Journal of Theoretical and Applied Information Technology
15th March 2012. Vol. 37 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

SOFT GENERALIZED CLOSED SETS IN SOFT TOPOLOGICAL SPACES

K. KANNAN
Asstt Prof., Department of Mathematics, Srinivasa Ramanujan Centre, SASTRA University,
Kumbakonam-612 001, Tamil Nadu, INDIA
E-mail: anbukkannan@rediffmail.com

ABSTRACT

In the present paper, we introduce soft generalized closed sets in soft topological spaces which are defined over an initial universe with a fixed set of parameters. A sufficient condition for a soft g-closed set to be a soft closed is also presented. Moreover, the union and intersection of two soft g-closed sets are discussed. Finally, the new soft separation axiom, namely soft $T_{\frac{1}{2}}$-space is introduced and its basic properties are investigated.

Keywords: *Soft Topological Spaces, Soft Closed, Soft Generalized Closed, Soft* $T_{\frac{1}{2}}$-Space, *Soft* T_0*-Space, Soft* T_1*-Space.*
2000 Mathematics Subject Classification: 06D72, 54A40.

1. INTRODUCTION

Several set theories can be considered as tools for dealing with uncertainties, for example theory of fuzzy sets [13], theory of intuitionistic fuzzy sets [1], theory of vague sets, theory of interval mathematics [2, 5] and theory of rough sets [11], but all these theories have their own difficulties. The reason for these difficulties is, possibly, the inadequacy of the parametrization tool of the theory as it was mentioned by Molodtsov in [9]. He initiated the concept of soft set theory as a new mathematical tool which is free from the problems mentioned above. In his paper [9], he presented the fundamental results of the new theory and successfully applied it to several directions such as smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, theory of probability etc.

A soft set is a collection of approximate descriptions of an object. He also showed how soft set theory is free from the parametrization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory and game theory. Soft systems provide a very general framework with the involvement of parameters. Research works on soft set theory and its applications in various fields are progressing rapidly. Maji et al. [7, 8] presented an application of soft sets in decision making problems that is based on the reduction of parameters to keep the optimal choice objects. Chen [4] presented a new definition of soft set parametrization reduction and a comparison of it with attribute reduction in rough set theory. Pei and Miao [12] showed that soft sets are a class of special information systems.

Muhammad Shabir and Munazza Naz introduced soft topological spaces which are defined over an initial universe with a fixed set of parameters. The notions of soft open sets, soft closed sets, soft closure, soft interior points, soft neighborhood of a point and soft separation axioms are also introduced and their basic properties are investigated by them [10]. Levine [6] introduced generalized closed sets in topological spaces.

2. PRELIMINARIES

Let U be an initial universe and E be a set of parameters. Let P(U) denote the power set of U and A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A $\rightarrow$ P(U). In other words, a soft set over U is a parametrized family of subsets of the universe U. For $\varepsilon \in$ A, F(ε) may be considered as the set of ε-approximate elements of the soft set (F, A). Clearly, a soft set is not a set. For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if (1) A $\subseteq$ B and (2) for all e $\in$ A, F(e) and G(e) are identical approximations. We write (F, A) $\tilde{\subset}$ (G, B). (F, A)

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

is said to be a soft superset of (G, B), if (G, B) is a soft subset of (F, A). We denote it by (F, A) $\tilde{\supseteq}$ (G, B). Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

The union of two soft sets of (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$, H(e) = F(e) if $e \in A - B$, G(e) if $e \in B - A$ and $F(e) \cup G(e)$ if $e \in A \cap B$. We write $(F, A) \cup (G, B) = (H, C)$. The following is the definition of the intersection of two soft sets. The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe U, denoted $(F, A) \cap (G, B)$, is defined as $C = A \cap B$, and $H(e) = F(e) \cap G(e)$ for all $e \in C$.

Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if (1) ϕ, $\tilde{X}$ belong to τ, (2) the union of any number of soft sets in τ belongs to τ, (3) the intersection of any two soft sets in τ belongs to τ. The triplet (X, τ, E) is called a soft topological space over X. Let (X, τ, E) be a soft space over X, then the members of τ are said to be soft open sets in X. The relative complement of a soft set (F, A) is denoted by (F, A)' and is defined by (F, A)' = (F', A) where F': $A \to P(X)$ is a mapping given by $F'(e) = X - F(e)$ for all $e \in A$. Let (X, τ, E) be a soft space over X. A soft set (F, E) over X is said to be a soft closed set in X, if its relative complement (F, E)' belongs to τ. Let X be an initial universe set, E be the set of parameters and $\tau = \{\phi, \tilde{X}\}$. Then τ is called the soft indiscrete topology on X and (X, τ, E) is said to be a soft indiscrete space over X. Let X be an initial universe set, E be the set of parameters and let τ be the collection of all soft sets which can be defined over X. Then τ is called the soft discrete topology on X and (X, τ, E) is said to be a soft discrete space over X. Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X. Then, the soft closure of (F, E), denoted by $\overline{(F, E)}$ is the intersection of all soft closed supersets of (F, E). Clearly (F, E) is the smallest soft closed set over X which contains (F, E). The soft interior of (F, E), denoted by $(F, E)^o$ is the union of all soft open subsets of (F, E). Clearly (F, E) is the largest soft open set over X which is contained in (F, E).

3. SOFT GENERALIZED CLOSED SETS

Now we shall define the new relatively closed set in the following definition.

DEFINITION 3.1

A soft set (A, E) is called a soft generalized closed (soft g-closed) in a soft topological space (X, τ, E) if $\overline{(A, E)} \tilde{\subseteq} (U, E)$ whenever $(A,E) \tilde{\subseteq} (U,E)$ and (U, E) is soft open in X.

EXAMPLE 3.2

Let X = { a, b, c }, E = $\{e_1, e_2\}$. Let A, B, C be the mappings from E to P(X) defined by $A(e_1) = \{b\}$, $A(e_2) = \{a\}$, $B(e_1) = \{b, c\}$, $B(e_2) = \{a, b\}$, $C(e_1) = \{a, b\}$, $C(e_2) = \{a, c\}$. Then (A, E), (B, E), (C, E) are soft sets over X and $\tau = \{ \phi, \tilde{X}, (A, E), (B, E), (C, E) \}$ is the soft topology over X and therefore, (X, τ, E) is a soft topological space over X. Clearly, (B, E) and (C, E) are soft g-closed in (X, τ, E), but (A, E) is not soft g-closed in (X, τ, E).

Since for every soft closed set (A, E), $(A, E) = \overline{(A, E)}$, we have every soft closed set is soft g-closed. But the converse is not true in general. The following example supports our claim.

EXAMPLE 3.3

In Example 3.2, (B, E) and (C, E) are soft g-closed in (X, τ, E), but not soft closed over X.

THEOREM 3.4

If (A, E) is soft g-closed in X and $(A, E) \tilde{\subseteq} (B, E) \tilde{\subseteq} \overline{(A, E)}$, then (B, E) is soft g-closed.

PROOF

Suppose that (A, E) is soft g-closed in X and $(A, E) \tilde{\subseteq} (B, E) \tilde{\subseteq} \overline{(A, E)}$. Let $(B, E) \tilde{\subseteq} (U, E)$ and (U, E) is soft open in X. Since $(A, E) \tilde{\subseteq} (B, E)$ and $(B, E) \tilde{\subseteq} (U, E)$, we have $(A, E) \tilde{\subseteq} (U, E)$. Hence $\overline{(A, E)} \tilde{\subseteq} (U, E)$ {Since (A, E) is soft g-closed}. Since $(B, E) \tilde{\subseteq} \overline{(A, E)}$, we have $\overline{(B, E)} \tilde{\subseteq} \overline{(A, E)} \tilde{\subseteq} (U, E)$. Therefore, B is soft g-closed

THEOREM 3.5

If (A, E) and (B, E) are soft g-closed sets then so is $(A, E) \cup (B, E)$.

PROOF

Suppose that (A, E) and (B, E) are soft g-closed sets. Let $(A, E) \cup (B, E) \tilde{\subseteq} (U, E)$ and (U, E) is soft

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

open in X. Since $(A, E) \cup (B, E) \tilde{\subseteq} (U, E)$, we have $(A, E) \tilde{\subseteq} (U, E)$ and $(B, E) \tilde{\subseteq} (U, E)$. Since (U, E) is soft open in X and (A, E) and (B, E) are soft g-closed sets, we have $\overline{(A, E)} \tilde{\subseteq} (U, E)$ and $\overline{(B, E)} \tilde{\subseteq} (U, E)$. Therefore, $\overline{(A, E) \cup (B, E)} = \overline{(A, E)} \cup \overline{(B, E)} \tilde{\subseteq} (U, E)$. This completes the proof.

THEOREM 3.6

If a set (A, E) is soft g-closed in X if and only if $\overline{(A, E)} \setminus (A,E)$ contains only null soft closed set.

PROOF

Suppose that (A, E) is soft g-closed in X . Let (F, E) be soft closed and $(F, E) \tilde{\subseteq} \overline{(A, E)} \setminus (A, E)$. Since F is soft closed, we have its relative complement F′ is soft open. Since $(F, E) \tilde{\subseteq} \overline{(A, E)} \setminus (A, E)$, we have $(F,E) \tilde{\subseteq} \overline{(A, E)}$ and $(F, E) \tilde{\subseteq} (A, E)'$. Hence $(A, E) \tilde{\subseteq} (F, E)'$. Consequently $\overline{(A, E)} \tilde{\subseteq} (F, E)'$ {Since (A, E) is soft g-closed in X}. Therefore, $(F, E) \tilde{\subseteq} \overline{(A, E)}'$. Hence $(F, E) = \phi$. Hence $\overline{(A, E)} \setminus (A,E)$ contains only null soft closed set. One can easily prove the converse part.

COROLLARY 3.7

A soft g-closed (A,E) is soft closed if and only if $\overline{(A, E)} \setminus (A,E)$ is soft closed.

PROOF

If (A,E) is soft closed, then $\overline{(A, E)} \setminus (A,E) = \phi$. Conversely, suppose that $\overline{(A, E)} \setminus (A,E)$ is soft closed. Since (A, E) is soft g-closed, $\overline{(A, E)} \setminus (A,E) = \phi$ {by Theorem 3.6}. Hence (A, E) is soft closed.

The intersection of two soft g-closed sets is generally not a soft g-closed set. Example 3.2 supports our claim. In this example, (B, E) and (C, E) are soft g-closed in (X, τ, E), but $(B, E) \cap (C, E) = (A, E)$ is not soft g-closed in (X, τ, E).

THEOREM 3.8

Let (A, E) be a soft g-closed set and suppose that (F, E) is a soft closed set. Then $(A \cap F, E)$ is a soft g-closed set.

PROOF

The proof is left to the reader.

4. SOFT GENERALIZED OPEN SETS

DEFINITION 4.1

A soft set (A, E) is called a soft generalized open (soft g-open) in a soft topological space (X, τ, E) if the relative complement (A, E)' is soft g-closed in X.

Equivalently, a soft set (A, E) is called a soft generalized open (soft g-open) in a soft topological space (X, τ, E) if and only if $(F, E) \tilde{\subseteq} (A, E)^o$ whenever $(F, E) \tilde{\subseteq} (A, E)$ and (F, E) is soft closed in X.

EXAMPLE 4.2

In Example 3.2, the soft sets (B, E)', (C, E)' are soft g-open sets over X where $B'(e_1) = \{a\}$ and $B'(e_2) = \{c\}$ and $C'(e_1) = \{c\}$ and $C'(e_2) = \{b\}$.

Since for every soft open set (A, E), $(A, E) = (A, E)^o$, we have every soft open set is soft g-open. But the converse is not true in general. The following example supports our claim.

EXAMPLE 4.3

In Example 3.2, (B, E)' and (C, E)' are soft g-open in (X, τ, E), but not soft open over X.

THEOREM 4.4

If (A, E) is soft g-open in X and $(A, E)^o \tilde{\subseteq} (B, E) \tilde{\subseteq} (A, E)$, then (B, E) is soft g-open.

PROOF

Suppose that (A, E) is soft g-open in X and $(A, E)^o \tilde{\subseteq} (B, E) \tilde{\subseteq} (A, E)$. Let $(F, E) \tilde{\subseteq} (B, E)$ and (F, E) is soft closed in X. Since $(B, E) \tilde{\subseteq} (A, E)$ and $(F, E) \tilde{\subseteq} (B, E)$, we have $(F, E) \tilde{\subseteq} (A, E)$. Hence $(F, E) \tilde{\subseteq} (A, E)^o$ {Since (A, E) is soft g-open}. Since $(A, E)^o \tilde{\subseteq} (B, E)$, we have $(F, E) \tilde{\subseteq} (A, E)^o \tilde{\subseteq} (B, E)^o$. Therefore, B is soft g-open.

THEOREM 4.5

If (A, E) and (B, E) are soft g-open sets then so is $(A, E) \cap (B, E)$.

PROOF

Suppose that (A, E) and (B, E) are soft g-open sets. Let $(F, E) \tilde{\subseteq} (A, E) \cap (B, E)$ and (F, E) is soft closed in X. Since $(F, E) \tilde{\subseteq} (A, E) \cap (B, E)$, we have $(F, E) \tilde{\subseteq} (A, E)$ and $(F, E) \tilde{\subseteq} (B, E)$. Since (F, E) is soft closed in X and (A, E) and (B, E) are soft g-open sets, we have $(F, E) \tilde{\subseteq} (A, E)^o$ and $(F, E) \tilde{\subseteq} (B, E)^o$. Therefore, $(F, E) \tilde{\subseteq} (A, E)^o \cap (B, E)^o$. This completes the proof.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

The union of two soft g-open sets is generally not a soft g-open set.

5. SOFT $T_{\frac{1}{2}}$-SPACES

In this section, we introduce the new separation axiom, namely soft $T_{\frac{1}{2}}$-space in soft topological space with the help of soft g-closed sets.

DEFINITION 5.1

A soft topological space (X, τ, E) is a soft $T_{\frac{1}{2}}$-space if every soft g-closed set is soft closed in X.

THEOREM 5.2

Every soft $T_{\frac{1}{2}}$-space is a soft T_0-space.

PROOF

The proof is left to the reader.

THEOREM 5.3

Every soft T_1-space is a soft $T_{\frac{1}{2}}$-space.

PROOF

Suppose that (X, τ, E) is a soft T_1-space. Let (A, E) be a soft set such that (A, E) is not soft closed. Let $x \in \overline{(A, E)} \setminus (A, E)$. Then $(x, E) \in \overline{(A, E)} \setminus (A, E)$ and (x, E) is soft closed in (X, τ, E) since (X, τ, E) is a soft T_1-space. Hence (A, E) is not soft g-closed {by Theorem 3.6}. It completes the proof.

COROLLARY 5.4

The property of soft $T_{\frac{1}{2}}$-space is strictly between soft T_0 and soft T_1.

6. CONCLUSION

We have introduced soft generalized closed sets in soft topological spaces which are defined over an initial universe with a fixed set of parameters. A sufficient condition for a soft g-closed set to be a soft closed is also presented. Moreover, the union and intersection of two soft g-closed sets are discussed. Finally, the new soft separation axiom, namely soft $T_{\frac{1}{2}}$-space is introduced and its basic properties are investigated. In particular very few ideas are studied and hence it is the initiative to the study of soft generalized closed sets. In future many types of soft generalized closed sets may be defined and using of them more soft separation axioms may be developed. Further research is being carried out in the framework of practical applications.

REFRENCES:

[1] K. Atanassov, "Intuitionistic fuzzy sets", *Fuzzy Sets and Systems*, Vol. 20, 1986, pp. 87-96.

[2] K. Atanassov, "Operators over interval valued intuitionistic fuzzy sets", *Fuzzy Sets and Systems*, Vol. 64, 1994, pp. 159-174.

[3] C.E. Aull and W.J. Thron, "Separation axioms between T_0 and T_1", *Indag. Math*, Vol. 24, 1963, pp. 26-37.

[4] D. Chen, "The parametrization reduction of soft sets and its applications", *Computers and Math. with Appl.*, Vol. 49, 2005, pp. 757-763.

[5] M.B. Gorzalzany, "A method of inference in approximate reasoning based on interval-valued fuzzy sets", *Fuzzy Sets and Systems*, Vol. 21, 1987, pp. 1-17.

[6] N. Levine, "Generalized closed sets in topology", *Rend. Circ. Mat. Palermo*, Vol. 19, No. 2, 1970, pp. 89-96.

[7] P.K. Maji, R. Biswas, R. Roy, "An application of soft sets in a decision making problem", *Comput. Math. Appl.*, Vol. 44, 2002, pp. 1077-1083.

[8] P. K. Maji, R. Biswas, R. Roy, "Soft set theory", *Comput. Math. Appl.*, Vol. 45, 2003, pp. 555-562.

[9] D. Molodtsov, "Soft set theory first results", *Comput. Math. Appl.*, Vol. 37, 1999, pp. 19-31.

[10] Muhammad Shabir, Munazza Naz, "On soft topological spaces", *Comput. Math. Appl.*, Vol. 61, 2011, pp. 1786-1799.

[11] Z. Pawlak, "Rough sets", *Int. J. Comput. Sci.*, Vol. 11, 1982, pp. 341-356.

[12] D. Pie, D. Miao, "From soft sets to information systems", *Granular computing, 2005 IEEE Inter. Conf.* Vol. 2, pp. 617-621.

[13] L.A. Zadeh, "Fuzzy sets", *Inf. Control*, Vol. 8, 1965, pp. 338-353.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

AUTHOR PROFILES:

K. Kannan received the M. Sc degree in Mathematics from Kamaraj College, Tuticorin (Manonmaniam Sundaranar University, Tirunelveli) in 1997 and M. Phil. degree in Mathematics from Loyola College (Madras University), Chennai, in 1998. He is a research student of Dr. K. Chandrasekhara Rao and submitted his Ph. D thesis on “Contribution to the study on some generalized closed sets in bitopological spaces” to SASTRA University, Thanjavur. Currently, he is an Assistant Professor at Srinivasa Ramanujan Centre, SASTRA University, Kumbakonam. His interests are in topological spaces, soft topological spaces and bitopological spaces. He published more than 20 research articles in national and international journals.