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Soft-Decision Metrics for Coded Orthogonal Signaling in Symmetric Alpha-Stable Noise

Michael R. Souryal, *Member, IEEE*, Erik G. Larsson, *Member, IEEE*, Bojan Peric, and Branimir R. Vojcic, *Senior Member, IEEE*

***Abstract*—This paper derives new soft-decision metrics for coded orthogonal signaling in impulsive noise, more specifically symmetric α-stable noise. For the case of a known channel amplitude and known noise dispersion, exact metrics are derived both for Cauchy and Gaussian noise. For the case that the channel amplitude or the dispersion is unknown, approximate metrics are obtained in closed-form based on a generalized-likelihood ratio approach. The performance of the new metrics is compared numerically for a turbo-coded system, and the sensitivity to side information of the optimum receiver for Cauchy noise is considered. The gain that can be achieved by using a properly chosen decoding metric is quantified, and it is shown that this gain is significant. The application of the results to frequency hopping ad hoc networks is also discussed.**

***Index Terms*—Cauchy, generalized likelihood ratio, impulsive noise, stable distribution.**

I. Introduction

WHILE noise in communication links is often modeled as a Gaussian process, many systems are subject to noise with an impulsive (heavy-tailed) character. One way of modeling impulsive noise is via the general class of symmetric α-stable (SαS) distributions, of which the Gaussian distribution is a special case. An important example of an environment with impulsive noise, and where the noise is well modeled by an SαS distribution, is a wireless ad hoc network with a Poisson field of co-channel users [1]. In this case, the characteristics of the noise are directly related to the path loss exponent and the spatial density of interferers; see Section V.

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The motivation for our work is that communication systems designed for Gaussian noise can perform very poorly in impulsive noise. We address this problem by deriving optimum and suboptimum receivers (more precisely, soft-decision metrics) for systems that use coded M-ary orthogonal signaling and which experience SαS noise for two special cases—$\alpha = 1$ (Cauchy noise) and $\alpha = 2$ (Gaussian noise)—and show that some of these receivers perform well for a wide range of α. The metrics we derive vary in complexity and in the amount of side information they require; also, in some cases we resort to approximations in order to find closed-form solutions. While previous work on noncoherent receivers in SαS noise addressed only uncoded systems [2], [3], our work applies to coded systems using soft-decision decoding. Numerical results are given for a turbo coded system.

Section II describes the system model for the analysis. Section III derives the optimum soft-decision metrics for Gaussian and Cauchy ($\alpha = 1$) noise, as well as simpler metrics requiring less side information using the generalized likelihood ratio paradigm. Section IV discusses numerical results, including an analysis of the sensitivity to side information. The application of these results to frequency hopping ad hoc networks is discussed in Section V, and conclusions are summarized in Section VI.

This paper is an extension of our work previously published in [4] with additional contributions as follows: we i) provide a sensitivity analysis of the Cauchy metric to dispersion and amplitude estimation errors, ii) evaluate the performance of the Cauchy receiver with a practical amplitude estimator, iii) give numerical results for nonbinary signals, and iv) discuss the application of our results to frequency hopping ad hoc networks.

II. System Model

Referring to Fig. 1, encoded bits are mapped $\log_2 M$ bits at a time onto one of M orthogonal waveforms, such as M-ary frequency-shift keying (FSK) signals, and transmitted over a channel that injects additive noise. The received waveform is correlated, in-phase and quadrature, with each one of the M possible transmitted signals. The output of the ith correlator, $0 \le i \le M-1$, is modeled as

$$\mathbf{z}_i = a_i \mathbf{s}_i + \mathbf{y}_i \tag{1}$$

where all vectors are two-dimensional, representing the in-phase and quadrature components. In (1), the following is held.

- a_i is the amplitude of the received signal.

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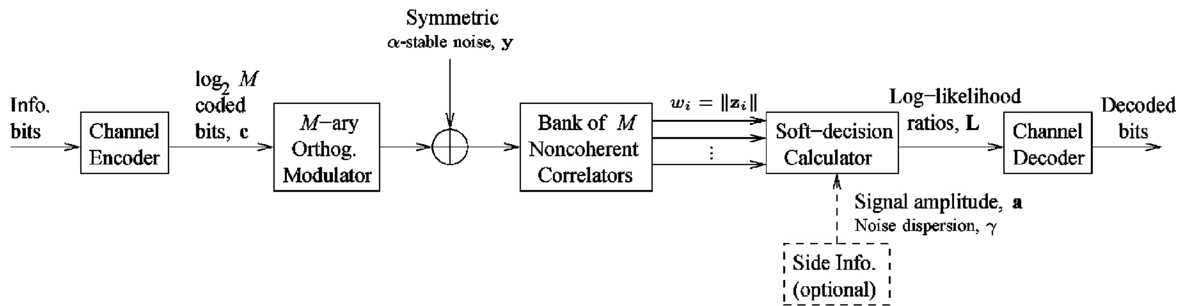

Fig. 1. System model.

- $\mathbf{s}_i$ is the desired signal component associated with the ith waveform (correlator). Assuming that the i'th waveform was transmitted, $\mathbf{s}_i$ may be expressed as

$$\mathbf{s}_i = \begin{cases} [\cos\theta_i \quad \sin\theta_i] & ; i = i' \\ 0 & ; i \neq i' \end{cases}$$

where θ_i is the relative phase of the signal. This model allows the amplitude and phase to be different for different i, which would be the case if M-FSK modulation is used on a frequency-selective channel, for example.

- $\mathbf{y}_i$ is the additive noise at the output of the ith correlator. We shall assume that $\mathbf{y}_i$, $0 \le i \le M-1$, are independent identically distributed (i.i.d.) SαS. This is the case if the noise results from independent Poisson field processes [1], [3], for example.

An SαS random vector has characteristic function

$$\Phi(\boldsymbol{\omega}) = \exp\left(-\gamma \|\boldsymbol{\omega}\|^{\alpha}\right)$$

where the *index of stability* α satisfies $0 < \alpha \le 2$, and the *dispersion* $\gamma > 0$ [5]. The smaller α is, the heaver tail the density function has and the more impulsive the noise is. Closed forms for the density of an SαS random vector exist only for the cases of $\alpha = 1$ and $\alpha = 2$, which correspond to the Cauchy ($\alpha = 1$) and Gaussian ($\alpha = 2$) distributions, respectively. In the Gaussian case, the variance σ^2 of each component is related to the dispersion through $\sigma^2 = 2\gamma$. Note that for $\alpha < 2$, the second-order moment (variance) of an SαS random variable does not exist; this is one reason why SαS noise can present problems in communication systems.

Soft decisions of coded symbols are generated by the demodulator, and passed to the decoder in the form of log-likelihood ratios (LLRs). The LLR of the jth coded bit c_j is defined as

$$L_j(\mathbf{z}, \mathbf{a}, \boldsymbol{\theta}) \triangleq \log\left(\frac{\Pr[c_j = 1|\mathbf{z}, \mathbf{a}, \boldsymbol{\theta}]}{\Pr[c_j = 0|\mathbf{z}, \mathbf{a}, \boldsymbol{\theta}]}\right) \quad (2)$$

where $\mathbf{z} = [\mathbf{z}_0 \quad \mathbf{z}_1 \quad \ldots \quad \mathbf{z}_{M-1}]$ is a vector of the outputs of all M in-phase and quadrature correlators, and likewise, $\mathbf{a} = [a_0 \quad a_1 \quad \ldots \quad a_{M-1}]$ and $\boldsymbol{\theta} = [\theta_0 \quad \theta_1 \quad \ldots \quad \theta_{M-1}]$. Since each transmitted signal represents a length-$\log_2 M$ sequence of coded bits $\mathbf{c}$, (2) can be expressed as

$$L_j(\mathbf{z}, \mathbf{a}, \boldsymbol{\theta}) = \log\left(\frac{\sum_{\mathbf{c}:c_j=1} p(\mathbf{c}|\mathbf{z}, \mathbf{a}, \boldsymbol{\theta})}{\sum_{\mathbf{c}:c_j=0} p(\mathbf{c}|\mathbf{z}, \mathbf{a}, \boldsymbol{\theta})}\right) \quad (3)$$

where $p(\mathbf{c})$ is the probability that sequence $\mathbf{c}$ was transmitted. Using Bayes' rule and assuming all coded sequences are equiprobable,[1] (3) can be written as

$$L_j(\mathbf{z}, \mathbf{a}, \boldsymbol{\theta}) = \log\left(\frac{\sum_{\mathbf{c}:c_j=1} f(\mathbf{z}|\mathbf{c}, \mathbf{a}, \boldsymbol{\theta})}{\sum_{\mathbf{c}:c_j=0} f(\mathbf{z}|\mathbf{c}, \mathbf{a}, \boldsymbol{\theta})}\right) \quad (4)$$

where $f(\mathbf{z}|\mathbf{c}, \mathbf{a}, \boldsymbol{\theta})$ is the conditional density of the correlator outputs

$$f(\mathbf{z}|\mathbf{c}, \mathbf{a}, \boldsymbol{\theta}) = f_{\mathbf{y}}(\mathbf{z}_i - a_i \mathbf{s}_i(\theta_i)) \prod_{k \neq i} f_{\mathbf{y}}(\mathbf{z}_k); \mathbf{c} \to i. \quad (5)$$

Here, the notation $\mathbf{c} \to i$ means that the coded bit sequence $\mathbf{c}$ results in transmission of the ith signal.

We shall assume throughout that the receiver does not have phase information, so the components of $\boldsymbol{\theta}$ are i.i.d., uniform on $(0, 2\pi]$.

III. Decision Metrics

A. *Decision Metrics for Known* $\mathbf{a}, \gamma$

To detect i without knowing the phase $\boldsymbol{\theta}$ one must average the conditional density (5) over $\boldsymbol{\theta}$. By doing so, we can obtain optimum decision metrics for Gaussian and Cauchy noise in closed form. Since the averaging over $\boldsymbol{\theta}$ performed here explicitly relies on the *a priori* assumption on $p(\boldsymbol{\theta})$, we call the resulting decoding metrics "Bayesian metrics."

[1]This assumption can be relaxed by inserting $p(\mathbf{c})$ in (4).

1) Gaussian Noise ($\alpha = 2$): Under the assumption of S2S noise, the random vectors $\mathbf{y}_i, 0 \le i \le M-1$, are i.i.d. bivariate Gaussian with density

$$f_{\mathbf{y}}(\mathbf{y}) = \frac{1}{4\pi\gamma} \exp\left(-\frac{\|\mathbf{y}\|^2}{4\gamma}\right). \quad (6)$$

Using (6) in (5), the conditional density of $\mathbf{z}$ is

$$\begin{aligned} f(\mathbf{z}|\mathbf{c},\mathbf{a},\boldsymbol{\theta}) &= \frac{1}{(4\pi\gamma)^M} \exp\left[-\frac{\|\mathbf{z}_i - a_i\mathbf{s}_i(\theta_i)\|^2 + \sum_{k\neq i}\|\mathbf{z}_k\|^2}{4\gamma}\right] \quad (7)\\ &= \frac{1}{(4\pi\gamma)^M} \exp\left[-\frac{a_i^2 - 2a_iw_i\cos(\theta_i - \phi) + \|\mathbf{z}\|^2}{4\gamma}\right] \quad (8)\end{aligned}$$

where in the last line we let w_i be the magnitude of $\mathbf{z}_i$ and ϕ its phase, i.e., $\mathbf{z}_i = w_i[\cos\phi \quad \sin\phi]$.

For noncoherent detection, (8) is averaged over θ_i, giving

$$\begin{aligned} f(\mathbf{z}|\mathbf{c},\mathbf{a}) &= \frac{1}{2\pi}\int_0^{2\pi} f(\mathbf{z}|\mathbf{c},\mathbf{a},\boldsymbol{\theta})\,\mathrm{d}\theta_i\\ &= \frac{\exp\left(-\frac{a_i^2+\|\mathbf{z}\|^2}{4\gamma}\right)}{(4\pi\gamma)^M}\frac{1}{2\pi}\int_0^{2\pi}\exp\left[\frac{a_iw_i\cos(\theta_i-\phi)}{2\gamma}\right]\mathrm{d}\theta_i\\ &= \frac{\exp\left(-\frac{a_i^2+\|\mathbf{z}\|^2}{4\gamma}\right)}{(4\pi\gamma)^M}\mathrm{I}_0\left(\frac{a_iw_i}{2\gamma}\right) \quad (9)\end{aligned}$$

where $\mathrm{I}_0(x) \triangleq (1/2\pi)\int_0^{2\pi} e^{x\cos\theta}\,\mathrm{d}\theta$ is the zeroth-order modified Bessel function of the first kind.

Using (9), and after canceling terms, the Gaussian LLR for noncoherent detection with known amplitudes and dispersion is

$$\begin{aligned} L_j(\mathbf{z},\mathbf{a})|_{\alpha=2,\gamma\text{ known}} &= \log\frac{\sum_{\mathbf{c}:c_j=1} f(\mathbf{z}|\mathbf{c},\mathbf{a})}{\sum_{\mathbf{c}:c_j=0} f(\mathbf{z}|\mathbf{c},\mathbf{a})}\\ &= \log\frac{\sum_{i:c_j=1} e^{-a_i^2/4\gamma}\mathrm{I}_0\left(\frac{a_iw_i}{2\gamma}\right)}{\sum_{i:c_j=0} e^{-a_i^2/4\gamma}\mathrm{I}_0\left(\frac{a_iw_i}{2\gamma}\right)} \quad (10)\end{aligned}$$

where in the last line the summations are over all $M/2$ signals to which are mapped coded sequences for which $c_j = 1$ and $c_j = 0$, respectively.

2) Cauchy Noise ($\alpha = 1$): Under the assumption of Cauchy (i.e., S1S) noise, the density of the noise vector is

$$f_{\mathbf{y}}(\mathbf{y}) = \frac{\gamma}{2\pi(\gamma^2 + \|\mathbf{y}\|^2)^{3/2}}. \quad (11)$$

The conditional density of $\mathbf{z}$ averaged over θ_i is

$$f(\mathbf{z}|\mathbf{c},\mathbf{a}) = \mathrm{E}_{\theta_i}[f_{\mathbf{y}}(\mathbf{z}_i - a_i\mathbf{s}_i(\theta_i))]\prod_{k\neq i} f_{\mathbf{y}}(\mathbf{z}_k). \quad (12)$$

The expectation in (12) evaluates to

$$\begin{aligned} \mathrm{E}_{\theta_i}&[f_{\mathbf{y}}(\mathbf{z}_i - a_i\mathbf{s}_i(\theta_i))]\\ &= \frac{1}{2\pi}\int_0^{2\pi}\frac{\gamma\,\mathrm{d}\theta_i}{2\pi[\gamma^2 + w_i^2 + a_i^2 - 2a_iw_i\cos(\theta_i - \phi)]^{3/2}}\\ &= \frac{\gamma}{\pi^2(\beta_i - \delta_i)\sqrt{\beta_i + \delta_i}}\mathrm{E}\left(\sqrt{\frac{2\delta_i}{\beta_i+\delta_i}}\right) \quad (13)\end{aligned}$$

where $\mathrm{E}(k) \triangleq \int_0^{\pi/2}\sqrt{1 - k^2\sin^2\phi}\,\mathrm{d}\phi$ is the complete elliptic integral of the second kind, $\beta_i = \gamma^2 + w_i^2 + a_i^2$, $\delta_i = 2a_iw_i$, and where [6, (2.575.4)] was used to obtain (13).[2]

Using (12) together with (11) and (13), and after simplifying, the Cauchy LLR for noncoherent detection with known amplitudes and dispersion is

$$L_j(\mathbf{z},\mathbf{a})|_{\alpha=1,\gamma\text{ known}} = \log\frac{\sum_{i:c_j=1} g(w_i,a_i)}{\sum_{i:c_j=0} g(w_i,a_i)} \quad (14)$$

where

$$g(w_i,a_i) = \mathrm{E}\left(\sqrt{\frac{2\delta_i}{\beta_i+\delta_i}}\right)\frac{(\gamma^2+w_i^2)^{3/2}}{(\beta_i-\delta_i)\sqrt{\beta_i+\delta_i}}.$$

B. Decision Metrics for Known γ but Unknown $\mathbf{a}$

In practice, the amplitudes $\mathbf{a}$ are typically unknown to the receiver. These amplitudes are also generally difficult to estimate accurately in practice, for example if the system is frequency hopping, and also due to the impulsive nature of the noise. If $\mathbf{a}$ is unknown, one would like to average $f(\mathbf{z}|\mathbf{c},\mathbf{a},\boldsymbol{\theta})$ over both $\mathbf{a}$ and $\boldsymbol{\theta}$. There are two difficulties with this, at least. First, the distribution of $p(\mathbf{a})$ may not be known: Knowing $p(\mathbf{a})$ means quantitatively knowing the fading statistics, and this is difficult to estimate in the environment we consider since only one, or possibly a few, observations of each fading realization is available, and since the noise is heavy-tailed. Second, averaging $f(\mathbf{z}|\mathbf{c},\mathbf{a},\boldsymbol{\theta})$ does not seem possible in closed form even for "simple" prior densities $p(\mathbf{a})$. As a remedy, we propose to approximate the averaged density $f(\mathbf{z}|\mathbf{c})$ with $f\left(\mathbf{z}|\mathbf{c},\hat{\mathbf{a}},\hat{\boldsymbol{\theta}}\right)$ where $\hat{\mathbf{a}},\hat{\boldsymbol{\theta}}$ are the instantaneous maximum-likelihood estimates of $\mathbf{a},\boldsymbol{\theta}$. Equivalently, $f(\mathbf{z}|\mathbf{c})$ is simply approximated by jointly maximizing it with respect to $\hat{\mathbf{a}},\hat{\boldsymbol{\theta}}$. The resulting detector is effectively a generalized-likelihood ratio (GLR) test, and therefore we call the metrics derived here "GLR" metrics. Though GLR tests are more common in detection theory [7], they have been used before with success in communication theory [8]–[10].

1) Gaussian Noise ($\alpha = 2$): Rewriting (7), and explicitly noting the dependence on γ, we have that

$$f(\mathbf{z}|\mathbf{c},\gamma,\mathbf{a},\boldsymbol{\theta}) = \frac{1}{(4\pi\gamma)^M}\exp\left(-\frac{\|\mathbf{z}_i - a_i\mathbf{s}_i(\theta_i)\|^2}{4\gamma}\right) \times \exp\left(-\sum_{k=0,k\neq i}^{M-1}\frac{w_k^2}{4\gamma}\right). \quad (15)$$

[2]A previously published evaluation of this integral in [2], and later used again in [3], contained an error. The square-root in the denominator of (7) in [2] should instead be $(\cdot)^{3/2}$, which can then be shown to be equivalent to (13) above.

Jointly maximizing (15) with respect to $\mathbf{a}$ and $\boldsymbol{\theta}$ gives

$$\hat{f}(\mathbf{z}|\mathbf{c},\gamma) \triangleq \max_{\mathbf{a},\boldsymbol{\theta}} f(\mathbf{z}|\mathbf{c},\gamma,\mathbf{a},\boldsymbol{\theta}) = \frac{1}{(4\pi\gamma)^M} \exp\left(-\sum_{k=0,k\neq i}^{M-1} \frac{w_k^2}{4\gamma}\right). \quad (16)$$

(We use $\hat{(\cdot)}$ to denote the likelihood after maximizing it with respect to $\mathbf{a},\boldsymbol{\theta}$. Such a maximized likelihood is sometimes called a *profile likelihood* or *concentrated likelihood*.) The resulting metric after simplifying is

$$L_j(\mathbf{z})|_{\text{GLR},\alpha=2,\gamma \text{ known}} = \log \frac{\sum_{i:c_j=1} e^{w_i^2/4\gamma}}{\sum_{i:c_j=0} e^{w_i^2/4\gamma}}. \quad (17)$$

For biorthogonal signaling ($M=2$), (17) reduces to $L_j(\mathbf{z}) = (w_1^2 - w_0^2)/(4\gamma)$.

2) Cauchy Noise ($\alpha = 1$): For $\alpha = 1$, we have from (11) that

$$f(\mathbf{z}|\mathbf{c},\gamma,\mathbf{a},\boldsymbol{\theta}) = \frac{\gamma}{2\pi(\gamma^2 + \|\mathbf{z}_i - a_i\mathbf{s}_i(\theta_i)\|^2)^{3/2}} \times \prod_{k=0,k\neq i}^{M-1} \frac{\gamma}{2\pi(\gamma^2+\|\mathbf{z}_k\|^2)^{3/2}}. \quad (18)$$

Maximizing (18) with respect to $\mathbf{a}$ and $\boldsymbol{\theta}$ yields

$$\hat{f}(\mathbf{z}|\mathbf{c},\gamma) = \frac{1}{2\pi\gamma^2} \prod_{k=0,k\neq i}^{M-1} \frac{\gamma}{2\pi(\gamma^2+w_k^2)^{3/2}} \quad (19)$$

and the resulting metric after simplification is

$$L_j(\mathbf{z})|_{\text{GLR},\alpha=1,\gamma \text{ known}} = \log \frac{\sum_{i:c_j=1} \left(\gamma^2+w_i^2\right)^{3/2}}{\sum_{i:c_j=0} \left(\gamma^2+w_i^2\right)^{3/2}}. \quad (20)$$

For $M=2$, (20) reduces to $L_j(\mathbf{z}) = (3/2)\log\left((\gamma^2+w_1^2)/(\gamma^2+w_0^2)\right)$.

It should be noted that the computational complexity of evaluating (17) and (20) is much lower than that of evaluating (10) and (14); however, the expressions for the metrics could be implemented via table lookups—in which case there would be no difference.

C. *Metrics for Unknown* $\mathbf{a}$ *and* γ

If both $\mathbf{a}$ and γ are unknown, then one must eliminate $\mathbf{a},\boldsymbol{\theta}$, and γ from $f(\mathbf{z}|\mathbf{c},\gamma,\mathbf{a},\boldsymbol{\theta})$. Like in Section III-B, we shall take a GLR test approach to this.

1) Gaussian Noise ($\alpha = 2$): Consider the profile likelihood (16). By maximizing this likelihood with respect to γ, we obtain

$$\hat{f}(\mathbf{z}|\mathbf{c}) \triangleq \max_{\gamma} \hat{f}(\mathbf{z}|\mathbf{c},\gamma) = \frac{e^{-M}}{\pi^M} \frac{1}{\left(\frac{1}{M}\sum_{k=0,k\neq i}^{M-1} w_k^2\right)^M}$$

and the associated bit metric

$$L_j(\mathbf{z})|_{\text{GLR},\alpha=2,\gamma \text{ unknown}} = \log \frac{\sum_{i:c_j=1}\left(\sum_{k=0,k\neq i}^{M-1} w_k^2\right)^{-M}}{\sum_{i:c_j=0}\left(\sum_{k=0,k\neq i}^{M-1} w_k^2\right)^{-M}}. \quad (21)$$

For $M=2$, (21) simplifies to $L_j(\mathbf{z}) = 2\log(w_1^2/w_0^2)$.

2) Cauchy Noise ($\alpha = 1$): Unfortunately, for Cauchy noise, the profile likelihood (19) is monotonically decreasing as γ increases for $M=2$, and the GLR metric for unknown γ does not exist in this case.

D. *Summary*

Table I summarizes the derived metrics. The general expression for the LLR in each case is obtained by substituting the fourth column (g_i) in

$$L_j = \log \frac{\sum_{i:c_j=1} g_i}{\sum_{i:c_j=0} g_i}.$$

IV. Quantitative Results

A. *Performance Comparison of Metrics*

Numerical performance results for the different metrics are obtained through Monte Carlo simulation of a coded binary FSK system in SαS noise. Information bits are encoded with a rate-1/2 binary parallel concatenated convolutional (turbo) code with interleaver size of 1024 bits and constituent encoder constraint length of four. Coded bits are mapped to binary FSK channel symbols with unit amplitude. After noncoherent detection and LLR computation, decoding is performed iteratively by a pair of soft-input/soft-output MAP decoders and is terminated after eight iterations. Generation of the bivariate SαS noise is straightforward for $\alpha = 1$ and 2. For other values of α, we approximate the noise with the first 100 terms in a bivariate version of the series representation of an SαS random variable given in [5, Theorem 1.4.2].

1) Performance in Gaussian Noise: Fig. 2 compares the performance of the metrics developed in Section III on an additive white Gaussian noise (AWGN) channel ($\alpha = 2$) in terms of bit error rate (BER) versus the inverse of the dispersion, $1/\gamma$, in decibels. Recall that in AWGN, $1/\gamma$ is proportional to the conventional signal-to-noise ratio (SNR). As expected, the optimal receiver for Gaussian noise which uses knowledge of the signal amplitudes and noise dispersion performs best, as it is perfectly matched to the noise distribution. The Bayesian Cauchy metric performs within 1 dB of the optimum Gaussian metric; that is, the penalty for assuming that the noise is impulsive when it is in fact not, is small. Also, the GLR metrics perform within 0.5 dB of the optimum Gaussian metric. It is notable that the GLR metrics achieve this performance without knowledge of the amplitudes (and, in one case, without knowing the dispersion).

2) Performance in Cauchy Noise: Fig. 3 compares performance on a Cauchy noise ($\alpha = 1$) channel. Here, as expected,

TABLE I
SUMMARY OF SOFT-DECISION METRICS

Side Info.	Strategy	Noise Assumption	g_i	LLR for $M = 2$
$\mathbf{a}$, γ	Bayesian (optimal)	Gaussian	$e^{-a_i^2/4\gamma}\, \mathrm{I}_0\left(\frac{a_i w_i}{2\gamma}\right)$	$\frac{a_0^2-a_1^2}{4\gamma} + \log \frac{\mathrm{I}_0\left(\frac{a_1 w_1}{2\gamma}\right)}{\mathrm{I}_0\left(\frac{a_0 w_0}{2\gamma}\right)}$
	Bayesian (optimal)	Cauchy*	$\mathrm{E}\left(\sqrt{\frac{2\delta_i}{\beta_i+\delta_i}}\right) \frac{(\gamma^2+w_i^2)^{3/2}}{(\beta_i-\delta_i)\sqrt{\beta_i+\delta_i}}$	$\frac{3}{2}\log\frac{\gamma^2+w_1^2}{\gamma^2+w_0^2} + \log \frac{(\beta_0-\delta_0)\sqrt{\beta_0+\delta_0}\,\mathrm{E}\left(\sqrt{\frac{2\delta_1}{\beta_1+\delta_1}}\right)}{(\beta_1-\delta_1)\sqrt{\beta_1+\delta_1}\,\mathrm{E}\left(\sqrt{\frac{2\delta_0}{\beta_0+\delta_0}}\right)}$
γ	GLR (approx.)	Gaussian	$e^{w_i^2/4\gamma}$	$\frac{w_1^2-w_0^2}{4\gamma}$
	GLR (approx.)	Cauchy	$\left(\gamma^2+w_i^2\right)^{3/2}$	$\frac{3}{2}\log\frac{\gamma^2+w_1^2}{\gamma^2+w_0^2}$
None	GLR (approx.)	Gaussian	$\left(\sum_{k=0,k\neq i}^{M-1} w_k^2\right)^{-M}$	$2\log\frac{w_1^2}{w_0^2}$

*See Section III-A for definitions of β_i and δ_i.

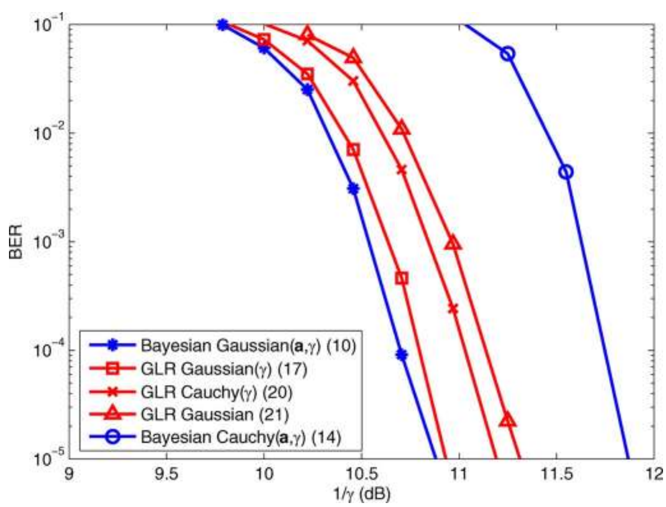

Fig. 2. BER performance of metrics in $\mathrm{S}(\alpha = 2)\mathrm{S}$ (Gaussian) noise with binary ($M = 2$) signals (side information required by each metric, and equations, indicated in ( )'s in legend).

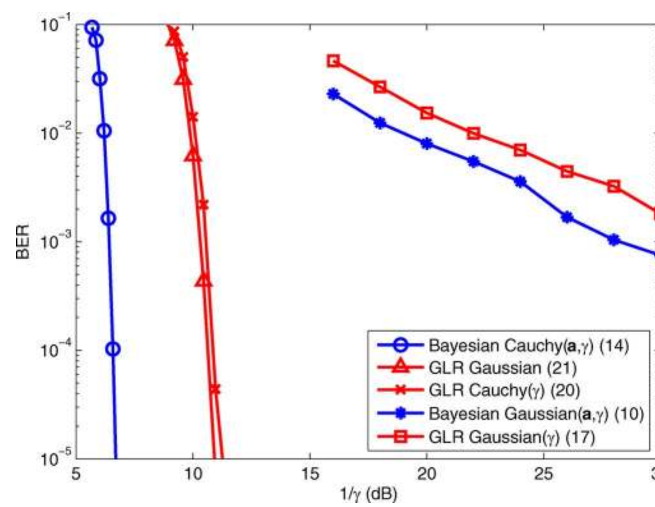

Fig. 3. BER performance of metrics in $\mathrm{S}(\alpha = 1)\mathrm{S}$ (Cauchy) noise; $M = 2$.

the Bayesian Cauchy metric outperforms the other metrics, and by much larger margins than those in the Gaussian channel (more than 4 dB here). Interestingly, the GLR Gaussian metric performs very well even in this impulsive noise. The surprisingly good performance of the GLR Gaussian metric for unknown γ can be understood from the sub-Gaussian nature of $\mathrm{S}\alpha\mathrm{S}$ variates, which informally means that any $\mathrm{S}\alpha\mathrm{S}$ random vector can be viewed as conditionally Gaussian with random variance [5, Sec. 2.5]. The GLR Gaussian detector which is not supplied by γ implicitly chooses the "variance" of the underlying Gaussian noise that maximizes the profile likelihood (16) for every instance of the "variance." Not surprisingly, the GLR Gaussian metric supplied with the "true" γ performs much worse because it treats the noise as simply Gaussian with fixed variance and, thus, is severely mismatched to the actual noise distribution.

3) Performance in Noise With $\alpha = 1.5$ and 0.5: Similar trends are observed on channels with $\mathrm{S}(\alpha = 1.5)\mathrm{S}$ noise (Fig. 4) and $\mathrm{S}(\alpha = 0.5)\mathrm{S}$ noise (Fig. 5), but with smaller and larger margins, respectively, as might be expected in these less and more impulsive noise environments. In general, the Bayesian Cauchy metric shows better performance than the Bayesian Gaussian metric in this type of noise. Furthermore, the GLR Gaussian metric (without knowledge of γ) provides substantial gains relative to the Bayesian Gaussian metric over this range, with no side information and lower complexity than either of the Bayesian metrics.

B. Nonbinary Signaling

Focusing for the remainder on the better performing metrics, Bayesian Cauchy with side information and GLR Gaussian without side information, Fig. 6 shows results with higher order signal sets ($M = 4$ and $M = 8$) in Cauchy noise. We observe that the required increases in the signal-to-noise-dispersion ratio (SNDR, i.e., $1/\gamma$) to accommodate larger signal sets are smaller for the Bayesian Cauchy metric than for the GLR Gaussian metric. In addition, the Bayesian Cauchy metric with 8-FSK signaling exhibits a BER that is comparable to the GLR Gaussian metric with BFSK. In other words, the Bayesian Cauchy metric provides a threefold ($\log_2 8$) increase in data rate at the same

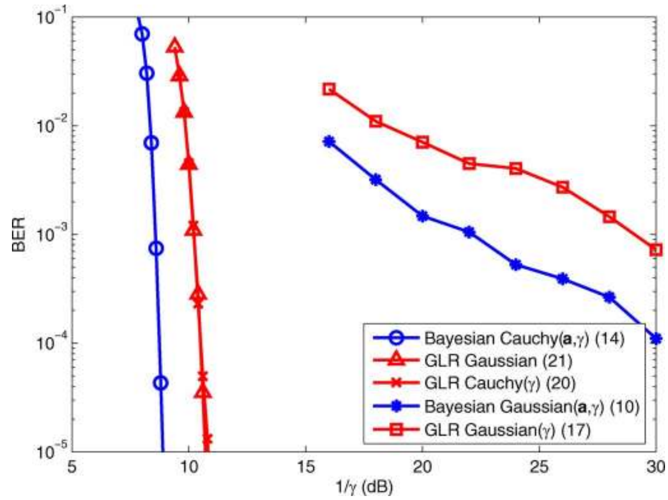

Fig. 4. BER performance of metrics in $\mathrm{S}(\alpha = 1.5)\mathrm{S}$ noise; $M = 2$.

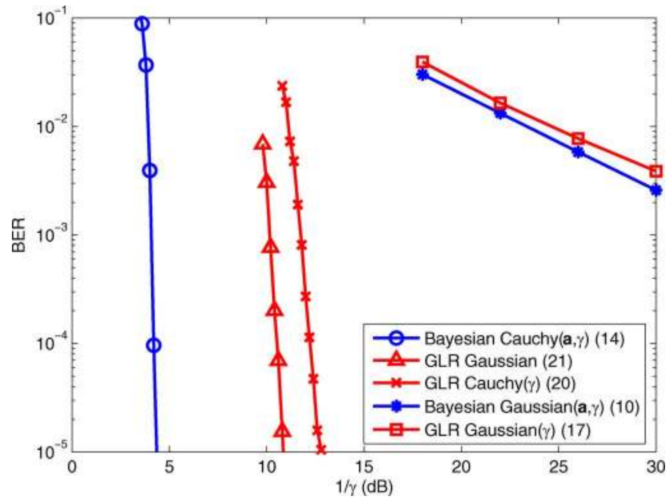

Fig. 5. BER performance of metrics in $\mathrm{S}(\alpha = 0.5)\mathrm{S}$ noise; $M = 2$.

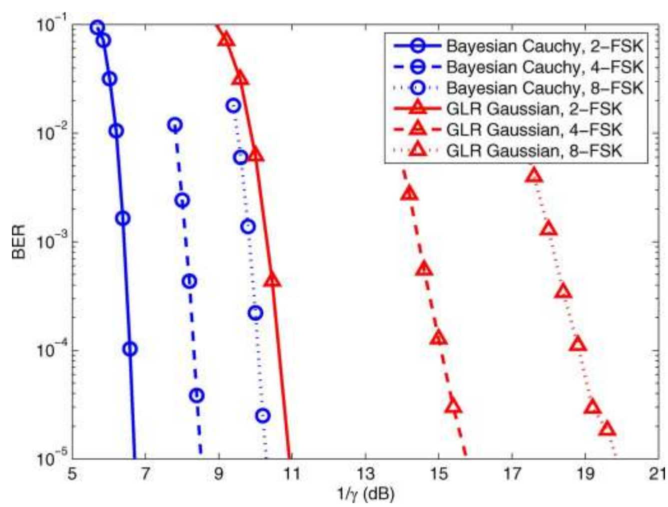

Fig. 6. BER performance of Bayesian Cauchy and GLR Gaussian metrics in Cauchy noise; $M = 2$, 4, 8.

noise dispersion. This improvement can be viewed as the benefit of the side information used by the Bayesian Cauchy metric.[3]

[3]Of course, increasing the FSK constellation size is normally a spectrally inefficient means of increasing throughput, but in certain spread spectrum applications (such as the one described in Section V), spreading bandwidth can be traded for signal bandwidth.

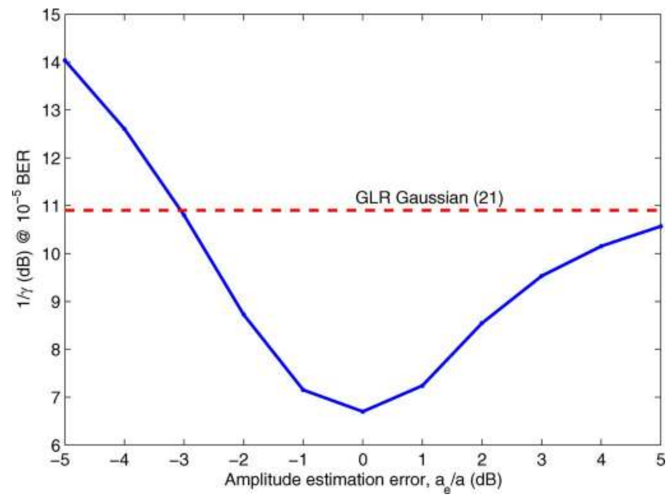

Fig. 7. Sensitivity of Bayesian Cauchy metric to amplitude estimation error; Cauchy noise.

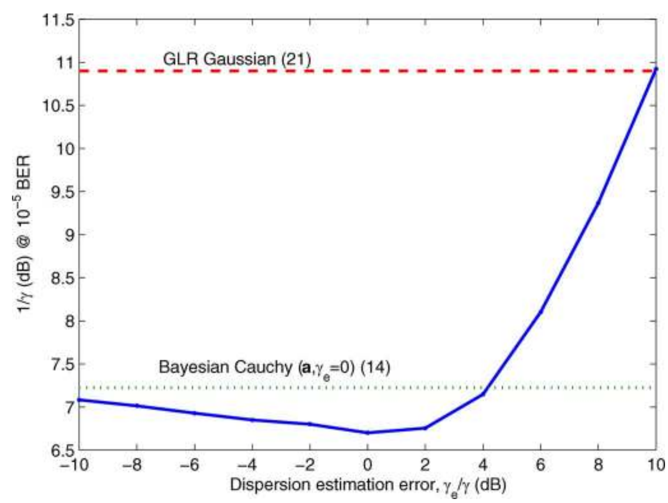

Fig. 8. Sensitivity of Bayesian Cauchy metric to dispersion estimation error; Cauchy noise.

C. Sensitivity of Bayesian Cauchy Metric to Imperfect CSI

In practice, perfect knowledge of the signal amplitude and noise dispersion, which are required by the Bayesian Cauchy metric, is unavailable. To investigate the sensitivity of this metric to imperfect side information, simulations were repeated in which the side information was perturbed by fixed errors. Results are shown in Figs. 7 and 8 as a function of the fixed amplitude and dispersion error, respectively. The performance metric, here, is the required SNDR to achieve a BER of 10^{-5}. In both cases, the noise is Cauchy ($\alpha = 1$).

One observes that the performance of the Cauchy metric is more sensitive to amplitude errors than dispersion errors. Furthermore, it is more sensitive to dispersion over-estimates than under-estimates. In fact, substituting zero for the dispersion parameter when evaluating the metric results in only a 1/2-dB penalty in SNDR (dotted line in Fig. 8). The performance of the GLR Gaussian metric (which requires no side information) is shown in both figures as the dashed line for comparison.

The preceding results suggest that a practical receiver based on the Bayesian Cauchy metric can provide an advantage over the GLR Gaussian metric if the noise dispersion is unknown and

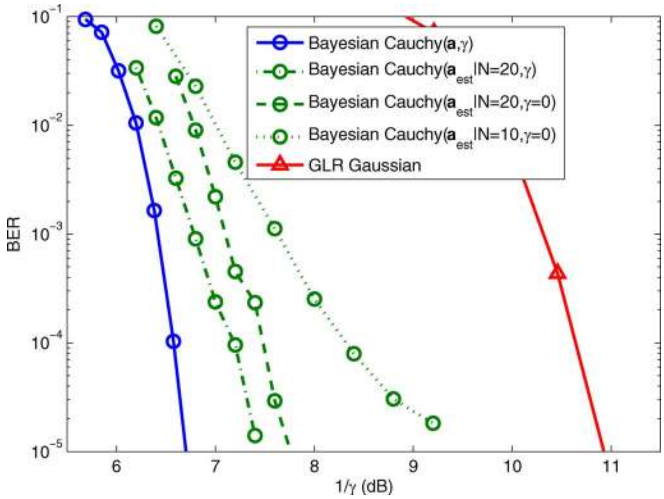

Fig. 9. BER performance of Bayesian Cauchy metric with median amplitude estimator in Cauchy noise; $M = 2$.

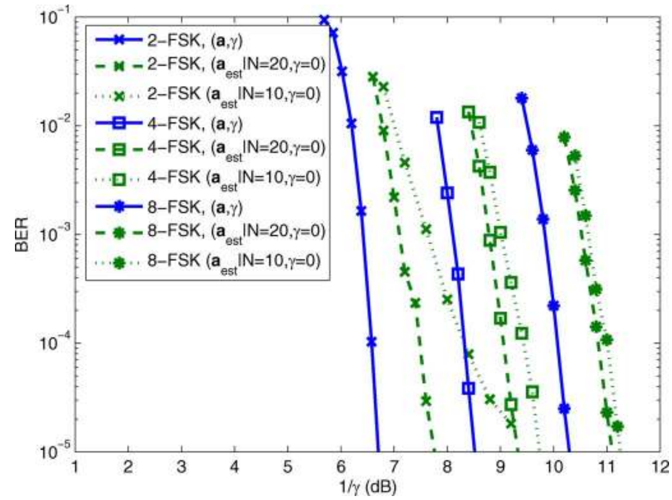

Fig. 10. BER performance of Bayesian Cauchy metric with median amplitude estimator in Cauchy noise; $M = 2$, 4, 8.

provided the amplitude estimation error is sufficiently bounded. To test this hypothesis, simulations with the Bayesian Cauchy metric were performed in which the signal amplitude information was estimated by median filtering of pilot-symbol amplitude measurements [11, Sec. 9.7]. Specifically, into the channel symbol sequence of each transmitted frame, we inject N pilot symbols carrying fixed energy in each signal dimension (or on each frequency in FSK). The receiver evaluates the median amplitude of this sequence in each frequency, resulting in M amplitude estimates $a_{i,\text{est}}$, $i = 1, \ldots, M$, which are then used by the soft-decision decoder.

Fig. 9 illustrates the BER performance of the Bayesian Cauchy metric with BFSK signaling and the pilot-assisted median amplitude estimator. The performance with $N = 20$ pilots and unknown noise dispersion (γ set to zero in evaluating the metric) is about 1 dB worse than with perfect CSI, at 10^{-5} BER, and is still over 3 dB better than the GLR Gaussian metric, which requires no side information. The performance penalty of unknown dispersion is 0.25 dB, and the penalty of reducing the number of pilots to $N = 10$ is around 1.5 dB. For the frame sizes used in these examples (2060 channel symbols), the pilot symbol overhead is less than 1%.

Fig. 10 shows additional results for 4-FSK and 8-FSK signaling. The performance gap between the $N = 10$ and $N = 20$ median amplitude filters narrows for these schemes since they operate in higher SNDR regions.

V. Application to FH Ad Hoc Networks

One application of our results is receiver design and radio link performance analysis for frequency hopping (FH) ad hoc networks using M-FSK signaling. In this context, we are interested in the performance between a given transmitter–receiver pair in the presence of interference generated by other terminals in the network. If we model the FH interferers as being randomly positioned on an infinite plane with a two-dimensional Poisson distribution and an average density of λ nodes per unit area, and if the received signal strength decays with the mth power of distance (i.e., m is the path loss exponent), then the total interference at the receiver can be shown to have an SαS distribution with $\alpha = 4/m$ [1]. (That is, the faster the power is attenuated with distance, the more impulsive is the noise.) Furthermore, when hopping over q frequencies, the dispersion of the SαS interference can be shown to be [3]

$$\gamma = \frac{\pi\lambda\Gamma\left(1 - \frac{\alpha}{2}\right)}{qM2^{\alpha}\Gamma\left(1 + \frac{\alpha}{2}\right)} \tag{22}$$

where $\Gamma(\cdot)$ is the Gamma function. Thus, the parameters of a given FH system (m, λ, q, and M) can be explicitly related to the noise parameters α and γ. Using results similar to those in Section IV we can predict the system's turbo-coded link performance with the various soft decision metrics.

From (22) it is clear that tolerating SαS noise with greater dispersion is equivalent to either tolerating higher interferer density λ, a lower FH processing gain q, or some combination of the two. The preceding numerical results allow us to quantify these gains. Consider, for example, a channel model with a path loss exponent of $m = 4$, which corresponds to an index of stability of $\alpha = 1$. From Fig. 3, we observe that, at BERs of practical interest, the Cauchy metric with knowledge of the received signal amplitude allows the system to tolerate several orders of magnitude higher interferer density or to operate at an equivalent factor less spreading bandwidth compared to the Bayesian Gaussian metric. Using a practical estimator of the signal amplitude results in a 1-dB penalty, and the absence of side information altogether can be compensated by a 4-dB increase in the processing gain. Similar observations can be made for channel propagation environments with other path loss exponents or, in other words, different values of α.

VI. Conclusion

New soft-decision metrics were derived for coded orthogonal signaling with noncoherent detection in symmetric α-stable noise (SαS). In addition to the optimum metrics for Gaussian and Cauchy noise, a class of generalized-likelihood ratio metrics was derived requiring less (or no) side information (i.e., signal amplitudes, noise dispersion). Performance was evaluated by Monte Carlo simulation for a system using turbo codes. While all the studied metrics perform closely (within 1 dB) for

$\alpha = 2$ (Gaussian noise), the Bayesian-derived Cauchy metric significantly outperforms the other metrics for a wide range of $\alpha < 2$. This observation is consistent with findings in [2] and [3] for uncoded systems, and it shows that one can design a receiver which is robust to impulsive noise and at the same time gives only a very small loss in sensitivity in the event that the noise is *not* impulsive. While the Cauchy receiver requires knowledge of the noise dispersion and signal amplitudes, a simpler GLR metric has been found that provides a substantial performance improvement over the Gaussian metric over a range of α while requiring no side information. However, even when the Cauchy receiver is used with a simple pilot-assisted amplitude estimator and without knowledge of the noise dispersion, it retains most of its performance advantage over the GLR metric. These results have direct application to receiver design in frequency hopping ad hoc networks for which the self-interference is well modeled as $\mathrm{S}\alpha\mathrm{S}$.

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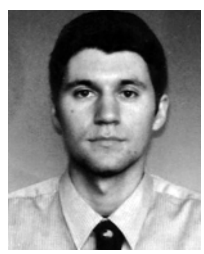

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