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Soft Maps via Soft Somewhere Dense Sets

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Abstract. The concept of soft sets was proposed as an effective tool to deal with uncertainty and vagueness. Topologists employed this concept to define and study soft topological spaces. In this paper, we introduce the concepts of soft *SD*-continuous, soft *SD*-open, soft *SD*-closed and soft *SD*-homeomorphism maps by using soft somewhere dense and soft *cs*-dense sets. We characterize them and discuss their main properties with the help of examples. In particular, we investigate under what conditions the restriction of soft *SD*-continuous, soft *SD*-open and soft *SD*-closed maps are respectively soft *SD*-continuous, soft *SD*-open and soft *SD*-closed maps. We logically explain the reasons of adding the null and absolute soft sets to the definitions of soft *SD*-continuous and soft *SD*-closed maps, respectively, and removing the null soft set from the definition of a soft *SD*-open map.

1. Introduction

The soft set theory, initiated by Molodtsov [25] in 1999, is one of the mathematical methods that aims to describe phenomena and concepts of ambiguous, undefined and imprecise meaning. This theory is applicable and represents a vital alternative tool of fuzzy and rough set theories whose they have some difficulties. Emergence of many papers concerning soft sets is due to their rich potential for applications in several research areas such as computer science, engineering and medical sciences and decision making problems (see, for example [8, 15, 21, 29]).

As we know, maps and their properties play an important role in topology and its applications. Two of their main roles are: The first is study what topological properties are preserved under certain families of maps?, and the second is investigating classification of maps by spaces and classification of spaces by maps. This makes exploring of soft maps was a great interest to scholars who mainly interested in studying soft setting. The fundamental contributions of this theme were done in 2011 and 2013 by [22] and [28], respectively. In [22], the authors studied soft maps by using two crisp maps, one of them between the sets of parameters and the second one between the universal sets. However, [28] defined soft maps by using

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the concept of soft points. Some significant applications of different types of soft maps were the goal of some articles [22, 24, 28].

Soft sets theory received the attention of the topologists who always seeking to generalize and apply the topological notions on different structures. This path of study began in 2011, by Shabir and Naz [27]. Since that, many papers concerning soft topologies have been published (see, for example [9, 12, 26, 31]). Generalization of soft open sets is one of important and common areas of the soft topological studies. They were utilized to construct wider classes of soft continuity, soft compactness and soft separation axioms.

In this field, Al-shami [6] extended his earlier work [5] by defining a new class of generalized soft open sets, namely soft somewhere dense sets. He studied main properties and pointed out that this class contains all non-null soft j-open sets, where $j \in \{\alpha, semi, pre, b, \beta\}$. Al-shami and Noiri [10] explored new types of crisp maps using somewhere dense and *cs*-dense sets. Recently, [18] has applied soft somewhere dense sets to introduce some operators of a soft set and to investigate new type of soft connectedness.

This paper aims to introduce new types of soft maps, namely soft *SD*-continuous, soft *SD*-irresolute, soft *SD*-open, soft *SD*-closed and soft *SD*-homeomorphism maps. These soft maps are characterized and main properties are studied. Furthermore, Some results related to graph and restriction of soft maps under a condition of soft hyperconnectedness are investigated. Some examples and counterexamples are given.

2. Preliminaries

This section is allocated to recall some notions and definitions which are needed in our subsequent discussions.

Definition 2.1. ([25]) A pair (G, E) is said to be a soft set over a non-empty set X provided that G is a mapping of a set of parameters E into 2^X.
A soft set is identified as a set of ordered pairs: $(G, E) = \{(e, G(e)) : e \in E \text{ and } G(e) \in 2^X\}$.

Definition 2.2. ([19]) A soft set (G, E) is said to be a subset of a soft set (H, E), denoted by $(G, E)\widetilde{\subseteq}(H, E)$, if $G(e) \subseteq H(e)$ for all $e \in E$.

The soft sets (G, E) and (H, E) are said to be soft equal if each one of them is a subset of the other.

Definition 2.3. ([4]) The relative complement of a soft set (G, E), denoted by $(G, E)^c$, is given by $(G, E)^c = (G^c, E)$, where $G^c : E \to 2^X$ is a mapping defined by $G^c(e) = X \setminus G(e)$ for each $e \in E$.

Definition 2.4. ([17, 23, 26]) A soft set (G, E) over X is said to be:

(i) null soft set, denoted by $\widetilde{\Phi_X}$, if $G(e) = \emptyset$ for each $e \in E$.

(ii) absolute soft set, denoted by $\widetilde{X}$, if its relative complement is null soft set.

(iii) soft point P_e^x if there are $e \in E$ and $x \in X$ such that $G(e) = \{x\}$ and $G(\alpha) = \emptyset$ for each $\alpha \in E \setminus \{e\}$. We write that $P_e^x \in (G, E)$ if $x \in G(e)$.

(iv) stable if there is a subset S of X such that $G(e) = S$ for each $e \in E$.

Definition 2.5. ([23]) The union of two soft sets (G, E) and (H, E) over X, denoted by $(G, E)\widetilde{\bigcup}(H, E)$, is a soft set (U, E), where a mapping $U : E \to 2^X$ is given by $U(e) = G(e) \bigcup H(e)$

Definition 2.6. ([4]) The intersection of two soft sets (G, E) and (H, E) over X, denoted by $(G, E)\widetilde{\bigcap}(H, E)$, is a soft set (U, E), where a mapping $U : E \to 2^X$ is given by $U(e) = G(e) \bigcap H(e)$.

Many types of union and intersection between soft sets has been studied in the literature. For more details about these types, see [1, 7] and the references mentioned therein.

Definition 2.7. ([13]) Let (G, E) and (H, F) be soft sets over X and Y, respectively. Then the cartesian product of (G, E) and (H, F), denoted by $(G \times H, E \times F)$, is defined as $(G \times H)(e, f) = G(e) \times H(f)$ for each $(e, f) \in E \times F$.

Definition 2.8. ([28]) Let (X, E) and (Y, F) be two soft sets. A soft relation $g_\varphi \widetilde{\subseteq} (X, E) \times (Y, F)$ is called a soft mapping from (X, E) to (Y, F), which is denoted by $g_\varphi : (X, E) \to (Y, F)$, if for each soft point $P_e^x \in (X, E)$, there exists only one a soft point $P_f^y \in (Y, F)$ such that $P_e^x \, g_\varphi \, P_f^y$ (which will be noted as $g_\varphi(P_e^x) = P_f^y$).

It is worth noting that there is a different definition of a soft map introduced in [22].

Theorem 2.9. ([28]) *Consider a soft map $g_\varphi : (X, E) \to (Y, F)$ and let $(G_i, E) \widetilde{\subseteq} (X, E)$ and $(H_i, F) \widetilde{\subseteq} (Y, F)$ for $i = 1, 2$. Then we have the following results.*

(*i*) *If $(G_1, E) \widetilde{\subseteq} (G_2, E)$, then $g_\varphi(G_1, E) \widetilde{\subseteq} g_\varphi(G_2, E)$.*

(*ii*) *If $(H_1, F) \widetilde{\subseteq} (H_2, F)$, then $g_\varphi^{-1}(H_1, F) \widetilde{\subseteq} g_\varphi^{-1}(H_2, F)$.*

(*iii*) *$(G_1, E) \widetilde{\subseteq} g_\varphi^{-1}(g_\varphi(G_1, E))$ and $(H_1, F) \widetilde{\subseteq} g_\varphi(g_\varphi^{-1}(H_1, F))$.*

(*iv*) *$g_\varphi[(G_1, E) \widetilde{\bigcup} (G_2, E)] = g_\varphi(G_1, E) \widetilde{\bigcup} g_\varphi(G_2, E)$.*

(*v*) *$g_\varphi[(G_1, E) \widetilde{\bigcap} (G_2, E)] \widetilde{\subseteq} g_\varphi(G_1, E) \widetilde{\bigcap} g_\varphi(G_2, E)$.*

(*vi*) *$g_\varphi^{-1}[(H_1, F) \widetilde{\bigcup} (H_2, F)] = g_\varphi^{-1}(H_1, F) \widetilde{\bigcup} g_\varphi^{-1}(H_2, F)$.*

(*vii*) *$g_\varphi^{-1}[(H_1, F) \widetilde{\bigcap} (H_2, F)] = g_\varphi^{-1}(H_1, F) \widetilde{\bigcap} g_\varphi^{-1}(H_2, F)$.*

Definition 2.10. ([27]) The class of soft sets, defined over a non-empty set X with a fixed parameters set E, is said to be a soft topology τ if satisfies the following:

(i) The absolute and null soft sets belong to τ.

(ii) τ is closed under arbitrarily soft union and finite soft intersection.

Then the triple (X, τ, E) is called a soft topological space. An element in τ is termed soft open and its relative complement is termed soft closed.

Throughout this paper, the two triples (X, τ, E) and (Y, θ, F) indicate soft topological spaces on which no soft separation axiom is assumed unless otherwise stated.

Definition 2.11. ([26]) The triple (A, τ_A, E) is said to be a soft subspace of (X, τ, E) provided that $A \subseteq X$ and $\tau_A = \{\widetilde{A} \widetilde{\bigcap} (G, E) : (G, E) \in \tau\}$.

Definition 2.12. ([6]) A soft subset (H, E) of (X, τ, E) is said to be soft somewhere dense if $int(cl(H, E)) \neq \widetilde{\Phi}$. The relative complement of a soft somewhere dense set is said to be soft *cs*-dense.

Definition 2.13. ([18, 26]) Let (H, E) be a soft subset of (X, τ, E). Then:

(i) $int(H, E)$ (resp. $Sint(H, E)$) is the union of all soft open (resp. soft somewhere dense) sets contained in (H, E).

(ii) $cl(H, E)$ (resp. $Scl(H, E)$) is the intersection of all soft closed (resp. soft *cs*-dense) sets containing (H, E).

Theorem 2.14. ([6]) *A soft subset (B, E) of (X, τ, E) is soft cs-dense if and only if there is a proper soft closed set (H, E) such that $int(B, E) \widetilde{\subseteq} (H, E)$.*

Theorem 2.15. ([6]) *Every soft subset of (X, τ, E) is soft somewhere dense or soft cs-dense.*

Theorem 2.16. ([6]) *Every superset of a soft somewhere dense set is soft somewhere dense.*

Theorem 2.17. ([6]) *The union of an arbitrary non-empty collection of soft somewhere dense subsets of* (X, τ, E) *is soft somewhere dense; and the intersection of an arbitrary non-empty collection of soft cs-closed subsets of* (X, τ, E) *is soft cs-closed.*

Definition 2.18. (X, τ, E) is said to be:

(i) soft hyperconnected [20] provided that $\widetilde{X}$ and $\widetilde{\Phi}$ are the only soft clopen sets.

(ii) strongly soft hyperconnected [6] provided that a soft subset of $\widetilde{X}$ is soft dense if and only if it is non-null soft open set.

Theorem 2.19. ([6]) *The intersection of soft open and soft somewhere dense subsets of a soft hyperconnected space is soft somewhere dense. And the intersection of two soft somewhere dense subsets of a strongly soft hyperconnected space is soft somewhere dense.*

Definition 2.20. ([17]) Let $\{(X_i, \tau_i, E_i) : i = 1, 2, ..., n\}$ be the collection of soft topological spaces. Then $\prod_{i=1}^{n} \tau_i = \{\prod_{i=1}^{n}(G, E_i) : (G, E_i) \in \tau_i\}$ defines a base for a soft topology T on $\prod_{i=1}^{n} X_i$ under a parameters set $\prod_{i=1}^{n} E_i$. T is called a finite product soft topology and $(\prod_{i=1}^{n} X_i, T, \prod_{i=1}^{n} E_i)$ is called a finite product soft space.

Theorem 2.21. ([6]) *The product of two soft somewhere dense sets is soft somewhere dense*

Definition 2.22. ([2, 3, 11, 16, 30]) Let $j \in \{semi, pre, b, \alpha, \beta\}$. A soft mapping $f_\varphi : (X, \tau, E) \to (Y, \theta, F)$ is said to be:

(i) *Soft j-continuous* if the inverse image of each soft open subset of (Y, θ, F) is a soft j-open subset of (X, τ, E).

(ii) *Soft j-open* (resp. *soft j-closed*) if the image of each soft open (resp. soft closed) subset of (X, τ, E) is a soft j-open (resp. soft j-closed) subset of (Y, θ, F).

(iii) *Soft j-homeomorphism* if it is bijective, soft j-continuous and soft j-open.

Definition 2.23. ([14]) (X, τ, E) is called soft T_2 if for every distinct soft points $e_x \neq f_y \in \widetilde{X}$, there are disjoint soft open sets (U, E) and (V, E) containing e_x and f_y, respectively.

3. Soft *SD*-Continuous Maps

In this section, we introduce and characterize the concepts of soft *SD*-continuous and soft *SD*-irresolute maps, where *SD* denotes "somewhere dense". We investigate some results which associate soft *SD*-continuous maps with the restriction and graph of a soft map.

Definition 3.1. A soft map $g_\varphi : (X, \tau, E) \to (Y, \theta, F)$ is said to be soft somewhere dense continuous (briefly, soft *SD*-continuous) at $P_e^x \in \widetilde{X}$ if for any soft open set (U, F) containing $g_\varphi(P_e^x)$, there is a soft somewhere dense set (G, E) containing P_e^x such that $g_\varphi(G, E) \widetilde{\subseteq} (U, F)$.

Definition 3.2. A soft map $g_\varphi : (X, \tau, E) \to (Y, \theta, F)$ is said to be soft *SD*-continuous if it is soft *SD*-continuous for each $P_e^x \in \widetilde{X}$.

Theorem 3.3. *A soft map* $g_\varphi : (X, \tau, E) \to (Y, \theta, F)$ *is soft SD-continuous if and only if the inverse image of each soft open set is the null soft set or a soft somewhere dense set.*

Proof. To prove the "*if*" part, let (U, F) be a soft open subset of (Y, θ, F). Then we have the following two cases:

(i) Either $g_\varphi^{-1}(U, F) = \widetilde{\Phi_X}$. Then the result holds.

(ii) Or $g_\varphi^{-1}(U, F) \neq \widetilde{\Phi_X}$, then it follows by hypothesis that for each $g_\varphi(P_e^x) \in (U, F)$, there is a soft somewhere dense subset (V_{xe}, E) of $\widetilde{X}$ containing P_e^x such that $g_\varphi(V_{xe}, E)\widetilde{\subseteq}(U, F)$. Therefore $P_e^x \in (V_{xe}, E)\widetilde{\subseteq}g_\varphi^{-1}(U, F)$. Now, $\widetilde{\bigcup}\{(V_{xe}, E) : P_e^x \in g_\varphi^{-1}(U, F)\} = g_\varphi^{-1}(U, F)$. Hence, $g_\varphi^{-1}(U, F)$ is soft somewhere dense, as required.

To prove the "*only if*" part, assume that $P_e^x \in \widetilde{X}$ and (U, F) is a soft open set containing $g_\varphi(P_e^x)$. Then $g_\varphi^{-1}(U, F) \neq \widetilde{X}$ is a soft somewhere dense set containing P_e^x and satisfies that $g_\varphi(g_\varphi^{-1}(U, F))\widetilde{\subseteq}(U, F)$. So that, g_φ is soft SD-continuous at P_e^x. Since P_e^x is chosen arbitrary, then g_φ is soft SD-continuous. □

Corollary 3.4. *A surjective soft map $g_\varphi : (X, \tau, E) \to (Y, \theta, F)$ is soft SD-continuous if and only if the inverse image of each non-null soft open set is soft somewhere dense.*

Proposition 3.5. *If $g_\varphi : (X, \tau, E) \to (Y, \theta, F)$ is a surjective soft continuous map and $h_\phi : (Y, \theta, F) \to (Z, \mu, T)$ is a soft SD-continuous map, then $h_\phi \circ g_\varphi$ is soft SD-continuous.*

Proof. Let (U, T) be a non-null soft open subset of $\widetilde{Z}$. Then $h_\phi^{-1}(U, T)$ is the null soft set or a soft somewhere dense set. Now, we have the following two cases:

1. Either $h_\phi^{-1}(U, T) = \widetilde{\Phi_Y}$, then $g_\varphi^{-1}(h_\phi^{-1}(U, T)) = \widetilde{\Phi_X}$.

2. Or $h_\phi^{-1}(U, T) \neq \widetilde{\Phi_Y}$, then there is a non-null soft open subset (H, F) of $\widetilde{Y}$ such that $(H, F)\widetilde{\subseteq}cl(h_\phi^{-1}(U, T))$. It is clear that $g_\varphi^{-1}(H, F)\widetilde{\subseteq}g_\varphi^{-1}(cl(h_\phi^{-1}(U, T)))$. Since g_φ is surjective and soft continuous, then $g_\varphi^{-1}(H, F)$ is a non-null soft open subset of $\widetilde{X}$. Also, the soft continuity of g_φ implies that $g_\varphi^{-1}(cl(h_\phi^{-1}(U, T)))\widetilde{\subseteq}cl(g_\varphi^{-1}(h_\phi^{-1}(U, T)))$. Therefore $g_\varphi^{-1}(H, F)\widetilde{\subseteq}cl(g_\varphi^{-1}(h_\phi^{-1}(U, T)))$. This means that $int(cl(g_\varphi^{-1}(h_\phi^{-1}(U, T)))) \neq \widetilde{\Phi_X}$. Thus $g_\varphi^{-1}(h_\phi^{-1}(U, T))$ is soft somewhere dense.

Hence $h_\phi \circ g_\varphi$ is soft SD-continuous. □

Proposition 3.6. *Every soft j-continuous map is soft SD-continuous for each $j \in \{\beta, b, semi, pre, \alpha\}$.*

Proof. Consider that $g_\varphi : (X, \tau, E) \to (Y, \theta, F)$ is a soft j-continuous map and let (U, F) be a soft open subset of $\widetilde{Y}$. Then $g_\varphi^{-1}(U, F)$ is a soft j-open subset of $\widetilde{X}$ for all $j \in \{\beta, b, semi, pre, \alpha\}$. Since any soft somewhere dense set contains all non-null soft j-open set for all $j \in \{\beta, b, semi, pre, \alpha\}$, then it follows from Theorem 3.3 that $g_\varphi^{-1}(U, F)$ is the null soft set or a soft somewhere dense set. Hence, g_φ is soft SD-continuous. □

The next example shows that the above proposition is not conversely.

Example 3.7. Consider the following soft sets defined on the universal set $X = \{1, 2, 3, 4\}$ with a set of parameters $E = \{e_1, e_2\}$ as follows:

$(G_1, E) = \{(e_1, X), (e_2, \emptyset)\}$;
$(G_2, E) = \{(e_1, \emptyset), (e_2, X)\}$;
$(G_3, E) = \{(e_1, \{1\}), (e_2, \emptyset)\}$ and
$(G_4, E) = \{(e_1, \{1\}), (e_2, X)\}$.

Then $\tau = \{\widetilde{\Phi}, \widetilde{X}, (G_i, E) : i = 1, 2, 3, 4\}$ is a soft topology on X. For a soft set $(H, E) = \{(e_1, \{2, 3\}), (e_2, \{1, 4\})\}$, we have $cl(H, E) = \{(e_1, \{2, 3, 4\}), (e_2, X)\}$. Owing to $int[cl(H, E)] = \{(e_1, \emptyset), (e_2, X)\} \in \tau$, we obtain (H, E) is a soft somewhere dense set. On the other hand, $(H, E) \not\subseteq cl[int[cl(H, E)]] = \{(e_1, \emptyset), (e_2, X)\}$. So that, (H, E) is not a soft β-open set. This automatically means that (H, E) is not a soft j-open set for each $j \in \{b, semi, pre, \alpha\}$.

Proposition 3.8. *Let $g_\varphi : (X, \tau, E) \to (Y, \theta, F)$ be a surjective soft i-continuous map and let $h_\phi : (Y, \theta, F) \to (Z, \mu, T)$ be a soft j-continuous map, where $i \in \{\beta, b, semi, pre, \alpha\}$ and $j \in \{semi, \alpha\}$. Then a soft map $h_\phi \circ g_\varphi$ is soft SD-continuous.*

Proof. It suffices to prove the proposition in the cases of $i = \beta$ and $j = semi$, and the other cases follow similarly.

Let (G, T) be a soft open subset of $\widetilde{Z}$. Then $h_\phi^{-1}(G, T)$ is a soft *semi*-open subset of Y. Now, we have two cases:

1. Either $h_\phi^{-1}(G, T) = \widetilde{\Phi_Y}$, then $g_\varphi^{-1}(h_\phi^{-1}(G, T)) = \widetilde{\Phi_X}$.

2. Or $h_\phi^{-1}(G, T) \neq \widetilde{\Phi_Y}$, then there is a non-null soft open subset (H, F) of $\widetilde{Y}$ satisfies that $(H, F)\widetilde{\subseteq}h_\phi^{-1}(G, T)$. It is clear that $g_\varphi^{-1}(H, F) \subseteq g_\varphi^{-1}(h_\phi^{-1}(G, T))$. Since g_φ is surjective soft β-continuous, then $g_\varphi^{-1}(H, F)$ is a non-null soft β-open subset of $\widetilde{X}$. Therefore $g_\varphi^{-1}(H, F)$ is soft somewhere dense. Thus, $g_\varphi^{-1}(h_\phi^{-1}(G, T))$ is soft somewhere dense.

The above two cases imply that $h_\phi \circ g_\varphi$ is soft *SD*-continuous. □

The next result investigates the equivalent conditions for the concept of soft *SD*-continuity.

Theorem 3.9. *Let $g_\varphi : (X, \tau, E) \to (Y, \theta, F)$ be a soft map. Then the following properties are equivalent:*

(*i*) *g_φ is soft SD-continuous.*

(*ii*) *The inverse image of every soft closed subset of (Y, θ, F) is $\widetilde{X}$ or soft cs-dense.*

(*iii*) *$Scl(g_\varphi^{-1}(K, F))\widetilde{\subseteq}g_\varphi^{-1}(cl(K, F))$ for each $(K, F)\widetilde{\subseteq}\widetilde{Y}$.*

(*iv*) *$g_\varphi(Scl(H, E))\widetilde{\subseteq}cl(g_\varphi(H, E))$ for each $(H, E)\widetilde{\subseteq}\widetilde{X}$.*

(*v*) *$g_\varphi^{-1}(int(K, F))\widetilde{\subseteq}Sint(g_\varphi^{-1}(K, F))$ for each $(K, F)\widetilde{\subseteq}\widetilde{Y}$.*

Proof. (*i*) ⇒ (*ii*): Suppose that (H, F) is a soft closed subset of (Y, θ, F). Then (H^c, F) is soft open. Therefore $g_\varphi^{-1}(H^c, F) = \widetilde{X} - g_\varphi^{-1}(H, F)$ is the null soft set or a soft somewhere dense set. So $g_\varphi^{-1}(H, F)$ is the absolute soft set $\widetilde{X}$ or a soft cs-dense set.

(*ii*) ⇒ (*iii*): We have two cases for any soft set $(K, F)\widetilde{\subseteq}\widetilde{Y}$:

1. Either $g_\varphi^{-1}(cl(K, F)) = \widetilde{X}$. Then $Scl(g_\varphi^{-1}(K, F))\widetilde{\subseteq}\widetilde{X} = g_\varphi^{-1}(cl(K, F))$.

2. Or $g_\varphi^{-1}(cl(K, F))$ is soft cs-dense. Then $Scl(g_\varphi^{-1}(K, F))\widetilde{\subseteq}Scl(g_\varphi^{-1}(cl(K, F))) = g_\varphi^{-1}(cl(K, F))$.

From 1 and 2. the desired result is obtained.

(*iii*) ⇒ (*iv*): It is obvious that $Scl(H, E)\widetilde{\subseteq}Scl(g_\varphi^{-1}(g_\varphi(H, E)))$ for each $(H, E)\widetilde{\subseteq}\widetilde{X}$. By (iii), $Scl(g_\varphi^{-1}(g_\varphi(H, E)))\widetilde{\subseteq} g_\varphi^{-1}(cl(g_\varphi(H, E)))$. Therefore $g_\varphi(Scl(H, E)\widetilde{\subseteq}g_\varphi(g_\varphi^{-1}(cl(g_\varphi(H, E)))) \widetilde{\subseteq} cl(g_\varphi(H, E))$.

(*iv*) ⇒ (*v*): Let (K, F) be an arbitrary soft set in (Y, θ, F). Then $g_\varphi(Scl(g_\varphi^{-1}(K^c, F))\widetilde{\subseteq}cl(g_\varphi(g_\varphi^{-1}(K^c, F)))\widetilde{\subseteq}cl(K^c, F)$. So that, $Scl((g_\varphi^{-1}(K, F))^c)\widetilde{\subseteq}g_\varphi^{-1}((int(K, F))^c = (int(K, F)^c)$. Hence, $g_\varphi^{-1}(int(K, F))\widetilde{\subseteq}Sint(g_\varphi^{-1}(K, F))$.

(*v*) ⇒ (*i*): Suppose that (K, F) is a soft open subset of Y. By (v), we obtain $g_\varphi^{-1}(K, F) = g_\varphi^{-1}(int(K, F))\widetilde{\subseteq} Sint(g_\varphi^{-1}(K, F))$. Since $Sint(g_\varphi^{-1}(K, F))\widetilde{\subseteq}g_\varphi^{-1}(K, F)$, then $g_\varphi^{-1}(K, F) = Sint(g_\varphi^{-1}(K, F))$. Therefore $g_\varphi^{-1}(K, F)$ is the null soft set or a soft somewhere dense set. Thus g_φ is soft *SD*-continuous. □

Definition 3.10. Let $g_\varphi : (X, E) \to (Y, F)$ be a soft map and $\widetilde{A}$ be a subset of (X, E). Then the soft restriction of g_φ to $\widetilde{A}$ is a soft map $g_\varphi|_A: (A, E) \to (Y, F)$ defined by $g_\varphi|_A (P_e^x) = g_\varphi(P_e^x)$ for each $P_e^x \in \widetilde{A}$.

Theorem 3.11. *If $g_\varphi : (X, \tau, E) \to (Y, \theta, F)$ is soft SD-continuous and $\widetilde{A}$ is a soft open dense subset of (X, τ, E), then the restricted soft map $g_\varphi|_A: (A, \tau_A, E) \to (Y, \theta, F)$ is soft SD-continuous.*

Proof. Let (G, F) be a soft open subset of $\widetilde{Y}$. Then $g_\varphi^{-1}(G, F)$ is the null soft set or a soft somewhere dense set. If the soft set $g_\varphi^{-1}(G, F)$ is the null soft set, then the theorem holds. If the soft set $g_\varphi^{-1}(G, F)$ is soft somewhere dense, then there is a non-null soft open set (U, E) such that $(U, E)\widetilde{\subseteq}cl(g_\varphi^{-1}(G, F))$. Now,

$(U,E)\widetilde{\bigcap}\widetilde{A}\widetilde{\subseteq}cl(g_\varphi^{-1}(G,F))\widetilde{\bigcap}\widetilde{A} = cl_A(g_\varphi^{-1}(G,F)\widetilde{\bigcap}\widetilde{A})$. Since $\widetilde{A}$ is soft open dense, then $(U,E)\widetilde{\bigcap}\widetilde{A}$ is a non-null soft open set in (A,τ_A,E). Therefore $g_\varphi^{-1}(G,F)\widetilde{\bigcap}\widetilde{A}$ is a soft somewhere dense subset of (A,τ_A,E). Thus $g_\varphi \mid_A$ is soft SD-continuous. □

Corollary 3.12. *If $\widetilde{A}$ is a soft open subset of a hyperconnected space (X,τ,E) and $g_\varphi : (X,\tau,E) \to (Y,\theta,F)$ is soft SD-continuous, then $g_\varphi \mid_A$: $(A,\tau_A,E) \to (Y,\theta,F)$ is soft SD-continuous.*

Proof. From the fact that every every soft open subset of a hyperconnected space is soft dense, the above result is immediate. □

Definition 3.13. Let $h_\phi : (X,\tau,E) \to (Y,\theta,F)$ be a soft map. Then:

(i) The graph of h_ϕ, usually denoted by $G(h_\phi)$, is the subset $\{(P_e^x, h_\phi(P_e^x)) : P_e^x \in \widetilde{X}\}$ of the product soft space $X \times Y$.

(ii) The graph of h_ϕ is called soft cs-dense if it is a soft cs-dense subset of the product soft spaces $X \times Y$.

Theorem 3.14. *Let f_ϕ be a soft map of a soft hyperconnected space (X,τ,E) into (Y,θ,F) and let $g_\varphi : (X,\tau,E) \to (X \times Y, T, E \times F)$ be the graph of f_ϕ, where T is the product soft topology on $X \times Y$. Then f_ϕ is soft SD-continuous if and only if g_φ is soft SD-continuous.*

Proof. Necessity: Let $P_e^x \in \widetilde{X}$ and $g_\varphi(P_e^x) \in (W, E \times F) \in T$. Then there exist $(G,E) \in \tau$ and $(H,F) \in \theta$ such that $g_\varphi(P_e^x) = (P_e^x, f_\phi(P_e^x)) \in (G,E) \times (H,F)\widetilde{\subseteq}(W, E \times F)$. Now, $P_e^x \in (G,E)$ and $f_\phi(P_e^x) \in (H,F)$. Since f_ϕ is soft SD-continuous, then there is a soft somewhere dense subset (V,E) of $\widetilde{X}$ containing P_e^x such that $f_\phi(V,E)\widetilde{\subseteq}(H,F)$. By Theorem 2.19, $(G,E)\widetilde{\bigcap}(V,E)$ is a somewhere dense set containing P_e^x. Therefore $g_\varphi((G,E)\widetilde{\bigcap}(V,E)) = ((G,E)\widetilde{\bigcap}(V,E), f_\phi((G,E)\widetilde{\bigcap}(V,E)))\widetilde{\subseteq}((G,E), f_\phi(V,E))\widetilde{\subseteq}(G,E) \times (H,F)\widetilde{\subseteq}(W, E \times F)$. Thus g_φ is soft SD-continuous at P_e^x. Since P_e^x is an arbitrary soft point, then g_φ is soft SD-continuous.

Sufficiency: Let P_e^x be an arbitrary soft point in $\widetilde{X}$ and $f_\phi(P_e^x) \in (V,F) \in \theta$. Then $(P_e^x, f_\phi(P_e^x)) \in \widetilde{X} \times (V,F) \in T$. Since g_φ is soft SD-continuous, then there is a soft somewhere dense subset (H,E) of $\widetilde{X}$ containing P_e^x such that $g_\varphi(H,E)\widetilde{\subseteq}\widetilde{X} \times (V,F)$. Now, $g_\varphi(H,E) = ((H,E), f(H,E))$. Thus $f_\phi(H,E)\widetilde{\subseteq}(V,F)$. Hence, f_ϕ is soft SD-continuous. □

Theorem 3.15. *Let $h_\phi : (X,\tau,E) \to (Y,\theta,F)$ be a soft SD-continuous map and (Y,θ,F) be a soft T_2-space. Then the graph of h_ϕ is a soft cs-dense subset of $X \times Y$.*

Proof. Let $(P_e^x, P_f^y) \in (G(h_\phi))^c$. Then $P_f^y \neq h_\phi(P_e^x)$. Since (Y,θ,F) is a soft T_2-space, then there exist two disjoint soft open subsets (H,F) and (W,F) of $\widetilde{Y}$ containing P_f^y and $h_\phi(P_e^x)$, respectively. By hypothesis, h_ϕ is soft SD-continuous. Then there is a soft somewhere dense subset (U,E) containing P_e^x such that $h_\phi(U,E)\widetilde{\subseteq}(W,F)$. From Theorem 2.21, $(U,E) \times (H,F)$ is a soft somewhere dense set. Since $h_\phi(U,E)\widetilde{\bigcap}(H,F) = \widetilde{\Phi_Y}$, then $((U,E) \times (H,F)) \bigcap G(h_\phi) = \widetilde{\Phi_{X \times Y}}$. Therefore $((U,E) \times (H,F))\widetilde{\subseteq}(G(h_\phi))^c$. Since a soft point (P_e^x, P_f^y) is chosen arbitrarily, then $(G(h_\phi))^c$ is a soft somewhere dense subset of $X \times Y$. This completes the proof. □

Definition 3.16. A soft map $g_\varphi : (X,\tau,E) \to (Y,\theta,F)$ is said to be soft SD-irresolute provided that the inverse image of each soft somewhere dense set is the null soft set or a soft somewhere dense set.

Proposition 3.17. *Every soft SD-irresolute map is soft SD-continuous.*

Proof. The proposition follows from the fact that every soft open set is soft somewhere dense. □

We present the next example to illustrate that the above proposition is not conversely.

Example 3.18. Let (X, τ, E) be the same as in Example 3.7. Consider the following soft sets over X under a parameters set E given as follows:

$(H_1, E) = \{(e_1, X), (e_2, \{1\})\}$;
$(H_2, E) = \{(e_1, \{2, 3\}), (e_2, \{1\})\}$ and
$(H_3, E) = \{(e_1, X), (e_2, \{1\})\}$.

Then $\theta = \{\widetilde{\Phi}, \widetilde{X}, (H_i, E) : i = 1, 2, 3\}$ is another soft topology on X. Let a soft map $g_\varphi : (X, \tau, E) \to (X, \theta, E)$ defined as follows:

$$g_\varphi(P_e^x) = P_e^x, \text{ for each } P_e^x \in \widetilde{X}.$$

It is clear that g_φ is a soft SD-continuous map. On the other hand, $(H, E) = \{(e_1, \{2, 3\}), (e_2, \emptyset)\}$ is a soft somewhere dense subset of (X, θ, E). Now, $g_\varphi^{-1}(H, E) = (H, E)$. In (X, τ, E), we find that $int[cl(H, E)] = \widetilde{\Phi}$. So that, (H, E) is not a soft somewhere dense subset of (X, τ, E). Hence, g_φ is not a soft SD-irresolute map.

The proof of the following theorem is similar to that of Theorem 3.9.

Theorem 3.19. *For a soft map $g_\varphi : (X, \tau, E) \to (Y, \theta, F)$, the following statements are equivalent:*

(*i*) *g_φ is soft SD-irresolute.*

(*ii*) *The inverse image of each soft cs-dense subset of (Y, θ, F) is the absolute soft set or a soft cs-dense set.*

(*iii*) *$Scl(g_\varphi^{-1}(A, F)) \widetilde{\subseteq} g_\varphi^{-1}(Scl(A, F))$ for each $(A, F) \widetilde{\subseteq} \widetilde{Y}$.*

(*iv*) *$g_\varphi(Scl(H, E)) \widetilde{\subseteq} Scl(g_\varphi(H, E))$ for each $(H, E) \widetilde{\subseteq} \widetilde{X}$.*

(*v*) *$g_\varphi^{-1}(Sint(A, F)) \widetilde{\subseteq} Sint(g_\varphi^{-1}(A, F))$ for each $(A, F) \widetilde{\subseteq} \widetilde{Y}$.*

Theorem 3.20. *A soft map $g_\varphi : (X, \tau, E) \to (Y, \theta, F)$ is soft SD-irresolute if one of the following conditions holds.*

(*i*) *$cl(g_\varphi^{-1}(K, F)) \widetilde{\subseteq} g_\varphi^{-1}(Scl(K, F))$ for each $(K, F) \widetilde{\subseteq} \widetilde{Y}$.*

(*ii*) *$g_\varphi(cl(H, E)) \widetilde{\subseteq} Scl(g_\varphi(H, E))$ for each $(H, E) \widetilde{\subseteq} \widetilde{X}$.*

(*iii*) *$g_\varphi^{-1}(Sint(K, F)) \widetilde{\subseteq} int(g_\varphi^{-1}(K, F))$ for each $(K, F) \widetilde{\subseteq} \widetilde{Y}$.*

Proof. (i) It is clear that $Scl(K, F) \widetilde{\subseteq} cl(K, F)$ for each $(K, F) \widetilde{\subseteq} \widetilde{Y}$. If the condition **(i)** holds, then $Scl(g_\varphi^{-1}(K, F)) \widetilde{\subseteq} cl(g_\varphi^{-1}(K, F)) \widetilde{\subseteq} g_\varphi^{-1}(Scl(K, F))$. By (iii) of Theorem 3.19, we have g_φ is soft SD-irresolute.

(ii) It is clear that $Scl(H, E) \widetilde{\subseteq} cl(H, E)$ for each $(H, E) \widetilde{\subseteq} \widetilde{X}$. If the condition (ii) holds, then $g_\varphi(Scl(H, E)) \widetilde{\subseteq} g_\varphi(cl(H, E)) \widetilde{\subseteq} Scl(g_\varphi(H, E))$. By (iv) of Theorem 3.19, we have g_φ is soft SD-irresolute.

(iii) It is clear that $int(K, F) \widetilde{\subseteq} Sint(K, F)$ for each $(K, F) \widetilde{\subseteq} \widetilde{Y}$. If the condition (iii) holds, then $g_\varphi^{-1}(Sint(K, F)) \widetilde{\subseteq} int(g_\varphi^{-1}(K, F)) \widetilde{\subseteq} Sint(g_\varphi^{-1}(K, F))$. By (v) of Theorem 3.19, we have g_φ is soft SD-irresolute. □

To see that the above theorem is not conversely, we present the following example.

Example 3.21. Let $E = \{e_1, e_2\}$ be a parameters set. Consider the two soft sets $(U, E), (G, E)$ over $X = \{1, 2\}$ and a soft set (H, E) over $Y = \{a, b\}$ given as follows:

$(U, E) = \{(e_1, X), (e_2, \emptyset)\}$;
$(G, E) = \{(e_1, \emptyset), (e_2, X)\}$ and
$(H, E) = \{(e_1, \emptyset), (e_2, \{b\})\}$.

Then $\tau = \{\widetilde{\Phi}, \widetilde{X}, (U, E), (G, E)\}$ and $\theta = \{\widetilde{\Phi}, \widetilde{Y}, (H, E)\}$ are soft topologies on X and Y, respectively. Let a soft map $g_\varphi : (X, \tau, E) \to (Y, \theta, E)$ be defined as follows:

$$g_\varphi(P_{e_1}^1) = P_{e_1}^a,\ g_\varphi(P_{e_2}^1) = g_\varphi(P_{e_1}^2) = P_{e_1}^b \text{ and } g_\varphi(P_{e_2}^2) = P_{e_2}^b.$$

Then g_φ is soft SD-irresolute, whereas the three conditions which mentioned in the above theorem are not satisfied as pointed out in the following:

(i) Let $(N,E) = \{(e_1, Y), (e_2, \emptyset)\}$. Then $cl(g_\varphi^{-1}(N,E)) = \widetilde{X}$ and $g_\varphi^{-1}(Scl(N,E)) = \{(e_1, X), (e_2, \{1\})\}$. Therefore $cl(g_\varphi^{-1}(N,E)) \nsubseteq g_\varphi^{-1}(Scl(N,E))$.

(ii) Let $(M,E) = \{(e_1, \{2\}), (e_2, \{1\})\}$. Then $g_\varphi(cl(M,E)) = \{(e_1, Y), (e_2, \{b\})\}$ and $Scl(g_\varphi(M,E)) = \{(e_1, \{b\}), (e_2, \emptyset)\}$. Therefore $g_\varphi(cl(M,E)) \nsubseteq Scl(g_\varphi(M,E))$.

(iii) Let $(O,E) = \{(e_1, \{a\}), (e_2, \{b\})\}$. Then $g_\varphi^{-1}(Sint(O,E)) = \{(e_1, \emptyset), (e_2, \{2\})\}$ and $int(g_\varphi^{-1}(O,E)) = \widetilde{\Phi_X}$. Therefore $g_\varphi^{-1}(Sint(O,E)) \nsubseteq int(g_\varphi^{-1}(O,E))$.

4. Soft *SD*-Homeomorphism Maps

We devote this section to introduce and to discuss the concepts of soft *SD*-open, soft *SD*-closed and soft *SD*-homeomorphism maps, where *SD* denotes "somewhere dense". We explore their main properties, in particular, we study under what conditions the restriction of soft *SD*-open (resp. soft *SD*-closed) is also soft *SD*-open (resp. soft *SD*-closed).

Definition 4.1. A soft map $g_\varphi : (X, \tau, E) \to (Y, \theta, F)$ is said to be:

(i) soft *SD*-open provided that the image of each non-null soft open set is soft somewhere dense.

(ii) soft *SD*-closed provided that the image of each soft closed set is the absolute soft set or a soft cs-dense set.

In the above definition, we define a soft *SD*-open map with respect to all soft open sets except for the null soft set because the image of the null soft set is itself which is not soft somewhere dense. This procedure is necessary to guarantee the existence such soft maps. On the other hand, we require the absolute soft set as a probability image for some soft closed sets because the image of the absolute soft set under a surjective soft map is itself which is not soft *cs*-dense. This procedure is necessary to guarantee the existence of a surjective soft *SD*-closed map. Furthermore, these omitting and adding keep the systematic relations among different types of generalized soft open and closed maps and soft *SD*-open and *SD*-closed maps, see, Proposition 4.7.

Theorem 4.2. *A soft map $g_\varphi : (X, \tau, E) \to (Y, \theta, F)$ is soft SD-open if and only if $g_\varphi(int(K,E)) \widetilde{\subseteq} Sint(g_\varphi(K,E))$ for every subset (K,E) of $\widetilde{X}$.*

Proof. Necessity: Assume that g_φ is a soft *SD*-open map and let (K,E) be a soft subset of $\widetilde{X}$. Then we have two cases:

1. Either $int(K,E) = \widetilde{\Phi_X}$. Then the necessary part holds.

2. Or $int(K,E) \neq \widetilde{\Phi_X}$. Then $g_\varphi(int(K,E))$ is a soft somewhere dense set. Since $g_\varphi(int(K,E)) \subseteq g_\varphi(K,E)$, then $g_\varphi(int(K,E)) \subseteq Sint(g_\varphi(K,E))$.

Sufficiency: Assume that (K,E) is a non-null soft open subset of $\widetilde{X}$. Then $g_\varphi(int(K,E)) = g_\varphi(K,E) \widetilde{\subseteq} Sint(g_\varphi(K,E))$. Therefore $g_\varphi(K,E) = Sint(g_\varphi(K,E))$. Thus g_φ is soft *SD*-open. □

Theorem 4.3. *A soft map $g_\varphi : (X, \tau, E) \to (Y, \theta, F)$ is soft SD-closed if and only if $Scl(g(K,E)) \widetilde{\subseteq} g(cl(K,E))$ for each soft subset (K,E) of $\widetilde{X}$.*

Proof. Necessity: Assume that g_φ is a soft *SD*-closed map and let (K,E) be a soft subset of $\widetilde{X}$. Since $cl(K,E)$ is a soft closed set, then $g_\varphi(cl(K,E))$ is the absolute soft set or a soft *cs*-dense set. In both cases, we obtain $Scl(g(K,E)) \widetilde{\subseteq} g(cl(K,E))$.

Sufficiency: Assume that (K,E) is a soft closed subset of $\widetilde{X}$. By hypothesis, $g_\varphi(K,E) \widetilde{\subseteq} Scl(g_\varphi(K,E)) \widetilde{\subseteq} g_\varphi(cl(K,E)) = g_\varphi(K,E)$. Therefore $g_\varphi(K,E) = Scl(g_\varphi(K,E))$. This shows that $g_\varphi(K,E)$ is the absolute soft set or a soft cs-dense set. So g_φ is soft *SD*-closed. □

Proposition 4.4. *A bijective soft map $g_\varphi : (X, \tau, E) \to (Y, \theta, F)$ is soft SD-open if and only if it is soft SD-closed.*

Proof. To prove the *'if'* part, let (G, E) be a soft closed subset of (X, τ, E). Since g_φ is soft SD-open, then $g_\varphi(G^c, E)$ is the null soft set or a soft somewhere dense set. Since g_φ is bijective, then $g_\varphi(G^c, E) = (g_\varphi(G, E))^c$. So that, $g_\varphi(G, E)$ is the absolute soft set or a soft *cs*-dense set. Hence, g_φ is soft SD-closed.
The proof is similar for the *'only if'* part. □

Proposition 4.5. *Let $g_\varphi : (X, \tau, E) \to (Y, \theta, F)$ be a soft SD-closed map and $\widetilde{A}$ be a soft closed subset of $\widetilde{X}$. Then $g_\varphi |_A : (A, \tau_A, E) \to (Y, \theta, F)$ is soft SD-closed.*

Proof. Suppose that (H, E) is a soft closed subset of (A, τ_A, E). Then there exists a soft closed subset (L, E) of (X, τ, E) such that $(H, E) = (L, E) \widetilde{\bigcap} \widetilde{A}$. Since $\widetilde{A}$ is a soft closed subset of (X, τ, E), then (H, E) is also a soft closed subset of (X, τ, E). Since $g_\varphi |_A (H, E) = g_\varphi(H, E)$, then $g_\varphi |_A (H, E)$ is the absolute soft set or a soft cs-dense set. Thus, $g_\varphi |_A$ is a soft SD-closed map. □

The proof of the following proposition is omitted because it is similar to that of Proposition 4.5.

Proposition 4.6. *Let $g_\varphi : (X, \tau, E) \to (Y, \theta, F)$ be a soft SD-open map and $\widetilde{A}$ be a soft open subset of $\widetilde{X}$. Then $g_\varphi |_A : (A, \tau_A, E) \to (Y, \theta, F)$ is soft SD-open.*

The proof of the next proposition is easy and thus it is omitted.

Proposition 4.7. *Every soft j-open (resp. soft j-closed) map is soft SD-open (resp. soft SD-closed) for each $j \in \{\beta, b, semi, pre, \alpha\}$.*

Proposition 4.8. *The next four statements hold for the two soft maps $g_\varphi : (X, \tau, E) \to (Y, \theta, F)$ and $h_\psi : (Y, \theta, F) \to (Z, \sigma, T)$.*

(i) *If g_φ is soft i-open for $i = \{\alpha, semi\}$ and h_ψ is soft j-open for $j = \{\beta, b, semi, pre, \alpha\}$, then $h_\psi \circ g_\varphi$ is soft SD-open.*

(ii) *If $h_\psi \circ g_\varphi$ is soft SD-open and g_φ is surjective soft continuous, then h_ψ is soft SD-open.*

(iii) *If $h_\psi \circ g_\varphi$ is soft open and h_ψ is injective soft SD-continuous, then g_φ is soft SD-open.*

(iv) *If $h_\psi \circ g_\varphi$ is soft SD-open and h_ψ is injective soft SD-irresolute map, then g_φ is soft SD-open.*

Proof. (i) We merely prove the proposition in the cases of $i = semi$ and $j = \beta$, and the other cases follow similarly. In doing so, let $(G, E) \neq \widetilde{\Phi_X}$ be a soft open subset of $\widetilde{X}$. Then $g_\varphi(G, E) \neq \widetilde{\Phi_Y}$ is a soft semi open subset of $\widetilde{Y}$. Therefore there is a non-null soft open subset (L, F) of $\widetilde{Y}$ such that $(L, F) \subseteq g_\varphi(G, E)$. Now, $h_\psi(L, F) \subseteq h_\psi(g_\varphi(G, E))$. Since h_ψ is soft β-open, then $h_\psi(L, F)$ is a non-null soft β-open subset of $\widetilde{Z}$. Therefore $h_\psi(L, F)$ is soft somewhere dense. This automatically means that $h_\psi(g_\varphi(G, E))$ is a soft somewhere dense set. Thus. $h_\psi \circ g_\varphi$ is SD-open.

(ii) Suppose that $(G, F) \neq \widetilde{\Phi_Y}$ is a soft open subset of $\widetilde{Y}$. Then $g_\varphi^{-1}(G, F) \neq \widetilde{\Phi_X}$ is a soft open subset of $\widetilde{X}$. Therefore $(h_\psi \circ g_\varphi)(g_\varphi^{-1}(G, F))$ is a soft somewhere dense subset of $\widetilde{Z}$. Since g_φ is surjective, then $(h_\psi \circ g_\varphi)(g_\varphi^{-1}(G, F)) = h_\psi(g_\varphi(g_\varphi^{-1}(G, F))) = h_\psi(G, F)$. Thus h_ψ is a soft SD-open map.

(iii) Let $(G, E) \neq \widetilde{\Phi_X}$ be a soft open subset of $\widetilde{X}$. Then $(h_\psi \circ g_\varphi)(G, E) \neq \widetilde{\Phi_Z}$ is a soft open subset of $\widetilde{Z}$. Therefore $h_\psi^{-1}(h_\psi \circ g_\varphi(G, E))$ is soft somewhere dense. Since h_ψ is injective, then $h_\psi^{-1}(h_\psi \circ g_\varphi(G, E)) = (h_\psi^{-1} h_\psi)(g_\varphi(G, E)) = g_\varphi(G, E)$. Thus, g_φ is a soft SD-open map.

(iv) The proof is similar to that of (iii). □

The proof of the next proposition is similar with the proof of the above proposition.

Proposition 4.9. *The following four statements hold for the soft maps $f_\phi : (X, \tau, E) \to (Y, \theta, F)$ and $g_\varphi : (Y, \theta, F) \to (Z, \sigma, T)$.*

(*i*) *If f_ϕ is soft i-closed for $i = \{\alpha, semi\}$ and g_φ is soft j-closed for $j = \{\beta, b, semi, pre, \alpha\}$, then $g_\varphi \circ f_\phi$ is soft SD-open.*

(*ii*) *If $g_\varphi \circ f_\phi$ is soft SD-closed and f_ϕ is surjective soft continuous, then g_φ is soft SD-closed.*

(*iii*) *If $g_\varphi \circ f_\phi$ is soft closed and g_φ is injective soft SD-continuous, then f_ϕ is soft SD-closed.*

(*iv*) *If $g_\varphi \circ f_\phi$ is soft SD-closed and g_φ is injective soft SD-irresolute map, then f_ϕ is soft SD-closed.*

Definition 4.10. A bijective soft map g_φ in which is soft *SD*-continuous and soft *SD*-open is called a soft *SD*-homeomorphism.

Theorem 4.11. *For a bijective soft map $g_\varphi : (X, \tau, E) \to (Y, \theta, F)$, the following properties are equivalent:*

(*i*) *g_φ is a soft SD-homeomorphism.*

(*ii*) *g_φ and g_φ^{-1} is soft SD-continuous.*

(*iii*) *g_φ is soft SD-closed and soft SD-continuous.*

Proof. Straightforward. □

Theorem 4.12. *A bijective soft map $g_\varphi : (X, \tau, E) \to (Y, \theta, F)$ is a soft SD-homeomorphism if and only if one of the following conditions holds.*

(*i*) *$g_\varphi(Scl(G, E)) \widetilde{\subseteq} cl(g_\varphi(G, E))$ and $Scl(g_\varphi(G, E)) \widetilde{\subseteq} g_\varphi(cl(G, E))$ for each $(G, E) \widetilde{\subseteq} \widetilde{X}$.*

(*ii*) *$g_\varphi(int(G, E)) \widetilde{\subseteq} Sint(g_\varphi(G, E))$ and $g_\varphi^{-1}(int(L, F)) \widetilde{\subseteq} Sint(g_\varphi^{-1}(L, F))$ for each $(G, E) \widetilde{\subseteq} \widetilde{X}$ and $(L, F) \widetilde{\subseteq} \widetilde{Y}$.*

Proof. (i) Since $g_\varphi(Scl(G, E)) \widetilde{\subseteq} cl(g_\varphi(G, E))$, then it follows from (iii) of Theorem 3.9 that g_φ is *SD*-continuous and since $Scl(g_\varphi(G, E)) \widetilde{\subseteq} g_\varphi(cl(G, E))$, then it follows from Theorem 4.3 that g_φ is *SD*-closed. From Theorem 4.11, we obtain g_φ is a soft *SD*-homeomorphism.

(ii) Since $g_\varphi(int(G, E)) \widetilde{\subseteq} Sint(g_\varphi(G, E))$, then it follows from (v) of Theorem 3.9 that g_φ is *SD*-continuous and since $g_\varphi^{-1}(int(L, F)) \widetilde{\subseteq} Sint(g_\varphi^{-1}(L, F))$, then it follows from Theorem 4.2 that g_φ is *SD*-open. From Definition 4.10, we obtain g_φ is a soft *SD*-homeomorphism. □

5. Conclusion

In 2018, Al-shami [6] enlarged the class of soft β-open sets by introducing a class of soft somewhere dense sets. This article completes that study by defining new types of soft maps, namely soft *SD*-continuous, soft *SD*-open, soft *SD*-closed and soft *SD*-homeomorphism maps. Their definitions are formulated depend on the soft somewhere dense and soft *cs*-dense sets. To preserve some symmetric relations between these concepts, some additional conditions are imposed on their definitions. In general, we characterize these concepts and establish some results related to graph and restriction of soft maps under a condition of soft hyperconnectedness. Some examples are furnished to show our obtained results.

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