8}$$

is the usual gauge-covariant derivative acting on adjoint-valued fields.  $\iota_\varphi$  denotes an interior product,

$$\iota_\varphi(v_M dx^M) = \varphi^i v_i, \quad \iota_\varphi^2 \left( \frac{1}{2} v_{MN} dx^M dx^N \right) = \frac{1}{2} [\varphi^j, \varphi^i] v_{ij}. \tag{3.9}$$

Note that due to the non-Abelian nature of the theory,  $\iota_\varphi^2 \neq 0$ . A bulk field appearing in the D-brane action is to be interpreted as a non-Abelian Taylor expansion,

$$\Psi(\varphi) = \sum_{n=0}^{\infty} \frac{\ell_s^{2n}}{n!} \varphi^{i_1} \dots \varphi^{i_n} \left[ \partial_{i_1} \dots \partial_{i_n} \Psi(\varphi) \right]_{\varphi=0}. \tag{3.10}$$

Finally,  $\text{Str}$  denotes a particular trace prescription [55]: before tracing over gauge indices, the expression is symmetrized over factors of  $f_{\alpha\beta}$ ,  $D_\alpha\varphi^i$ ,  $[\varphi^i, \varphi^j]$  and the  $\varphi^i$  appearing in the Taylor expansion. Note that this allows us to treat these objects as commuting.

Our goal is to deduce the effective action to leading order in  $\ell_s$ . That is, the bosonic action consists of an infinite series of irrelevant operators that can be thought of as arising from integrating out massive string modes. Since our interest is the long-wavelength theory, we will consider only the relevant and marginal operators. Furthermore, the coefficients of these operators generically have expansions of the schematic form

$$c \sim \sum_n \ell_s^n \partial^n \Psi, \tag{3.11}$$

in which  $\Psi$  indicates a bulk field and  $\partial^n$  indicates  $n$  derivatives. Since we wish to work in the supergravity regime, we must consider backgrounds where  $\ell_s$  corrections to the supergravity action (A.4) can be neglected, and thus we must have that the derivatives of bulk fields are small with respect to the string scale, at least when evaluated near the position of the probe branes. Therefore we can truncate the sum (3.11) after a certain number of terms. Note that for both expansions, we are comparing energies to  $\ell_s^{-1}$  and so, although it's dimensionful, we can expand in powers of  $\ell_s$  as a proxy for the double expansion in powers of open-string fields and closed-string curvatures. As evidenced by explicit examples [26, 37–42, 61–64] and discussed in the previous section, the closed-string background will generically be divergent at the position of the  $\overline{\text{D3}}$ s (at least in the approximation of linearized supergravity) and so we will work far from the anti-branes.

For the case of interest  $p = 3$  and we can replace the worldvolume indices  $\alpha, \beta$  with the usual  $R^{3,1}$  indices  $\mu, \nu$  and the transverse indices with the internal indices of  $X^6$   $m, n$ . Then

$$\hat{M}_{\mu\nu} = e^{2A(\varphi)} \eta_{\mu\nu} + e^{-\phi(\varphi)/2} \ell_s^2 f_{\mu\nu} + \ell_s^4 (e^{-2A(\varphi)} g_{mn}(\varphi) + e^{-\phi(\varphi)/2} B_{mn}(\varphi)) D_\mu \varphi^m D_\nu \varphi^n, \tag{3.12}$$

since  $B_{\mu\nu} = 0$ ,  $\hat{E}_{\mu m} = 0$  and  $Q = 1 + \mathcal{O}(\ell_s^2)$ . Then, making use of the identity

$$\sqrt{\det(1 + M)} = 1 + \frac{1}{2} \text{tr}(M) - \frac{1}{4} \text{tr}(M^2) + \frac{1}{8} [\text{tr}(M)]^2 + \dots, \tag{3.13}$$

we have

$$\sqrt{-\det(\hat{M}_{\mu\nu})} = e^{4A(\varphi)} + \frac{\ell_s^4}{2} g_{mn}(\varphi) D_\mu \varphi^m D^\mu \varphi^n + \frac{\ell_s^4 e^{-\phi(\varphi)}}{4} f_{\mu\nu} f^{\mu\nu}, \tag{3.14}$$

where we have made use of the anti-symmetry of  $f_{(2)}$  and  $B_{(2)}$ . From this expression, we see that we are interested in an expansion through  $\mathcal{O}(\ell_s^4)$ . Performing the Taylor expansions,

$$\begin{aligned} \sqrt{-\det(\hat{M}_{\mu\nu})} &= \frac{1}{2} (\Phi_+ + \Phi_-) + \frac{\ell_s^4}{2} g_{mn} D_\mu \varphi^m D^\mu \varphi^n + \frac{\ell_s^4 \text{Im } \tau}{4} f_{\mu\nu} f^{\mu\nu} \\ &\quad + \frac{\ell_s^2}{2} \partial_m (\Phi_+ + \Phi_-) \varphi^m + \frac{\ell_s^4}{4} \partial_m \partial_n (\Phi_+ + \Phi_-) \varphi^m \varphi^n. \end{aligned} \tag{3.15}$$

Our notation is such that closed-string fields without an expressed  $\varphi$  dependence are to be evaluated at  $\varphi = 0$  and external indices are contracted only with  $\eta^{\mu\nu}$ . Note that the Taylor

expansion of the warp factor demonstrates the point discussed regarding (3.11): further expansion leads to operators that are relevant and marginal (as well as irrelevant), but their coefficients are suppressed by higher orders in derivatives of the warp factor which we take to be small compared to the string scale.

Similarly,

$$\begin{aligned} \sqrt{\det(Q^m_n)} &= 1 - \frac{i\ell_s^2}{2} B_{mn}(\varphi) [\varphi^m, \varphi^n] - \frac{\ell_s^4}{4} e^{-4A(\varphi)} e^{\phi(\varphi)} g_{mn}(\varphi) g_{pq}(\varphi) [\varphi^m, \varphi^p] [\varphi^n, \varphi^q] \\ &\quad - \frac{\ell_s^4}{8} B_{mn}(\varphi) B_{pq}(\varphi) [\varphi^m, \varphi^n] [\varphi^p, \varphi^q], \end{aligned} \quad (3.16)$$

where we have made use of the symmetry properties of  $g_{mn}$  and  $B_{mn}$ . We can choose a gauge so that  $B_{(2)} = 0$  at  $\varphi = 0$ , and so this result simplifies to

$$\sqrt{\det(Q^m_n)} = 1 - \frac{i\ell_s^4}{2} \partial_p B_{mn} \varphi^p [\varphi^m, \varphi^n] - \frac{\ell_s^4}{2(\Phi_+ + \Phi_-) \text{Im } \tau} g_{mn} g_{pq} [\varphi^m, \varphi^p] [\varphi^n, \varphi^q]. \quad (3.17)$$

Using that the trace is cyclic we can write

$$\partial_m B_{np} \text{tr} \{ \varphi^m [\varphi^n, \varphi^p] \} = \frac{2}{3} H_{mnp} \text{tr} (\varphi^m \varphi^n \varphi^p). \quad (3.18)$$

Putting things together,

$$\begin{aligned} S_{\text{D3}}^{\text{DBI}} &= -\tau_{\text{D3}} \ell_s^4 \int d^4x \text{tr} \left\{ \frac{\Phi_+ + \Phi_-}{2\ell_s^4} + \frac{\text{Im } \tau}{4} f_{\mu\nu} f^{\mu\nu} + \frac{1}{2} g_{mn} D_\mu \varphi^m D^\mu \varphi^n \right. \\ &\quad + \frac{1}{2\ell_s^2} \partial_m (\Phi_+ + \Phi_-) \varphi^m + \frac{1}{4} \partial_m \partial_n (\Phi_+ + \Phi_-) \varphi^m \varphi^n \\ &\quad + \frac{i(\Phi_+ + \Phi_-)}{24 \text{Im } \tau} (G_+ - G_- + \bar{G}_+ - \bar{G}_-)_{mnp} \varphi^m \varphi^n \varphi^p \\ &\quad \left. - \frac{1}{4 \text{Im } \tau} g_{mn} g_{pq} [\varphi^m, \varphi^p] [\varphi^n, \varphi^q] \right\}. \end{aligned} \quad (3.19)$$

Let's now turn to the CS action (3.4b). In type-IIB,  $n$  takes on even values,  $n = 0, 2, 4, 6, 8$  where  $C_{(6)}$  and  $C_{(8)}$  are the redundant magnetic duals of  $C_{(2)}$  and  $C_{(0)}$  respectively.

For  $n = 0$ , we write the contribution to the action as

$$\begin{aligned} S_{\text{D3}}^0 &= \pm \tau_{\text{D3}} \int \text{Str} \left\{ \text{P} \left[ \left( 1 + i\ell_s^2 \iota_\varphi^2 - \frac{\ell_s^4}{2} \iota_\varphi^4 \right) \left( C_{(0)}(\varphi) \wedge e^{B_{(2)}(\varphi)} \right) \right] \wedge \right. \\ &\quad \left. \left( 1 + \ell_s^2 f_{(2)} + \frac{\ell_s^4}{2} f_{(2)} \wedge f_{(2)} \right) \right\}, \end{aligned} \quad (3.20)$$

where we have omitted terms that will only contribute at  $\mathcal{O}(\ell_s^6)$  or higher. The only terms that contribute to the action are those that, after expanding  $e^{B_{(2)}}$ , are 4-forms. Since  $B_{(2)}$  has no legs on  $R^{3,1}$ , it contributes  $\ell_s^4$  from the pullback and then another factor of  $\ell_s^2$  from the fact that we chosen the gauge such the potential vanishes at the position of the probe D3-branes. Thus terms in which  $B_{(2)}$  contributes to “soak up” the legs of the integral are

higher order in  $\ell_s^2$  and so the 4-form must be formed entirely from  $f_{(2)} \wedge f_{(2)}$ . Any scalars resulting from the interior product acting on  $e^{B_{(2)}}$  are again higher order. The result for the  $n = 0$  contribution through  $\mathcal{O}(\ell_s^4)$  is then

$$S_{D3}^0 = \mp \frac{\ell_s^4 \tau_{D3}}{8} \int d^4x \operatorname{tr} \left\{ \operatorname{Re} \tau \epsilon^{\mu\nu\rho\sigma} f_{\mu\nu} f_{\rho\sigma} \right\}. \quad (3.21)$$

We can perform a similar argument for the  $n = 2$  contribution but since  $C_{(2)}$  has no legs on the non-compact directions, there is no contribution through  $\mathcal{O}(\ell_s^4)$ .

For the  $n = 4$  contribution, we write  $C_{(4)} = C_{(4)}^{\text{ext}} + C_{(4)}^{\text{int}}$ , where from (2.1)  $C_{(4)}^{\text{ext}} = \alpha \operatorname{dvol}_{R^{3,1}}$  while  $C_{(4)}^{\text{int}}$  has all four legs on the internal manifold. Using the same reasoning as above, we find that  $C_{(4)}^{\text{int}}$  does not contribute to action at  $\ell_s^4$  order. For  $C_{(4)}^{\text{ext}}$ , we have

$$S_{D3}^{\text{ext}} = \pm \tau_{D3} \int \operatorname{Str} \left\{ \operatorname{P} \left[ \left( 1 + i\ell_s^2 \iota_\varphi^2 - \frac{\ell_s^4}{2} \iota_\varphi^4 \right) \left( C_{(4)}^{\text{ext}}(\varphi) \wedge e^{B_{(2)}(\varphi)} \right) \right] \wedge \left( 1 + \ell_s^2 f_{(2)} + \frac{\ell_s^4}{2} f_{(2)} \wedge f_{(2)} \right) \right\}. \quad (3.22)$$

Now, since  $\iota_\varphi C_{(4)}^{\text{ext}} = 0$ ,  $C_{(4)}^{\text{ext}}$  soaks up all of the legs and this becomes

$$S_{D3}^{\text{ext}} = \pm \tau_{D3} \int \operatorname{Str} \left\{ C_{(4)}^{\text{ext}}(\varphi) \wedge \left[ 1 + i\ell_s^2 \iota_\varphi^2 B_{(2)}(\varphi) - \frac{\ell_s^4}{4} \iota_\varphi^4 \left( B_{(2)}(\varphi) \wedge B_{(2)}(\varphi) \right) \right] \right\}. \quad (3.23)$$

We can make another gauge choice to set the constant part of  $C_{(4)}^{\text{ext}}$  to zero, and so combining this with the similar gauge choice for  $B_{(2)}$ , the Taylor expansion gives

$$S_{D3}^{\text{ext}} = \pm \tau_{D3} \ell_s^4 \int d^4x \operatorname{tr} \left\{ \frac{1}{2\ell_s^2} \partial_m (\Phi_+ - \Phi_-) \varphi^m + \frac{1}{4} \partial_m \partial_n (\Phi_+ - \Phi_-) \varphi^m \varphi^n \right\}. \quad (3.24)$$

For  $n = 6$ , the corresponding potential is defined by

$$F_{(7)} = dC_{(6)} + C_{(4)} \wedge H_{(3)} = -\hat{\star}^{(s)} F_{(3)}, \quad (3.25)$$

in which  $\hat{\star}^{(s)}$  is the 10d Hodge-\* in the string frame. We find<sup>12</sup>

$$F_{(7)} = -e^{4A+\phi} \operatorname{dvol}_{R^{3,1}} \wedge *F_{(3)}, \quad (3.26)$$

and thus  $C_{(6)}$  has four legs on  $R^{3,1}$  and two legs on the internal space. Setting the constant part of  $C_{(6)}$  to be a constant and applying reasoning similar to the  $C_{(4)}^{\text{ext}}$  part gives the leading order contribution

$$S_{D3}^6 = \pm i \tau_{D3} \ell_s^2 \int \operatorname{Str} \left\{ \iota_\varphi^2 C_{(6)}(\varphi) \right\}. \quad (3.27)$$

Writing  $C_{(6)} = \operatorname{dvol}_{R^{3,1}} \wedge \tilde{C}_{(2)}$ , the leading-order contribution is

$$S_{D3}^6 = \mp \frac{i \tau_{D3} \ell_s^4}{2} \int d^4x \operatorname{tr} \left\{ \partial_m \tilde{C}_{np} \varphi^m [\varphi^n, \varphi^p] \right\}. \quad (3.28)$$

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<sup>12</sup>Recall that our notation is that unadorned \* means the Hodge-\* built from the 6d unwarped metric  $g_{mn}$ .

Following the same steps that lead to (3.18) and using  $d\tilde{C}_{(2)} = -e^{4A+\phi} * F_{(3)}$  this term becomes

$$S_{\text{D3}}^6 = \pm \frac{i\tau_{\text{D3}}\ell_s^4}{24} \int d^4x \text{tr} \left\{ \frac{\Phi_+ + \Phi_-}{\text{Im } \tau} (G_+ + G_- + \bar{G}_+ + \bar{G}_-)_{mnp} \varphi^m \varphi^n \varphi^p \right\}. \quad (3.29)$$

Finally, we consider  $n = 8$  where the potential is defined via

$$F_{(9)} = dC_{(8)} + C_{(6)} \wedge H_{(3)} = \hat{*}^{(s)} F_{(1)}. \quad (3.30)$$

Setting the constant part of  $C_{(8)}$  to vanish, the potential does not contribute to the action at this order as there is a factor of  $\ell_s^4$  coming just from the interior product.

Combining these, we find

$$S_{\text{D3}}^{\text{CS}} = \pm \tau_{\text{D3}} \ell_s^4 \int d^4x \text{tr} \left\{ -\frac{\text{Re } \tau}{8} \epsilon^{\mu\nu\rho\sigma} f_{\mu\nu} f_{\rho\sigma} + \frac{1}{2\ell_s^2} \partial_m (\Phi_+ - \Phi_-) \varphi^m \right. \\ \left. + \frac{1}{4} \partial_m \partial_n (\Phi_+ - \Phi_-) \varphi^m \varphi^n \right. \\ \left. + \frac{i(\Phi_+ + \Phi_-)}{24 \text{Im } \tau} (G_+ + G_- + \bar{G}_+ + \bar{G}_-)_{mnp} \varphi^m \varphi^n \varphi^p \right\}. \quad (3.31)$$

Adding this with (3.19), we get the 4d Lagrangian for the bosonic sector

$$\mathcal{L}^{\text{B}} = \text{tr} \left\{ -\frac{1}{4g^2} f_{\mu\nu} f^{\mu\nu} - \frac{\vartheta}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} f_{\mu\nu} f_{\rho\sigma} - \frac{1}{2} K_{mn} D_\mu \varphi^m D^\mu \varphi^n - V_0 - T_m \varphi^m \right. \\ \left. - \frac{1}{2} m_{\text{B},mn}^2 \varphi^m \varphi^n - \frac{i}{3!} C_{mnp} \varphi^m \varphi^n \varphi^p + \frac{g^2}{4} K_{mn} K_{pq} [\varphi^m, \varphi^p] [\varphi^n, \varphi^q] \right\}, \quad (3.32)$$

in which

$$K_{mn} = \frac{2\pi}{g_s} g_{mn}, \quad (3.33a)$$

$$g^{-2} = \frac{2\pi}{g_s} \text{Im } \tau, \quad (3.33b)$$

$$\vartheta = \pm \frac{16\pi^3}{g_s} \text{Re } \tau, \quad (3.33c)$$

$$V_0 = \frac{\pi}{\ell_s^4 g_s} (\Phi_+ + \Phi_-), \quad (3.33d)$$

$$T_m = \frac{2\pi}{\ell_s^2 g_s} \partial_m \Phi_{\mp}, \quad (3.33e)$$

$$m_{\text{B},mn}^2 = \frac{2\pi}{g_s} \partial_m \partial_n \Phi_{\mp}, \quad (3.33f)$$

$$C_{mnp} = \mp \frac{\pi}{g_s} \frac{\Phi_+ + \Phi_-}{\text{Im } \tau} (G_{\mp} + \bar{G}_{\mp})_{mnp}, \quad (3.33g)$$

where again the upper (lower) sign applies for D3-branes ( $\overline{\text{D3}}$ -branes).

## 3.2 Fermionic action

In this subsection, we consider the fermionic modes on the D3. We begin with the Dirac-like action of [68–70] (see also [71]). Although this action is applicable only in the Abelian case of a single Dp-brane, it is enough to deduce the kinetic terms and mass terms. The analogous action in the non-Abelian case is not well-understood; however we will make use of a portion of the action that follows from consistency with T-duality to determine the Yukawa couplings.

### 3.2.1 Abelian case

To leading order in  $\ell_s$ , the fermionic action for a single Dp-brane in the Einstein frame is [70]

$$S_{Dp}^F = i\tau_{Dp}\ell_s^4 \int d^{p+1}\xi e^{\frac{p-3}{4}\phi} \sqrt{-\det(\hat{M}_{\alpha\beta})} \bar{\Theta} P_{\pm}^{Dp} P \left[ (\hat{\mathcal{M}}^{-1})^{\alpha\beta} \hat{\Gamma}_{\beta} \left( \hat{\mathcal{D}}_{\alpha} + \frac{1}{4} \hat{\Gamma}_{\alpha} \hat{\mathcal{O}} \right) - \hat{\mathcal{O}} \right] \Theta, \quad (3.34)$$

in which  $\Theta$  is a double 10d Majorana-Weyl spinor (see appendix A) and again the upper (lower) sign applies to a Dp-brane ( $\overline{Dp}$ -brane).<sup>13</sup> Note that in (3.34) we have redefined  $\Theta$  with respect to [70] so that an explicit power of  $\ell_s$  appears in order to match the one that appears in the bosonic action (3.32).  $\hat{M}_{\alpha\beta}$  is given by (3.3) (taken in the limit  $\ell_s \rightarrow 0$ ) while in IIB<sup>14</sup>

$$\hat{M}_{\alpha\beta} = P[\hat{g}_{\alpha\beta}] + e^{-\phi/2} \mathcal{F}_{\alpha\beta} \Gamma_{(10)} \otimes \sigma^3, \quad (3.35)$$

where

$$\mathcal{F}_{(2)} = P[B_{(2)}] + \ell_s^2 f_{(2)}. \quad (3.36)$$

$\Gamma_{(10)}$  is the 10d-chirality operator while  $\sigma^3$  acts on the extension space as discussed in appendix A. The projection operator takes the form

$$P_{\pm}^{Dp} = \frac{1}{2} \begin{pmatrix} 1 & \pm \check{\Gamma}_{Dp}^{-1} \\ \pm \check{\Gamma}_{Dp} & 1 \end{pmatrix}, \quad (3.37a)$$

in which

$$\check{\Gamma}_{Dp} = i^{(p-2)(p-3)} \Gamma_{Dp}^{(0)} \Lambda(\mathcal{F}), \quad (3.37b)$$

$$\Gamma_{Dp}^{(0)} = \frac{1}{(p+1)!} \hat{\varepsilon}_{\alpha_1 \dots \alpha_{p+1}} \hat{\Gamma}^{\alpha_1 \dots \alpha_{p+1}}, \quad (3.37c)$$

$$\Lambda(\mathcal{F}) = \frac{\sqrt{-\det(P[\hat{g}_{\alpha\beta}])}}{\sqrt{-\det(\hat{M}_{\alpha\beta})}} \sum_q \frac{e^{-q\phi/2}}{2^q q!} \mathcal{F}_{\alpha_1 \beta_1} \dots \mathcal{F}_{\alpha_q \beta_q} \hat{\Gamma}^{\alpha_1 \beta_1 \dots \alpha_q \beta_q}. \quad (3.37d)$$

The operators  $\hat{\mathcal{D}}_M$  and  $\hat{\mathcal{O}}$  are related to the supersymmetry transformations of the gravitino and dilatino (A.7).

For a D3 probing (2.1) with  $\langle f_{\mu\nu} \rangle = 0$ , we have to leading order in  $\ell_s$   $\mathcal{F}_{(2)} = 0$  and  $\hat{M}_{\mu\nu} = e^{2A} \eta_{\mu\nu}$ . This latter fact implies that  $\hat{\mathcal{O}}$  cancels out of the action. Also to this order,

<sup>13</sup>The sign difference in the projection operator with respect to [70] is a consequence of our different convention for the Levi-Civita tensor.

<sup>14</sup>In IIA, we make the replacement  $\sigma^3 \rightarrow \mathbb{I}_2$ .



only the leading term in  $\Lambda(\mathcal{F})$  contributes and so we take  $\Lambda(\mathcal{F}) = 1$ , giving  $\check{\Gamma}_{\text{D3}} = -i\Gamma_{(4)}$ . Furthermore, in the background (2.1), we have

$$\hat{\mathcal{D}}_\mu = \hat{\nabla}_\mu - \frac{1}{16}e^{\phi/2}\hat{\Gamma}_\mu\hat{\mathcal{G}}^+ + \frac{1}{16}\hat{\mathcal{F}}_{(5)}\hat{\Gamma}_\mu(i\sigma^2), \quad (3.38)$$

where, as in (A.8),

$$\mathcal{G}^\pm = \hat{\mathcal{F}}_{(3)}\sigma^1 \pm e^{-\phi}\hat{\mathcal{H}}_{(3)}\sigma^3, \quad (3.39)$$

and  $\hat{\nabla}_\mu$  is the covariant derivative. As is familiar from the Green-Schwarz superstring, the fermionic action (3.34) is subject to a gauge redundancy known as  $\kappa$ -symmetry

$$\Theta \sim \Theta + P_\pm^{\text{D}p}\kappa, \quad (3.40)$$

in which  $\kappa$  is an arbitrary double Majorana-Weyl spinor. We can use this to set

$$\Theta = \begin{pmatrix} \theta \\ 0 \end{pmatrix}, \quad (3.41)$$

in which  $\theta$  is an ordinary Majorana-Weyl spinor. With this choice of  $\kappa$ -fixing, we find

$$S_{\text{D3}}^{\text{F}} = \frac{i\tau_{\text{D3}}\ell_s^4}{2} \int d^4x e^{4A}\bar{\theta} \left\{ \hat{\Gamma}^\mu\hat{\nabla}_\mu \mp \frac{i}{16}\Gamma_{(4)}\hat{\Gamma}^\mu\hat{\mathcal{F}}_{(5)}\hat{\Gamma}_\mu - \frac{e^{\phi/2}}{4}(\pm i\Gamma_{(4)}\hat{\mathcal{F}}_{(3)} + e^{-\phi}\hat{\mathcal{H}}_{(3)}) \right\} \theta. \quad (3.42)$$

From (A.14) we have

$$\hat{\nabla}_\mu = \partial_\mu + \frac{1}{2}\partial_m A \hat{\Gamma}_\mu \hat{\Gamma}^m = \partial_\mu + \frac{e^{-2A}}{16}\partial_m(\Phi_+ + \Phi_-)(\gamma_\mu\gamma_{(4)} \otimes \gamma^m), \quad (3.43)$$

where we have used the decomposition (A.26) and  $\gamma_\mu$  and  $\gamma_m$  are the unwarped  $\gamma$ -matrices. On the other hand, from (2.1) we have

$$F_{\mu\nu\rho\sigma m} = \varepsilon_{\mu\nu\rho\sigma}\partial_m\alpha, \quad F_{mnpqr} = -e^{-8A}\varepsilon_{mnpqr}{}^s\partial_s\alpha, \quad (3.44)$$

where  $\varepsilon_{123456} = \sqrt{\det(g_{mn})}$  and similarly for  $\varepsilon_{\mu\nu\rho\sigma}$ . Hence,

$$\hat{\mathcal{F}}_{(5)} = -ie^{-3A}\partial_m\alpha(\mathbb{I}_4 \otimes \gamma^m)(1 - \Gamma_{(10)}). \quad (3.45)$$

Using that  $\Gamma_{(10)}\theta = +\theta$ , we find

$$\hat{\mathcal{F}}_{(5)}\hat{\Gamma}_\mu\theta = -ie^{-2A}\partial_m(\Phi_+ - \Phi_-)(\gamma_\mu \otimes \gamma^m)\theta. \quad (3.46)$$

Thus, the action becomes

$$S_{\text{D3}}^{\text{F}} = \frac{i\tau_{\text{D3}}\ell_s^4}{2} \int d^4x \bar{\theta} \left\{ e^{3A}\not{\partial} \otimes \mathbb{I}_8 + \frac{e^A}{2}\partial_m\Phi_\mp\gamma_{(4)} \otimes \gamma^m \mp \frac{ie^{7A+\phi/2}}{8}[(\mathbb{I}_4 \mp \gamma_{(4)}) \otimes \mathcal{G}_{(3)} + (\mathbb{I}_4 \pm \gamma_{(4)}) \otimes \overline{\mathcal{G}}_{(3)}] \right\} \theta, \quad (3.47)$$

where, for example,  $\mathcal{G}'_{(3)} = \frac{1}{3!}G_{mnp}\gamma^{mnp}$  involves only unwarped SO(6)  $\gamma$ -matrices and similarly  $\not{\partial} = \gamma^\mu\partial_\mu$ . If  $\eta_\pm$  is a 6d Weyl spinor satisfying  $\gamma_{(6)}\eta_\pm = \pm\eta_\pm$ , then

$$\gamma^{mnp}\eta_\pm = \pm\frac{i}{3!}\epsilon^{mnp}{}_{stl}\gamma^{stl}\eta_\pm, \tag{3.48}$$

and so for a 3-form  $X_{(3)}$ ,

$$\not{X}_{(3)}\eta_\pm = \mp i\tilde{\not{X}}_{(3)}\eta_\pm, \tag{3.49}$$

in which  $\tilde{X}_{(3)} = *X_{(3)}$ . Since  $\Gamma_{(10)}\theta = +\theta$ , we have  $\Gamma_{(4)}\theta = \Gamma_{(6)}\theta$  and so

$$S_{D3}^F = \frac{i\tau_{D3}\ell_s^4}{2} \int d^4x \bar{\theta} \left\{ e^{3A} \not{\partial} \otimes \mathbb{I}_8 + \frac{e^A}{2} \gamma_{(4)} \otimes \not{\partial} \Phi_\mp + \frac{e^{7A+\phi/2}}{16} [(\mathbb{I}_4 \mp \gamma_{(4)}) \otimes \mathcal{G}'_\mp - (\mathbb{I}_4 \pm \gamma_{(4)}) \otimes \bar{\mathcal{G}}'_\mp] \right\} \theta. \tag{3.50}$$

The fermionic modes on the D3 can be decomposed into a gaugino  $\lambda$  and a number of modulini  $\psi^m$ , the fermionic partners of the transverse deformations of the worldvolume,

$$\theta = \theta_g + \theta_m. \tag{3.51}$$

Following [50], we can determine how to extract these modes by considering the supersymmetry transformations. To this end, we consider the case where the metric and fluxes satisfy the conditions for  $\mathcal{N}_4 = 1$  supersymmetry. Then the solution to the Killing spinor equations

$$\hat{D}_M \hat{\epsilon} = 0, \quad \hat{\mathcal{O}} \hat{\epsilon} = 0, \tag{3.52}$$

takes the form

$$\hat{\epsilon} = \begin{pmatrix} \hat{\epsilon}_1 \\ \hat{\epsilon}_2 \end{pmatrix}, \tag{3.53}$$

where [54]

$$\begin{aligned} \hat{\epsilon}_1 &= e^{A/2} \begin{pmatrix} 0 \\ \epsilon_\alpha \end{pmatrix} \otimes \eta_- - e^{A/2} \begin{pmatrix} i\bar{\epsilon}^{\dot{\alpha}} \\ 0 \end{pmatrix} \otimes \eta_+, \\ \hat{\epsilon}_2 &= -ie^{A/2} \begin{pmatrix} 0 \\ \epsilon_\alpha \end{pmatrix} \otimes \eta_- - ie^{A/2} \begin{pmatrix} i\bar{\epsilon}^{\dot{\alpha}} \\ 0 \end{pmatrix} \otimes \eta_+, \end{aligned} \tag{3.54}$$

in which  $\epsilon_\alpha$  is an arbitrary constant spinor,  $\eta_-$  is a negative chirality spinor satisfying

$$0 = \nabla_m \eta_- + \frac{i}{4} e^\phi F_m \eta_-, \tag{3.55}$$

and  $\eta_+ := B_\delta^* \eta_-^*$ . In the string frame the supersymmetry transformations of the D3 bosonic fields take the schematic forms

$$\delta_\epsilon A_\mu \sim \bar{\Theta}^{(s)} \hat{\Gamma}_\mu^{(s)} \hat{\epsilon}^{(s)}, \quad \delta_\epsilon \Phi^m \sim \bar{\Theta}^{(s)} \hat{\Gamma}^{m(s)} \hat{\epsilon}^{(s)} \tag{3.56}$$

Moving to the Einstein frame,  $\hat{g}_{MN} = e^{-\phi/2} \hat{g}_{MN}^{(s)}$ ,  $\hat{\epsilon} = e^{-\phi/8} \hat{\epsilon}^{(s)}$ ,  $\Theta = e^{-\phi/8} \Theta^{(s)}$ , we have

$$\delta_\epsilon A_\mu \sim e^{\phi/2} \bar{\Theta} \hat{\Gamma}_\mu \hat{\epsilon}, \quad \delta_\epsilon \Phi^m \sim \bar{\Theta} \hat{\Gamma}^m \hat{\epsilon}. \tag{3.57}$$

We wish to recover the usual  $\mathcal{N}_4 = 1$  supersymmetry transformations

$$\delta A_\mu \sim \lambda \gamma_\mu \epsilon, \quad \delta \varphi^i \sim \psi^i \epsilon, \quad (3.58)$$

where we have used  $\eta_-$  to define a complex structure characterized by the (3, 0) form

$$\Omega_{mnp} = \eta_-^\dagger \gamma_{mnp} \eta_+, \quad (3.59)$$

and have denoted holomorphic and anti-holomorphic indices by  $i$  and  $\bar{i}$ . Then (3.58) is recovered from (3.57) by taking

$$\begin{aligned} \theta_g &= a e^{-3A/2 - \phi/2} \begin{pmatrix} 0 \\ \lambda_\alpha \end{pmatrix} \otimes \eta_- - a e^{-3A/2 - \phi/2} \begin{pmatrix} i \bar{\lambda}^{\dot{\alpha}} \\ 0 \end{pmatrix} \otimes \eta_+, \\ \theta_m &= b e^{-3A/2} \begin{pmatrix} 0 \\ \psi_\alpha^i \end{pmatrix} \otimes \Omega_{ijk} \gamma^{jk} \eta_- - b e^{-3A/2} \begin{pmatrix} i \bar{\psi}^{\bar{i}\dot{\alpha}} \\ 0 \end{pmatrix} \otimes \bar{\Omega}_{\bar{i}\bar{j}\bar{k}} \gamma^{\bar{j}\bar{k}} \eta_+, \end{aligned} \quad (3.60)$$

in which  $a$  and  $b$  are normalization constants. Note that the form is taken to ensure that  $\theta$  is Majorana-Weyl.

Consider now the non-supersymmetric case. As discussed in the previous section, generically, the addition of  $\overline{D3}$ -branes will cause the metric to no longer be Hermitian with respect to the complex structure. However, at least away from the  $\overline{D3}$ , the spinor  $\eta_-$  defines an  $SU(3)$  structure<sup>15</sup> and from this we can construct an almost complex structure  $J_m^n$  and a pre-symplectic structure  $\omega_{mn}$ . The existence of the former is equivalent to the existence of a 3-form  $\Omega$  and we have

$$\Omega_{mnp} = \eta_-^\dagger \gamma_{mnp} \eta_+, \quad \omega_{mn} = i \eta_+^\dagger \gamma_{mn} \eta_+. \quad (3.61)$$

We emphasize that, since in the non-supersymmetric case there is no natural spinor to define them, these structures are defined by the spinor satisfying (3.55) where the derivative is built from the unperturbed Kähler metric of the supersymmetric solution that we are perturbing. We also note that we are no longer guaranteed that  $\eta_-$  is well-defined and non-vanishing everywhere in the internal space, and so these structures may only be locally defined. By construction, these structures satisfy the compatibility condition

$$\Omega \wedge \omega = 0, \quad (3.62)$$

which ensures that the metric that defines the Clifford algebra is Hermitian and we have  $\omega_{mn} = J_{mn}$ . However, in general we are not ensured that either  $\omega$  nor  $\Omega$  is closed and so the space is not immediately Kähler or indeed even complex. Therefore, we will not, for now, explicitly denote indices that are holomorphic or anti-holomorphic with respect to this perturbed almost complex structure and write

$$\theta_m = b e^{-3A/2} \begin{pmatrix} 0 \\ \psi_\alpha^m \end{pmatrix} \otimes \Omega_{mnp} \gamma^{np} \eta_- - b e^{-3A/2} \begin{pmatrix} i \bar{\psi}^{m\dot{\alpha}} \\ 0 \end{pmatrix} \otimes \Omega_{mnp}^* \gamma^{np} \eta_+. \quad (3.63)$$

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<sup>15</sup>See, e.g., [72, 73] for reviews on  $G$ -structures.

Note that although the notation suggests that there are now six independent Weyl fermions in 4d, the fact that  $\eta_-$  is Weyl, and therefore pure in the sense that it is annihilated by half of the  $\gamma$ -matrices, implies that only three of them are independent. In the supersymmetric case, the analogous statement is  $\psi^{\bar{i}} = 0$  (since  $\Omega_{\bar{i}mn} = 0$ ) where  $\psi^{\bar{i}}$  should not be confused with  $\bar{\psi}^i = (\psi^i)^*$ .

Consider now (3.50). The first operator that appears gives rise to the 4d kinetic terms and we have

$$e^{3A}\bar{\theta}_g \not{\partial} \otimes \mathbb{I}_8 \theta_g = -a^2 e^{-\phi} \left\{ \bar{\lambda} \bar{\sigma}^\mu \partial_\mu \lambda \eta_-^\dagger \eta_- + \lambda \sigma^\mu \partial_\mu \bar{\lambda} \eta_+^\dagger \eta_+ \right\}. \quad (3.64)$$

We normalize  $\eta_-$  so that at the position of the D3,

$$\eta_-^\dagger \eta_- = \eta_+^\dagger \eta_+ = 1. \quad (3.65)$$

The factor of  $e^{-\phi}$  is what is expected from the kinetic term of  $A_\mu$  appearing in (3.32) and so we get a properly normalized term by setting  $a = 1$ .

Next, we consider

$$e^{3A}\bar{\theta}_g \not{\partial} \otimes \mathbb{I}_8 \theta_m = -ab e^{-\phi/2} \left\{ \bar{\lambda} \bar{\sigma}^\mu \partial_\mu \psi^m \Omega_{mnp} \eta_-^\dagger \gamma^{np} \eta_- + \lambda \sigma^\mu \partial_\mu \bar{\psi}^m \Omega_{mnp}^* \eta_+^\dagger \gamma^{np} \eta_+ \right\}. \quad (3.66)$$

Using (3.61), we see that these terms depend on  $\Omega_{mnp} \omega^{np}$  which vanishes as a consequence of compatibility (3.62) and the fact that  $\Omega$  is IASD (using (3.61) and (3.49)). We then have

$$e^{3A}\bar{\theta}_g \not{\partial} \otimes \mathbb{I}_8 \theta_m = e^{3A}\bar{\theta}_m \not{\partial} \otimes \mathbb{I}_8 \theta_g = 0. \quad (3.67)$$

The last kinetic term is

$$\begin{aligned} & e^{3A}\bar{\theta}_m \not{\partial} \otimes \mathbb{I}_8 \theta_m \\ &= -b^2 \left\{ \bar{\psi}^m \bar{\sigma}^\mu \partial_\mu \psi^n \Omega_{mpq}^* \Omega_{nst} \eta_-^\dagger \gamma^{qp} \gamma^{st} \eta_- + \psi^m \sigma^\mu \partial_\mu \bar{\psi}^n \Omega_{mpq} \Omega_{nst}^* \eta_+^\dagger \gamma^{qp} \gamma^{st} \eta_+ \right\}. \end{aligned} \quad (3.68)$$

Making use of the Clifford algebra, the fact that  $*\omega = \frac{1}{2}\omega \wedge \omega$ , the compatibility of the almost complex and pre-symplectic structures, and the identity

$$\gamma^{mnpq} \eta_\pm = \mp \frac{i}{2!} \epsilon^{mnpq}{}_{st} \gamma^{st} \eta_\pm, \quad (3.69)$$

we have

$$\Omega_{mpq}^* \Omega_{nst} \eta_-^\dagger \gamma^{qp} \gamma^{st} \eta_- = 8 \Omega_m^*{}^{pq} \Omega_{npq} = 8 |\Omega|^2 (g_{mn} - i\omega_{mn}), \quad (3.70)$$

where we use the notation (2.4). Thus, setting

$$b = \frac{1}{4|\Omega|}, \quad (3.71)$$

we get

$$e^{3A}\bar{\theta}_m \not{\partial} \otimes \mathbb{I}_8 \theta_m = -\frac{1}{2} (g_{mn} - i\omega_{mn}) \bar{\psi}^m \bar{\sigma}^\mu \partial_\mu \psi^n - \frac{1}{2} (g_{mn} + i\omega_{mn}) \psi^m \sigma^\mu \partial_\mu \bar{\psi}^n. \quad (3.72)$$

Summarizing, after integrating by parts the kinetic terms are

$$-i\tau_{D3}\ell_s^4 \int d^4x \left\{ \text{Im } \tau \bar{\lambda} \bar{\sigma}^\mu \partial_\mu \lambda + \frac{1}{2} (g_{mn} - i\omega_{mn}) \bar{\psi}^m \bar{\sigma}^\mu \partial_\mu \psi^n \right\}. \quad (3.73)$$

The next operator in (3.50) is the coupling to  $\Phi_\pm$ . However, since  $\theta$  is Majorana-Weyl, any bilinear of the type

$$\bar{\theta} \hat{\Gamma}^{M_1 \dots M_n} \theta, \quad (3.74)$$

automatically vanishes unless  $n$  is 3 or 7 and hence this coupling vanishes.

The masses therefore come only from the 3-form contribution. For the D3 case,

$$S_{D3}^3 = \frac{i\tau_{D3}\ell_s^4}{32} \int d^4x e^{7A+\phi/2} \bar{\theta} \left\{ (\mathbb{I}_4 - \gamma_{(4)}) \otimes \mathcal{G}_- - (\mathbb{I}_4 + \gamma_{(4)}) \otimes \bar{\mathcal{G}}_- \right\} \theta. \quad (3.75)$$

Consider

$$\frac{e^{7A+\phi/2}}{32} \bar{\theta}_g (\mathbb{I}_4 - \gamma_{(4)}) \otimes \mathcal{G}_- \theta_g = -\frac{ie^{4A-\phi/2} a^2}{16} \lambda \lambda \eta_+^\dagger \mathcal{G}_- \eta_-. \quad (3.76)$$

From (3.61), we have

$$\eta_+^\dagger \gamma_{mnp} \eta_- = -\Omega_{mnp}^* \quad (3.77)$$

so this becomes

$$\frac{ie^{4A-\phi/2} a^2}{16} G_- \cdot \bar{\Omega} \lambda \lambda, \quad (3.78)$$

where again we recall (2.4). Note that if the complex structure were not perturbed this would provide a coupling to the (3,0) part of  $G_{(3)}$  alone. However, in general this will couple also to other (unperturbed) Hodge-types. The term in the action is

$$- \tau_{D3} \ell_s^4 \int d^4x \frac{(\Phi_+ + \Phi_-) (\text{Im } \tau)^{1/2}}{32} \left\{ G_- \cdot \bar{\Omega} \lambda \lambda + \bar{G}_- \cdot \Omega \bar{\lambda} \bar{\lambda} \right\}. \quad (3.79)$$

Next, we consider the terms that mix the gaugino and the modulini in the mass matrix

$$\frac{e^{7A+\phi/2}}{32} \bar{\theta}_g (\mathbb{I}_4 - \gamma_{(4)}) \otimes \mathcal{G}_- \theta_m = -\frac{ie^{4A} ab}{16} \lambda \psi^m \eta_+^\dagger \mathcal{G}_- \Omega_{mnp} \gamma^{np} \eta_-. \quad (3.80)$$

One can show  $\eta_+^\dagger \gamma_m \eta_- = 0$  which implies that  $\eta_+^\dagger \gamma_{mnpqr} \eta_- = 0$  and hence, using the Clifford algebra, we find

$$\eta_+^\dagger \mathcal{G}_- \Omega_{mnp} \gamma^{np} \eta_- = -\Omega_{mnp} \Omega_{tl}^* {}^p G_-^{ntl}. \quad (3.81)$$

Using

$$\Omega_{tl}^* {}^p \Omega_{mnp} = \frac{|\Omega|^2}{4} \left[ (g_{tm} - i\omega_{tm})(g_{ln} - i\omega_{ln}) - (g_{lm} - i\omega_{lm})(g_{tn} - i\omega_{tn}) \right], \quad (3.82)$$

we find

$$\eta_+^\dagger \mathcal{G}_- \Omega_{mnp} \gamma^{np} \eta_- = \frac{i|\Omega|^2}{2} (g_{ml} + i\omega_{ml}) G_{-nt}^l \omega^{nt}. \quad (3.83)$$

Hence, this coupling corresponds to the non-primitive part of the (2,1)-flux in the case in which the complex structure is not perturbed.<sup>16</sup> We get the same result coming from  $\bar{\theta}_m(\mathbb{I}_4 - \gamma_{(4)}) \otimes \mathcal{G}'_{-}\theta_m$  and hence the gaugino-modulino mass-mixing is

$$i\tau_{D3}\ell_s^4 \int d^4x \frac{(\Phi_+ + \Phi_-)|\Omega|}{128} \left\{ \lambda\psi^m(g_{ml} + i\omega_{ml})G_{-nt}^l\omega^{nt} - \bar{\lambda}\bar{\psi}^m(g_{ml} - i\omega_{ml})\bar{G}_{-nt}^l\omega^{nt} \right\}. \quad (3.84)$$

The final contribution to the mass matrix is the modulino-modulino part. We have

$$\frac{e^{7A+\phi/2}}{32}\bar{\theta}_m(\mathbb{I}_4 - \gamma_{(4)}) \otimes \mathcal{G}'_{-}\theta_m = -\frac{ie^{4A+\phi/2}b^2}{16}\psi^m\psi^n\eta_+^\dagger\Omega_{mpq}\gamma^{qp}\mathcal{G}'_{-}\Omega_{nst}\gamma^{st}\eta_-. \quad (3.85)$$

Since  $\psi^m\psi^n$  is symmetric in  $m$  and  $n$  this becomes

$$\psi^m\psi^n\eta_+^\dagger\Omega_{mpq}\gamma^{qp}\mathcal{G}'_{-}\Omega_{nst}\gamma^{st}\eta_- = -4|\Omega|^2(g_{l(m} - i\omega_{l(m)}\Omega_{n)pq}G_{-}^{lpq}\psi^m\psi^n, \quad (3.86)$$

and so the corresponding part of the action is

$$-\tau_{D3}\ell_s^4 \int d^4x \frac{\Phi_+ + \Phi_-}{128(\text{Im}\tau)^{1/2}} \left\{ \psi^m\psi^n[g_{l(m} - i\omega_{l(m)}\Omega_{n)pq}G_{-}^{lpq} + \bar{\psi}^m\bar{\psi}^n[g_{l(m} + i\omega_{l(m)}\bar{\Omega}_{n)pq}\bar{G}_{-}^{lpq} \right\}. \quad (3.87)$$

This couples to the primitive (1,2) flux in the case in which the complex structure is not perturbed.

### 3.2.2 Non-abelian case

The previous analysis in the Abelian case suffices to determine the kinetic terms and mass terms in the fermionic action, but in order to determine the Yukawa couplings we need to move to the non-Abelian case. Unfortunately, the non-Abelian version of the fermionic action (3.34) is not currently well-understood. However, we can argue from T-duality and supersymmetry how the action (3.34) will be modified to leading order in  $\ell_s$  for our backgrounds of interest.<sup>17</sup>

To do so, we again consider a stack of  $N$  Dp-branes. As discussed in section 3.1, this involves the promotion of the gauge symmetry to  $U(N)$  and the corresponding modification to the connection  $A_{(1)}$  and its curvature. This of course must be accompanied by the modification of the ordinary derivative to the gauge-covariant derivative. However, since we have taken  $\langle f_{(2)} \rangle = 0$ , there are no other modifications to marginal or relevant operators from this modification to  $A_{(1)}$ . A further change is the promotion of the transverse fluctuations to adjoint-valued fields and the Taylor expansion to non-Abelian Taylor expansions. But, as even the usual Taylor expansion in (3.34) will lead only to  $\ell_s$ -corrections, this again will not be relevant. The fermionic variables themselves are promoted to adjoint-valued

<sup>16</sup>Recall however that generic Calabi-Yaus and other simply connected spaces have  $b_1 = b_5 = 0$  (where  $b_i$  are the Betti numbers) and so do not support non-primitive flux since  $\omega \wedge X_{(3)} = 0$  automatically.

<sup>17</sup>In addition to the term that we consider here, there may be  $\ell_s$ -suppressed Yukawa couplings arising from, for example, performing a Taylor expansion of closed-string fields.

fields and the non-Abelian action must contain a trace that is symmetrized according to some procedure. Fortunately, since all of the operators discussed in the previous section are quadratic in the fermions and, to this order in  $\ell_s$ , the closed string fields are proportional to the identity, this becomes a simple trace. As a result, part of the action is (c.f. (3.50))

$$S_{D3}^F \ni \frac{i\tau_{D3}\ell_s^4}{2} \int d^4x \operatorname{tr} \left[ \bar{\theta} \left\{ e^{3A} \not{D} \otimes \mathbb{I}_8 + \frac{e^A}{2} \gamma_{(4)} \otimes \not{D} \Phi_{\mp} + \frac{e^{7A+\phi/2}}{16} [(\mathbb{I}_4 \mp \gamma_{(4)}) \otimes \not{G}_{\mp} - (\mathbb{I}_4 \pm \gamma_{(4)}) \otimes \bar{\not{G}}_{\mp}] \right\} \theta \right]. \quad (3.88)$$

A further modification, required by gauge invariance, is the replacement of the ordinary derivative  $\partial_\mu$  with the gauge-covariant derivative

$$D_\alpha = \partial_\alpha - i[A_\alpha, \cdot]. \quad (3.89)$$

This leads to an additional term in the action and in the absence of fluxes, the  $\kappa$ -fixed string-frame action includes, for any  $p$

$$\frac{\tau_{Dp}\ell_s^4}{2} \int d^{p+1}\xi e^{-\phi} \operatorname{tr} \left\{ \bar{\theta}^{(s)} \hat{\Gamma}^{(s)\alpha} [A_\alpha, \theta^{(s)}] \right\}. \quad (3.90)$$

For this generalization to be consistent with T-duality under which  $A_\alpha$  is exchanged with transverse deformations  $\varphi^i$ , we must include the term

$$\frac{\tau_{Dp}\ell_s^4}{2} \int d^{p+1}\xi e^{-\phi} \operatorname{tr} \left\{ \bar{\theta}^{(s)} \hat{\Gamma}_i^{(s)} [\varphi^i, \theta^{(s)}] \right\}. \quad (3.91)$$

We can confirm that at this level no symmetrization prescription is required since these couplings agree with the expectation from supersymmetry (see also [74]). In the presence of fluxes, it is natural, given the bosonic action (3.4), to expect that the worldvolume indices ought to be contracted with  $\hat{M}_{\alpha\beta}$  (or  $\hat{\mathcal{M}}_{\alpha\beta}$  before  $\kappa$ -fixing) while transverse indices ought to be contracted with  $\hat{E}_{mn}$  and its inverse. However, taking the gauge choice  $B_{(2)} = 0$  at the position of the D3s, these effects do not contribute at this order in  $\ell_s$ .

In summary, to leading order in  $\ell_s$ , the effect of moving to the non-Abelian case in our background is to replace (3.50) with

$$S_{D3}^F = \frac{i\tau_{D3}\ell_s^4}{2} \int d^4x \operatorname{tr} \left[ \bar{\theta} \left\{ e^{3A} \not{D} \otimes \mathbb{I}_8 + \frac{e^A}{2} \gamma_{(4)} \otimes \not{D} \Phi_{\mp} - i e^{3A+\phi/2} (\gamma_{(4)} \otimes \gamma_m) [\varphi^m, \cdot] + \frac{e^{7A+\phi/2}}{16} [(\mathbb{I}_4 \mp \gamma_{(4)}) \otimes \not{G}_{\mp} - (\mathbb{I}_4 \pm \gamma_{(4)}) \otimes \bar{\not{G}}_{\mp}] \right\} \theta \right]. \quad (3.92)$$

The factors of  $e^\phi$  arise from moving to the 10d Einstein frame. The same sign for the Yukawa applies for both the D3 case and the  $\overline{D3}$  case since it results from the supersymmetrization of the DBI part of the bosonic action which is independent of the sign of the D3-brane charge.

For the kinetic and mass terms, the modification from the Abelian case is minimal since, in our normalization, the generators satisfy  $\operatorname{tr}(T^a T^b) = \delta^{ab}$ . However, the Yukawa

couplings involve further analysis. Note that because of the non-trivial gauge structure, the bilinear doesn't automatically vanish even though there is only a single  $\hat{\Gamma}$ -matrix present ( $\hat{\Gamma}_m = e^{-A}\gamma_{(4)} \otimes \gamma_m$ ). However, for the term arising when  $\theta$  is pure gaugino, we have

$$\begin{aligned}
 & -ie^{3A+\phi/2} \text{tr} \left\{ \bar{\theta}_g(\gamma_{(4)} \otimes \gamma_m) [\varphi^m, \theta_g] \right\} \\
 & = a^2 e^{-\phi/2} \text{tr} \left\{ \lambda[\varphi^m, \lambda] \right\} \eta_+^\dagger \gamma_m \eta_- - a^2 e^{-\phi/2} \text{tr} \left\{ \bar{\lambda}[\varphi^m, \bar{\lambda}] \right\} \eta_-^\dagger \gamma_m \eta_+, \quad (3.93)
 \end{aligned}$$

which, on account of the fact that  $\eta_+^\dagger \gamma_m \eta_- = 0$ , does vanish.

For terms involving the gaugino and the modulino, we have

$$\begin{aligned}
 & -ie^{3A+\phi/2} \text{tr} \left\{ \bar{\theta}_g(\gamma_{(4)} \otimes \gamma_m) [\varphi^m, \theta_m] + \bar{\theta}_m(\gamma_{(4)} \otimes \gamma_m) [\varphi^m, \theta_g] \right\} \\
 & = ab \text{tr} \left\{ \lambda[\varphi^m, \psi^n] \right\} \Omega_{npq} \eta_+^\dagger \{ \gamma_m, \gamma^{pq} \} \eta_- - ab \text{tr} \left\{ \bar{\lambda}[\varphi^m, \bar{\psi}^n] \right\} \Omega_{npq}^* \eta_-^\dagger \{ \gamma_m, \gamma^{pq} \} \eta_+ \\
 & = \frac{i|\Omega|}{2} \omega_{mn} \text{tr} \left\{ \lambda[\varphi^m, \psi^n] - \bar{\lambda}[\varphi^m, \bar{\psi}^n] \right\}. \quad (3.94)
 \end{aligned}$$

where we have made use of the cyclicity of the trace and (3.70).

Finally, for the modulini Yukawas, we have

$$\begin{aligned}
 & -ie^{3A+\phi/2} \text{tr} \left\{ \bar{\theta}_m(\gamma_{(4)} \otimes \gamma_m) [\varphi^m, \theta_m] \right\} \\
 & = b^2 e^{\phi/2} \text{tr} \left\{ \psi^n [\varphi^m, \psi^r] \right\} \Omega_{npq} \Omega_{rst} \eta_+^\dagger \gamma^{qp} \gamma_m \gamma^{st} \eta_- \\
 & \quad - b^2 e^{\phi/2} \text{tr} \left\{ \bar{\psi}^n [\varphi^m, \bar{\psi}^r] \right\} \Omega_{npq}^* \Omega_{rst}^* \eta_-^\dagger \gamma^{qp} \gamma_m \gamma^{st} \eta_+. \quad (3.95)
 \end{aligned}$$

Making use of (3.70) and the fact that  $\Omega_{mnp} \Omega_{st}^p = 0$ , this becomes

$$\begin{aligned}
 & -ie^{3A+\phi/2} \text{tr} \left\{ \bar{\theta}_m(\gamma_{(4)} \otimes \gamma_m) [\varphi^m, \theta_m] \right\} \\
 & = -\frac{e^{\phi/2}}{2} \Omega_{mnp} \text{tr} \left\{ \psi^m [\varphi^n, \psi^p] \right\} - \frac{e^{\phi/2}}{2} \Omega_{mnp}^* \text{tr} \left\{ \bar{\psi}^m [\varphi^n, \bar{\psi}^p] \right\}. \quad (3.96)
 \end{aligned}$$

We can now put things together, and the fermionic Lagrangian for a D3 is

$$\begin{aligned}
 \mathcal{L}^F = \text{tr} \left\{ & -i\tilde{K}_{mn} \bar{\psi}^m \bar{\sigma}^\mu \partial_\mu \psi^n - \frac{i}{g^2} \bar{\lambda} \bar{\sigma}^\mu \partial_\mu \lambda - m_{1/2} \lambda \lambda - m_{1/2}^* \bar{\lambda} \bar{\lambda} \right. \\
 & - m_{F,m} \lambda \psi^m - m_{F,m}^* \bar{\lambda} \bar{\psi}^m - \frac{1}{2} m_{F,mn} \psi^m \psi^n - \frac{1}{2} m_{F,mn}^* \bar{\psi}^m \bar{\psi}^n \\
 & \left. - i h_{mn} \lambda \psi^m \varphi^n - i h_{mn}^* \bar{\lambda} \bar{\psi}^m \varphi^n - i h_{mnp} \psi^m \psi^n \varphi^p - i h_{mnp}^* \bar{\psi}^m \bar{\psi}^n \varphi^p \right\}, \quad (3.97)
 \end{aligned}$$



with

$$\tilde{K}_{mn} = \frac{\pi}{g_s} (g_{mn} - i\omega_{mn}), \tag{3.98a}$$

$$m_{1/2} = \frac{\pi}{16g_s} (\Phi_+ + \Phi_-) (\text{Im } \tau)^{1/2} G_- \cdot \bar{\Omega}, \tag{3.98b}$$

$$m_{F,m} = -\frac{i\pi}{64g_s} (\Phi_+ + \Phi_-) |\Omega| (g_{ml} + i\omega_{ml}) G_{-nt}^l \omega^{nt}, \tag{3.98c}$$

$$m_{F,mn} = \frac{\pi}{32g_s} \frac{\Phi_+ + \Phi_-}{(\text{Im } \tau)^{1/2}} [g_{l(m} - i\omega_{l(m)}] \Omega_{n)pq} G_{-}^{lpq}, \tag{3.98d}$$

$$h_{mn} = \frac{2\pi i}{g_s} |\Omega| \omega_{mn}, \tag{3.98e}$$

$$h_{mnp} = \frac{\pi}{g_s} \frac{1}{(\text{Im } \tau)^{1/2}} \Omega_{mnp}. \tag{3.98f}$$

For the case of  $\overline{\text{D3}}$  branes, we can define the fermionic degrees of freedom in the same way. The Lagrangian takes the same form with the masses modified according to

$$m_{1/2} = -\frac{\pi}{16g_s} (\Phi_+ + \Phi_-) (\text{Im } \tau)^{1/2} \overline{G}_+ \cdot \bar{\Omega}, \tag{3.99a}$$

$$m_{F,m} = \frac{i\pi}{64g_s} (\Phi_+ + \Phi_-) |\Omega| (g_{ml} + i\omega_{ml}) \overline{G}_{+nt}^l \omega^{nt}, \tag{3.99b}$$

$$m_{F,mn} = -\frac{\pi}{32g_s} \frac{\Phi_+ + \Phi_-}{(\text{Im } \tau)^{1/2}} [g_{l(m} - i\omega_{l(m)}] \Omega_{n)pq} \overline{G}_+^{lpq}. \tag{3.99c}$$

## 4 The soft Lagrangian

Consider now a general  $\mathcal{N}_4 = 1$  theory. Such a theory consists of the supergravity multiplet, vector multiplets giving rise to a gauge group  $G$ , and chiral multiplets transforming under various representations of the gauge group. The theory is specified by the Kähler function  $\mathcal{K}$ , which is a real function of the chiral superfields, and the superpotential  $W$  and gauge kinetic functions  $f$ , which are holomorphic in the chiral superfields. The purpose of this section is in part to review how the theory for a stack of probe D3s discussed in the previous section can be expressed in terms of these data in the supersymmetric case. Additionally, we will argue that in the non-supersymmetric case, the resulting Lagrangian is consistent with the spontaneous breaking of supersymmetry.

As discussed in section 2, an  $\mathcal{N}_4 = 1$  theory is obtained by taking  $G_{(3)}$  to be (2, 1) primitive (and hence  $G_- = 0$ ),  $\Phi_- = 0$ , the internal metric  $g_{mn}$  to be Kähler, and  $\tau$  to vary holomorphically over the internal space. In this case, all of the masses appearing in (3.32) and (3.97) vanish. Since our focus is on the interaction of the open strings with themselves and not the interactions of open strings with closed-string fluctuations (though such interactions can be important), we take  $\tau$  and the metric to be constant. After a constant rescaling of the fields, the low-energy Lagrangian following from (3.32) and (3.97)

takes the form

$$\begin{aligned} \mathcal{L} = \text{tr} \left\{ -\frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{\vartheta g^2}{32\pi^2} f_{\mu\nu} \tilde{f}^{\mu\nu} - i \bar{\lambda} \bar{\sigma}^\mu D_\mu \lambda - g_{i\bar{j}} D_\mu \varphi^i D^\mu \bar{\varphi}^{\bar{j}} - i g_{i\bar{j}} \bar{\psi}^{\bar{j}} \bar{\sigma}^\mu D_\mu \psi^i \right. \\ \left. - i \sqrt{2} g g_{i\bar{j}} ([\varphi^{\bar{j}}, \psi^i] \lambda + [\varphi^i, \psi^{\bar{j}}] \bar{\lambda}) + i g (\Omega_{ijk} \psi^i \psi^j \varphi^k + \bar{\Omega}_{\bar{i}\bar{j}\bar{k}} \bar{\psi}^{\bar{i}} \bar{\psi}^{\bar{j}} \bar{\varphi}^{\bar{k}}) \right. \\ \left. + \frac{g^2}{2} g_{i\bar{j}} g_{k\bar{l}} ([\varphi^i, \varphi^k] [\bar{\varphi}^{\bar{j}}, \bar{\varphi}^{\bar{l}}] + [\varphi^i, \bar{\varphi}^{\bar{l}}] [\bar{\varphi}^{\bar{j}}, \varphi^k]) \right\}, \end{aligned} \quad (4.1)$$

in which  $\tilde{f}_{(2)} := *_4 f_{(2)}$  and we have made use of the complex structure to separate holomorphic and anti-holomorphic indices and have used that in our conventions  $|\Omega|^2 = 8$ . Here the gauge-covariant derivative is now  $D_\mu = \partial_\mu - ig [A_\mu, \cdot]$  due to the field redefinition.

Let's now compare this to the Lagrangian following from the usual data of  $\mathcal{N}_4 = 1$  supergravity. Our interest is in the Lagrangian only through marginal order and in the rigid supersymmetry limit. In this case, the Lagrangian takes the form

$$\begin{aligned} \mathcal{L}_{\mathcal{N}_4=1} = -\frac{1}{4} f_{\mu\nu}^a f^{a\mu\nu} - \frac{\vartheta g^2}{32\pi^2} f_{\mu\nu}^a \tilde{f}^{a\mu\nu} - i \bar{\lambda}^a \bar{\sigma}^\mu D_\mu \lambda^a - \mathcal{K}_{I\bar{J}} D_\mu \varphi^I D^\mu \bar{\varphi}^{\bar{J}} - i \mathcal{K}_{I\bar{J}} \bar{\psi}^{\bar{J}} \bar{\sigma}^\mu D_\mu \psi^I \\ - i \sqrt{2} g \mathcal{K}_{I\bar{J}} \left( (\bar{\varphi}^{\bar{J}} T_r^a \psi^I) \lambda^a + (\bar{\psi}^{\bar{J}} T_r^a \varphi^I) \bar{\lambda}^a \right) - \frac{g^2}{2} (\mathcal{K}_{I\bar{J}} \bar{\varphi}^{\bar{J}} T_r^a \varphi^I)^2 \\ - \mathcal{K}^{I\bar{J}} W_I \bar{W}_{\bar{J}} - \frac{1}{2} (W_{IJ} \psi^I \psi^J + \bar{W}_{\bar{I}\bar{J}} \bar{\psi}^{\bar{I}} \bar{\psi}^{\bar{J}}), \end{aligned} \quad (4.2)$$

in which  $W_I = \partial_I W$ , and  $W_{IJ} = \partial_I \partial_J W$  when treating  $W$  as a function of the scalar components and  $T_r^a$  indicates the generators in the representation  $r$ . Here  $\mathcal{K}_{I\bar{J}}$  is assumed to be non-singular at  $\varphi^I = 0$  and is evaluated at that point.

Comparing to (4.1), we immediately make the well-known identification of the matter-field metric with the internal metric  $\mathcal{K}_{I\bar{J}} \rightarrow g_{i\bar{j}}$ . To deduce the superpotential that corresponds to (4.1), we note that we can write

$$ig \Omega_{ijk} \text{tr} \{ \psi^i \psi^j \varphi^k \} = \frac{ig}{2} \Omega_{ijk} \text{tr} \{ \psi^i [\psi^j, \varphi^k] \} = -\frac{1}{2} f^{abc} \Omega_{ijk} \psi_a^i \psi_b^j \varphi_c^k, \quad (4.3)$$

where we have normalized the generators according to  $\text{tr} (T^a T^b) = \delta^{ab}$  and defined the structure constants  $[T^a, T^b] = i f^{abc} T^c$ . Thus the Yukawa couplings not involving the gaugino follow from

$$W_{\mathcal{N}_4=4} = \frac{g}{3!} f^{abc} \Omega_{ijk} \varphi_a^i \varphi_b^j \varphi_c^k = -\frac{ig}{3} \Omega_{ijk} \text{tr} \{ \varphi^i \varphi^j \varphi^k \}, \quad (4.4)$$

which is the usual superpotential used to describe  $\mathcal{N}_4 = 4$  super Yang-Mills theory in  $\mathcal{N}_4 = 1$  language. From this superpotential, we find the  $F$ -term potential

$$V_F = g^{i\bar{j}} W_i \bar{W}_{\bar{j}} = -g^2 g_{i\bar{j}} g_{k\bar{l}} \text{tr} \{ [\varphi^i, \varphi^k] [\bar{\varphi}^{\bar{j}}, \bar{\varphi}^{\bar{l}}] \}. \quad (4.5)$$

Adding this to the  $D$ -term potential

$$V_D = -\frac{g^2}{2} (g_{i\bar{j}} [\varphi^i, \bar{\varphi}^{\bar{j}}])^2, \quad (4.6)$$

and making use of the Jacobi identity we recover the scalar potential appearing in (4.1).

We now turn to the more general case in which the geometry no longer satisfies the conditions for supersymmetry. An important distinction between the supersymmetric and non-supersymmetric cases, as discussed in section 2, is that once non-supersymmetric fluxes are introduced to the geometry, the equations of motion imply that the internal metric  $g_{mn}$  will generically no longer be Hermitian with respect to the unperturbed complex structure. Indeed, there is no guarantee at this level that the internal metric is even either complex or symplectic. However, we can make use of the almost complex structure and pre-symplectic structures that are defined, at least locally, by the Killing spinor of the non-perturbed geometry (3.61). Note that although the same spinor is used, it will not generically satisfy the Killing spinor equations of the perturbed geometry. Furthermore, since the internal gamma matrices are defined in terms of the vielbein

$$\gamma_m = e_m^{\underline{n}} \gamma_{\underline{n}}, \quad (4.7)$$

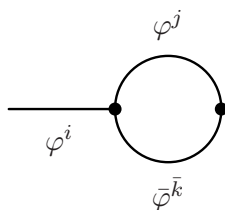
and these vielbein are perturbed according to the perturbation of the metric, the almost complex structure and pre-symplectic structure are not equal to their non-perturbed counterparts. In what follows, we will make use of this almost complex structure to locally define holomorphic and anti-holomorphic indices, keeping in mind that the structure is not expected to be integrable.

Before discussing the Lagrangian resulting from the stack of D3s, let us review the impact that the breaking of supersymmetry can have on the system. As briefly mentioned in the introduction, there are two ways that supersymmetry can be broken in a theory. The first is explicit breaking in which the theory is altered by changing the action (which may be accompanied by changing the field content) such that it no longer respects any supersymmetry transformations. The second way is by spontaneous or dynamical supersymmetry breaking in which the theory is invariant under supersymmetry transformations, but the supercharges do not annihilate the state being considered, typically a meta-stable false vacuum. This latter case is, from a phenomenological standpoint, more interesting since spontaneous breaking restricts the sorts of terms that can appear in the resulting effective field theory so that certain operators, such as scalar masses, are protected from large quantum corrections. In the case of spontaneous breaking, the effective field theory may not have manifest supersymmetry, but instead supersymmetry may be realized only non-linearly. This is similar to the case of the spontaneous breaking of bosonic symmetries. The effective low-energy Lagrangian in such a case is a non-linear  $\Sigma$ -model in which the symmetry, though spontaneously broken and realized only non-linearly, greatly restricts low-energy physics.

In a typical model of supersymmetry breaking<sup>18</sup> supersymmetry is broken spontaneously in a particular sector of a theory by a non-vanishing expectation value for an  $F$ -term or a  $D$ -term (other possibilities exist if one is willing to give up Lorentz invariance). In order to avoid a phenomenologically unacceptable spectrum, supersymmetry breaking is typically assumed to occur in a “hidden” sector, rather than in the visible sector of interest. The effects of the breaking are then mediated by a (not necessarily distinct)

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<sup>18</sup>See [75, 76] for reviews and [77–81] for early treatments of gravity mediation which is the mechanism most relevant for the discussion here.



**Figure 1.** A Feynman diagram demonstrating hard breaking of supersymmetry. Integration over the internal momentum gives rise to quadratic dependence on the UV regulator. However, if  $\varphi^i$  is not a gauge singlet, then gauge invariance forces this diagram to vanish. Note that the holomorphic  $a$ -terms do not give rise to such hard breaking since kinetic terms, schematically represented by the insertion in the loop, do not mix holomorphic with holomorphic fields.

sector known as the messenger sector. Upon integrating out the hidden and messenger sectors, the resulting visible sector does not possess manifest supersymmetry. However, the Lagrangian is non-generic in that only the relevant operators do not obey supersymmetry relations. That is, after breaking supersymmetry, the visible sector Lagrangian in the rigid limit takes the form

$$\mathcal{L}_{\text{vis}} = \mathcal{L}_{\text{susy}} + \mathcal{L}_{\text{soft}}, \tag{4.8}$$

where  $\mathcal{L}_{\text{susy}}$  linearly preserves supersymmetry, while  $\mathcal{L}_{\text{soft}}$  does not, but has no operators of dimension greater than three. In general,  $\mathcal{L}_{\text{soft}}$  takes the schematic form

$$\mathcal{L}_{\text{soft}} \sim t_i \varphi^i + b_{ij} \varphi^i \varphi^j + m_{ij}^2 \varphi^i \bar{\varphi}^j + m_{1/2} \lambda \lambda + m_i \lambda \psi^i + a_{ijk} \varphi^i \varphi^k \varphi^k + c_{ij\bar{k}} \varphi^i \varphi^j \bar{\varphi}^{\bar{k}} + \text{h.c.} \tag{4.9}$$

Other operators can be shuffled into a redefinition of the superpotential as we will do below. We also note the existence of the potentially unfamiliar term  $m_i \lambda \psi^i$  that can be present for adjoint-valued fields, while in more phenomenologically viable constructions, such fields typically do not exist and so this term is absent. The operators are denoted “soft” since although the absence of supersymmetry implies less protection against quantum corrections, most of the operators in  $\mathcal{L}_{\text{soft}}$  only depend logarithmically on the scale of ultraviolet physics. However, some of these operators may break supersymmetry in a hard manner. In particular, the operator  $c_{ij\bar{k}} \varphi^i \varphi^j \bar{\varphi}^{\bar{k}}$  will produce quadratically divergent tadpole graphs such as that appearing in figure 1. Fortunately, gauge invariance implies that such a graph can only be non-vanishing if one of the fields is a singlet and it is easy to argue, as we do below, that these couplings are absent for the singlets in the theory.

The structure of the Lagrangian in the case of spontaneous breaking is to be contrasted with the generic Lagrangian in the case of explicit breaking. In the latter case, one would expect from a Wilsonian standpoint that the resulting Lagrangian would have no special structure and instead consist of all scalar operators consistent with gauge invariance. In particular, generic explicit breaking should also lead to non-supersymmetric marginal deformations of the Lagrangian.

We now return to the case of D3s probing a flux compactification. As can be seen from the results of the previous section, a general perturbation to a supersymmetric flux compactification alters the Lagrangian (4.2) in a more drastic way than the simple addition

of (4.9). In particular, a general perturbation from  $\mathcal{N}_4 = 1$  GKP will modify the marginal couplings, namely the kinetic terms, Yukawa couplings and  $\varphi^4$ . However, by making use of the perturbed structures (3.61), it follows immediately that the marginal operators take the same form as they do in (4.1) when written in terms of these structures. As stated previously, in general the structures are not expected to be integrable and so  $g_{i\bar{j}}$  is not expected to be Kähler. Insofar as we are interested only in relevant and marginal operators, this will not affect the Lagrangian and so still the marginal operators follow from the superpotential (4.4). We return to this point of non-Kählerity in section 6.

In terms of these renormalized fields and the local almost complex structure, we can rewrite the Lagrangians (3.32) and (3.97) as

$$\begin{aligned} \mathcal{L} = \text{tr} \left\{ & -\frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{\vartheta g^2}{32\pi^2} f_{\mu\nu} \tilde{f}^{\mu\nu} - i\bar{\lambda}^a \bar{\sigma} D_\mu \lambda^a - g_{i\bar{j}} D_\mu \varphi^i D^\mu \bar{\varphi}^{\bar{j}} + i g_{i\bar{j}} \bar{\psi}^{\bar{j}} \bar{\sigma}^\mu D_\mu \psi^i \right. \\ & - i\sqrt{2} g g_{i\bar{j}} ([\varphi^{\bar{j}}, \psi^i] \lambda + [\varphi^i, \psi^{\bar{j}}] \bar{\lambda}) + i g (\Omega_{ijk} \psi^i \psi^j \varphi^k + \bar{\Omega}_{i\bar{j}\bar{k}} \bar{\psi}^{\bar{i}} \bar{\psi}^{\bar{j}} \bar{\varphi}^{\bar{k}}) \\ & + \frac{g^2}{2} g_{i\bar{j}} g_{k\bar{l}} ([\varphi^i, \varphi^k] [\bar{\varphi}^{\bar{j}}, \bar{\varphi}^{\bar{l}}] + [\varphi^i, \bar{\varphi}^{\bar{l}}] [\bar{\varphi}^{\bar{j}}, \varphi^k]) \\ & - (t_i \varphi^i + t_{\bar{i}} \bar{\varphi}^{\bar{i}}) - \frac{1}{2} (b_{ij} \varphi^i \varphi^j + b_{i\bar{j}}^* \bar{\varphi}^{\bar{i}} \bar{\varphi}^{\bar{j}}) - m_{i\bar{j}}^2 \varphi^i \bar{\varphi}^{\bar{j}} \\ & - \frac{i}{3!} (a_{ijk} \varphi^i \varphi^j \varphi^k + a_{i\bar{j}\bar{k}}^* \bar{\varphi}^{\bar{i}} \bar{\varphi}^{\bar{j}} \bar{\varphi}^{\bar{k}}) - \frac{i}{2!} (c_{ijk} \varphi^i \varphi^j \bar{\varphi}^{\bar{k}} + c_{i\bar{j}\bar{k}}^* \bar{\varphi}^{\bar{i}} \bar{\varphi}^{\bar{j}} \varphi^k) \\ & \left. - (m_{1/2} \lambda \lambda + m_{1/2}^* \bar{\lambda} \bar{\lambda}) - (m_i \lambda \psi^i + m_i^* \bar{\lambda} \bar{\psi}^{\bar{i}}) - \frac{1}{2} (\mu_{ij} \psi^i \psi^j + \mu_{i\bar{j}}^* \bar{\psi}^{\bar{i}} \bar{\psi}^{\bar{j}}) \right\}, \quad (4.10) \end{aligned}$$

in which for the D3 case (recall  $g^{-2} = \frac{2\pi}{g_s} \text{Im } \tau$ )

$$t_i = \sqrt{\frac{2\pi}{g_s}} \frac{1}{\ell_s^2} \partial_i \Phi_-, \quad (4.11a)$$

$$b_{ij} = \partial_i \partial_j \Phi_-, \quad (4.11b)$$

$$m_{i\bar{j}}^2 = \partial_i \bar{\partial}_{\bar{j}} \Phi_-, \quad (4.11c)$$

$$a_{ijk} = -\frac{g^2}{2} \sqrt{\frac{2\pi}{g_s}} (G_- + \bar{G}_-)_{ijk}, \quad (4.11d)$$

$$c_{i\bar{j}\bar{k}} = -\frac{g^2}{2} \sqrt{\frac{2\pi}{g_s}} (G_- + \bar{G}_-)_{i\bar{j}\bar{k}}, \quad (4.11e)$$

$$m_{1/2} = g \sqrt{\frac{2\pi}{g_s}} \frac{\Phi_+ + \Phi_-}{32} G_- \cdot \bar{\Omega}, \quad (4.11f)$$

$$m_i = \frac{g}{8\sqrt{2}} \sqrt{\frac{2\pi}{g_s}} (\Phi_+ + \Phi_-) G_{-ij}{}^j, \quad (4.11g)$$

$$\mu_{ij} = g \sqrt{\frac{2\pi}{g_s}} \frac{\Phi_+ + \Phi_-}{32} G_{-(i}{}^{kl} \Omega_{j)kl}. \quad (4.11h)$$

As mentioned above, the general Lagrangian contains a holomorphic mass term for the fermions. Such a term can be absorbed into a superpotential as discussed in [43],

$$W = W_{\mathcal{N}_4=4} + \frac{1}{2} \mu_{ij} \varphi^i \varphi^j, \quad (4.12)$$

in which  $W_{\mathcal{N}_4=4}$  is given by (4.4).

The gauge group on the D3 branes is a semi-direct product  $U(N) = SU(N) \times U(1)$ . Since the adjoint of  $U(1)$  is trivial, this implies the existence of gauge singlets on the worldvolume (for example, the center-of-mass of the D3 branes in the internal space) and hence we must check if the terms like  $c_{ij\bar{k}}$  lead to the hard breaking of supersymmetry. To this end, we expand the open-string fields in terms of the generators of the gauge group, e.g.  $\varphi^i = \varphi_a^i T^a$ , and denote the  $U(1)$  generator by  $T^0$ .  $c_{ij\bar{k}}$  is anti-symmetric in  $i$  and  $j$ , so the terms  $\varphi_0^i$  or  $\varphi_0^j$  automatically vanish as  $T^0$  commutes with everything. The term involving  $\varphi_0^{\bar{k}}$  also vanishes since we have, for  $a, b \neq 0$ ,

$$c_{ij\bar{k}} \varphi_a^i \varphi_b^j \bar{\varphi}_0^{\bar{k}} \text{tr}\{T_a T_b T_0\} \sim c_{ij\bar{k}} \varphi_a^i \varphi_b^j \bar{\varphi}_0^{\bar{k}} f^{abc} \text{tr}\{T^c T^0\} = 0, \tag{4.13}$$

which follows from  $T^c \neq T^0$ . Therefore the couplings of the type  $c_{ij\bar{k}}$  vanish when they involve singlets and so do not introduce any hard breaking.

Let us pause to emphasize that many of these operators have appeared elsewhere in the literature. The relevant operators as well as the  $\varphi^4$  operator appeared in the weak-warping limit in [43]. These operators also appeared with more general warping in the Abelian case in [50] as did a subset of them in [51]. Finally, some of these operators can be deduced via worldsheet methods [52, 53]. However, in these works, the expression of such terms in terms of softly-broken  $\mathcal{N}_4 = 1$  language made use of the existence of an underlying complex structure (i.e. when the underlying metric is Kähler) while here we have expressed the Lagrangian in terms of a softly broken supersymmetric Lagrangian (including Yukawa couplings that were not considered in some previous work) in more general cases.

The superpotential (4.12) is of course holomorphic, but it is holomorphic with respect to a perturbed complex structure. Said differently, (4.12) is not holomorphic in the fields of the D3 probing the non-perturbed GKP compactification but instead holomorphic in fields after a non-holomorphic field redefinition. This implies that although the Lagrangian (4.10) describes a theory of a spontaneously broken rigid supersymmetry, this supersymmetry is not the same as the supersymmetry preserved by GKP. Instead the supercharges that are treated as spontaneously broken in (4.10) is some linear combination of the supercharges preserved by GKP and those that are not,

$$Q_\alpha \sim Q_\alpha^{\text{GKP}} + \sum c_a Q_\alpha^a, \tag{4.14}$$

where the coefficients  $c_a$  are of the same order as the perturbation to GKP and we have kept the spinor index explicit. We note that a similar phenomenon must occur even with certain supersymmetric perturbations. Changes to the complex structure (which of course must involve either complex structure moduli that are not fixed by fluxes or a modification of the fluxes as well) implies that a different spinor  $\eta_+$  is annihilated by the new anti-holomorphic  $\gamma$ -matrices and so correspondingly the preserved supercharges is shifted.

Another way to see the shift in supercharges is in terms of the gravitini. Type-IIB is a theory with 32 supercharges, and so a toroidal compactification to 4d gives eight gravitini  $\psi_{\mu\alpha}^I$  where  $I = 1, \dots, 8$ . On a Calabi-Yau with strictly  $SU(3)$  holonomy, only two of these gravitini remain light, and the other six can be thought of as being lifted to the Kaluza-Klein. Once supersymmetric fluxes have been added, the remaining gravitino is (at least in

generic cases) lifted by that flux, leaving a single light gravitino  $\psi_{\mu\alpha}^{\text{GKP}}$  (without fluxes, the geometry cannot distinguish between D3-branes and  $\overline{\text{D3}}$ -branes, so the gravitino lifted by the fluxes must correspond to the supercharges preserved by an  $\overline{\text{D3}}$ -brane). Finally, once non-supersymmetric fluxes have been added, the remaining gravitino will also be lifted (see, for example [82]). Schematically, and neglecting warping effects, the mass is similar to that for the gaugino discussed above

$$m_{3/2} \sim \int \eta_+^\dagger \overline{G} \eta_-, \tag{4.15}$$

which follows from the 10d gravitino action and depends on the (0, 3) ISD flux. However, generically all Hodge types of fluxes are sourced and this will give rise to mixing between  $\psi_{\mu\alpha}^{\text{GKP}}$  and the gravitini lifted by the geometry and flux. Due to this mixing, the lightest gravitino will not be the GKP gravitino, but instead will include an admixture of the gravitini lifted by GKP itself. Although we leave a more precise treatment (for example, the incorporation of the compactification effects required to ensure a finite Kaluza-Klein scale and that (4.15) is well-defined) for future work, this mixture of gravitini is another way of understanding the physics of why (4.12) is not holomorphic in the unperturbed fields.

If supersymmetry is broken by the addition of  $\overline{\text{D3}}$ -branes, then the supersymmetric state obtained by the system after the decay of such branes may not be the same as supersymmetric state to which the anti-branes were originally added. For example, in the KS system the  $\overline{\text{D3}}$ s decay, via NS5s, into flux and D3-branes that were not present in the original KS geometry [25]. This system has, due to the change in flux, a different complex structure and hence a different supersymmetry than the one preserved by the geometry before the addition of the anti-branes. Generically, one would again expect that the lightest gravitino is not quite the gravitino gauging the supersymmetry in this final state, but it would be worthwhile to understand this in detail.<sup>19</sup>

We close this section by noting that the softness of the D3-action is independent of the background that the D3-branes are probing. In the approximation scheme of our analysis, the marginal operators are controlled exclusively by the internal metric. Although in the above analysis we considered small perturbations away from GKP, we can always perform a local field redefinition so that the matter-field metric always takes a form proportional to  $\delta_{i\bar{j}}$ . This field redefinition will also cause the Yukawa-couplings and  $\varphi^4$  potentials to take the form that they do in (4.2).

## 5 An anti-brane goldstino

The result of the previous section is that through marginal order, a stack of D3-branes probing a perturbation of an  $\mathcal{N}_4 = 1$  GKP compactification in the supergravity limit experiences the breaking of supersymmetry softly. Although soft breaking and spontaneous breaking are not equivalent (indeed, as discussed previously non-zero  $c_{ijk}$  which may be present will introduce hard breaking in certain other models), soft breaking is a very non-generic feature of models of explicit breaking. We thus take the softness of the D3 action as

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<sup>19</sup>We thank T. Wrase for discussions related to this point.

evidence that the non-supersymmetric flux, and therefore what is giving rise to that flux, may break supersymmetry spontaneously (although other explanations may be possible). If supersymmetry is indeed spontaneously broken, then there must exist a fermion that is massless in the  $m_p \rightarrow \infty$  limit. In this section, we consider the case where the fluxes result as a backreaction of an  $\overline{\text{D3}}$  and argue for the presence of such a goldstino in the spectrum of  $\overline{\text{D3}}$ -fluctuations.

To this end, we consider the effective Lagrangians (3.32) and (3.97) for the case of an  $\overline{\text{D3}}$ -brane probing a GKP compactification that exhibits  $\mathcal{N}_4 = 1$  before the addition of the  $\overline{\text{D3}}$ . In that case, the Lagrangian again takes the form (4.10) with

$$g^{-2} = \frac{2\pi}{g_s} \text{Im } \tau, \quad (5.1a)$$

$$\vartheta = -\frac{16\pi^3}{g_s} \text{Re } \tau, \quad (5.1b)$$

$$t_i = \sqrt{\frac{2\pi}{g_s}} \frac{1}{\ell_s^2} \partial_i \Phi_+, \quad (5.1c)$$

$$b_{ij} = \partial_i \partial_j \Phi_+, \quad (5.1d)$$

$$m_{ij}^2 = \partial_i \bar{\partial}_j \Phi_+, \quad (5.1e)$$

$$a_{ijk} = -\frac{g^2}{2} \sqrt{\frac{2\pi}{g_s}} (G_+ + \bar{G}_+)_{ijk}, \quad (5.1f)$$

$$c_{ij\bar{k}} = -\frac{g^2}{2} \sqrt{\frac{2\pi}{g_s}} (G_+ + \bar{G}_+)_{ij\bar{k}}, \quad (5.1g)$$

$$m_{1/2} = -g \sqrt{\frac{2\pi}{g_s}} \frac{\Phi_+ + \Phi_-}{32} \bar{G}_+ \cdot \bar{\Omega}, \quad (5.1h)$$

$$m_i = -\frac{g}{8\sqrt{2}} \sqrt{\frac{2\pi}{g_s}} (\Phi_+ + \Phi_-) \bar{G}_+{}^{ij}{}_j, \quad (5.1i)$$

$$\mu_{ij} = -g \sqrt{\frac{2\pi}{g_s}} \frac{\Phi_+ + \Phi_-}{32} \bar{G}_+{}^{(i}{}^{kl} \Omega_{j)kl}. \quad (5.1j)$$

In what follows, we will for simplicity consider a single  $\overline{\text{D3}}$ -brane so that  $a$  and  $c$  both vanish.<sup>20</sup> Here  $g_{i\bar{j}}$  is the Kähler metric of the unperturbed geometry and  $\Omega_{ijk}$  is the form associated with the complex structure. Now, in addition to an  $\mathcal{N}_4 = 1$  GKP compactification having  $G_- = 0$ , the non-vanishing  $G_+$  part is restricted to be  $(2, 1)$  and primitive and as a consequence,

$$m_{1/2} = 0, \quad m_i = 0. \quad (5.2)$$

That is, the gaugino on the  $\overline{\text{D3}}$  is massless. Note that it was important that both  $m_{1/2}$  and  $m_i$  vanished; even if  $m_{1/2}$  vanished but  $m_i$  were non-vanishing then upon diagonalization of the mass matrix, there would generically not be any massless mode.

The massless fermions on a  $Dp$ -brane in flat space can be considered as goldstini associated with the spontaneous breaking of 16 supercharges. However, just as the action

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<sup>20</sup>In the case of multiple  $\overline{\text{D3}}$ s, the goldstino is most likely related to the  $U(1)$  part of the fermionic mode that we identify below, just as for multiple D-branes the goldstone is the center-of-mass.



for a goldstone boson is restricted, the action for a goldstino  $\chi$  in  $R^{3,1}$  takes the Akulov-Volkov form [46, 47],

$$S_{AV} = -\frac{f^2}{2} \int d^4x \det \left[ \delta_\nu^\mu + \frac{i}{f^2} (\bar{\chi} \bar{\sigma}^\mu \partial_\nu \chi + \chi \sigma^\mu \partial_\nu \chi) \right], \quad (5.3)$$

in which  $f$  is related to the scale of the breaking of supersymmetry. Although this action does not contain a full multiplet, it is invariant under the transformation

$$\delta_\epsilon \chi = f\epsilon - \frac{i}{f} (\chi \sigma^\mu \bar{\epsilon} - \epsilon \sigma^\mu \bar{\chi}) \partial_\mu \chi, \quad (5.4)$$

in which  $\epsilon$  is an arbitrary constant spinor. Moreover, this transformation reproduces the usual supersymmetry algebra and so (5.3) realizes supersymmetry non-linearly.

If the fermionic modes on a  $Dp$  are to be interpreted as goldstini, then their action must be similarly constrained. The action for a single  $Dp$ -brane in flat space can be expanded out to higher order in fermions. In flat space, the  $\kappa$ -fixed action takes the form [48] (matching to our conventions)

$$S_{Dp} = -\tau_{Dp} \int d^{p+1}\xi \sqrt{-\det(M_{\alpha\beta})}, \quad (5.5)$$

$$M_{\alpha\beta} = \eta_{\alpha\beta} + \ell_s^2 f_{\alpha\beta} + \ell_s^4 \partial_\alpha \varphi^i \partial_\beta \varphi^i - i\ell_s^4 \bar{\theta} (\Gamma_\alpha + \ell_s^2 \Gamma_i \partial_\alpha \varphi^i) \partial_\beta \theta - \frac{1}{4} \ell_s^8 (\bar{\theta} \Gamma^M \partial_\alpha \theta) (\bar{\theta} \Gamma_M \partial_\beta \theta).$$

The term that is quadratic order in fermions is the action (3.34). In addition to the linearized supersymmetry transformations corresponding to the supercharges that the  $Dp$  preserves, it also realizes another set of supersymmetries non-linearly, as detailed in [48]. Unlike (5.3), the action for a  $Dp$  brane realizes some supersymmetry in a linear way and therefore we should not expect to recover precisely (5.3) and indeed, (5.5) is closed under the non-linearly realized supersymmetry only once the bosonic terms are included [48]. Nevertheless, the corresponding supersymmetry transformations realize the supersymmetry algebra and  $\theta$ , which appears non-linearly in the supersymmetry transformations, ought to be identified with the goldstini associated with the spontaneous breaking of 16 supercharges by the  $Dp$ -brane.

When moving to flux backgrounds, the extension of the action to higher-order in fermions becomes more complicated (see e.g. [83] for a review of related issues). However, the physics of the situation remains the same: D-branes spontaneously break supersymmetry and the worldvolume fermions are the corresponding goldstini. We are not aware of a presentation of the higher-order fermionic action in a flux background that is as readily applicable as (3.34) or (5.5), but from the higher-order terms presented in [48] and the expansion of the Ramond-Ramond superfields as presented in, for example, [68, 69], it is clear that it must involve products of bilinears of the type that appear in (3.34), at least in the absence of scalar fluctuations. When  $\theta$  is pure gaugino then the Hodge-types of background fluxes and the property that  $\bar{\theta} \hat{\Gamma}_{M_1 \dots M_p} \theta$  vanishes if  $p \neq 0, 3, 7, 10$  imply that the only non-vanishing bilinears are the derivative terms. Therefore, the higher-order fermionic action is expected to take the form (5.5) when  $\theta$  is pure gaugino, with some modifications

due to warping. By comparing the scale of the constant term in the action to that of the kinetic term of the gaugino, we find

$$f^2 = \tau_{D3} e^{4A} \sim \ell_s^{-4} e^{4A}, \tag{5.6}$$

which is the familiar statement that the scale of supersymmetry is warped down from the string scale [8, 10].

Finally, let's consider the case in which, instead of probing an  $\mathcal{N}_4 = 1$  GKP compactification, the  $\overline{D3}$  probes a compactification with non-vanishing  $(0, 3)$  or primitive  $(1, 2)$  flux. Although such flux is still ISD and so the internal space is Kähler, supersymmetry is no longer preserved by the background, and so the  $\overline{D3}$  is not, by itself, responsible for the breaking of the supersymmetry preserved by the primitive  $(2, 1)$  flux. From (5.1) it is clear that in this case the gaugino will no longer be massless, which is consistent with the fact that the goldstino cannot be exclusively an  $\overline{D3}$  mode and consistent also with the interpretation of the  $\overline{D3}$  gaugino as the goldstino when probing a supersymmetric compactification. A possible objection to this line of reasoning comes from considering the limit in which all of the flux vanishes. Then according to (5.1), all of the fermions on the  $\overline{D3}$  are massless, yet the goldstino cannot be purely an open-string mode since the geometry itself breaks three of the supercharges preserved by the  $\overline{D3}$  and so one should expect some closed-string component to the goldstino (note that the same issue arises for D3-branes as the geometry itself cannot distinguish between the charges). However, one can imagine going to a region in moduli space where the internal volume is very large and flat and so the  $\overline{D3}$  is, to good approximation, probing flat 10d space, in which case the interpretation of the  $\overline{D3}$  moduli modes as goldstini is appropriate. It is therefore not surprising that the moduli will be massless in other regions of moduli space. Presumably, the goldstino is at all points in moduli space a mixture of open- and closed-string modes. It would be interesting to confirm this fact by understanding the super-Higgs mechanism in such cases. Note that this is again entirely analogous to what occurs in the bosonic sector: the Calabi-Yau itself generically has no isometries and yet there are still massless bosons that are neatly associated with goldstone modes associated with the spontaneous breaking of translational symmetry in the large-radius limit.

To summarize this section, we have identified the  $\overline{D3}$  gaugino as a candidate for the goldstino associated with the spontaneous breaking of supersymmetry. Although more work is required to rigorously demonstrate this, the gaugino is massless when an  $\overline{D3}$  probes  $\mathcal{N}_4 \geq 1$  GKP compactifications and massive when probing  $\mathcal{N}_4 = 0$  GKP compactifications, as is expected from such a goldstino.

## 6 Discussion and concluding remarks

In this work, we have presented some circumstantial evidence that  $\overline{D3}$ s spontaneously break supersymmetry in a flux compactification, contrary to some common folklore which claims that they are an explicit source of breaking. Although this evidence is not conclusive, it approaches a coherent story about the breaking of supersymmetry by anti-branes. In this final section, we summarize these arguments, discuss some possible objections, and lay out

some directions for future work. Although many of the arguments here refer explicitly to  $\overline{\text{D3}}$ -branes, they apply to other sources of supersymmetry breaking as well. However, since we have been able to identify a candidate goldstino in the case of  $\overline{\text{D3}}$ s, we will largely limit our discussion to that case.

As mentioned previously, the common wisdom is that  $\overline{\text{D3}}$ -branes break supersymmetry explicitly. However, there are two possible meanings to “explicit” breaking: either the breaking is spontaneous but the scale of breaking is so high so that the low-energy action effectively exhibits explicit breaking after truncating the operators beyond a certain mass dimension (this is, for example in the AV Lagrangian (5.3) where the marginal operator alone does not exhibit supersymmetry), or the breaking is truly explicit in that it exhibits  $\mathcal{N} = 0$  at arbitrarily high energies. In the absence of warping, the scale of breaking is naturally expected to be the compactification or string scale<sup>21</sup> and so the distinction between explicit and spontaneous breaking is perhaps not important. However, for the case of an  $\overline{\text{D3}}$ -brane which is naturally attracted to regions of large redshift, the scale of supersymmetry breaking may be warped down and so the distinction may be relevant. Before reviewing the circumstantial evidence in this paper, let us first review some heuristic reasoning for why the  $\overline{\text{D3}}$ s might be expected to break supersymmetry spontaneously.

The first is simply the statement that an  $\overline{\text{D3}}$ -brane represents a particular state in a supersymmetric theory, namely string theory. That is, whatever the fundamental description of string theory is, it admits configurations, such as flat 10d/11d space, that preserves 32 supercharges and therefore the theory itself has 32 supercharges. Any other state in the theory that preserves fewer supercharges is still a state in a supersymmetric theory and so those supercharges are, by definition spontaneously broken, though, as mentioned previously, the scale of breaking may be beyond the scale at which field theory is applicable.<sup>22</sup> Furthermore, as stated previously in this work, the breaking of supersymmetry by a D-brane should be entirely analogous to the breaking of translational symmetry and the latter is an example of spontaneous breaking. It is occasionally argued that the  $\overline{\text{D3}}$ s in a GKP geometry break supersymmetry explicitly because they “project out” the supercharges preserved by the background. However, they again do so in a way that is completely analogous to the projecting out of the translational symmetries associated to translating the brane. Another way of stating this argument is that the  $\overline{\text{D3}}$  couples anti-holomorphically to some fields when supersymmetry demands holomorphic couplings (e.g. the gauge kinetic function for a  $\overline{\text{D3}}$ -brane is proportional to  $\bar{\tau}$  rather than  $\tau$ ). However, the coupling is of course holomorphic with respect to the conjugate complex structure. That is, while the action for a D3-brane will have actions that are expressible as  $\int d^4x d^2\theta \dots$ , for an  $\overline{\text{D3}}$ -brane, the same term will be  $\int d^4x d^2\theta'$  for some other fermionic coordinate  $\theta'$ . This integral over a part of the  $\mathcal{N}_4 = 8$  superspace of type-IIB that is different than the part integrated over

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<sup>21</sup>An exception to this is of course dynamical supersymmetry breaking in which the scale of breaking can naturally be much lower.

<sup>22</sup>One possible exception to this is an orientifold plane which in a perturbative treatment literally removes fields from the spectrum. However orientifolds, like D-branes, ultimately map to dynamical objects (M-branes and gravitational monopoles) in M-theory and so ought be treated on the same footing as D-branes in this sense, though the scale of breaking is expected to be non-perturbatively high.

by a D3-brane is entirely analogous to the integration over a particular part of bosonic space (namely the worldvolume) for the  $Dp$ -brane action. That is, integrating over part of superspace is, in terms of the breaking of supersymmetry, on the same footing as only integrating over part of the bosonic space. A problem very similar in spirit, involving the spontaneous breaking of  $\mathcal{N}_4 = 2$  to  $\mathcal{N}_4 = 1$  was considered in [84].

In this work, our first line of evidence towards the spontaneous breaking was from a stack of D3-branes probing a non-supersymmetric perturbation to an  $\mathcal{N}_4 = 1$  GKP compactification, a system that has been considered previously [43, 50, 51]. A supersymmetric GKP compactification is characterized by (among other criteria)  $G_- = 0$  and  $\Phi_- = 0$ . A small non-zero perturbation to the latter, which is sourced “directly” by  $\overline{D3}$ -branes, perturbs the geometry and fluxes in many directions, at least when  $G_+ \neq 0$  before the perturbation. Among these is a perturbation to the internal unwarped metric such that the metric is no longer Kähler, at least with respect to the unperturbed structures. Since the internal metric is identified with the matter-field metric for a D3 probing the geometry, this corresponds to a marginal deformation of the effective field theory describing the open-string fluctuations of the D3s. As all soft terms are relevant operators, naively this would imply a hard breaking of supersymmetry. Despite this fact, we found that when a very natural, albeit non-holomorphic, field redefinition is performed, the marginal operators are related by supersymmetry and thus the breaking is soft. Although one must be careful to not conflate soft breaking with spontaneous breaking and hard breaking with explicit breaking, from the Wilsonian point of view, explicit breaking is generically expected to be hard<sup>23</sup> while spontaneous breaking is soft (so long as complete multiplets remain in the low-energy theory and even then the Lagrangian is soft only up to some potentially hard relevant operators, which were absent for the probe D3s). We thus take the non-generic non-supersymmetric Lagrangian of the probe D3s as an indication that breaking of supersymmetry may be spontaneous.

From the field theory point of view, the breaking is a little unusual in that spontaneous breaking of supersymmetry is usually accomplished by way of some non-vanishing  $F$ -term or  $D$ -term which do not themselves alter kinetic terms. In contrast, the probe D3s do experience such a deformation. Further, the field redefinition discussed above is non-holomorphic in the original set of fields. This suggests that the “least” broken supersymmetry is not that of the original GKP, but instead some linear combination of this supersymmetry and others broken by GKP. More precisely, an  $\mathcal{N}_4 = 1$  GKP compactification breaks 28 of the 32 supercharges of type-IIB. These supercharges can be thought of as being spontaneously broken, but since this breaking occurs at a much higher scale than many scales of interest, we can for the most part ignore these charges. That is, one should in principle be able to treat the theory as having non-linearly realized supersymmetries, but as discussed above, this gains us very little in terms of practical value (for example, it tells us about the higher-order terms in the fermionic action for  $Dp$ -branes, but the scale of suppression will typically be the string scale). When an  $\overline{D3}$ -brane is added to a GKP

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<sup>23</sup>However, see [85] for an interesting example of a string construction in which the field-theory dual exhibits soft explicit breaking.

compactification, the remaining four supercharges are also broken, but the  $\mathcal{N}_4 = 1$  that is most conveniently thought of as being spontaneously broken is not quite the one preserved by GKP but instead includes an admixture of the charges broken by GKP. This is reflected in both the non-holomorphic field redefinition and mixing between the gravitino that gauges the supersymmetry preserved by GKP and those that are lifted in GKP. The lightest gravitino should not be  $\psi_\mu^{\text{GKP}}$  but should instead include a linear combination of  $\psi_\mu^{\text{GKP}}$  and the gravitini lifted by the fluxes and curvature.

A gap in this perspective is the extension of the action for the D3s to irrelevant order, as supersymmetry restricts more than just the relevant and marginal operators that we considered here. This was reflected even in the supersymmetric case as the Lagrangian (4.1) exhibits  $\mathcal{N}_4 = 4$  through marginal order while the irrelevant operators coming, for example, from the non-trivial Kähler metric reveal it to be  $\mathcal{N}_4 = 1$ . In the case of spontaneously broken  $\mathcal{N}_4 = 1$ , supergravity imposes that the target space metric is Kähler, while the internal metric (which as stated previously is the target space metric for probe D3s) resulting from an  $\overline{\text{D3}}$  appears to be generically non-Kähler (at least when expressed in the complex structure of the unperturbed geometry). One possibility is that the backreaction is in fact still Kähler, though the corresponding structures would likely differ from (3.61). We do not supply any evidence in favor of this possibility which would be a very non-trivial consequence of the supergravity equations of motion.<sup>24</sup> Another possibility is that the open-string moduli on the D3 are not the correct Kähler coordinates, but instead the correct coordinates are combinations of the open-string fields on the D3 and closed-string fields, which occurs even in the supersymmetric case (see, e.g. [50, 86]). This would be a very interesting case to check more precisely, but requires more work as even the theory for closed strings alone is not wholly understood in flux compactifications, whether or not supersymmetry is present. Note that since the structures (3.61) are not integrable, we have not demonstrated that the supersymmetry that is softly broken in (4.10) is globally well-defined. It would be important to work out whether such a globally well-defined supersymmetry exists.

From the point of view of the D3s and setting aside the fluctuations of the  $\overline{\text{D3}}$  for a moment, it may seem almost obvious that the breaking is spontaneous. The DBI and CS actions for the D3-branes describes the interaction between the light open strings and light closed strings and placing the D3 brane in the  $\overline{\text{D3}}$  background amounts to just setting certain expectation values for these closed string fields at a single point in the D3-brane moduli space. It may be that the soft structure of the Lagrangian is a consequence of the locality of the D3s. Indeed it would be valuable to repeat this analysis for, for example, D7-branes or closed strings. However, as mentioned previously, even in supersymmetric cases the incorporation of warping and fluxes into the effective action for strings that are not localized at a point in the internal space can be involved,<sup>25</sup> and we leave such analyses for future work (though see, e.g., [27, 51, 98] for some progress in this direction).

If the non-supersymmetric fluxes do break supersymmetry spontaneously, then there must exist a fermion that is massless in the  $m_p \rightarrow \infty$  limit. The easiest case to consider

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<sup>24</sup>In fact it is quite unlikely as there are even supersymmetric compactifications that are not Kähler.

<sup>25</sup>See, for example [51, 82, 87–97].

is when the fluxes result from the backreaction of an  $\overline{D3}$ , and we argued for the existence of such a massless fermion, namely the gaugino on the  $\overline{D3}$ . If this identification is correct, then the  $\overline{D3}$  brane should be thought of as  $D$ -term breaking, though  $F$ -term breaking in the closed-string sector would consequently result. This possibility was also raised in [43]. However, to make a solid case for identification of this mode as the goldstino, there is still work to be done. The first, and most important, would be to demonstrate that supersymmetry is still realized non-linearly on the anti-branes. Here, we primarily made reference to previous work (e.g. [48]), but it would be worthwhile to see this explicitly in the case at hand. It would then be interesting to see how the super-Higgs mechanism is realized in this setup, and to show precisely which gravitino is lightest. Finally, although the discussion above focused on the case in which anti-branes were the source of the breaking of supersymmetry, many of the points go through for other backgrounds as well. Indeed, the softness of the D3 Lagrangian is a consequence of the fact that all of the marginal operators are (to leading order in  $\ell_s$ ) controlled by the same closed-string field, namely the internal metric, and therefore the D3 Lagrangian will apparently be soft in any background. If non-supersymmetric fluxes can always be interpreted as spontaneous breaking, then one should be able to identify the goldstino and understand its physics even when the fluxes do not result from the backreaction of an anti-brane. We hope to return to these questions in the near future.

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### A Conventions

We work with the type-IIB superstring in the supergravity limit and largely follow the conventions of [70]. In the 10d Einstein frame, the bosonic pseudo-action is

$$S_{\text{IIB}} = S_{\text{IIB}}^{\text{NS}} + S_{\text{IIB}}^{\text{R}} + S_{\text{IIB}}^{\text{CS}}, \tag{A.1a}$$

$$S_{\text{IIB}}^{\text{NS}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-\det(\hat{g})} \left[ \hat{R} - \frac{1}{2} \hat{g}^{MN} \partial_M \phi \partial_N \phi - \frac{1}{2} e^{-\phi} \hat{H}_{(3)}^2 \right], \quad (\text{A.1b})$$

$$S_{\text{IIB}}^{\text{R}} = - \frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{-\det(\hat{g})} \left[ e^{2\phi} \hat{F}_{(1)}^2 + e^{\phi} \hat{F}_{(3)}^2 + \frac{1}{2} \hat{F}_{(5)}^2 \right], \quad (\text{A.1c})$$

$$S_{\text{IIB}}^{\text{CS}} = \frac{1}{4\kappa_{10}^2} \int C_{(4)} \wedge H_{(3)} \wedge F_{(3)}, \quad (\text{A.1d})$$

in which the 10d gravitational constant is  $2\kappa_{10}^2 = \frac{1}{2\pi} \ell_s^8 g_s^2$  where  $\ell_s = 2\pi\sqrt{\alpha'}$  is the string length.  $\hat{R}$  is the Ricci scalar built from the 10d Einstein-frame metric  $\hat{g}_{MN}$  which is related to the 10d string-frame metric by  $\hat{g}_{MN} = e^{-\phi/2} \hat{g}_{MN}^{(s)}$ .  $\phi$  is the dilaton defined so that the string coupling is  $g_s e^{\phi}$ . The NS-NS 2-form potential is  $B_{(2)}$  and the R-R potentials are  $C_{(p)}$  for  $p = 0, 2, 4$ . The gauge-invariant field strengths are

$$H_{(3)} = dB_{(2)}, \quad F_{(1)} = dC_{(0)}, \quad F_{(3)} = dC_{(2)} + C_{(0)} \wedge H_{(3)}, \quad F_{(5)} = dC_{(4)} + C_{(2)} \wedge H_{(3)}. \quad (\text{A.2})$$

$F_{(5)}$  is constrained at the level of the equations of motion to satisfy the self-duality constraint  $F_{(5)} = \hat{*}F_{(5)}$  in which  $\hat{*}$  is the 10d Hodge-\*,  $(\hat{*}F)_{MNPQR} = \frac{1}{5!} \hat{\epsilon}_{MNPQR}{}^{STLKI} F_{STLKI}$ . We use the convention that in flat space  $\hat{\epsilon}_{01\dots 9} = +1$ . For a  $p$ -form we define

$$\hat{\Omega}_{(p)}^2 = \frac{1}{p!} \hat{g}^{M_1 N_1} \dots \hat{g}^{M_p N_p} \Omega_{M_1 \dots M_p N_1 \dots N_p}. \quad (\text{A.3})$$

More generally,  $\hat{\phantom{x}}$  will indicate objects pertaining to the 10d metric  $\hat{g}_{MN}$ .

The equations of motion that follow from (A.1) are (see, e.g. [99])

$$0 = \hat{R}_{MN} - \frac{1}{2} \partial_M \phi \partial_N \phi - \frac{1}{2} e^{2\phi} F_M F_N - \frac{1}{2 \cdot 2!} e^{-\phi} H_{MPQ} \hat{H}_N{}^{PQ} - \frac{1}{2 \cdot 2!} e^{\phi} F_{MPQ} \hat{F}_N{}^{PQ} - \frac{1}{4 \cdot 4!} F_{MPQRS} \hat{F}_N{}^{PQRS} + \frac{1}{8} \hat{g}_{MN} \left[ e^{-\phi} \hat{H}_{(3)}^2 + e^{\phi} \hat{F}_{(3)}^2 \right], \quad (\text{A.4a})$$

$$0 = \hat{\nabla}^2 \phi - e^{2\phi} \hat{F}_{(1)}^2 - \frac{1}{2} e^{\phi} \left[ \hat{F}_{(3)}^2 - e^{-2\phi} \hat{H}_{(3)}^2 \right], \quad (\text{A.4b})$$

$$0 = \hat{\nabla}^M (e^{2\phi} F_M) - \frac{e^{\phi}}{3!} H_{MNP} \hat{F}^{MNP}, \quad (\text{A.4c})$$

$$0 = d\hat{*}(e^{-\phi} H_{(3)} + C_{(0)} e^{\phi} F_{(3)}) - F_{(5)} \wedge F_{(3)}, \quad (\text{A.4d})$$

$$0 = d\hat{*}(e^{\phi} F_{(3)}) + F_{(5)} \wedge H_{(3)}, \quad (\text{A.4e})$$

$$0 = d\hat{*}F_{(5)} + H_{(3)} \wedge F_{(3)}. \quad (\text{A.4f})$$

Here,  $\hat{R}_{MN}$  is the Ricci tensor and we have imposed self-duality on  $F_{(5)}$ . In addition, we have the Bianchi identities

$$dH_{(3)} = 0, \quad dF_{(1)} = 0, \quad dF_{(3)} = F_{(1)} \wedge H_{(3)}, \quad dF_{(5)} = F_{(3)} \wedge H_{(3)}. \quad (\text{A.4g})$$

Along with these bosonic modes, type-IIB supergravity contains a pair of 32-component Majorana-Weyl dilatini  $\hat{\chi}^{1,2}$  and a pair of Majorana-Weyl-Rarita-Schwinger gravitini  $\hat{\Psi}_M^{1,2}$ .

We take these modes to be right-handed in the sense that  $\Gamma_{(10)}\hat{\Psi}_M^i = \hat{\Psi}_M^i$  where the 10d-chirality operator  $\Gamma_{(10)}$  is defined by (A.28). These can be used to construct so-called double spinors

$$\hat{\chi} = \begin{pmatrix} \hat{\chi}^1 \\ \chi^2 \end{pmatrix}, \quad \hat{\Psi}^M = \begin{pmatrix} \hat{\Psi}_M^1 \\ \hat{\Psi}_M^2 \end{pmatrix}. \quad (\text{A.5})$$

The action for the closed-strings fermions will not be used here, but the combined action is invariant under  $\mathcal{N}_{10} = (2, 0)$  supersymmetry under which the fermions transform as

$$\delta_{\hat{\epsilon}}\hat{\chi} = \mathcal{O}\hat{\epsilon}, \quad \delta_{\hat{\epsilon}}\hat{\Psi}_M = \hat{\mathcal{D}}_M\hat{\epsilon}, \quad (\text{A.6})$$

in which  $\hat{\epsilon}$  is a double right-handed Majorana-Weyl spinor and

$$\hat{\mathcal{O}} = \frac{1}{2}\hat{\not{\partial}}\phi - \frac{1}{2}e^\phi\hat{\not{F}}_{(1)}i\sigma^2 - \frac{1}{4}e^{\phi/2}\mathcal{G}^-, \quad (\text{A.7a})$$

$$\hat{\mathcal{D}}_M = \hat{\nabla}_M + \frac{1}{4}e^\phi\partial_M C_{(0)}i\sigma^2 + \frac{1}{8}e^{\phi/2}\left(\mathcal{G}^+\hat{\Gamma}_M + \frac{1}{2}\hat{\Gamma}_M\mathcal{G}^+\right) + \frac{1}{16}\hat{\not{F}}_{(5)}\hat{\Gamma}_Mi\sigma^2, \quad (\text{A.7b})$$

in which

$$\mathcal{G}^\pm = \hat{\not{F}}_{(3)}\sigma^1 \pm e^{-\phi}\hat{\not{H}}_{(3)}\sigma^3. \quad (\text{A.8})$$

For a  $p$ -form,

$$\hat{\not{\mathcal{F}}}_{(p)} := \frac{1}{p!}\Omega_{M_1\dots M_p}\hat{\Gamma}^{M_1\dots M_p}, \quad (\text{A.9})$$

in which

$$\hat{\Gamma}^{M_1\dots M_p} = \hat{\Gamma}^{[M_1\dots M_p]}, \quad (\text{A.10})$$

where  $[\dots]$  denotes averaging over signed permutations, e.g.,

$$X^{(MPQ)} = \frac{1}{3!}\left(X^{MPQ} + X^{MQP} + \dots\right), \quad X^{[MPQ]} = \frac{1}{3!}\left(X^{MPQ} - X^{MQP} + \dots\right). \quad (\text{A.11})$$

Unless otherwise noted,  $\Gamma$ -matrices on double spinors as

$$\hat{\Gamma}_M\hat{\epsilon} = \begin{pmatrix} \hat{\Gamma}_M\hat{\epsilon}^1 \\ \hat{\Gamma}_M\hat{\epsilon}^2 \end{pmatrix}. \quad (\text{A.12})$$

The Pauli matrices appearing in (A.7) act on the so-called extension space. For example,

$$\sigma^1 \begin{pmatrix} \hat{\epsilon}^1 \\ \hat{\epsilon}^2 \end{pmatrix} = \begin{pmatrix} \hat{\epsilon}^2 \\ \hat{\epsilon}^1 \end{pmatrix}. \quad (\text{A.13})$$

$\hat{\nabla}$  is the covariant derivative defined by

$$\hat{\nabla}_M = \partial_M + \frac{1}{4}\hat{\omega}_M^{\underline{NP}}\hat{\Gamma}_{\underline{NP}}, \quad (\text{A.14a})$$

where  $\hat{\omega}_M^{\underline{NP}}$  are the components of the spin connection

$$\hat{\omega}_M^{\underline{NP}} = \frac{1}{2}\hat{e}_M^{\underline{Q}}\left(\hat{T}_{\underline{Q}}^{\underline{NP}} - \hat{T}^{\underline{NP}}_{\underline{Q}} - \hat{T}^{\underline{P}}_{\underline{Q}}^{\underline{N}}\right), \quad \hat{T}_{\underline{NP}}^{\underline{M}} = \left(\hat{e}^{\underline{Q}}_{\underline{N}}\hat{e}^{\underline{R}}_{\underline{P}} - \hat{e}^{\underline{Q}}_{\underline{P}}\hat{e}^{\underline{R}}_{\underline{N}}\right)\partial_{\underline{R}}\hat{e}_Q^{\underline{M}}, \quad (\text{A.14b})$$



in which  $\hat{e}_M^N$  are the vielbein defining the local frame  $\hat{e}^N = \hat{e}_M^N dx^M$  and  $\hat{e}^M_N$  are the inverse vielbein.

For the  $\hat{\Gamma}$ -matrices, we choose a basis that is useful for the decomposition  $\text{SO}(9,1) \rightarrow \text{SO}(3,1) \times \text{SO}(6)$ . In 3 + 1 dimensions, we take in a local frame

$$\gamma^\mu = \begin{pmatrix} 0 & -\bar{\sigma}^\mu \\ \sigma^\mu & 0 \end{pmatrix}, \tag{A.15}$$

in which

$$\sigma^\mu = (1, \boldsymbol{\sigma}), \quad \bar{\sigma}^\mu = (1, -\boldsymbol{\sigma}). \tag{A.16}$$

where  $\boldsymbol{\sigma}$  are the usual Pauli matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.17}$$

The  $\gamma$ -matrices then satisfy

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}. \tag{A.18}$$

The 4d chirality operator is

$$\gamma_{(4)} = \frac{i}{4!} \varepsilon_{\mu_1 \dots \mu_4} \gamma^{\mu_1 \dots \mu_4} = \begin{pmatrix} \mathbb{I}_2 & 0 \\ 0 & -\mathbb{I}_2 \end{pmatrix}, \tag{A.19}$$

in which  $\varepsilon_{0123} = +\sqrt{\det(g_{\mu\nu})}$ . Since  $\gamma^2$  is the only imaginary  $\gamma$ -matrix, the 4d Majorana matrix is

$$B_4 = \gamma_{(4)} \gamma^2 = \begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix}, \tag{A.20}$$

and satisfies  $B_4 B_4^* = \mathbb{I}_4$ ,  $\gamma^\mu B_4 = B_4 \gamma^{\mu*}$ , and  $\gamma_{(4)} B_4 = -B_4 \gamma_{(4)}^*$ .

We will make use the dotted, undotted notation of [100] and write a 4d Dirac spinor as

$$\psi = \begin{pmatrix} i\bar{\psi}_{\dot{R}\alpha} \\ \psi_{L\alpha} \end{pmatrix}, \tag{A.21}$$

where we raise and lower indices with  $\epsilon^{12} = \epsilon_{21} = 1$ .

In 6 dimensions, we take in an orthonormal frame<sup>26</sup>

$$\begin{aligned} \tilde{\gamma}^1 &= \sigma^1 \otimes \mathbb{I}_2 \otimes \mathbb{I}_2, & \tilde{\gamma}^4 &= \sigma^2 \otimes \mathbb{I}_2 \otimes \mathbb{I}_2, \\ \tilde{\gamma}^2 &= \sigma^3 \otimes \sigma^1 \otimes \mathbb{I}_2, & \tilde{\gamma}^5 &= \sigma^3 \otimes \sigma^2 \otimes \mathbb{I}_2, \\ \tilde{\gamma}^3 &= \sigma^3 \otimes \sigma^3 \otimes \sigma^1, & \tilde{\gamma}^6 &= \sigma^3 \otimes \sigma^3 \otimes \sigma^2. \end{aligned} \tag{A.22}$$

They satisfy

$$\{\tilde{\gamma}^m, \tilde{\gamma}^n\} = 2\delta^{mn}. \tag{A.23}$$

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<sup>26</sup>In the main text, we drop the  $\tilde{\cdot}$  appearing above these  $\gamma$ -matrices since context should hopefully make clear whether we mean  $\text{SO}(3,1)$  or  $\text{SO}(6)$   $\gamma$ -matrices.

The 6d chirality operator is then

$$\tilde{\gamma}_{(6)} = -\frac{i}{6!} \tilde{\varepsilon}_{m_1 \dots m_6} \tilde{\gamma}^{m_1 \dots m_6} = \sigma_3 \otimes \sigma_3 \otimes \sigma_3. \quad (\text{A.24})$$

The 6d Majorana matrix is

$$\tilde{B}_6 = \tilde{\gamma}^4 \tilde{\gamma}^5 \tilde{\gamma}^6 = i \sigma^2 \otimes \sigma^1 \otimes \sigma^2. \quad (\text{A.25})$$

It satisfies  $\tilde{B}_6 \tilde{B}_6^* = \mathbb{I}_8$ ,  $\tilde{\gamma}^m \tilde{B}_6 = -\tilde{B}_6 \tilde{\gamma}^{m*}$ , and  $\tilde{\gamma}_{(6)} \tilde{B}_6 = -\tilde{B}_6 \tilde{\gamma}_{(6)}^*$ .

From these, we define the 10d  $\Gamma$ -matrices,

$$\hat{\Gamma}^\mu = \gamma^\mu \otimes \mathbb{I}_8, \quad \hat{\Gamma}^m = \gamma_{(4)} \otimes \tilde{\gamma}^m, \quad (\text{A.26})$$

where the second equality should be read as  $\hat{\Gamma}^4 = \gamma_{(4)} \otimes \tilde{\gamma}^1$ , etc. They satisfy

$$\{\hat{\Gamma}^M, \hat{\Gamma}^N\} = 2\hat{\eta}^{MN}. \quad (\text{A.27})$$

The 10d chirality operator is then

$$\Gamma_{(10)} = \frac{1}{10!} \hat{\varepsilon}_{M_1 \dots M_{10}} \hat{\Gamma}^{M_1 \dots M_{10}} = \gamma_{(4)} \otimes \tilde{\gamma}_{(6)}. \quad (\text{A.28})$$

The 10d Majorana matrix

$$\hat{B}_{10} = \hat{\Gamma}^2 \hat{\Gamma}^7 \hat{\Gamma}^8 \hat{\Gamma}^9 = -B_4 \otimes \tilde{B}_6, \quad (\text{A.29})$$

satisfies  $\hat{B}_{10} \hat{B}_{10}^* = \mathbb{I}_{32}$ ,  $\hat{\Gamma}^M \hat{B}_{10} = \hat{B}_{10} \hat{\Gamma}_M^*$ , and  $\Gamma_{(10)} \hat{B}_{10} = \hat{B}_{10} \Gamma_{(10)}^*$ .

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