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Soft Multisets Theory

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Abstract

In 1999 Molodtsov introduced the concept of soft set theory as a general mathematical tool for dealing with uncertainty. The solutions of such problems involve the use of mathematical principles based on uncertainty and imprecision. In this paper we recall the definition of a soft set, its properties and its operations. As a generalization of Molodtsov's soft set we introduce the definitions of a soft multiset, its basic operations such as complement, union and intersection. We give examples for these concepts. Basic properties of the operations are also given.

Keywords: soft set; soft multiset

1 Introduction

Most of the problems in engineering, medical science, economics, environments etc. have various uncertainties. Molodtsov [6] initiated the concept of soft set theory as a mathematical tool for dealing with uncertainties which is free from the above difficulties. After Molodtsov's work, some different operations and application of soft sets were studied by Chen et al. [1] and Maji et al. [4, 5]. Furthermore Maji et al. [3] presented the definition of fuzzy soft set and Roy and Maji [7] presented the applications of this notion to decision making problems. In the definition of Molodtsov the soft set is a mapping from a set of parameters to a power set of the universe. That means we have one set of parameters and one universe, so the user can make a decision whether an element in the universe having a property (e), for example, is in the set of parameters or not. The question here is: Are all the problems of this type consists of one universe? The answer is definitely no. To solve problem with multiset of universes and as a generalization of Molodtsov's soft set we introduce the definition of a soft multiset, its basic operations such as complement, union and intersection. We give some examples for these concepts. Basic properties of the operations are also given.

2 Preliminary

In this section, we recall some basic notions in soft set theory. Molodtsov [6] defined soft set in the following way. Let U be a universe and E be a set of parameters. Let $P(U)$ denote the power set of U and $A \subseteq E$.

Definition 2.1 *[6]. A pair (F, E) is called a soft set over U, where F is a mapping given by $F : E \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U.*

Example 2.2 *Let us consider a soft set (F, E) which describes the "attractiveness of houses" that Mr. X is considering to purchase. Suppose that there are six houses in the universe $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ under consideration, and that $E = \{e_1, e_2, e_3, e_4, e_5\}$ is a set of decision parameters. The $e_i\,(i = 1, 2, 3, 4, 5)$ denotes the parameters "expensive", "beautiful", "wooden", "cheap", and "in green surroundings" respectively. Consider the mapping F given by "houses (.)", where (.) is to be filled in by one of the parameters $e_i \in E$. For instance, $F(e_1)$ means "houses (expensive)", and its functional value is the set $\{h \in U : h \text{ is an expensive house}\}$. Suppose that $F(e_1) = \{h_2, h_4\}$, $F(e_2) = \{h_1, h_3\}$, $F(e_3) = \emptyset$, $F(e_4) = \{h_1, h_3, h_5\}$ and $F(e_5) = \{h_1\}$. Then we can view the soft set (F, E) as consisting of the following collection of approximations:*

$$(F, E) = \{(\textit{expensive houses, } \{h_2, h_4\}),\ (\textit{beautiful houses, } \{h_1, h_3\}),$$

$$(\textit{wooden houses, } \emptyset),\ (\textit{cheap houses, } \{h_1, h_3, h_5\}),$$

(*in the green surroundings*, $\{h_1\}$))

Each approximation has two parts: a predicate and an approximate value set.

Definition 2.3 *[5]. For two soft sets* (F, A) *and* (G, B) *over* U*,* (F, A) *is called a soft subset of* (G, B) *if*

1. $A \subset B$ *and*

2. $\forall \varepsilon \in A, F(\varepsilon)$ *and* $G(\varepsilon)$ *are identical approximations.*

This relationship is denoted by $(F,\ A) \tilde{\subset} (G, B)$*. In this case,* (G, B) *is called a soft superset of* (F, A)*.*

Definition 2.4 *[5]. Two soft sets* $(F,\ A)$ *and* $(G,\ B)$ *over a common universe* U *are said to be soft equal if* (F, A) *is a soft subset of* (G, B) *and* (G, B) *is a soft subset of* (F, A)*.*

Definition 2.5 *[5]. Let* $E = \{e_1, e_2, ..., e_n\}$ *be a set of parameters. The NOT set of* E *denoted by* $\rceil E$ *is defined by* $\rceil E = \{\rceil e_1,\ \rceil e_2, ...,\ \rceil e_n\}$ *where* $\rceil e_i$ $=$ *not* e_i*,*$\forall i$*.*

Definition 2.6 *[5].The complement of a soft set* (F, A) *is denoted by* $(F, A)^c$ *and is defined by* $(F, A)^c = (F^c, \rceil A)$ *where* $F^c : \rceil A \to P(U)$ *is a mapping given by*

$$F^c(\alpha) = U - F(\rceil \alpha), \forall \alpha \in \rceil A.$$

Definition 2.7 *[5]. A soft set* (F, A) *over* U *is said to be a NULL soft set denoted by* $\emptyset$ *if* $\forall \varepsilon \in A, F(\varepsilon) = \emptyset$*, (null-set).*

Definition 2.8 *[5]. A soft set* (F, A) *over* U *is said to be an absolute soft set, denoted by* $\tilde{A}$*, if* $\forall \varepsilon \in A, F(\varepsilon) = U$ *.*

Definition 2.9 *[5]. The union of two soft sets* (F, A) *and* (G, B) *over a common universe* U *is the soft set* (H, C) *where* $C = A \cup B$*, and* $\forall \varepsilon \in C$*,*

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & if \quad \varepsilon \in A - B \\ G(\varepsilon), & if \quad \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & if \quad \varepsilon \in A \cap B. \end{cases}$$

.

Remark The definition of intersection of two soft sets given by Maji et al. [5] was not correct because they defined $H(\varepsilon) = F(\varepsilon) \operatorname{or} G(\varepsilon)$. This was pointed out by Irfan et al. [2]. In fact Irfan et al. also give the definition of restricted intersection.

Definition 2.10 *[2]. The extended intersection of two soft sets* (F, A) *and* (G, B) *over a common universe* U *is the soft set* (H, C) *where* $C = A \cup B$, *and* $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & if \quad \varepsilon \in A - B \\ G(\varepsilon), & if \quad \varepsilon \in B - A \\ F(\varepsilon) \cap G(\varepsilon), & if \quad \varepsilon \in A \cap B. \end{cases}$$

.

3 Soft Multisets

In this section, we introduce the definition of a soft multiset, and its basic operations such as complement, union and intersection. We give examples for these concepts. Basic properties of the operations are also given. Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i \in I} U_i = \emptyset$ and let $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i \in I} P(U_i)$ where $P(U_i)$ denotes the power set of U_i, $E = \prod_{i \in I} E_{U_i}$ and $A \subseteq E$.

Definition 3.1 *A pair* $(F, \ A)$ *is called a soft multiset over* U, *where* F *is a mapping given by* $F : A \to U$.

In other words, a soft multiset over U is a parameterized family of subsets of U. For $\varepsilon \subset A, F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft multiset (F, A). Based on the above definition, any change in the order of universes will produce a different soft multiset.

Example 3.2 *Suppose that there are three universes* U_1, U_2 *and* U_3. *Let us consider a soft multiset* (F, A) *which describes the "attractiveness of houses", "cars" and "hotels" that Mr. X is considering for accommodation purchase, transportation purchase, and venue to hold a wedding celebration respectively. Let* $U_1 = \{h_1, h_2, h_3, h_4, h_5, h_6\}$, $U_2 = \{c_1, c_2, c_3, c_4, c_5\}$ *and* $U_3 = \{v_1, v_2, v_3, v_4\}$. *Let* $E_U = \{E_{U_1}, E_{U_2}, E_{U_3}\}$ *be a collection of sets of decision parameters related to the above universes, where*

$$E_{U_1} = \{e_{U_1,1} = \textit{expensive}, e_{U_1,2} = \textit{cheap}, e_{U_1,3} = \textit{beautiful}, \\ e_{U_1,4} = \textit{wooden}, e_{U_1,5} = \textit{in green surroundings}\},$$

$$E_{U_2} = \{e_{U_2,1} = \textit{expensive}, \ e_{U_2,2} = \textit{cheap}, e_{U_2,3} = \textit{Model 2000 and above} , \\ e_{U_2,4} = \textit{Black}, \ e_{U_2,5} = \textit{Made in Japan}, \ e_{U_2,6} = \textit{Made in Malaysia}\},$$

$$E_{U_3} = \{e_{U_3,1} = \textit{expensive}, e_{U_3,2} = \textit{cheap}, e_{U_3,3} = \textit{majestic}, \\ e_{U_3,4} = \textit{in Kuala Lumpur}, e_{U_3,5} = \textit{in Kajang}\}.$$

Let $U = \prod_{i=1}^{3} P(U_i)$, $E = \prod_{i=1}^{3} E_{U_i}$ *and* $A \subseteq E$, *such that*

$$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}),$$

$$a_3 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1}), a_4 = (e_{U_1,5}, e_{U_2,4}, e_{U_3,2}),$$

$$a_5 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,3}), a_6 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,2}),$$

$$a_7 = (e_{U_1,3}, e_{U_2,1}, e_{U_3,1}), a_8 = (e_{U_1,1}, e_{U_2,3}, e_{U_3,2})\}.$$

Suppose that

$$F(a_1) = (\{h_2, h_3, h_6\}, \{c_2\}, \{v_2, v_3\}),$$

$$F(a_2) = (\{h_2, h_3, h_6\}, \{c_1, c_3, c_4, c_5\}, \{v_2\}),$$

$$F(a_3) = (\{h_1, h_4, h_5\}, \{c_1, c_3\}, \emptyset),$$

$$F(a_4) = (\{h_1, h_4, h_6\}, \emptyset, \{v_1, v_4\}),$$

$$F(a_5) = (\{h_1, h_4\}, \{c_1, c_3\}, \{v_1\}),$$

$$F(a_6) = (\{h_1, h_4, h_5\}, \{c_1, c_3\}, U_3),$$

$$F(a_7) = (\{h_1, h_4\}, \emptyset, \{v_3\}) \text{ and}$$

$$F(a_8) = (\{h_2, h_3, h_6\}, \{c_1, c_3\}, \{v_1, v_4\}).$$

Then we can view the soft multiset (F, A) *as consisting of the following collection of approximations:*

$$(F, A) = \{(a_1, (\{h_2, h_3, h_6\}, \{c_2\}, \{v_2, v_3\})),$$

$$(a_2, (\{h_2, h_3, h_6\}, \{c_1, c_3, c_4, c_5\}, \{v_2\})),$$

$$(a_3, (\{h_1, h_4, h_5\}, \{c_1, c_3\}, \emptyset)), (a_4, (\{h_1, h_4, h_6\}, \emptyset, \{v_1, v_4\})),$$

$$(a_5, (\{h_1, h_4\}, \{c_1, c_3\}, \{v_1\})), (a_6, (\{h_1, h_4, h_5\}, \{c_1, c_3\}, U_3)),$$

$$(a_7, (\{h_1, h_4\}, \emptyset, \{v_3\})), (a_8, (\{h_2, h_3, h_6\}, \{c_1, c_3\}, \{v_1, v_4\}))\}.$$

Each approximation has two parts: a predicate and an approximate value set.

We can logically explain the previous example as follows:

For $F(a_1) = (\{h_2, h_3, h_6\}, \{c_2\}, \{v_2, v_3\})$, IF $\{h_2, h_3, h_6\}$ is the set of expensive houses to Mr. X THEN the set of relatively expensive cars to him is $\{c_2\}$ and IF $\{h_2, h_3, h_6\}$ is the set of expensive houses to Mr. X and $\{c_2\}$ is the set of relatively expensive cars to him THEN the set of relatively expensive hotels to him is $\{v_2, v_3\}$. It is clear that the relation in soft multiset is a conditional relation.

Definition 3.3 *For any soft multiset* (F, A), *a pair* $\left(e_{U_i,j}, F_{e_{U_i,j}}\right)$ *is called a* U_i-*soft multiset part* $\forall e_{U_i,j} \in a_k$ *and* $F_{e_{U_i,j}} \subseteq F(A)$ *is an approximate value set, where* $a_k \in A, k = \{1, 2, 3, ..., n\}, i \in \{1, 2, 3, ..., m\}$ *and* $j \in \{1, 2, 3, ..., r\}$.

Example 3.4 *Consider Example 3.2. Then*

$$\begin{aligned}\left(e_{U_1,j}, F_{e_{U_1,j}}\right) = \{&(e_{U_1,1}, \{h_2, h_3, h_6\}), (e_{U_1,1}, \{h_2, h_3, h_6\}), (e_{U_1,2}, \{h_1, h_4, h_5\}), \\ &(e_{U_1,5}, \{h_1, h_4, h_6\}), (e_{U_1,4}, \{h_1, h_4\}), (e_{U_1,2}, \{h_1, h_4, h_5\}), \\ &(e_{U_1,3}, \{h_1, h_4\}), (e_{U_1,1}, \{h_2, h_3, h_6\})\}\end{aligned}$$

is a U_1-*soft multiset part of* (F, A).

Definition 3.5 *For two soft multisets* (F, A) *and* (G, B) *over* U, (F, A) *is called a soft multisubset of* (G, B) *if*

1. $A \subseteq B$ *and*
2. $\forall e_{U_i,j} \in a_k, \left(e_{U_i,j}, F_{e_{U_i,j}}\right) \subseteq \left(e_{U_i,j}, G_{e_{U_i,j}}\right)$

where $a_k \in A, k = \{1, 2, 3, ..., n\}, i \in \{1, 2, 3, ..., m\}$ *and* $j \in \{1, 2, 3, ..., r\}$.

This relationship is denoted by $(F, A) \widetilde{\subseteq} (G, B)$. In this case (G, B) is called a soft multisuperset of (F, A).

Definition 3.6 *Two soft multisets* (F, A) *and* (G, B) *over* U *are said to be equal if* (F, A) *is a soft multisubset of* (G, B) *and* (G, B)*is a soft multisubset of* (F, A).

Example 3.7 *Consider Example 3.2. Let*

$$\begin{aligned}A = \{&a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1}), \\ &a_3 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,3}), a_4 = (e_{U_1,3}, e_{U_2,1}, e_{U_3,1})\} \text{ and}\end{aligned}$$

$$B = \{b_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), b_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}),$$

$$b_3 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1}), b_4 = (e_{U_1,5}, e_{U_2,4}, e_{U_3,2}),$$

$$b_5 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,3}), b_6 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,2}),$$

$$b_7 = (e_{U_1,3}, e_{U_2,1}, e_{U_3,1}), b_8 = (e_{U_1,1}, e_{U_2,3}, e_{U_3,2})\}.$$

Clearly $A \subseteq B$. *Let* (F, A) *and* (G, B) *be two soft multisets over the same* U *such that*

$$(F, A) = \{(a_1, (\{h_2, h_3\}, \{c_2\}, \{v_2\})), (a_2, (\{h_1, h_5\}, \{c_1, c_3\}, \emptyset)),$$

$$(a_3, (\{h_1, h_4\}, \{c_1, c_3\}, \{v_1\})), (a_4, (\{h_4\}, \emptyset, \{v_3\}))\}.$$

$$(G, B) = \{(b_1, (\{h_2, h_3, h_6\}, \{c_2\}, \{v_2, v_3\})),$$

$$(b_2, (\{h_2, h_3, h_6\}, \{c_1, c_3, c_4, c_5\}, \{v_2\})),$$

$$(b_3, (\{h_1, h_4, h_5\}, \{c_1, c_3\}, \emptyset)), (b_4, (\{h_1, h_4, h_6\}, \emptyset, \{v_1, v_4\})),$$

$$(b_5, (\{h_1, h_4\}, \{c_1, c_3\}, \{v_1\})), (b_6, (\{h_1, h_4, h_5\}, \{c_1, c_3\}, U_2)),$$

$$(b_7, (\{h_1, h_4\}, \emptyset, \{v_3\})), (a_8, (\{h_2, h_3, h_6\}, \{c_1, c_3\}, \{v_1, v_4\}))\}.$$

Therefore $(F, A) \widetilde{\subseteq} (G, B)$.

Definition 3.8 *Let* $E = \prod_{i=1}^{m} E_{U_i}$ *where* E_{U_i} *is a set of parameters. The NOT set of* E *denoted by* $\rceil E$ *is defined by*

$$\rceil E = \prod_{i=1}^{m} \rceil E_{U_i}$$

where $\rceil E_{U_i} = \{\rceil e_{U_i,j} = not\ e_{U_i,j}, \forall i, j\}$.

Definition 3.9 *The complement of a soft multiset* (F, A) *is denoted by* $(F, A)^c$ *and is defined by* $(F, A)^c = (F^c, \rceil A)$ *where* $F^c : \rceil A \to U$ *is a mapping given by* $F^c(\alpha) = U - F(\rceil\alpha), \forall \alpha \in \rceil A$.

Example 3.10 *Consider Example 3.2. Here*

$$(F, A)^c = \{(\rceil a_1, (F(\rceil a_1))), (\rceil a_2, (F(\rceil a_2))), (\rceil a_3, (F(\rceil a_3))),$$

$$(\rceil a_4, (F\rceil(\rceil a_4))), (\rceil a_5, (F(\rceil a_5))), (\rceil a_6, (F(\rceil a_6))),$$

$$(\rceil a_7, (F(\rceil a_7))), (\rceil a_8, (F(\rceil a_8)))\}$$

$$= \{(\rceil a_1, (\{h_1, h_4, h_5\}, \{c_1, c_3, c_4, c_5\}, \{v_1, v_4\})),$$

$$(\rceil a_2, (\{h_1, h_4, h_5\}, \{c_2\}, \{v_1, v_4\})),$$
$$(\rceil a_3, (\{h_2, h_3, h_6\}, \{c_2, c_4, c_5\}, U_3)),$$
$$(\rceil a_4, (\{h_2, h_3, h_5\}, U_2, \{v_2, v_3\})),$$
$$(\rceil a_5, (\{h_2, h_3, h_5, h_6\}, \{c_2, c_4, c_5\}, \{v_2, v_3\})),$$
$$(\rceil a_6, (\{h_2, h_3, h_6\}, \{c_2, c_4, c_5\}, \emptyset)),$$
$$(\rceil a_7, (\{h_2, h_3, h_5, h_6\}, U_2, \{v_1, v_2, v_4\})),$$
$$(\rceil a_8, (\{h_1, h_4, h_5\}, \{c_2, c_4, c_5\}, \{v_2, v_3\}))\}.$$

Definition 3.11 *A soft multiset (F, A) over U is called a semi-null soft multiset denoted by $(F, A)_{\approx\Phi_i}$, if at least one of a soft multiset parts of (F, A) equals $\emptyset$.*

Example 3.12 *Consider Example 3.2 again, with a soft multiset (F, A) which describes the "attractiveness of stone houses", "cars" and "hotels". Let*

$$A = \{a_1 = (e_{U_1,4}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,1}),$$
$$a_3 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,3})\}.$$

The soft multiset (F, A) is the collection of approximations as given below:

$$(F, A)_{\approx\Phi_1} = \{(a_1, (\emptyset, \{c_2\}, \{v_2\})), (a_2, (\emptyset, \{c_1, c_3\}, \emptyset)),$$
$$(a_3, (\emptyset, \{c_1, c_3\}, \{v_1\}))\}.$$

Then $(F, A)_{\approx\Phi_1}$ is a semi-null multisoft set.

Definition 3.13 *A soft multiset (F, A) over U is called a null soft multiset denoted by $(F, A)_\Phi$, if all the soft multiset parts of (F, A) equals $\emptyset$.*

Example 3.14 *Consider Example 3.2 again, with a soft multiset (F, A) which describes the "attractiveness of stone houses", "red cars model 1999" and "hotels in Kajang". Let*

$A = \{a_1 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,4}), a_2 = (e_{U_1,4}, e_{U_2,4}, e_{U_3,4})\}$. *The soft multiset*

(F, A) is the collection of approximations as below:

$$(F, A)_\Phi = \{(a_1, (\emptyset, \emptyset, \emptyset)), (a_2, (\emptyset, \emptyset, \emptyset))\}.$$

Then $(F, A)_{\approx\Phi_1}$ is a null multisoft set.

Definition 3.15 *A soft multiset* (F, A) *over* U *is called a semi-absolute soft multiset denoted by* $(F, A)_{\approx A_i}$ *if* $\left(e_{U_i,j}, F_{e_{U_i,j}}\right) = U_i$ *for at least one* i, $a_k \in A$, $a_k \in A, k = \{1, 2, 3, ..., n\}, i \in \{1, 2, 3, ..., m\}$ *and* $j \in \{1, 2, 3, ..., r\}$.

Example 3.16 *Consider Example 3.2 again, with a soft multiset* (F, A) *which describes the "attractiveness of wooden houses", "cars" and "hotels". Let*

$$A = \{a_1 = (e_{U_1,4}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,1}),$$
$$a_3 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,3})\}.$$

The soft multiset (F, A) *is the collection of approximations as given below:*

$$(F, A)_{\approx A_i} = \{(a_1, (U_1, \{c_2\}, \{v_2\})), (a_2, (U_1, \{c_1, c_3\}, \emptyset)),$$
$$(a_3, (U_1, \{c_1, c_3\}, \{v_1\}))\}.$$

Then $(F, A)_{\approx \Phi_1}$ *is a semi-absolute soft multiset.*

Definition 3.17 *A soft multiset* (F, A) *over* U *is called an absolute soft multiset denoted by* $(F, A)_A$ *if* $\left(e_{U_i,j}, F_{e_{U_i,j}}\right) = U_i$, $\forall i$.

Example 3.18 *Consider Example 3.2 again, with a soft multiset* (F, A) *which describes the "attractiveness of wooden houses", "black cars model 2001" and "hotels in KL". Let*

$$A = \{a_1 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,4}), a_2 = (e_{U_1,4}, e_{U_2,4}, e_{U_3,4})\}.$$

The soft multiset (F, A) *is the collection of approximations as shown below:*

$$(F, A)_A = \{(a_1, (U_1, U_2, U_3)), (a_2, (U_1, U_2, U_3))\}.$$

Then $(F, A)_A$ *is an absolute soft multiset.*

Proposition 3.19 *If* (F, A) *is a soft multiset over* U, *then*

1. $((F, A)^c)^c = (F, A)$,
2. $(F, A)^c{}_{\approx \Phi_i} = (F, A)_{\approx A_i}$,
3. $(F, A)^c{}_{\Phi} = (F, A)_A$,
4. $(F, A)^c{}_{\approx A_i} = (F, A)_{\approx \Phi_i}$,
5. $(F, A)^c{}_A = (F, A)_\Phi$.

Proof: The proof is straightforward.

Definition 3.20 *The union of two soft multisets* (F,A) *and* (G,B) *over* U *denoted by* $(F,A)\,\widetilde{\cup}\,(G,B)$ *is the soft multiset* (H,C) *where* $C = A \cup B$, *and* $\forall \varepsilon \subset C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & if \varepsilon \subset A - B \\ G(\varepsilon), & if \varepsilon \subset B - A \\ F(\varepsilon) \cup G(\varepsilon), & if \varepsilon \subset A \cap B. \end{cases}$$

Example 3.21 *Consider Example 3.2. Let*

$$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1}), a_3 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,3}), \\ a_4 = (e_{U_1,3}, e_{U_2,1}, e_{U_3,1})\} \text{ and}$$

$$B = \{b_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), b_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}), b_3 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1}), \\ b_4 = (e_{U_1,5}, e_{U_2,4}, e_{U_3,2}), b_5 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,2}), b_6 = (e_{U_1,1}, e_{U_2,3}, e_{U_3,2})\}.$$

Suppose (F,A) *and* (G,B) *are two soft multisets over the same* U *such that*

$$(F,A) = \{(a_1, (\{h_2,h_3\}, \{c_2\}, \{v_2\})), (a_2, (\{h_1,h_5\}, \{c_1,c_3\}, \emptyset)),$$

$$(a_3, (\{h_1,h_4\}, \{c_1,c_3\}, \{v_1\})), (a_4, (\{h_4\}, \emptyset, \{v_3\}))\} \text{ and}$$

$$(G,B) = \{(b_1, (\{h_2,h_3,h_6\}, \{c_2\}, \{v_2,v_3\})),$$

$$(b_2, (\{h_2,h_3,h_6\}, \{c_1,c_3,c_4,c_5\}, \{v_2,v_3\})),$$

$$(b_3, (\{h_1,h_4,h_5\}, \{c_1,c_3\}, \emptyset)), (b_4, (\{h_1,h_4\}, \{c_1,c_3\}, \{v_1\})),$$

$$(b_5, (\{h_1,h_4,h_5\}, \{c_1,c_3\}, U_2)),$$

$$(b_6, (\{h_2,h_3,h_6\}, \{c_1,c_3\}, \{v_1,v_4\}))\}.$$

Therefore $(F,A)\,\widetilde{\cup}\,(G,B) = (H,C)$

$$= \{(c_1, (\{h_2,h_3,h_6\}, \{c_2\}, \{v_2,v_3\})), \\ (c_2, (\{h_2,h_3,h_6\}, \{c_1,c_3,c_4,c_5\}, \{v_2,v_3\})), \\ (c_3, (\{h_1,h_4,h_5\}, \{c_1,c_3\}, \emptyset)), (c_4, (\{h_1,h_5\}, \{c_1,c_3\}, \emptyset)), \\ (c_5, (\{h_1,h_4\}, \{c_1,c_3\}, \{v_1\})), (c_6, (\{h_1,h_4,h_5\}, \{c_1,c_3\}, U_2)), \\ (c_7, (\{h_4\}, \emptyset, \{v_3\})), (c_8, (\{h_2,h_3,h_6\}, \{c_1,c_3\}, \{v_1,v_4\}))\}.$$

Proposition 3.22 *If* (F,A), (G,B) *and* (H,C) *are three soft multisets over* U, *then*

1. $(F,A)\widetilde{\cup}((G,B)\widetilde{\cup}(H,C)) = ((F,A)\widetilde{\cup}(G,B))\widetilde{\cup}(H,C)$,

2. $(F,A)\widetilde{\cup}(F,A) = (F,A)$,

3. $(F,A)\,\widetilde{\cup}(G,A)_{\approx\Phi_i} = (R,A)\,,$

4. $(F,A)\,\widetilde{\cup}(G,A)_{\Phi} = (F,A)\,,$

5. $(F,A)\,\widetilde{\cup}(G,B)_{\approx\Phi_i} = (R,D)\,,$

6. $(F,A)\,\widetilde{\cup}(G,B)_{\Phi} = \begin{cases} (F,A) & if A = B, \\ (R,D) & otherwise \end{cases},$ *where* $D = A \cup B$,

7. $(F,A)\,\widetilde{\cup}(G,A)_{\approx A_i} = (R,A)_{\approx A_i}$,

8. $(F,A)\,\widetilde{\cup}(G,A)_{A} = (G,A)_{A}$,

9. $(F,A)\,\widetilde{\cup}(G,B)_{\approx A_i} = \begin{cases} (R,D)_{\approx A_i} & if A = B, \\ (R,D) & otherwise \end{cases},$ *where* $D = A \cup B$,

10. $(F,A)\,\widetilde{\cup}(G,B)_{A} = \begin{cases} (G,B)_{A} & if\ \ A \subseteq B, \\ (R,D) & otherwise \end{cases}$,*where* $D = A \cup B$.

Proof: The proof is straightforward.

Definition 3.23 *The intersection of two soft multisets* (F,A) *and* (G,B) *over* U *denoted by* $(F,A)\,\widetilde{\cap}\,(G,B)$ *is the soft multiset* (H,C) *where* $C = A \cup B$, *and* $\forall \varepsilon \subset C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon)\,, & if \varepsilon \subset A - B \\ G(\varepsilon)\,, & if \varepsilon \subset B - A \\ F(\varepsilon) \cap G(\varepsilon)\,, & if \varepsilon \subset A \cap B. \end{cases}$$

Example 3.24 *Consider Example 3.2. Let*

$$\begin{aligned} A = \{ & a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1})\,, a_2 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1})\,, a_3 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,3})\,, \\ & a_4 = (e_{U_1,3}, e_{U_2,1}, e_{U_3,1})\} \text{ and} \end{aligned}$$

$$\begin{aligned} B = \{ & b_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1})\,, b_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1})\,, b_3 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1})\,, \\ & b_4 = (e_{U_1,5}, e_{U_2,4}, e_{U_3,2})\,, b_5 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,2})\,, b_6 = (e_{U_1,1}, e_{U_2,3}, e_{U_3,2})\}\,. \end{aligned}$$

Suppose (F,A) *and* (G,B) *are two soft multisets over the same* U *such that*

$$(F,A) = \{(a_1, (\{h_2, h_3\}, \{c_2\}, \{v_2\}))\,, (a_2, (\{h_1, h_5\}, \{c_1, c_3\}, \emptyset))\,,$$

$$(a_3, (\{h_1, h_4\}, \{c_1, c_3\}, \{v_1\}))\,, (a_4, (\{h_4\}, \emptyset, \{v_3\}))\} \text{ and}$$

$$(G,B) = \{(b_1, (\{h_2,h_3,h_6\}, \{c_2\}, \{v_2,v_3\})),$$

$$(b_2, (\{h_2,h_3,h_6\}, \{c_1,c_3,c_4,c_5\}, \{v_2,v_3\})),$$

$$(b_3, (\{h_1,h_4,h_5\}, \{c_1,c_3\}, \emptyset)), (b_4, (\{h_1,h_4\}, \{c_1,c_3\}, \{v_1\})),$$

$$(b_5, (\{h_1,h_4,h_5\}, \{c_1,c_3\}, U_2)),$$

$$(b_6, (\{h_2,h_3,h_6\}, \{c_1,c_3\}, \{v_1,v_4\}))\}.$$

Therefore $(F,A)\,\widetilde{\cap}\,(G,B) = (H,C)$

$$\begin{aligned}= \{&(c_1, (\{h_2,h_3\}, \{c_2\}, \{v_2\})), (c_2, (\{h_2,h_3,h_6\}, \{c_1,c_3,c_4,c_5\}, \{v_2,v_3\})),\\ &(c_3, (\{h_1,h_4,h_5\}, \{c_1,c_3\}, \emptyset)), (c_4, (\{h_1,h_5\}, \{c_1,c_3\}, \emptyset)),\\ &(c_5, (\{h_1,h_4\}, \{c_1,c_3\}, \{v_1\})), (c_6, (\{h_1,h_4,h_5\}, \{c_1,c_3\}, U_2)),\\ &(c_7, (\{h_4\}, \emptyset, \{v_3\})), (c_8, (\{h_2,h_3,h_6\}, \{c_1,c_3\}, \{v_1,v_4\}))\}.\end{aligned}$$

Proposition 3.25 *If* (F,A), (G,B) *and* (H,C) *are three soft multisets over* U, *then*

1. $(F,A)\,\widetilde{\cap}\,\big((G,B)\,\widetilde{\cap}\,(H,C)\big) = \big((F,A)\,\widetilde{\cap}\,(G,B)\big)\,\widetilde{\cap}\,(H,C)$,
2. $(F,A)\,\widetilde{\cap}\,(F,A) = (F,A)$,
3. $(F,A)\,\widetilde{\cap}(G,A)_{\approx\Phi_i} = (R,A)_{\approx\Phi_i}$,
4. $(F,A)\,\widetilde{\cap}(G,A)_{\Phi_i} = (G,A)_{\Phi_i}$,
5. $(F,A)\,\widetilde{\cap}(G,B)_{\approx\Phi_i} = \begin{cases} (R,D)_{\approx\Phi_i} & if\ A\subseteq B \\ (R,D) & otherwise \end{cases}$, *where* $D = A\cup B$,
6. $(F,A)\,\widetilde{\cap}(G,B)_{\Phi} = \begin{cases} (R,D)_{\Phi} & if\ A\subseteq B \\ (R,D) & otherwise \end{cases}$, *where* $D = A\cup B$,
7. $(F,A)\,\widetilde{\cap}(G,A)_{\approx A_i} = (R,D)$,
8. $(F,A)\,\widetilde{\cap}(G,A)_{A} = (F,A)$,
9. $(F,A)\,\widetilde{\cap}(G,B)_{\approx A_i} = (R,D)$,
10. $(F,A)\,\widetilde{\cap}(G,B)_{A} = \begin{cases} (F,A) & if A\subseteq B \\ (R,D) & otherwise \end{cases}$,*where* $D = A\cup B$.

Proof: The proof is straightforward.

Proposition 3.26 *If* (F, A), (G, B) *and* (H, C) *are three soft multisets over* U, *then*

$$1.\ (F, A)\,\widetilde{\cup}\,\big((G, B)\,\widetilde{\cap}\,(H, C)\big) = \big((F, A)\,\widetilde{\cup}\,(G, B)\big)\,\widetilde{\cap}\,\big((F, A)\,\widetilde{\cup}\,(H, C)\big),$$

$$2.\ (F, A)\,\widetilde{\cap}\,\big((G, B)\,\widetilde{\cup}\,(H, C)\big) = \big((F, A)\,\widetilde{\cap}\,(G, B)\big)\,\widetilde{\cup}\,\big((F, A)\,\widetilde{\cap}\,(H, C)\big).$$

Proof: The proof is straightforward.

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