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Soft Modes Associated with Chiral Symmetry Breaking

—— *The Use of a QCD-Motivated Effective Interaction* ——

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Possible critical phenomena associated with chiral symmetry breaking and restoration are studied in the mean field approximation with the use of a QCD-motivated effective Lagrangian. Collective excitations in a system with zero temperature and zero chemical potential are examined extensively: It is shown that (i) there exists a precursory mode in the Wigner phase, which softens as the system approaches the critical point for chiral symmetry breaking, and (ii) the σ-meson mode in the Nambu-Goldstone (NG) phase softens being associated with symmetry restoration. The observabilities of these critical phenomena in experiment such as ultra-relativistic heavy ion collisions are discussed. We also calculate the critical temperature T_χ and critical chemical potential μ_χ where chiral symmetry gets restored and we obtain T_χ $=164$ MeV for ($N_c=3$, $N_f=2$)-case without free parameters.

§ 1. Introduction

As is implied by the asymptotic freedom of QCD and is explicitly shown by the lattice Monte-Carlo simulations,[1] the chirally symmetric phase (the Wigner phase) is sure to be realized in the systems with large-momentum scale, at high temperature and/or high density; for example, the interior of the hadron bags, the intermediate stages of ultra-relativistic heavy ion collisions and the deconfined quark matter in the early universe. In this paper, we investigate the critical phenomena associated with chiral symmetry breaking (CSB); it should provide us with some characteristic features of such systems.

A study on this subject has been briefly reported by the present authors in Ref. 2): They have shown the existence of soft modes*) in the Wigner phase, which are the precursory modes of CSB, by using the Nambu-Jona-Lasinio[3] (NJL) type of effective Lagrangian. In this paper, we examine the nature of the modes extensively and show that the existence of the soft modes is a direct reflection of an enhancement of the long range correlations of the specific pair operators in the Wigner phase. Thus one finds that the softening is a characteristic feature of the systems which are near the critical point of CSB. A critical phenomenon seen in the system in the Nambu-Goldstone (NG) phase near the critical point has been discussed in a phenomenological analysis[6] (see also Ref. 7)). We will also investigate such critical phenomena associated with the restoration of chiral symmetry (CS).

Although it is now controversial what is the most essential mechanism of chiral transition in QCD, there have been some proposals for it; the perturbative gluon ex-

*) Soft modes are seen in some fields of physics. A soft mode for the magnetic phase transition to the ferromagnet is known as the paramagnon, which is observed as a bump of the cross section of the neutron scattering in the low energy transfer region.[4] The pairing vibration in nuclei in normal phase, which causes an enhancement of the cross section of a two-particle transfer reaction is a soft mode for the phase transition to the superconducting phase.[5]

change,[8] the instanton effect,[9] the confining force,[10] the large N_c limit[11] and the strong coupling limit.[12] In this paper, we adopt the following Lagrangian as an effective one which works well in the intermediate length scale between the asymptotic free and confinement region,

$$\mathcal{L}=\mathcal{L}_0+\mathcal{L}_1\,, \tag{1·1a}$$

$$\mathcal{L}_0=\sum_i \bar{\psi}_i{}^a(i\gamma\cdot\partial-m_i)\psi_i{}^a\,, \tag{1·1b}$$

$$\mathcal{L}_1=2K[\det\{\bar{\psi}_i{}^a(1+\gamma_5)\psi_j{}^a\}+\text{h.c.}]\,, \tag{1·1c}$$

where $a=1,\cdots,N_c$ is a color index, $i,j=1,\cdots,N_f$ are flavor indices, m_i's are the current mass of quarks, and det denotes a determinant with respect to the flavor indices. The interaction $\mathcal{L}_1$, which has chiral $SU(N_f)\otimes SU(N_f)$ symmetry, has a form of a local approximation to the instanton-induced effective interaction[15),16),*)] and is also derived from the chiral Lagrangian incorporating the $U_A(1)$ anomaly term.[17] We will show that the following two-scale picture for CSB is consistent with our effective Lagrangian: A chiral invariant force in the intermediate length scale is responsible for the dynamical breaking of CS and the effect of confinement can be treated as a small perturbation in this region.[**]

In the following, we confine ourselves to the $N_f=2$ case. Then, $\mathcal{L}_1$ can be rewritten in a form of an NJL type interaction,[***]

$$\mathcal{L}_1=K[(\bar{\psi}\psi)^2+(\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)^2-(\bar{\psi}\boldsymbol{\tau}\psi)^2-(\bar{\psi}i\gamma_5\psi)^2]\,, \tag{1·1d}$$

where color and flavor indices are suppressed. In our model, there are two free parameters; the coupling constant K and a momentum-cutoff Λ which must be introduced because of the nonrenormalizability of the interaction. Hereafter, we use a dimensionless quantity $a_{\text{eff}}=gN_cN_f\Lambda^2/(2\pi^2)$ instead of K, with g being $K(1+1/(2N_c))$. The cutoff procedure implies that the interaction has an effective vertex,

$$\mathcal{V}(\Lambda^{-1};p,q)=a_{\text{eff}}\theta(2\Lambda-p)\theta(2\Lambda-q) \tag{1·2}$$

in the momentum space, where θ is a step function and (p,q) are the total and relative momentum of the interacting quarks. (The effective vertex may be considered to simulate the instanton induced vertex[9a)] by the step function.) Then Λ has the following physical meaning; only the two particles with the relative momentum smaller than 2Λ can contribute to the dynamical breaking of CS. Λ and dynamically generated mass of quarks will be determined to be about 1 GeV and 240 MeV respectively by putting the experimental values of the pion decay constant f_π and pion mass m_π into the calculated physical quantities. Here we note that the obtained value for Λ is consistent with the picture mentioned above; the typical scale of CSB ($\Lambda\simeq$1GeV) is greater than that of

*) It has been explicitly shown that instanton-induced nonlocal interaction is sufficiently strong to break the CS and produces the Nambu-Goldstone bosons.[9b)] For the relation between our effective Lagrangian and the $U_A(1)$ problem, see the review[13)] and Ref. 14).

**) A phenomenological justification of this picture is made in Ref. 18) and the consistency with the non-relativistic quark model and the low-energy chiral Lagrangian is argued in Ref. 18a~c). The instanton approach also gives this picture.[9a)]

***) Possible modifications of our Lagrangian are discussed in the Appendix.

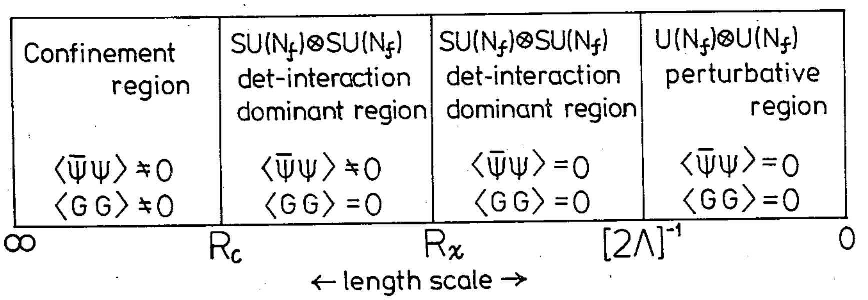


Fig. 1. A schematic figure which represents our picture. R_c and R_χ are the characteristic length scale of confinement and chiral symmetry breaking. Λ is the cutoff of our effective interaction. The $SU(N_f)\otimes SU(N_f)$ determinantial interaction denotes Eqs.(1·1 a~c) in the text.

confinement ($\Lambda_{\rm QCD}\simeq 200$ MeV). This result is also consistent with the other analyses of CSB.[8a)~d),9a),18)] For definiteness, the relevant regions to this paper are shown schematically in Fig. 1.

In this paper, we deal with the deconfined vacuum with zero temperature ($T=0$) and zero chemical potential ($\mu=0$). We assume that the cutoff Λ is independent of the phases of the system and investigate the collective excitations in the system by varying the dimensionless coupling constant $\alpha_{\rm eff}$ by hand. The qualitative features of the results obtained by this prescription survive when we perform the calculation at finite T and μ with fixed $\alpha_{\rm eff}$.[19)] It may be noticed here that, in our model interaction (1·1d), chiral transition is second order at least within the Hartree-Fock approximation. Although there is a possibility that the order of the transition becomes first order if we consider the higher order correlations[20)] or the larger number of flavors,[21)] the soft modes are expected to exist for the weak first order transitions.[19)]

This paper is composed as follows. In § 2, we give field equations of the collective excitations on the basis of the self-consistent mean field theory. Section 3 is devoted to the determination of Λ and the consistency check mentioned above. In § 4, collective excitations in the Wigner phase are analyzed. It will be shown that the very precursory modes with $J^P=0^\pm$ exist and they cause an enhancement of the total cross section of the q-$\bar{q}$ annihilation process or the strength function. In § 5, collective excitations in the NG phase are analyzed and we discuss a typical critical phenomenon associated with the restoration of CS. In § 6, we examine how the current mass of quarks modifies the results obtained in the preceding sections, and also discuss the critical temperature and chemical potential of chiral transition.

§ 2. Preliminaries

In this section, using a self-consistent mean field (SCMF) method,[22)] we give necessary equations and expressions for the later sections.

In the SCMF method, the system is approximated by an assembly of non-interacting quasi-particles moving in the MF's which are generated self-consistently by the particles. Let $|C\rangle\equiv|[\sigma, \boldsymbol{\pi}, \eta, \boldsymbol{\delta}, F^0_{\mu\nu}, \boldsymbol{F}^1_{\mu\nu}]\rangle$ be a state where the Lorentz scalar $\bar{\psi}\psi(\bar{\psi}\boldsymbol{\tau}\psi)$, pseudo scalar $\bar{\psi}i\gamma_5\psi(\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)$ and anti-symmetric tensor $\bar{\psi}\sigma_{\mu\nu}\psi(\bar{\psi}\sigma_{\mu\nu}\boldsymbol{\tau}\psi)$ operators have the following expectation values

$$-\mu^2/G\cdot\begin{bmatrix}\sigma(x)\\ \boldsymbol{\pi}(x)\\ \eta(x)\\ \boldsymbol{\delta}(x)\\ F^0_{\mu\nu}(x)\\ \boldsymbol{F}^1_{\mu\nu}(x)\end{bmatrix}=\langle C|\bar{\psi}(x)\begin{bmatrix}1\\ i\gamma_5\boldsymbol{\tau}\\ i\gamma_5\\ \boldsymbol{\tau}\\ \sigma_{\mu\nu}/\sqrt{4N_c+2}\\ \sigma_{\mu\nu}\boldsymbol{\tau}/\sqrt{4N_c+2}\end{bmatrix}\psi(x)|C\rangle\,, \quad (2\cdot1)$$

where μ with a mass dimension has been introduced to make the MF's σ, $\boldsymbol{\pi}$, η, $\boldsymbol{\delta}$ have unit mass dimension and G is a dimensionless quantity defined by

$$G^2/\mu^2=2K(1+1/2N_c)\equiv 2g\,. \quad (2\cdot2)$$

In the mean-field approximation (MFA), the Lagrangian (1·1) becomes

$$\mathcal{L}\to\mathcal{L}_{\mathrm{MFA}}=\bar{\psi}[i\gamma\cdot\partial-\hat{m}-G\{\sigma+i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}-i\gamma_5\eta-\boldsymbol{\tau}\cdot\boldsymbol{\delta}$$
$$+(\sigma^{\mu\nu}F^0_{\mu\nu}-\sigma^{\mu\nu}\boldsymbol{\tau}\cdot\boldsymbol{F}^1_{\mu\nu})/\sqrt{4N_c+2})\}]\psi$$
$$-\mu^2/2\cdot(\sigma^2+\boldsymbol{\pi}^2-\eta^2-\boldsymbol{\delta}^2+F^{02}_{\mu\nu}-\boldsymbol{F}^{12}_{\mu\nu})\,, \quad (2\cdot3)$$

where we have assumed $m_u=m_d=\hat{m}$. We note that the Hartree and Fock terms of the interaction produce the scalar and pseudo scalar fields, while only the Fock terms which are next to leading in the $1/N_c$-expansion[22] produce the vector fields ($F^0_{\mu\nu}$ and $\boldsymbol{F}^1_{\mu\nu}$).

Let us define the Green's function $S_F(x, y; C)$ by

$$S_F(x,y;C)=-i\langle C|T\psi(x)\bar{\psi}(y)|C\rangle\,, \quad (2\cdot4)$$

and also define the following notation for MF's and matrices:

$$B_\alpha(x)=(\sigma(x),\,\boldsymbol{\pi}(x),\,\eta(x),\,\boldsymbol{\delta}(x),\,F^0_{\mu\nu}(x),\,\boldsymbol{F}^1_{\mu\nu}(x)) \quad (2\cdot5\mathrm{a})$$

and

$$\Gamma_\alpha=(1,\,i\gamma_5\boldsymbol{\tau},\,i\gamma_5,\,\boldsymbol{\tau},\,\sigma_{\mu\nu}/\sqrt{4N_c+2},\,\sigma_{\mu\nu}\boldsymbol{\tau}/\sqrt{4N_c+2})\,, \quad (2\cdot5\mathrm{b})$$

where Lorentz and isospin structures are labeled by α. Then (2·1) (we call it the self-consistency condition (SCC)) reads

$$-\mu^2/G\cdot B_\alpha(x)=-i\lim_{y\to x^+}\mathrm{Tr}^{(cfs)}[\Gamma_\alpha S_F(x,y;C)]\,, \quad (2\cdot6)$$

where $\mathrm{Tr}^{(cfs)}$ denotes the trace taken over the color, flavor and spin indices. It can be shown that (i) the SCC (2·1) or (2·6) is reduced to the "gap equation" which determines the dynamical quark mass M when $|C\rangle$ is the true vacuum (ground state), and (ii) SCC is reduced to the field equations which govern the space-time dependence of the MF's when $|C\rangle$ is a collective state.

If $|C\rangle$ is the ground state, one can put

$$\sigma(x)=\text{constant}\equiv\sigma_M \text{ and other MF's}=0\,, \quad (2\cdot7)$$

on account of the symmetry properties of the state. If we express $|C\rangle$ as $|\sigma_M\rangle$, the gap equation reads

$$-\mu^2/G\cdot\sigma_M=\langle\sigma_M|\bar{\psi}\psi|\sigma_M\rangle\,, \tag{2·8a}$$

$$=-2iN_c\mathrm{Tr}^{(s)}\int\frac{d^4p}{(2\pi)^4}\frac{1}{\not p-M+i\epsilon}\,, \tag{2·8b}$$

$$=2N_cM\Lambda^2I(M^2/\Lambda^2)\,, \tag{2·8c}$$

where

$$I(M^2/\Lambda^2)=-(1/4\pi^2)[1-x^2\ln(1+x^{-2})]_{x=M/\Lambda}\,, \tag{2·8d}$$

and M is the sum of current mass and dynamical mass; $M=M_D+\hat{m}$ with $M_D=G\sigma_M$. Here we have used an invariant cutoff at Λ. If we neglect the current quark mass $\hat{m}$, a simple calculation shows that for $a_{\mathrm{eff}}\leq 1$, $\sigma_M=0$ (the vacuum is in the Wigner phase), and for $a_{\mathrm{eff}}>1$, σ_M is nonvanishing (chiral symmetry is dynamically broken, hence the vacuum is in the NG phase).[22] For the case $\hat{m}\neq 0$, see § 6.1.

Collective excitations in the system can be described by the space-time dependent MF's fluctuating around the vacuum expectation values. In the small amplitude approximation for the MF's, the SCC becomes

$$-\mu^2/G\cdot\tilde{B}_\alpha(x)=-i\lim_{y\to x^+}\mathrm{Tr}^{(cfs)}[\Gamma_\alpha\int d^4zS_F(x-z;M)\mathcal{M}(z;\sigma_M)S_F(z-y;M)]\,, \tag{2·9a}$$

where

$$\tilde{B}_\alpha(x)\equiv(s(x),\boldsymbol{\pi}(x),\eta(x),\boldsymbol{\delta}(x),F^0_{\mu\nu}(x),\boldsymbol{F}^1_{\mu\nu}(x))\,, \tag{2·9b}$$

$$\begin{aligned}\mathcal{M}(z;\sigma_M)=G[&s(z)+i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}(z)-i\gamma_5\eta(z)-\boldsymbol{\tau}\cdot\boldsymbol{\delta}(z)\\&+\sigma^{\mu\nu}(F^0_{\mu\nu}(z)-\boldsymbol{\tau}\cdot\boldsymbol{F}^1_{\mu\nu}(z))/\sqrt{4N_c+2}]\end{aligned} \tag{2·9c}$$

with $s(z)=\sigma(z)-\sigma_M$ and

$$S_F(x-y;M)=\int\frac{d^4p}{(2\pi)^4}\frac{e^{-ip\cdot(x-y)}}{\not p-M+i\epsilon}\,. \tag{2·9d}$$

A simple evaluation of the r. h. s. of (2·9a) gives the following equations for the respective modes,

$$[1\pm 4gN_cJ_s(-\partial^2)]\begin{bmatrix}s(x)\\ \boldsymbol{\delta}(x)\end{bmatrix}=0\,, \tag{2·10a}$$

$$[1\pm 4gN_cJ_{ps}(-\partial^2)]\begin{bmatrix}\boldsymbol{\pi}(x)\\ \eta(x)\end{bmatrix}=0\,, \tag{2·10b}$$

$$[1\pm 4gN_c/(2N_c+1)\cdot J_V(-\partial^2)]\begin{bmatrix}\boldsymbol{V}_\mu{}^1(x)\\ V_\mu{}^0(x)\end{bmatrix}=0\,, \tag{2·10c}$$

$$[1\pm 4gN_c/(2N_c+1)\cdot J_A(-\partial^2)]\begin{bmatrix}\boldsymbol{A}_\mu{}^1(x)\\ A_\mu{}^0(x)\end{bmatrix}=0\,, \tag{2·10d}$$

where

$$J_k(q^2)=-4i\int_0^1 d\alpha\int\frac{d^4p}{(2\pi)^4}\frac{j_k}{[p^2+\alpha(1-\alpha)q^2-M^2+i\epsilon]^2}\,,\quad(k=s,\,ps,\,V,\,A) \tag{2·11a}$$

with

$$j_k=\begin{cases} p^2-\alpha(1-\alpha)q^2+M^2\,, & (k=s) \\ p^2-\alpha(1-\alpha)q^2-M^2\,, & (k=ps) \\ p^2+\alpha(1-\alpha)q^2-3M^2\,, & (k=V) \\ p^2+3\alpha(1-\alpha)q^2-3M^2\,. & (k=A) \end{cases} \tag{2·11b}$$

Here we have decomposed anti-symmetric tensor fields into independent vector ($V_\mu{}^0$, $\boldsymbol{V}_\mu{}^1$) and axial vector ($A_\mu{}^0$, $\boldsymbol{A}_\mu{}^1$) fields;

$$F^0_{\mu\nu}=\mu^{-1}[\partial_\mu V_\nu{}^0-\partial_\nu V_\mu{}^0+\epsilon_{\mu\nu\lambda\rho}\partial^\lambda A^{\rho 0}]\,,$$

$$\boldsymbol{F}^1_{\mu\nu}=\mu^{-1}[\partial_\mu \boldsymbol{V}_\nu{}^1-\partial_\nu \boldsymbol{V}_\mu{}^1+\epsilon_{\mu\nu\lambda\rho}\partial^\lambda \boldsymbol{A}^{\rho 1}]\,. \tag{2·12}$$

§ 3. Determination of Λ

3.1. *Some physical quantities in the NG phase*

In the NG phase, the collective mode in the isovector-pseudoscalar channel appears as the NG boson. Although we are considering the deconfined phase, it is plausible that this mode is essentially the same as pion in real world since our interaction is enough strong to bind q-$\bar{q}$ in this channel and the confining force gives only the small modification to the q-$\bar{q}$ wave function.[8c] In this subsection we evaluate some physical quantities related with the mode (we call it pion in the following) in the lowest order of the chiral perturbation and determine cutoff Λ by employing the real world's value of pion decay constant f_π and pion mass m_π in the next subsection. For later convenience, let us introduce the pion propagator

$$D_\pi(q^2)=-[\mu^2(1+4gN_cJ_{ps}(q^2))]^{-1}\,. \tag{3·1}$$

Then the pion mass m_π is given by the pole of $D_\pi(q^2)$ and the pion-quark coupling constant is given by[3]

$$G^2_{\pi q}=G^2\lim_{q^2\to m_\pi{}^2}(q^2-m_\pi{}^2)D_\pi(q^2)\,, \tag{3·2a}$$

$$=-[2N_c\frac{d}{dq^2}J_{ps}(q^2)]^{-1}_{q^2=m_\pi{}^2}\,. \tag{3·2b}$$

The pion decay constant is defined by

$$\langle\sigma_M|\bar{\psi}(0)\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi(0)|\pi^b(q)\rangle=i\delta_{ab}f_\pi q_\mu\,, \tag{3·3}$$

where $|\sigma_M\rangle$ is the vacuum in the NG phase and $\pi^b(q)$ the pion-state with the energy-

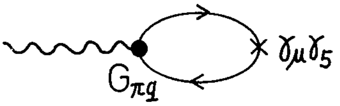


Fig. 2. The matrix element of Eq. (3·3). The wavy line represents the external line of pion with momentum q_μ. $G_{\pi q}$ is given by Eq.(3·2).

momentum $q_\mu(q_\mu{}^2=m_\pi{}^2)$. The l.h.s. of (3·3) can be evaluated according to Fig.2; then f_π becomes

$$f_\pi=N_c G_{\pi q}[-4i\int_0^1 d\alpha\int\frac{d^4p}{(2\pi)^4}\frac{M}{\{p^2+\alpha(1-\alpha)m_\pi{}^2-M^2+i\epsilon\}^2}], \qquad (3\cdot4\text{a})$$

$$=(N_c G_{\pi q}/2M)\cdot(J_s(m_\pi{}^2)-J_{ps}(m_\pi{}^2)) . \qquad (3\cdot4\text{b})$$

In the chiral perturbation, simple relations among the physical quantities are obtained: If we expand $D_\pi{}^{-1}(m_\pi{}^2)$ and $G^2_{\pi q}$ by $m_\pi{}^2$, we get

$$m_\pi{}^2=\frac{\hat{m}}{M}\frac{G^2_{\pi q}}{2g}, \qquad (3\cdot5)$$

up to $O(m_\pi{}^2/\Lambda^2)$ which corresponds to the expansion up to $O(\hat{m}/\Lambda)$. Here we note that $m_\pi{}^2$ is proportional to the current mass of quark. By expanding f_π into power series of $m_\pi{}^2$ and taking the lowest order, we get the Goldberger-Treiman relation in the quark level,

$$G_{\pi q}f_\pi=M_D . \qquad (3\cdot6)$$

Furthermore, if one recalls the SCC for dynamical mass

$$-(M_D/2g)=\langle\sigma_M|\bar{\psi}\psi|\sigma_M\rangle , \qquad (3\cdot7)$$

and combines (3·5) and (3·6), one can reach one of the current algebra relations

$$f_\pi{}^2 m_\pi{}^2=-\hat{m}\langle\sigma_M|\bar{\psi}\psi|\sigma_M\rangle , \qquad (3\cdot8)$$

which is valid up to $O(\hat{m}/\Lambda)$.

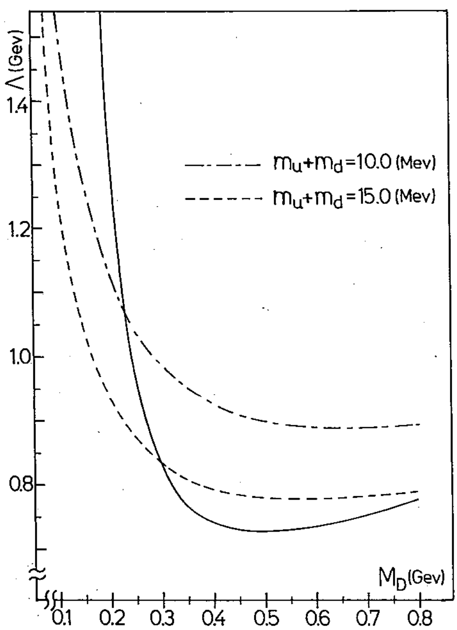


Fig. 3. The relation between cutoff Λ and dynamical mass M_D. The solid line is determined by (3·4) and the dashed line by (3·8).

3.2. *Determination of* Λ *and a consistency check*

In this subsection, we determine the value of Λ together with the dynamical mass M_D in the real world. For this sake, we first pay attention to (3·4) and (3·8): If we insert the experimental values $f_\pi(=93\text{ MeV})$, $m_\pi(\simeq 140\text{ MeV})$ and $2\hat{m}=m_u+m_d$ ($\simeq 10\sim 15$ MeV)[23] into these equations, we get two independent Λ-M_D relations. In the actual computations, we retain only the lowest order of $\hat{m}/\Lambda$ in each equation,

$$f_\pi=x[N_c dI(x^2)/dx^2]^{1/2}\cdot\Lambda \qquad (3\cdot9)$$

and

$$f_\pi{}^2 m_\pi{}^2=-2N_c(\hat{m}/\Lambda)\cdot xI(x^2)\cdot\Lambda^4, \qquad (3\cdot10)$$

where $x=M_D/\Lambda$ and $I(x^2)$ is defined in (2·8d). The resultant Λ-M_D relations are shown in Fig.3; thus, from the cross point we get $\Lambda\simeq 1$ GeV and $M_D\simeq 240$ MeV. The

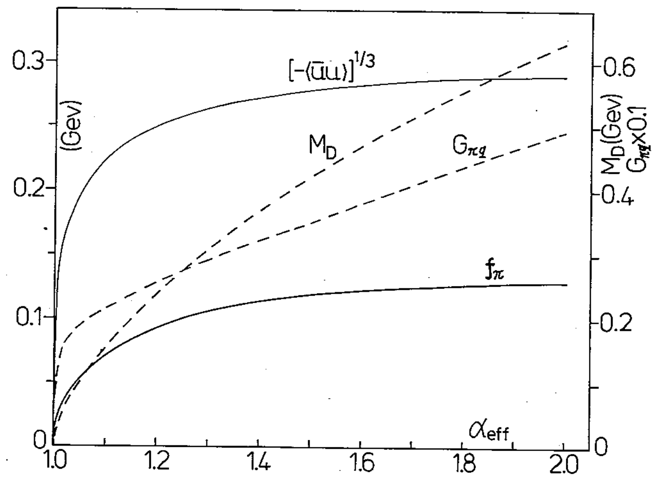


Fig. 4. α_{eff}-dependence of the physical quantities. f_π and $(-\langle\bar{u}u\rangle)^{1/3}$ should be read by the scale of l.h.s. The real world value of α_{eff} is 1.2.

value of $M_D=240$ MeV is smaller than the so-called constituent quark mass ($\simeq M_N/3$) but it coincides with the value obtained from other microscopic approach.[24] It is noteworthy that our $\Lambda\simeq 1$ GeV is consistent with the two-scale picture of CSB mentioned in § 1: Let R_χ and R_c be the length scales of CSB ($R_\chi\simeq\Lambda^{-1}$) and absolute confinement ($R_c\simeq\Lambda_{QCD}^{-1}$ $\simeq(200\,\text{MeV})^{-1}$) respectively, then $R_\chi<R_c$ must hold in the picture and this is the case in our model.*) In the following, we use $\Lambda=1007$ MeV which is obtained from the input $2\hat{m}$ $=11$ MeV: In this case, $M_D=240$ MeV, $\alpha_{eff}=1.2$ and the vacuum condensate becomes

$$\langle\bar{u}u\rangle=\langle\bar{d}d\rangle=(-249\,\text{MeV})^3\,. \tag{3·11}$$

One can see the coupling strength dependence of the physical quantities (M_D, f_π, $G_{\pi q}$ and $\langle\bar{u}u\rangle$) by varying α_{eff} by hand with fixed Λ. These are shown in Fig. 4 where the quantities are calculated in the chiral limit ($\hat{m}=0$).

In the proceeding sections, collective excitations will be examined within the chiral limit by using Λ determined above. Modifications of the results due to finite $\hat{m}$ will be discussed in § 6.

§ 4. Collective excitations in the Wigner phase
—— *Soft Modes Associated with CSB* ——

In this section, we examine the collective excitations in the Wigner phase. First, we solve the dispersion equations derived in § 2 to see what types of excitations appear. The dispersion equations in the Wigner phase can be obtained by setting $M=0$ in the momentum representation ($-\partial^2\to q^2$) of (2·10). On account of the chiral invariance of the vacuum in this phase, σ and $\boldsymbol{\pi}$ (η and $\boldsymbol{\delta}$) become energetically degenerate modes. We note that there is no real solution of the dispersion equations; this is rather reasonable because the system is in the Wigner phase which consists of massless quarks, so any collective mode is embedded in the continuum of q-$\bar{q}$ excitations and necessarily becomes

*) $\Lambda\cong 1$ GeV is also derived in other analyses.[8),18b)]

a damping mode with complex mass.*) To find out zeros of the dispersion equations corresponding to such damping modes, we must perform an analytic continuation of (2·10) to the lower half of the q^2-plane. For example, the results for the scalar and pseudo scalar modes are as follows:

$$1\pm\alpha_{\rm eff}[\frac{z}{2}{\rm Log}z-(2-z)\sqrt{\frac{z}{4-z}}{\rm Tan}^{-1}\sqrt{\frac{z}{4-z}}-1-z\cdot n\pi i]=0\,, \qquad (4\cdot1)$$

where $z=q^2/\Lambda^2$, n is the sheet number, and positive (negative) sign corresponds to $(\sigma, \boldsymbol{\pi})$ $((\eta, \boldsymbol{\delta}))$ mode.

A numerical evaluation shows;

(i) In $(\sigma, \boldsymbol{\pi})$ channels, a damping solution (Re $q^2>0$, Im $q^2<0$) having four-fold degeneracy appears on the $n=1$ sheet (see Fig. 5). This solution plays a special role in CSB; as $\alpha_{\rm eff}$ approaches unity from below, this excitation softens more and more, and when $\alpha_{\rm eff}$ exceeds unity, it turns into a tachyon solution (Re $q^2<0$, Im $q^2=0$) which appears on the negative axis of the $n=0$ sheet. The occurence of the tachyon solution for $\alpha_{\rm eff}>1$ indicates that the Wigner phase is unstable in such coupling strength. Therefore, our resonance-like solution for $\alpha_{\rm eff}<1$ is the precursory mode for CSB.

(ii) In the $(\eta, \boldsymbol{\delta})$ channels and vector channels, we have also found damping solution on

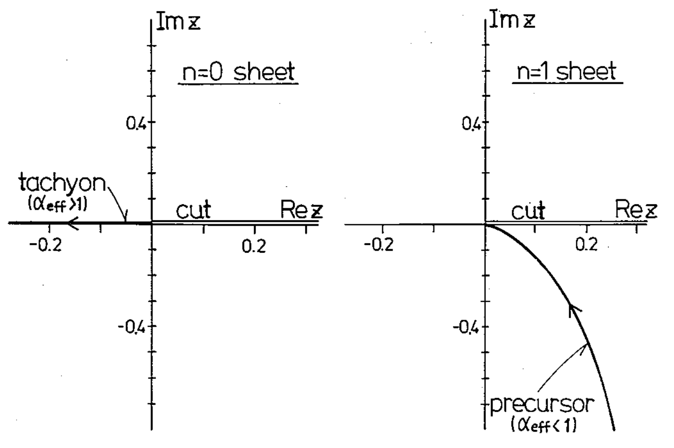


Fig. 5. A tachyon pole on the $n=0$ sheet and a damping mode on the $n=1$ sheet. These two are continuously connected as a function of $\alpha_{\rm eff}$. z denotes q^2/Λ^2.

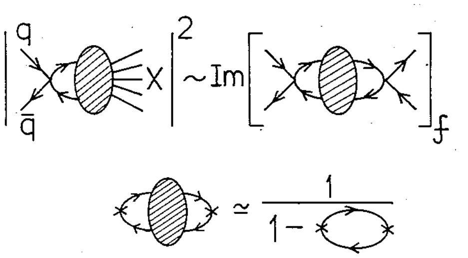


Fig. 6. Optical theorem for the q-$\bar{q}$ annihilation process and the ring approximation for it.

*) The damping mechanism is essentially the same as that known as the Landau damping[25] in the many body theory.

$n=1$ sheet. However they have large mass and width (Re $q^2>\Lambda^2$).

To see a physical relevance of the above excitations, let us consider the q-$\bar{q}$ annihilation process in the Wigner phase (see Fig. 6). Consider the case where the system is near the critical point and our effective interaction (1·1) dominates over the perturbative gluon exchange interaction. Then, because of the optical theorem, the total cross section (σ_{tot}) of the process is proportional to the imaginary part of the forward scattering amplitude. If we make an approximation in which only the bosonic excitations (σ, $\boldsymbol{\pi}$, η, $\boldsymbol{\delta}$, $F^0_{\mu\nu}$ and $\boldsymbol{F}^1_{\mu\nu}$) are taken into account as the intermediate states of the amplitude, σ_{tot} becomes proportional to the imaginary part of the polarization propagators (correlation function of the pair operators),

$$\sigma_{\text{tot}}(q^2)=\left(\frac{g}{2N_c}\right)^2\frac{1}{q^2}\text{Tr}^{(cfs)}[\not{p}_1\Gamma_\beta\not{p}_2\Gamma_\alpha]\text{Im}[i\int d^4x e^{iqx}\Pi_{\alpha\beta}(x)] \tag{4·2}$$

with

$$\Pi_{\alpha\beta}(x)\equiv\langle 0|T[\bar{\psi}(x)\Gamma_\alpha\psi(x)][\bar{\psi}(0)\Gamma_\beta\psi(0)]|0\rangle\,, \tag{4·3}$$

where p_1 and p_2 are incident four-momentum of quark and anti-quark respectively, $q=p_1+p_2$ and Γ_α's are the vertices defined in (2·5b). In the ring-approximation in which the polarization propagators are assumed to be dominated by the ones of the respective excitations as depicted in Fig. 6, $\Pi_{\alpha\beta}(x)$ can be written as

$$\text{Im}[2gi\Pi_{\alpha\beta}(x)_{F.T.}]$$

$$=\text{Im}\left[\frac{1}{1+4gN_cJ(q^2)}\right]\quad\text{for }(\sigma,\boldsymbol{\pi})\,, \tag{4·4a}$$

$$=\text{Im}\left[\frac{-1}{1-4gN_cJ(q^2)}\right]\quad\text{for }(\eta,\boldsymbol{\delta})\,, \tag{4·4b}$$

$$=\text{Im}\left[\frac{\pm(P_A)_{\mu\nu;\alpha\beta}}{1\mp4gN_c/(2N_c+1)\cdot J_A(q^2)}+\frac{\pm(P_V)_{\mu\nu;\alpha\beta}}{1\mp4gN_c/(2N_c+1)\cdot J_V(q^2)}\right]\quad\text{for}\begin{pmatrix}+:F^0_{\mu\nu}\\ -:\boldsymbol{F}^1_{\mu\nu}\end{pmatrix}, \tag{4·4c}$$

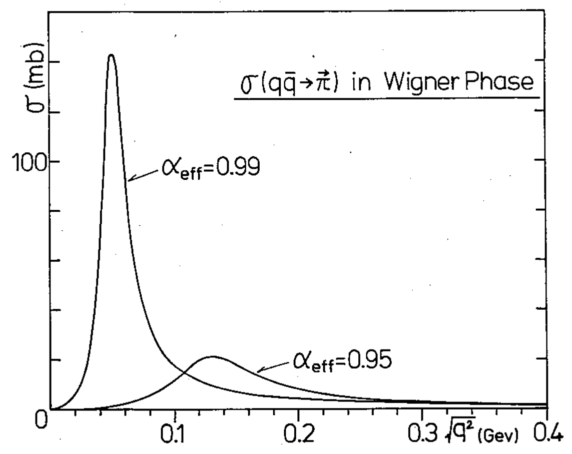
where $J(q^2)\equiv J_s(q^2)|_{M\to0}=J_{ps}(q^2)|_{M\to0}$ and (P_A, P_V) are the projection operators on the axial vector ($J^p=1^+$) and vector ($J^p=1^-$) channels;

$$(P_A)_{\mu\nu;\alpha\beta}=g_{\mu\alpha}(g_{\nu\beta}-\frac{1}{2}\hat{q}_\nu\hat{q}_\beta)+\begin{pmatrix}\mu\leftrightarrow\nu\\ \alpha\leftrightarrow\beta\end{pmatrix}-(\alpha\leftrightarrow\beta)-(\mu\leftrightarrow\nu)\,,$$

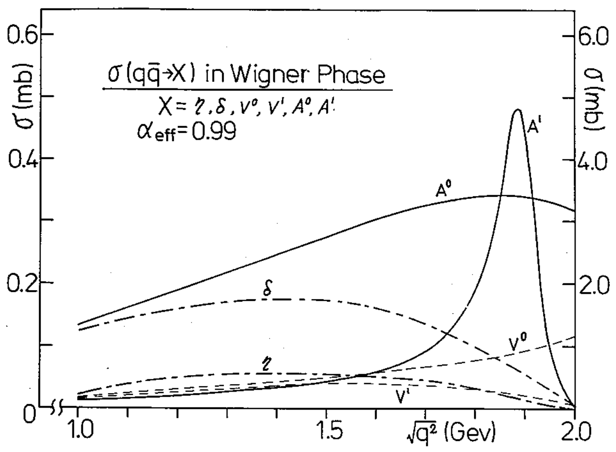
$$(P_B)_{\mu\nu;\alpha\beta}=\frac{1}{2}[g_{\mu\alpha}\hat{q}_\nu\hat{q}_\beta+\begin{pmatrix}\mu\leftrightarrow\nu\\ \alpha\leftrightarrow\beta\end{pmatrix}-(\alpha\leftrightarrow\beta)-(\mu\leftrightarrow\nu)] \tag{4·5}$$

with $\hat{q}_\mu\equiv q_\mu/\sqrt{q^2}$. σ_{tot} for the (σ, $\boldsymbol{\pi}$)channel and other channels are shown in Fig. 7(a) and (b). From the figures, we can see: (i) σ_{tot} has resonance-like peak in the (σ, $\boldsymbol{\pi}$)-channels the position and width of which correspond to the real and imaginary parts of the mass of the precursory modes respectively.

(ii) As α_{eff} increases, the peak position in (σ, $\boldsymbol{\pi}$) channel approaches the origin and the strength is dramatically enhanced, which indicates that the softening of the mode implies a development of a long range correlation of the pair operators $\psi\bar{\psi}$ and $\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi$.



(a)



(b)

Fig. 7 (a) Cross sections of annihilation processes in $(J^P, I)=(0^-, 1)$ channel in the Wigner phase. $\sqrt{q^2}$ is the q-$\bar{q}$ invariant mass. Cross sections in $(J^P, I)=(0^+, 0)$ channel i.e., $\sigma(q\bar{q}\to\sigma)$ equals to $1/3\cdot\sigma(q\bar{q}\to\boldsymbol{\pi})$.

(b) Cross sections of annihilation processes in η, $\boldsymbol{\delta}$ and vector channels in the Wigner phase. $\sigma(q\bar{q}\to A^1)$ should be read by the scale of r.h.s. and others by that of l.h.s.

(iii) In the $(\eta, \boldsymbol{\delta})$-channel and vector channels, there arise broad bumps with large mass, which indicates the small collectivity of the modes.

Recently it has been argued that an enhancement of the production rate of lepton-pairs via the thermal q-$\bar{q}$ annihilation process might be a signature of the formation of the quark-gluon plasma.[26] If there exists well-developed collective mode with a quantum number $J^P=1^-$ in the Wigner phase, it is expected that the mode causes a further enhancement of the production rate. However, the small collectivity in the $(V_\mu{}^0, \boldsymbol{V}_\mu{}^1)$-channels shown in our calculation implies that such a further enhancement is small, at least if the interaction (1·1) is dominated near the critical point.

Here we mention possible experiment to observe the precursors which shows the most dramatic change in the strength function near the critical point. One of such experiments is to see an enhancement of low energy pion production rate in ultrarelativistic heavy ion collisions. When the system reaches the Wigner phase in the intermediate stage, it may be approximated by a quasi-free gas composed of massless quarks, gluons and low energy precursors; the last one will gradually disappear as the incident energy of the projectile E increases, because the system approaches the perturbative region and α_{eff} tends to zero as E grows. Thus it is expected that near the critical point the enhancement of excitations having the same quantum numbers as $(\sigma, \boldsymbol{\pi})$ shown in Fig. 7(a) will cause an increase of the low energy pion production rate greatly or might be observed directly through the $\pi^0\to2\gamma$ process, and also they will affect the cooling of the droplet of quark-gluon plasma produced by the heavy ion collisions.[19),27)]

§ 5. Collective excitations in the NG phase

In this section, we briefly examine the collective excitations in the NG phase.

First, let us see the $\alpha_{\rm eff}$ dependence of the masses of the respective modes. These are obtained by solving the dispersion equations (2·10). The results are summarized as follows:

(i) As is well known,[3] π-mode becomes massless (NG-mode), irrespective of the value of $\alpha_{\rm eff}$.

(ii) As has been paid attention to by some authors,[3),28)] the mass of σ-mode (m_σ) is just twice the dynamical quark mass M_D, which also holds irrespective of the value of $\alpha_{\rm eff}$. Thus m_σ decreases as the system approaches the critical point. This behavior of σ-mode is a typical critical phenomenon seen when chiral symmetry becomes restored. We will discuss more on this point later.

(iii) Since our effective interaction breaks the $U_A(1)$ symmetry explicitly, η-mode becomes necessarily massive and does not appear as the NG-mode.

(iv) The masses of η, $\boldsymbol{\delta}$ and vector modes are found above the threshold ($2M$); so we must have sought the complex solutions of the dispersion equations and found that the real parts of the masses of all these modes lie above the cutoff Λ, which indicates the interactions between quark and anti-quark are weak in these channels.

Recall that we are now considering a deconfined system with broken CS. The above point (iv) suggests that the η, $\boldsymbol{\delta}$ and vector-particles in the real world where the confinement is complete and CS is broken are bound rather by the confining force than by our determinantial force. The small collectivity in these channels can be seen in Fig. 8 where the total cross sections of q-$\bar{q}$ annihilations in the NG phase within the ring approximation are shown: There are only broad bumps, the peaks of which all lie above 1.3 GeV.

In contrast with these modes, σ and π modes which are directly related to CSB can be considered to be bound by the force responsible for CSB; so they would not change their

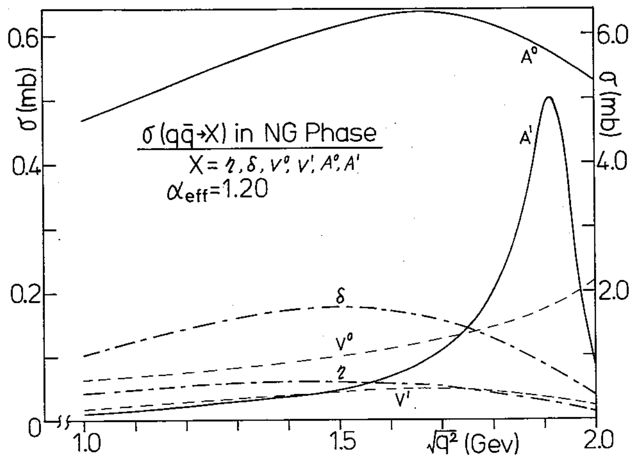


Fig. 8. Cross sections of annihilation processes in the NG phase. Only $\sigma(q\bar{q}\to A^1)$ should be read by the scale of r.h.s.

nature whether the confinement occurs or not. This picture is consistent with and supported by the point of view discussed in § 1.

Here, we discuss a possible critical phenomena associated with the restoration of CS, which may serve as a signature of the formation of the quark gluon plasma. Our argument is based on the relation

$$m_\sigma = 2M_D \quad \text{for all } \alpha_{\text{eff}} \geq 1\,, \tag{5·1}$$

which holds exactly in the small amplitude approximation (RPA) provided that the current quark mass is neglected. The relation tells us that the mass of σ-mode decreases as the system approaches the critical point and vanishes when the symmetry is restored. The relation and the above feature hold with some modifications for the systems at finite temperature.[19),29)] Therefore if deconfined NG phase is realized in the intermediate stage of ultra-relativistic heavy ion collisions and if we can observe the total cross section of 0^+ channel $\sigma_{\text{tot}}(0^+)$, the peak or bump of $\sigma_{\text{tot}}(0^+)$ will move to low invariant mass as the incident energy E is raised so that the system approaches the critical point ($\alpha_{\text{eff}} \to 1$).

§ 6. Discussion

6.1. *Modifications due to the current quark mass*

We have discussed the collective excitations both in the Wigner and NG phase, neglecting the current quark mass. In this subsection, we give some modifications of the results due to it.

It is noteworthy that there is no sharp phase transition when the current quark mass is included, which is shown in Fig.9 where α_{eff} dependence of m_σ and m_π are shown together with that of $M = M_D + \hat{m}$.*) As can be seen from the figure, pion mass increases as α_{eff} decreases; m_π is about 140 MeV at $\alpha_{\text{eff}} = 1.2$, and at $\alpha_{\text{eff}} = 0.98$ the pion pole enters the q-$\bar{q}$ continuum. The real part of the pole of σ-mode keeps lying in the continuum and m_σ takes the minimum value ($\simeq$260 MeV) at $\alpha_{\text{eff}} = 0.92$. Furthermore, as α_{eff} decreases, mass difference m_σ-m_π becomes smaller.

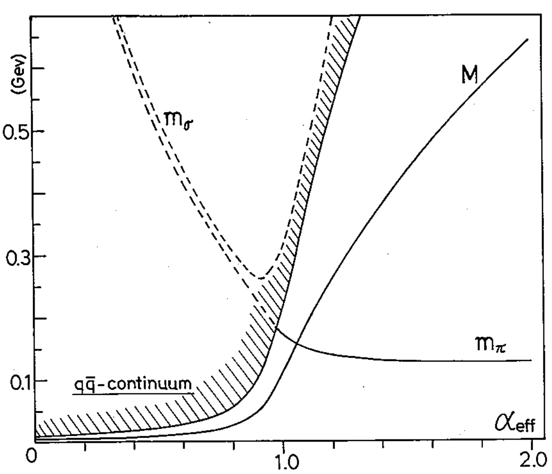


Fig. 9. α_{eff}-dependence of the real part of m_σ, m_π and quark mass $M = M_D + \hat{m}$. The dashed lines indicate that the modes are in the q-$\bar{q}$ continuum and have a damping width.

Thus, one sees that nearly degenerate modes which have the same quantum number with pion and σ-meson soften as α_{eff} increases from a small value ($<$0.92) to 0.92. The softening can be considered as a precursory phenomenon of chiral symmetry breaking. The increase of m_π and the decrease of m_σ which is seen when α_{eff} decreases from a large value to 0.92 can be also considered to be a precursory phenomenon of the restoration of CS. Therefore, we conclude that

*) We have neglected the higher orders of $(m_{\sigma,\pi}/\Lambda)^2$ in the dispersion equations (2·10) in the calculation of the masses of σ and π, but we have no approximation for evaluating M.

the results obtained by neglecting the current quark mass ($\hat{m}$) are not altered qualitatively but altered somewhat quantitatively with the inclusion of $\hat{m}$.

6.2. *Phase diagram in T-μ plane*

In the above sections, we have confined ourselves to the case of zero temperature ($T=0$) and zero chemical potential ($\mu=0$) and we have investigated the typical critical phenomena by varying the coupling strength $\alpha_{\rm eff}$ by hand. Here we briefly examine the system with finite T and μ in the chiral limit by using the parameters ($\alpha_{\rm eff}$, Λ) determined in $T=\mu=0$ system and show the critical temperature T_χ in our model is consistent with the estimates by different approaches.[30] Detailed examination of the soft modes in $T\neq 0$ and $\mu\neq 0$ system is given in Ref. 19).

The generalization of our SCC (Eq. (2·1)) to finite T and μ is straightforward; we need only to replace the ground state expectation value $\langle \mathcal{O} \rangle$ by the thermal averages taken by the thermal-equilibrium distributions $\langle\!\langle \mathcal{O} \rangle\!\rangle$. Then, the gap equation which determines a (T, μ)-dependent dynamical mass $M(T,\mu)$ can be written as

$$-\mu^2/G\cdot\sigma_M=\langle\!\langle \bar{\psi}(x)\psi(x)\rangle\!\rangle=-i\lim_{y\to x+}{\rm Tr}S_F(x-y;\,T,\mu)\,, \qquad (6\cdot1)$$

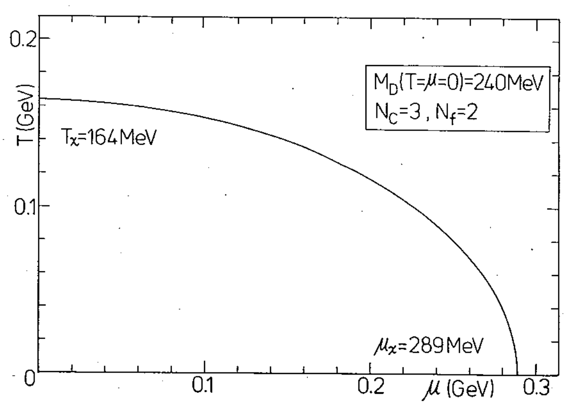
where $S_F(x-y;\,T,\mu)$ is a thermal Green's function in the real time formulation the Fourier transform of which can be expressed as[31]

$$S_F(\boldsymbol{p},\omega;\,T,\mu)=(\not p+M)\left[\frac{1}{p^2-M^2+i\epsilon}+2i\pi\delta(p^2-M^2)\{\theta(p^0)n(\boldsymbol{p})+\theta(-p^0)m(\boldsymbol{p})\}\right] \qquad (6\cdot2)$$

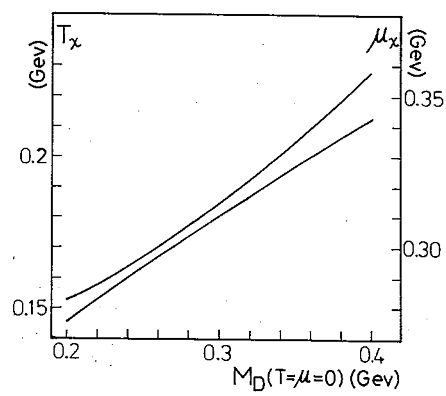
with

$$n(\boldsymbol{p})=[1+\exp\{\beta(E(\boldsymbol{p})-\mu)\}]^{-1} \quad \text{and} \quad m(\boldsymbol{p})=[1+\exp\{\beta(E(\boldsymbol{p})+\mu)\}]^{-1}\,, \qquad (6\cdot3)$$

where we use the notation; $\beta=1/(kT)$, $p^\mu=(p^0,\boldsymbol{p})=(\omega+\mu,\boldsymbol{p})$ and $E(\boldsymbol{p})=\sqrt{p^2+M^2}$. The critical line $T=T(\mu)$, which is a contour of the phase diagram, is defined as that on which the nontrivial solution $M_D(T,\mu)$ of (6·1) vanishes. The resulting equation to determine the critical line is



(a)



(b)

Fig. 10 (a) Phase diagram of chiral transition by the use of our effective interaction and the parameters determined at $T=\mu=0$ system.

(b) Critical temperature T_χ and critical chemical potential μ_χ as a function of dynamical mass in the $T=\mu=0$ vacuum. Upper line corresponds to T_χ.

$$\frac{1}{\alpha_{\rm eff}^{(3)}}=\frac{2}{\Lambda_3{}^2}\int_0^{\Lambda_3}d|\boldsymbol{p}|\cdot|\boldsymbol{p}|[1-n(\boldsymbol{p})-m(\boldsymbol{p})]_{M=0}\,, \tag{6·4}$$

where we have used the three-momentum cutoff Λ_3 for convenience. The values of Λ_3, $\alpha_{\rm eff}^{(3)}$(coupling constant in three-momentum cutoff scheme) in $T=\mu=0$ system are 825 MeV and 1.14 respectively, which are determined by the same procedure taken in § 3. The dynamical mass $M_D(T=\mu=0)$ is equal to 240 MeV in this case also. The critical line thus determined is shown in Fig. 10 (a). In Fig. 10 (b), the critical temperature T_χ and critical chemical potential μ_χ are shown as a function of $M_D(T=\mu=0)$ with fixed Λ. The critical temperature of chiral transition in our model is 164 MeV for ($N_c=3$, $N_f=2$) case, which is consistent with the estimates by different approaches.[30] Note that we have been dealing with chiral transition of the deconfined quark matter. As has been stated in § 1, an additional confinement transition occurs in the real world: It is an open question how to modelize this effect and incorporate it into our formalism.

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Appendix

We summarize here the various modifications of the effective Lagrangian which gives the same spectra for σ and $\boldsymbol{\pi}$ channels but gives the different spectra for other channels. Note that we consider only the color singlet channels and omit the non-singlet parts of the interaction in the text and in the following.

The determinantial interaction we adopted in the text is

$$\mathcal{L}_1{}^{(\rm det)}\propto(\bar\psi\psi)^2+(\bar\psi i\gamma_5\boldsymbol{\tau}\psi)^2-(\bar\psi i\gamma_5\psi)^2-(\bar\psi\boldsymbol{\tau}\psi)^2\,, \tag{A·1}$$

and its Fierz transform is

$$\frac{1}{2N_c}[(\bar\psi\psi)^2+(\bar\psi i\gamma_5\boldsymbol{\tau}\psi)^2-(\bar\psi i\gamma_5\psi)^2-(\bar\psi\boldsymbol{\tau}\psi)^2]+\frac{1}{4N_c}[(\bar\psi\sigma_{\mu\nu}\psi)^2-(\bar\psi\sigma_{\mu\nu}\boldsymbol{\tau}\psi)^2] \tag{A·2}$$

$\mathcal{L}_1{}^{(\rm det)}$ has $SU(2)\otimes SU(2)$ invariance but breaks the $U_A(1)$ symmetry explicitly. A similar type of Lagrangian having $U(2)\otimes U(2)$ symmetry is

$$\mathcal{L}_1{}^{(\rm sym)}\propto(\bar\psi\psi)^2+(\bar\psi i\gamma_5\boldsymbol{\tau}\psi)^2+(\bar\psi i\gamma_5\psi)^2+(\bar\psi\boldsymbol{\tau}\psi)^2\,, \tag{A·3}$$

and its Fierz transform reads

$$-\frac{1}{N_c}[(\bar\psi\gamma_\mu\psi)^2+(\bar\psi\gamma_5\gamma_\mu\psi)^2]\,. \tag{A·4}$$

In this Lagrangian, η channel becomes a Nambu-Goldstone mode in the NG phase. So it needs to add the $U_A(1)$ breaking term to give η a finite mass as is done in Ref. 18c). A Lagrangian adopted by Nambu and Jona-Lasinio[3] is a sum of $\mathcal{L}_1{}^{(\rm det)}$ and $\mathcal{L}_1{}^{(\rm sym)}$,

$$\mathcal{L}_1{}^{(\rm NJL)}=\mathcal{L}_1{}^{(\rm det)}+\mathcal{L}_1{}^{(\rm sym)}\propto(\bar\psi\psi)^2+(\bar\psi i\gamma_5\boldsymbol{\tau}\psi)^2\,, \tag{A·5}$$

which has $SU(2)\otimes SU(2)$ symmetry but breaks $U_A(1)$ symmetry. All the above interactions give not so much binding for the vector channels in the NG phase; the reason is that the vector modes are produced by the Fock terms if the above interactions and hence the coupling strength is reduced to $1/N_c$ times the Hartree terms. A $U(2)\otimes U(2)$ invariant Lagrangian which gives more binding to the vector modes is a local version of the gluon exchange interaction,

$$\mathcal{L}_1^{(\mathrm{OGE})} \propto (\bar{\psi}\gamma_\mu\lambda^a\psi)(\bar{\psi}\gamma^\mu\lambda^a\psi)\,, \tag{A·6}$$

where λ^a are the $SU_c(3)$ Gell-Mann matrices. The Fierz transform of (A·6) becomes

$$\frac{8}{9}[(\bar{\psi}\psi)^2+(\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)^2+(\bar{\psi}i\gamma_5\psi)^2+(\bar{\psi}\boldsymbol{\tau}\psi)^2$$
$$-\frac{1}{2}(\bar{\psi}\gamma_\mu\psi)^2-\frac{1}{2}(\bar{\psi}\gamma_\mu\boldsymbol{\tau}\psi)^2+\frac{1}{2}(\bar{\psi}\gamma_5\gamma_\mu\psi)^2+\frac{1}{2}(\bar{\psi}\gamma_5\gamma_\mu\boldsymbol{\tau}\psi)^2]\,. \tag{A·7}$$

Here one can see that (A·6) has the same particle contents as that of (A·1) and the coupling strength of the vector modes has no suppression factor being propotional to $1/N_c$; $1/2$ in (A·7) originates from the Fierz transform of the Dirac matrices.

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