SOFT PHYSICS : APPLICATIONS OF EFFECTIVE CHIRAL LAGRANGIANS TO NUCLEAR PHYSICS AND QUARK MODELS

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DISSERTATION COMMITTEE:

To Juliana and Laura.

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by

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The chiral lagrangian approach involving pions and nucleons is applied to a few problems in nuclear physics. A nucleon-nucleon potential is derived up to a certain order in chiral perturbation theory; a reasonable fit to phase shifts is obtained up to laboratory energies of about 100 MeV, and deuteron properties fall within 20% of observed values. Few-body forces are also considered to the same order, and some empirical features are recovered. Isospin violation from both the down-up quark mass difference and electromagnetism is shown to be generically small in low-energy hadronic phenomena as a consequence of the structure of the chiral lagrangian. Finally, an attempt is made at estimating two parameters that appear in such a lagrangian, the axial vector coupling and the anomalous magnetic moment of nucleons; this is done using sum rules and a chiral lagrangian involving pions and constituent quarks, in the limit of a large number of colors.

Table of Contents

1. Introduction

1.1 What is effective?

The concept of effective field theories [1] is over fifty years old, being probably born to Euler and Heisenberg in the context of QED [2]. It was raised during the late 60's by Weinberg [3], who noticed that a chiral symmetric lagrangian used at tree level was a copycat of current algebra. It finally matured during the 70's, mating with the renormalization group ideas of Wilson [4], and it was clearly exposed by the end of that decade as *the* rationale behind the use of field theory in particle physics phenomenology (see, for example, the views expressed by Weinberg on chiral lagrangians [5] and the Standard Model [6]).

Consider a field theory (which I call the "underlying theory") given by some lagrangian written in terms of some fields ("elementary fields"), which is found to adequately describe physics over a certain energy range. The effective theory for energies smaller than some scale Λ is the collection of operators that result from integrating out of the underlying theory those fields with momenta larger than Λ. The effective lagrangian can be very complicated, but has two

[1] For a better introduction than I will be able to deliver in what follows, see [1].

important properties. First, it is local, in the sense that it involves only fields at the same spacetime point—according to the uncertainty principle, particles with momentum $p < \Lambda$ can only probe distances $\frac{1}{p} > \frac{1}{\Lambda}$—but will contain arbitrary number of derivatives of such fields. Second, it transforms under various groups like the underlying theory: if the underlying lagrangian is symmetric under a transformation that is not anomalous, then so is the effective lagrangian, although the symmetry might be realized non-linearly if it is spontaneously broken; if the symmetry is explicitly broken either at a classical or a quantum level, operators will appear at low energies that break the symmetry in the same way.

The notion of an effective lagrangian is particularly useful when the underlying theory has (at least) one characteristic mass scale M. In this case, the effective lagrangian for $\Lambda < M$ is in general more conveniently written in terms of a different set of fields than the elementary ones. This reformulation ends up selecting those effects of the underlying theory that are more important at low energies.

The simplest case is the one where M is just the mass of a physical particle. Its production and decay involve large momenta and do not concern the effective theory. Effects of the virtual exchange of this particle, or of a particle-antiparticle pair, are of short range and thus included indirectly in the coefficients of operators of the effective lagrangian that involves only the low-energy degrees of freedom [7]. If the particle is not stable in the context of the underlying theory, then it does not appear at all at low energies. If, on the other hand, it is stable, then we are stuck with this heavy, non-relativistic particle that acts much like a static source of light fields.

Another case is where M is the scale associated with some (elementary or composed) scalar field acquiring a non-vanishing vacuum expectation value and breaking a continuous internal symmetry group G down to a subgroup H. Then, a massless spin-zero particle—a Goldstone boson— should appear [8] (or else be eaten by a vector gauge boson which becomes massive, if G is a local symmetry). It is convenient to introduce in the effective lagrangian a field for this particle, which is a parametrization of the coset space G/H at each spacetime point. There exist an infinite number of such parametrizations, but there is at least one in terms of which all interactions of the Goldstone boson are derivative [9]. If G is in some sense weakly, explicitly broken in the underlying theory, then non-derivative terms will appear—in particular, the particle might pick up a mass and we refer to it as a pseudo-Goldstone boson [10]. An effective description might be adequate as long as the dimensional parameters associated with the explicit breaking are small compared to M. Anyway, we get a nearly massless particle with interactions that are weak at low energy—an ideal situation for an effective lagrangian.

A third possibility is more complicated: M is a scale where some wild non-perturbative phenomena dominate and drastically affect the whole spectrum. This is the case of confinement in a non-abelian theory with the appropriate matter content. We do not know much about what happens then, but it seems adequate to restrict the effective lagrangian to fields that are singlets under the gauge group.

There are other possibilities (like the existence of a Fermi surface [1]), or combinations of various cases, but the point is, this mass M will provide a measure of the high energy effects that appear only indirectly in the effective description, if this description is natural. As terms with more and more derivatives are considered, they require coupling constants of larger inverse mass dimensions. If there is no symmetry or particular dynamical mechanism to suppress or enhance a coupling constant, we expect it to be of order 1 when made dimensionless by multiplication by the appropriate power of M. In other words, an effective lagrangian is infinitely complicated, and useless unless we turn this assumption of naturalness into an ordering of interactions.

1.2 The power of counting

The effective lagrangian will generate diagrams where all particles will have three-momenta $Q \ll M$, so there exists a natural expansion parameter Q/M. The importance of a graph will be estimated by counting powers of Q, that add to a number I call ν. Consider a generic diagram in D spacetime dimensions with $E_b(E_f)$ external and $I_b(I_f)$ internal boson (fermion) lines, L loops, C separately connected pieces and V_i vertices of type i, with $b_i(f_i)$ boson (fermion) lines and d_i (spatial) derivatives. These quantities satisfy the usual topological identities of counting lines,

$$\sum_i V_i b_i(f_i) = 2I_{b(f)} + E_{b(f)}, \tag{1}$$

and relating faces, sides and vertices of polyhedra,

$$L = I_b + I_f - \sum_i V_i + C, \tag{2}$$

A particle's mass smaller than M can roughly be considered of the same order as Q, so that its energy is also $\sim Q$. We can extend the definition of d_i above to include time derivatives and explicit factors of mass. Let me start with the case where all particles are like that. Then, to a generic diagram, internal boson (fermion) lines, loops, vertex derivatives and delta functions contribute $Q^{-2}(Q^{-1}), Q^D, Q$ and Q^{-D} respectively. Removing the overall momentum conservation delta function, and using the identities (1) and (2), we get

$$\nu = -\frac{1}{2}(D-2-r)E_b - \frac{1}{2}(D-1-r)E_f + D + r(L-C) + \sum_i V_i \Delta_i^{(r)}, \quad (3)$$

where $\Delta_i^{(r)}$ is hereby christened the r-index of a vertex,

$$\begin{aligned} \Delta_i^{(r)} &\equiv \frac{1}{2}(D-2-r)b_i + \frac{1}{2}(D-1-r)f_i + d_i - D + r \\ &= \delta_i^{(D)} - \frac{1}{2}r(b_i + f_i - 2) \end{aligned} \quad (4)$$

with

$$\delta_i^{(D)} = \frac{1}{2}(D-2)b_i + \frac{1}{2}(D-1)f_i + d_i - D \quad (5)$$

the usual mass dimension (in D dimensions) of an interaction in the action.

Here r is just a number introduced by adding and subtracting rL to ν; it is just a matter of convenience for the discussion that follows, ν being of course independent of r. The point is that, for a given process (fixed number of external legs), we can show the existence of a perturbative expansion if we can choose an $r \geq 0$ for which all $\Delta_i^{(r)} \geq 0$. For, then, C being bounded, ν only increases as graphs become more and more complicated.

When there are no super-renormalizable terms ($\delta_i^{(D)} < 0$), I choose

$$r = \frac{2\delta_0^{(D)}}{b_0 + f_0 - 2} \geq 0, \quad (6)$$

where $\delta_0^{(D)}$ is the dimension of the interaction of smallest $\delta_i^{(D)}$, which has $b_0(f_0)$ boson (fermion) fields. Then

$$\Delta_i^{(r)} = \delta_i^{(D)} - \delta_0^{(D)} \frac{b_i + f_i - 2}{b_0 + f_0 - 2} \geq 0. \tag{7}$$

There are two possibilities:

i) $\delta_0^{(D)} = 0$, that is, there are renormalizable interactions:

The index (7) is just the mass dimension and ν, the superficial degree of divergence of the graph. Eq. (3) simply tells us that the dominant graphs are expected to be those containing only renormalizable interactions, for which $\sum_i V_i \Delta_i^{(0)} = 0$. The number of vertices and loops is arbitrary, which means that the theory is still infinitely complicated, unless we find a further ordering of diagrams as provideed by, say, a small coupling or a large number of charges. On the other hand, all diagrams with a non-renormalizable interaction have $\sum_i V_i \Delta_i^{(0)} > 0$, so they will be small as long as Q is small compared to M. The most important of these suppressed graphs have the minimum number of non-renormalizable interactions.

ii) $\delta_0^{(D)} > 0$, that is, there are only non-renormalizable interactions:

Because in this case operators with different dimensions will contribute to the lowest order ν, $\Delta_i^{(r)}$ in (7) is more convenient than $\delta_i^{(D)}$ to classify the relative importance of operators. Eq. (3) tell us that the dominant graphs to any process are tree graphs with all vertices having index zero. Adding loops add also positive powers of Q, so a perturbative expansion indeed emerges, even if the couplings of the underlying theory are not small. Moreover, the more connected parts a graph has, the more important it is. Pairwise interactions are favored: many-body forces are relatively suppressed and impulse approximation holds.

Super-renormalizable terms are a problem, however: their couplings are proportional to positive powers of M, and that disturbs the power counting by introducing large factors. The most dramatic example is a large cosmological constant. Another is an interaction like φ^3 in $D = 4$, which would force a negative r in (6) and result in arbitrarily large contributions from diagrams of arbitrarily large number of loops. And there is also generically a problem with the masses of the various particles. All these large factors better be forbidden or at least suppressed; if this is achieved by a symmetry, one says that the effective theory is natural.

There is one case in which a super-renormalizable term must be faced but is tolerated at low-energies: a large mass for a stable fermion. We can integrate out fermion-antifermion pairs in loops, but the fermion might be there already as an external particle. In this case, it is represented by a line that simply goes through the diagrams, and it is necessarily non-relativistic, its energy being essentially its rest mass m plus a very small kinetic energy Q^2/m. Because this is a problem with three scales $m >> Q >> Q^2/m$, the above power counting needs adaptation. In order to separate the first scale, m, we have to formulate the effective lagrangian in a way that m does not appear explicitly. This can be done elegantly [11] by working with a field of definite velocity, in what is called a heavy fermion formalism. The same result in a non-covariant form can be achieved by eliminating time derivatives of the heavy fermion in interaction terms in favor of the value given by the Dirac equation. In order to separate the third scale, Q^2/m, we have to distinguish two sorts of diagrams [12]: irreducible diagrams that cannot be disconnected by cutting through an intermediate state only those lines corresponding to initial or final particles; and reducible diagrams that are formed by sewing together irreducible diagrams with several separately connected pieces. The power counting (3) and its consequences hold for the former. The latter will contain infrared divergences when the heavy fermion kinetic energy is neglected; when it is not, they will be bigger than naively expected by the power counting by a large factor m/Q; contrary to the irreducible diagrams themselves, their iteration has to be carried out to all orders.

The power counting done above provides the rationale to select a finite number of operators compatible with our desired degree of detail in describing the low energy physics: to a certain order in Q/M, all observables are known functions of a finite number of parameters in the effective lagrangian. One can only do better if the underlying theory and a way to solve it are known, in which case these effective parameters can be obtained explicitly in terms of the more fundamental parameters of the underlying theory.

1.3 The soft end of a hard theory

This dissertation is devoted to the application of these ideas to the soft limit of the strong interactions.

It does not seem to be an overstatement to say that the results from *all* experiments and observations made to date are explained by, or at least consistent with, the Standard Model: a theory of quarks and leptons with gravitational interactions given by general relativity plus renormalizable, gauge-invariant interactions to bosons of a $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge group, spontaneously broken to $SU(3)_c \times U(1)_{em}$ by an unspecified mechanism. This is a(n almost) natural effective theory at a scale $\sim$ 100GeV as long as the electroweak breaking sector has a custodial $SU(2)$ symmetry to ensure the correct relation between the W and Z masses, and whatever other symmetries necessary to suppress superrenormalizable terms.

As we go down to a scale of a few GeV, weak gauge bosons have been integrated out in favor of non-renormalizable terms suppressed by powers of the W, Z masses; what remains is QCD and QED. The part of the lagrangian containing quarks, gluons and photons is

$$\begin{aligned} \mathcal{L} &= -\sum_{i=1}^{n_f} \bar{q}_i(\partial\!\!\!/ - ig_s G\!\!\!/ - iz_i e A\!\!\!/ + m_i)q_i - \frac{1}{2}Tr[G_{\mu\nu}G^{\mu\nu}] - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &+\frac{\theta g_s^2}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}Tr[G^{\mu\nu}G^{\rho\sigma}] + \text{non-renormalizable terms,} \end{aligned} \tag{8}$$

where q_i $(i = 1, \ldots, n_f)$ is the quark field of mass m_i and charge z_i, G_μ (A_μ) is the gluon (photon) field of strength $G_{\mu\nu}$ $(F_{\mu\nu})$ and coupling g_s (e), and θ is a parameter.

I will ignore non-renormalizable terms in what follows. I will further restrict my attention to processes that do not involve explicitly strangeness, charm, beauty,...: most of what I will do in the rest of this work can be straightforwardly extended to include the s quark; c, b,... can be incorporated as heavy fermions in an effective low-energy theory the way I mentioned above, an approach that blossomed recently (see [13] for a review). The renormalizable terms in the lagrangian (8) for $n_f = 2$ have 5 parameters: the masses of the up (m_u)

and the down (m_d) quarks, the gauge couplings for strong (g_s) and electromagnetic (e) interactions and the strong CP (θ) parameter. The latter is found to be unnaturally small (strong CP problem), so I will neglect it.

What are the characteristic scales of such a theory? In the limit where e, m_u and m_d are zero, there is a global chiral $SU(2)_L \times SU(2)_R$ symmetry. A consistent picture emerges if we assume that two non-perturbative phenomena happen, the spontaneous breaking of chiral symmetry to $SU(2)_V$ of isospin at a scale $\Lambda_{\chi SB}$, and confinement at Λ_{QCD}. At energies small compared to these scales, thus, the effective lagrangian will contain Goldstone bosons—the pions—and possibly other hadrons. To describe nuclear processes, the less massive, stable baryons, the nucleons, can be included as heavy fermions; the Δ-isobar is not much heavier and can also be incorporated. The resulting lagrangian has only non-renormalizable interactions and is therefore amenable to a perturbative treatment as indicated above. Other mesons, however, can couple to pions and nucleons via renormalizable interactions that are not small, so I do not know of an expansion that justifies neglecting all but a finite number of contributions. I will stay below the ρ mass, in which case these other mesons are integrated out. Now, the pion mass is not zero, being mainly due to the explicit breaking of chiral symmetry from the isospin symmetric combination of quark mass terms. But it is small in the characteristic scales of QCD, so it can be incorporated in the effective lagrangian without destroying its momentum expansion.

In chapters 2 and 3 I will follow the pioneering work of Weinberg [14] and apply this effective lagrangian approach to the fundamental problem of nuclear physics—a derivation of the nuclear potential. The general chiral lagrangian up to second order is constructed; the nucleon-nucleon potential is derived in terms of lagrangian parameters, the Schrödinger equation is solved, and phase shifts and deuteron properties are fitted; and few-body forces are discussed. Most of the features present in more phenomenological approach are recovered.

In chapter 4, I look at the extra terms in the chiral lagrangian that arise from the quark mass and charge differences, and argue that the smallness of these effects can be naturally understood.

Can we do better and predict the parameters in this effective lagrangian? This is a difficult task because strong interactions are, well, strong; QCD is a hard theory to solve. A step in this direction can be made with a dose of goodwill. The two scales $\Lambda_{\chi SB}$ and Λ_{QCD} are related, but are not necessarily the same. Indeed, it has been argued [15] that there might be a range of energies below $\Lambda_{\chi SB}$ but above Λ_{QCD} where there are still colored degrees of freedom but chiral symmetry has been broken. An effective lagrangian would contain pions, constituent quarks and gluons, and provide a natural understanding of the successes of constituent quark models, from which parameters of the nucleonic chiral lagrangian can be predicted.

In chapter 5, I consider this chiral quark lagrangian in the limit of large number of colors. In order to determine parameters, I follow Weinberg once more [16] and impose that some amplitudes have good high energy behavior encoded in sum rules: the Adler-Weisberger sum rule provides information on the axial vector coupling, and the Drell-Hearn-Gerasimov sum rule on the anomalous magnetic moments.

In summary, my aim is to show that chiral effective lagrangians provide a link between QCD, quark models and nuclear physics, and in doing so, help to improve our understanding of the low energy hadronic nether world in terms of the symmetries of the Standard Model.

2. The nucleon-nucleon potential

2.1 Introduction

The problem of deriving the nuclear potential is as old as nuclear physics itself. After early field-theoretical work ran into all sorts of difficulties in the 50's and was followed by a much more phenomenological approach in the 60's, the last two decades saw a compromise being reached, with meson exchange potentials providing very good fits to nucleon-nucleon data. Why, then, should we look at this problem again?

It has been argued [17] that Regge phenomenology can be extended to low-energy nucleon-nucleon scattering, Regge poles leading to an one-boson-exchange (OBE) potential where i) the contributions of meson trajectories (including a scalar ε's) are dominated by the particles with lowest spin, which couple to nucleons with a gaussian form factor; and ii) gaussian potentials arise from the Pomeron and tensor trajectories. Such a potential in a non-relativistic expansion has been constructed by the Nijmegen group [18] and fits data well. However, Regge cuts are simply neglected. The Bonn group (for a review, see [19]) seriously attempted to include multi-boson exchange, in the framework of old-fashioned perturbation theory. To the OBE of known mesons, they added 2π and $\pi\rho$ exchange with nucleons and Δ isobars in intermediate states, "correlated" two-pion exchange in the form of a σ' scalar meson, and even $\pi\sigma_{OBE}$ (with σ_{OBE} an approximation to 2π, σ' and $\pi\rho$) and $\pi\omega$ exchange. Agreement with data is but impressive.

Nevertheless, the justification of such an approach from the known theory of strong interactions, QCD, remains mysterious. In particular, it is not clear how one can consistently deal with the exchange of mesons with masses not much smaller than the typical inverse hadronic radius set by the QCD scale Λ_{QCD}. This has led many people (see Ref.[20] for a review) to attempt derivations of nucleon-nucleon scattering from quark models (either constituent or baggy), formulated in terms of some effective degrees of freedom carrying the same quantum numbers as current quarks and gluons. Although such models are not currently derived from QCD, either, they usually have only a few parameters, most fixed by a fitting of one-nucleon properties. Generically [20] one produces adequate short-range interactions, but the long range potential is still formulated in terms of pion exchange.

It seems natural, therefore, to start a treatment of the nuclear force problem by recognizing the unique role played by the pion. Although we are largely ignorant of the non-perturbative dynamics of QCD at low energies, we know there exists an approximate chiral symmetry which is broken by the vacuum. This symmetry restricts the form of the allowed interactions of pions with themselves and with other particles. Consequences are i) the small pion mass in the scale set by Λ_{QCD}, responsible for its long range, and ii) theorems relating processes involving different numbers of pions, which yield some predictive power. The pion is indeed the most important character, besides the nucleon, of the nuclear physics drama.

Such distinguished status of the pion has, of course, been emphasized before, particularly by the Stony Brook and Paris groups. It resulted in a coordinate space potential by the latter [21], containing: i) a long range, "theoretical" part constructed through unitarity, analyticity and crossing relations from $\pi\pi$ and πN phase shifts, which includes one, two (continuum plus ρ, ε) and partially three (in the form of ω) pion exchange; ii) a short range, purely phenomenological part with several combinations of spin and isospin factors. Both groups later moved from this model-independent but parameter-crowded approach to the other extreme, the only vaguely justified but two-parameter Skyrme model. Semi-quantitative success results, except for the lack of central, intermediate range attraction [22].

What is fundamentally new in our approach is the framework of the general effective chiral lagrangian. By considering the most general lagrangian which involves the pion and the nucleon, and transforms under chiral symmetry as the QCD lagrangian, we can split the problem in two. One task is to deal with QCD and reformulate it in terms of the low-energy degrees of freedom. The result has to have the form of the general chiral lagrangian (because the latter contains *all* the interactions with the correct symmetry), but the coupling constants will be known functions of more fundamental quantities like Λ_{QCD} and the quark masses. In other words, the dynamics of QCD is buried in the couplings of the chiral lagrangian. Since different models of QCD are just different attempts to capture the essence of its dynamics, they will in general differ in the strenght of the low-energy parameters. The second part of the problem is to relate these parameters to the measured, low-energy quantities, like scattering phase shifts and deuteron properties.

We will not attempt to "solve" QCD here, so we will concentrate on the second task mentioned above. We start with the general chiral lagrangian with undetermined coefficients. Because chiral symmetry is manifest (contrary to most meson-exchange models—e.g. [18, 19]), our approach is *a priori* compatible both with QCD and with all known low-energy phenomenology: $\pi\pi$, πN, γN scattering, meson-exchange currents, etc. When a systematic analysis of such processes as πN scattering with a chiral lagrangian is carried out, a number of our undetermined coefficients will be fixed by fitting data from such processes. Meanwhile, by keeping such parameters free, our scheme is model independent: we do not commit to either a massive meson exchange picture or a particular quark model. We do have to make one assumption, that of naturalness, that the parameters can be estimated by naive dimensional analysis. But under this sole assumption, we can develop a perturbative treatment of the nuclear potential that is lacking in other approaches. Here the perturbative expansion is in powers of momenta small compared to a typical QCD scale. To a certain order, we know all the interactions and diagrams that should be included. Of course, because we have no choice of what to put in, it is not guaranteed that we will have all the necessary ingredients. If we get a good overall description of the problem, that tells us that we have carried out perturbation theory to the order the precision of the data generically requires. If, on the other hand, a particular ingredient (say, scalar isoscalar attraction) is missing, then that might be telling us that a certain operator or diagram is more important than naively expected. It could be included, but what is more important, this in turn would be indicative of some characteristic dynamic mechanism, and we would be learning something about QCD.

We would like to stress that our aim is not to get a better fit of nucleon-nucleon data than the already excellent fits by meson-exchange potentials. What we hope for is to establish a bridge between QCD and nuclear physics. In doing so, we might learn about QCD dynamics, and, at the same time, provide a sound nuclear potential whose off shell structure is fixed and may be used for other nuclear calculations. In short, the general chiral lagrangian is a useful way to parametrize both our ignorance of QCD and our knowledge of nuclear physics.

The rest of the chapter is devoted to the support of this claim. In Sect.2.2 some generalities are discussed regarding what ingredients an effective theory of nuclear processes should contain, and in Sect.2.3 we present the chiral lagrangian. The two-nucleon potential is derived to a certain order in chiral perturbation theory in momentum space in Sect.2.4, and in coordinate space—using a momentum space gaussian cut-off—in Sect.2.5. Sect.2.6 presents the results of the fitting of the coordinate space potential to nucleon-nucleon scattering and bound state data. Conclusions are the contents of Sect.2.7. Finally, some of the details regarding different aspects of the work are saved for Appendices A, B, C and D.

2.2 Power Counting

Typical three-momenta Q exchanged in nuclei can be estimated as the inverse of the rms eletromagnetic radius $< r_{ch}^2 >^{1/2}$ of a light nucleus. For example, for triton $< r_{ch}^2 >^{1/2} \simeq 1.75 fm$ and we find that $Q \sim m_\pi$, the pion mass. The theory of strong interactions, QCD, on the other hand, gets strong and is dominated by non-perturbative effects at a scale M that is roughly given by a typical hadronic mass, ~ 1 GeV. Whenever we face such a two-scale problem, it is useful to separate the corresponding physics by looking at an effective low-energy theory that involves only the relevant degrees of freedom, all with small three-momenta Q. It can be formulated with a Lagrangian that is local (in the sense that it involves only operators containing fields at the same spacetime point) and shares the symmetries of the underlying theory. The dynamical information of modes with momenta $\gtrsim M$ is contained in an (infinite) set of parameters.

What are the relevant degrees of freedom in this case? We do not expect to (and we do not indeed) see quarks and gluons with such low energy probes. Our fields will represent mesons and baryons. Clearly, the lightest stable particles in each sector should be included. The pion π has a mass that is small compared to M, and its pseudo-Goldstone boson nature makes it a fundamental ingredient. The nucleon N has a mass m_N which is not small but protons and neutrons are already there in the systems we are interested in, so they should, and they can, be included. (The explicit appearence of the nucleon mass m_N in the effective theory will require some care later.) What about higher mass states? Their effects in the pion-nucleon system will generically be suppressed by the inverse of their masses (in the case of mesons) or of the mass differences with the nucleon (in the case of baryons). We keep only those states for which this factor is bigger than $\sim 1/M$. In the meson sector, this implies we do not explicitly keep the ρ, ω, etc. whose masses $\gtrsim 5.5m_\pi$ are closer to M than to m_π. In the baryon sector, we retain the Δ isobar which has a mass $m_\Delta \sim m_N + 2m_\pi$; as for the N^* of mass $m_{N^*} \sim m_N + 3.5m_\pi$ and other states, we decide more or less arbitrarily not to include them explicitly, but this could be done in much the same way as the Δ is included below. Finally there are the other octet pseudo-Goldstone bosons and the hyperons. Again, for simplicity we consider only $SU(2) \times SU(2)$, our treatment being easily extended to $SU(3) \times SU(3)$ and hypernuclear physics.

The requirement that the low energy lagrangian incorporates the symmetries of QCD will restrict the form of possible interactions involving π , N and Δ, but we will still be left with an infinity of interactions i, which differ in the number of derivatives or powers of pion mass d_i, fermion fields f_i, etc. If we knew how to solve the QCD dynamics at such low energies, we would calculate the corresponding coupling constants g_i. We can resort to models which incorporate many explicit and implicit assumptions concerning that dynamics, but then we face quandaries like quark model vs. meson exchange.

In any case, there is no *a priori* reason for the couplings to be small. The situation might seem hopeless, because we want to avoid model dependent assumptions and yet, we know little more than perturbation theory.

We can proceed only by making an assumption of naturalness: that once a coupling constant g_i of mass dimension $-\delta_i$ is expressed as $g_i = \tilde{g}_i M^{-\delta_i}$, the dimensionless coupling $\tilde{g}_i$ is of order $O(1)$, unless suppressed by symmetry. Of course this might not be true for all the couplings, but then we will discover that sooner or later on phenomenological grounds. If a coupling constant is found to be anomalously large or small, it will require perhaps a special treatment at low energies, but will also possibly point towards a particular dynamical or symmetry effect at the level of QCD.

We now have a natural expansion parameter $\frac{Q}{M} \sim \frac{m_\pi}{M}$, the contribution of any diagram being characterized by the power ν of the soft momentum Q. We can count powers of Q in the same way we do it to get the superficial degree of divergence of a graph, but some care must be taken with baryons due to explicit factors of their large masses. Because in the effective theory all three-momenta $Q \ll m_N$, nucleons and Δ's are non-relativistic: they just sit there getting a little kick now and then from a soft pion or another slow baryon. The first task is to organize the expansion in such a way as to eliminate time-derivatives of the fermions in interaction terms, since they would contribute large factors. This can be done elegantly by redefining the fermion fields in terms of velocity eigenstates [23], but also more simply by directly replacing the time-derivatives of fermion fields by their expressions given by the equations of motion; in the process we generate interactions that have already been accounted for, thus just redefining their coefficients. The second thing is to realize that there is a class of diagrams involving at least two nucleons which are larger than one could naively think. These diagrams, that we call reducible, are such that they can be separated in two parts by cutting in an intermediate state all initial or final lines, but only those. This type of intermediate state produces infrared divergences in the limit when the baryon kinetic energy is ignored; when it is not, we find a small recoil energy denominator which makes the overall diagram bigger than expected by a factor $\frac{m_N}{Q} \gg 1$. The simplest way to isolate these diagrams is to work in the framework of old-fashioned, time-ordered perturbation theory. Irreducible diagrams are those that contain only intermediate states with energies that differ from the initial energy by an amount $O(Q)$. For such an irreducible diagram with V_i vertices of type i, L loops, C separately connected pieces and $E_f = 2A$ external fermion lines, the power of Q can be conveniently written as

$$\nu = 4 - E_f + 2L - 2C + \sum_i V_i \Delta_i \tag{9}$$

where

$$\Delta_i = d_i + \frac{n_i}{2} - 2 \tag{10}$$

is called the index of vertex i. A generic reducible diagram can then be built from irreducible diagrams with several connected pieces, and intermediate states with energies that differ from the initial energy by an amount $O(Q^2/m_N)$ or smaller.

We will here be dealing with process with external nucleons only. Irreducible diagrams are then A-nucleon irreducible: any intermediate state has at least one pion or one isobar. We define the nuclear potential as the sum of such irreducible diagrams, their contributions being ordered by (9). The amplitude for scattering can then be evaluated by iterating the nuclear potential in the Lippmann-Schwinger equation, or equivalently, by solving (numerically) the corresponding Schrödinger equation.

One last remark before we move on to the symmetries of the effective lagrangian. We do not know *a priori* what exactly the scale M is, so it is not clear how relativistic corrections (which are suppressed by $1/m_N$) compare to $1/M$ corrections. A rough idea of their relative importance can be obtained from the following naive dimensional argument. The nucleon-nucleon potential in momentum space can be written as $V(p,p') = \alpha I(p,p')$ where $I(p,p')$ is some dimensionless function of the initial and final c.m. momenta p, p', and $\alpha \sim 2\pi^2/M^2$ if we attempt to count powers of 2 and π *à la* Georgi and Manohar[24]. Putting this in the Lippmann-Schwinger equation we see that it is an expansion in $\alpha Q m_N/2\pi^2 \sim Q m_N/M^2$. A shallow bound-state indicates that this series barely diverges, so one might guess that $M^2 \sim Q m_N$. This estimate is admittedly crude and is not crucial for our approach, but it suggests that relativistic corrections $O(\frac{Q}{m_N})$ are $O(\frac{Q^2}{M^2})$. If M is actually larger, it only indicates that relativistic corrections are relatively a little larger than assumed here.

2.3 Chiral Lagrangian

One can see from (9) that it is essential for a perturbative expansion that $\Delta_i \geq 0$. For, in this case, there is a lower bound for ν, corresponding to diagrams with the maximum number of separately connected pieces, no loops and all vertices having $\Delta_i = 0$. Corrections with higher ν are obtained by inserting loops and interactions with $\Delta_i > 0$, and decreasing the number of connected pieces. We will now show that chiral symmetry enforces

$$\Delta_i \geq 0. \tag{11}$$

Here for simplicity we work with QCD with only two light flavors u and d, with masses m_u and m_d, but it is straightforward to include the strange quark. In the limit of vanishing quark masses there is an $SU(2) \times SU(2) \sim SO(4)$ symmetry which is spontaneously broken to $SU(2) \sim SO(3)$. As a result, there exist Goldstone bosons whose fields live in the three-sphere $S^3 \sim SO(4)/SO(3)$, of a radius that turns out to be the pion decay constant $F_\pi \simeq 190\text{MeV}$. Following Weinberg [25, 26] we use stereographic coordinates $\boldsymbol{\pi}$; their covariant derivative is then

$$\boldsymbol{D}_\mu = \frac{1}{1+\boldsymbol{\pi}^2/F_\pi^2}\frac{\partial_\mu \boldsymbol{\pi}}{F_\pi} \equiv D^{-1}\frac{\partial_\mu \boldsymbol{\pi}}{F_\pi}. \tag{12}$$

The baryons here considered provide the 1/2 and 3/2 representations of the spin and isospin $SU(2)$ groups. A nucleon N (isobar Δ) is described by a Pauli spinor (a 4-component spinor) in both spin and isospin spaces, the respective generators being denoted $\frac{1}{2}\vec{\sigma}(\frac{1}{2}\vec{\sigma}^{(3/2)})$ and $\boldsymbol{t}(\boldsymbol{t}^{(3/2)})$. There are also, of course, 2×4 transition operators $\frac{1}{2}\vec{S}$ and $\boldsymbol{T}$, satisfying

$$S_i S_j^+ = \frac{1}{3}(2\delta_{ij} - i\varepsilon_{ijk}\sigma_k) \tag{13}$$

$$T_a T_b^+ = \frac{1}{6}(\delta_{ab} - i\varepsilon_{abc}t_c), \tag{14}$$

which allow us to couple N and Δ in bilinears with spin and isospin 1, respectively.

The effective chiral lagrangian is now constructed out of the fields $\boldsymbol{D}_\mu$, N and Δ and their covariant derivatives

$$\mathcal{D}_\mu \boldsymbol{D}_\nu = \partial_\mu \boldsymbol{D}_\nu + i\boldsymbol{E}_\mu \times \boldsymbol{D}_\nu \tag{15}$$

$$\mathcal{D}_\mu N = (\partial_\mu + \boldsymbol{t}\cdot\boldsymbol{E}_\mu)N \tag{16}$$

$$\mathcal{D}_\mu \Delta = (\partial_\mu + \boldsymbol{t}^{(3/2)}\cdot\boldsymbol{E}_\mu)\Delta \tag{17}$$

where

$$E_\mu \equiv \frac{2i}{F_\pi}\boldsymbol{\pi} \times \boldsymbol{D}_\mu. \tag{18}$$

This is done by considering all possible isoscalar terms and imposing the discrete spacetime symmetries of QCD, parity and time-reversal.

That is not all, tough, because the quark masses break $SO(4)$ explicitly. They can be written as a linear combination of the fourth component of a chiral four-vector and the third component of another four-vector, with coefficients $\frac{1}{2}(m_u + m_d)$ and $\frac{1}{2}(m_u - m_d)$ respectively. We account for this explicit breaking by including in the chiral lagrangian all the terms built out of $\boldsymbol{\pi}$, N and Δ that transform under $SO(4)$ in the same way. Their coefficients will then be proportional to powers of those combinations of masses. That is the way the pion mass arises, $m_\pi^2 \propto (m_u + m_d)$, so each power of $m_u + m_d$ will count as Q^2. Here we will for simplicity neglect isospin breaking; when we incorporate its effects along similar lines we get an understanding of why they are so feeble in most nuclear phenomena [27].

(Appendix A presents more details regarding the transformation properties of the field representation we use.)

Now, when we try to write down operators that are chiral invariant or that break chiral invariance as the quark mass term, we immediately convince ourselves that all interaction terms have $\Delta_i \geq 0$: operators involving pions only have at least two derivatives or two powers of m_π and nucleon bilinears have at least one derivative. *Chiral symmetry guarantees a natural perturbative low-energy theory.*

The index of interaction Δ_i provides an useful ordering scheme for the chiral lagrangian. Below we denote by $\mathcal{L}^{(n)}$, and call it n-th order lagrangian, the collection of terms with indices $\Delta_i = n$. We show explicitly only those terms relevant to our results: since we will be considering the two-body potential up to one-loop, operators with more pion fields or isobars than those exhibited below will not contribute, although they are obviously there, in many cases to assure chiral invariance. Note also that we eliminate some redundant terms by integrating by parts, by using the equations of motion (e.g. to eliminate nucleon time-derivatives) and by applying Fierz reordering.

The lowest order lagrangian is

$$
\begin{aligned}
\mathcal{L}^{(0)} = & -\frac{1}{2}D^{-2}((\vec{\nabla}\boldsymbol{\pi})^2 - \dot{\boldsymbol{\pi}}^2) - \frac{1}{2}D^{-1}m_\pi^2\boldsymbol{\pi}^2 \\
& +\bar{N}[i\partial_0 - 2D^{-1}F_\pi^{-2}\boldsymbol{t}\cdot(\boldsymbol{\pi}\times\dot{\boldsymbol{\pi}}) - m_N]N \\
& -2D^{-1}F_\pi^{-1}g_A\bar{N}(\boldsymbol{t}\cdot\vec{\sigma}\cdot\vec{\nabla}\boldsymbol{\pi})N \\
& -\frac{1}{2}C_S\bar{N}N\bar{N}N - \frac{1}{2}C_T\bar{N}\vec{\sigma}N\cdot\bar{N}\vec{\sigma}N \\
& +\bar{\Delta}[i\partial_0 - 2D^{-1}F_\pi^{-2}\boldsymbol{t}^{(3/2)}\cdot(\boldsymbol{\pi}\times\dot{\boldsymbol{\pi}}) - m_\Delta]\Delta \\
& -2D^{-1}F_\pi h_A[\bar{N}\boldsymbol{T}\cdot(\vec{S}\cdot\vec{\nabla}\boldsymbol{\pi})\Delta + h.c.] \\
& +\ldots
\end{aligned}
\qquad (19)
$$

where g_A is the axial vector coupling of the nucleon, h_A is the $\Delta N\pi$ coupling and C_S and C_T are the parameters first introduced by Weinberg [25, 26].

In this work we will also employ terms with more derivatives and powers of m_π. The first-order lagrangian is

$$
\begin{aligned}
\mathcal{L}^{(1)} = & -\frac{B_1}{F_\pi^2}D^{-2}\bar{N}N[(\vec{\nabla}\boldsymbol{\pi})^2 - \dot{\boldsymbol{\pi}}^2] \\
& -\frac{B_2}{F_\pi^2}D^{-2}\varepsilon_{ijk}\varepsilon_{abc}\bar{N}\sigma_k t_c N\partial_i\pi_a\partial_j\pi_b
\end{aligned}
$$

$$-\frac{B_3}{F_\pi^2} m_\pi^2 D^{-1} \bar{N} N \boldsymbol{\pi}^2$$
$$+ \ldots \tag{20}$$

where the B_i's are coefficients of order $O(1/M)$; in particular, B_3 is the (in)famous σ-term. The second-order lagrangian is

$$\begin{aligned}
\mathcal{L}^{(2)} = & \frac{1}{2m_N}\bar{N}\vec{\nabla}^2 N - \frac{A_1'}{F_\pi}[\bar{N}(\boldsymbol{t}\cdot\vec{\sigma}\cdot\vec{\nabla}\boldsymbol{\pi})\vec{\nabla}^2 N + \overline{\vec{\nabla}^2 N}(\boldsymbol{t}\cdot\vec{\sigma}\cdot\vec{\nabla}\boldsymbol{\pi})N] \\
& -\frac{A_2'}{F_\pi}\overline{\vec{\nabla}N}(\boldsymbol{t}\cdot\vec{\sigma}\cdot\vec{\nabla}\boldsymbol{\pi})\cdot\vec{\nabla}N \\
& -C_1'[(\bar{N}\vec{\nabla}N)^2 + (\overline{\vec{\nabla}N}N)^2] - C_2'(\bar{N}\vec{\nabla}N)\cdot(\overline{\vec{\nabla}N}N) \\
& -C_3'\bar{N}N[\bar{N}\vec{\nabla}^2 N + \overline{\vec{\nabla}^2 N}N] \\
& -iC_4'[\bar{N}\vec{\nabla}N\cdot(\overline{\vec{\nabla}N}\times\vec{\sigma}N) + (\overline{\vec{\nabla}N})N\cdot(\bar{N}\vec{\sigma}\times\vec{\nabla}N)] \\
& -iC_5'\bar{N}N(\overline{\vec{\nabla}N}\cdot\vec{\sigma}\times\vec{\nabla}N) - iC_6'(\bar{N}\vec{\sigma}N)\cdot(\overline{\vec{\nabla}N}\times\vec{\nabla}N) \\
& -(C_7'\delta_{ik}\delta_{jl} + C_8'\delta_{il}\delta_{kj} + C_9'\delta_{ij}\delta_{kl})\times \\
& \qquad [\bar{N}\sigma_k\partial_i N\bar{N}\sigma_l\partial_j N + \overline{\partial_i N}\sigma_k N\overline{\partial_j N}\sigma_l N] \\
& -(C_{10}'\delta_{ik}\delta_{jl} + C_{11}'\delta_{il}\delta_{kj} + C_{12}'\delta_{ij}\delta_{kl})\bar{N}\sigma_k\partial_i N\overline{\partial_j N}\sigma_l N \\
& -(\frac{1}{2}C_{13}'(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{kj}) + C_{14}'\delta ij\delta_{kl})\times \\
& \qquad [\overline{\partial_i N}\sigma_k\partial_j N + \overline{\partial_j N}\sigma_k\partial_i N]\bar{N}\sigma_l N \\
& + \ldots
\end{aligned} \tag{21}$$

where the A_i', C_i' are still other undetermined coefficients of order $O(1/M^2)$.

From these lagrangians it is straightforward to read the rules to be used in diagrams. Because we got rid of time derivatives in all interaction terms but four (those that come together with the pion and fermion kinetic terms in $\mathcal{L}^{(0)}$, and the B_1 term in $\mathcal{L}^{(1)}$), and all of them involve at least two pion fields, the interaction hamiltonian is just (-1) times the interaction lagrangian, up to interactions with more pion fields that will not contribute to the order we will be working.

2.4 The two-nucleon potential in momentum - space

We are now in position to evaluate any process we want involving soft pions and non-relativistic nucleons. Equations (9), (10) and (11) guarantee that the dominant contributions to any such process come from tree graphs with the maximum number of connected pieces and constructed out of the lagrangian $\mathcal{L}^{(0)}$. When applied to processes with at most one nucleon, this is just what is given by current algebra. For example, one easily obtain the famous Weinberg [28] pion-pion and Tomozawa-Weinberg [29, 28] pion-nucleon S–wave scattering lenghts. But in the late 70's Weinberg [30] realized that chiral lagrangians go beyond that, and provide a framework to evaluate corrections to such dominant contributions. Chiral perturbation theory in the mesonic sector really began with the work of Gasser and Leutwyler [31] and has been extensively studied in the case of $SU(3) \times SU(3)$, up to $L = 1$ and $\Delta_i = 2$, and including electroweak effects (for an introduction, see Ref. [32]). A systematic study of the $SU(2) \times SU(2)$ chiral lagrangian for processes involving one nucleon was started by Gasser, Saino and Švarc [33] and is continuing with the work of Bernard, Meissner and Kaiser [34] (for a review see Ref.[35]). In principle, the coefficients g_A, h_A, B_i and A'_i can be determined from these processes once all contributions up to one loop are evaluated. Unfortunately, we do not know all their values yet. In Sect.2.6 we will fit these parameters to nucleon-nucleon data, but it should be kept in mind that the number of parameters in our potential could be reduced when sufficient information from the one-nucleon sector is gathered.

Here we carry out the analysis of the age-old problem of nuclear forces. As noted in Sect.2.2, (9) applies to the nuclear potential, which can then be calculated perturbatively and later iterated to all orders when solving the Schrödinger equation numerically. Eq. (9) tells us that, as it is well known by any nuclear physicist, three-(and more-)body forces are smaller than two body forces. They have already been considered in [36] and here we will restrict ourselves to the two-nucleon system where data are aplenty. So one can set $A = 2$ and $C = 1$ in (9),

$$\nu = 2L - 2 + \sum_i V_i \Delta_i \tag{22}$$

We will be working in the center of mass and denoting the initial energy by $2m_N + E$, initial (final) momentum by $\vec{p}(\vec{p}')$, with $\vec{q} \equiv \vec{p} - \vec{p}'$ being the transferred momentum and $\vec{k} \equiv \frac{1}{2}(\vec{p} + \vec{p}')$ the other independent combination of momenta; subscripts 1 and 2 on $\vec{\sigma}$ and $\boldsymbol{t}$ matrices refer to nucleons 1 and 2 respectively.

The leading order potential $V^{(0)}$ (with $\nu = -2$) is obtained from the graphs in Fig.1 and interactions given by $\mathcal{L}^{(0)}$ in (19). Note that to this order nucleons are static, so that their energies in intermediate states are simply m_N, and the Δ isobar does not contribute. One obtains [25] the well-known static one-pion-exchange (OPE) potential supplemented by contact interactions,

$$V^{(0)} = -(\frac{2g_A}{F_\pi})^2 \boldsymbol{t}_1 \cdot \boldsymbol{t}_2 \frac{\vec{\sigma}_1 \cdot \vec{q}\,\vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^2 + m_\pi^2} + C_S + C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2 \tag{23}$$

The OPE term provides the longest range force, and it is well established [37] that it accounts for the higher partial waves in nucleon-nucleon scattering and the bulk of the properties of the deuteron, like its quadrupole moment. But it is also known that the nuclear force has other sizeable components, like a spin-orbit force, a strong short-range repulsion and an intermediate range attraction. Clearly, the simple lowest order result (23) does not provide much room for them. The question naturally arises then, whether higher order contributions yield such features.

(Some of the results presented below have already been published in [36].)

First corrections ($\nu = -1$) would came from the same graphs of Fig.1, with one vertex from $\mathcal{L}^{(1)}$. But there are no appropriate terms in (20) and we conclude that there are no corrections to the leading order potential $V^{(0)}$ that are smaller by just one power of Q/M,

$$V^{(1)} = 0. \tag{24}$$

This is a direct consequence of parity invariance. Indeed, for these tree graphs, we could only add a power of momentum (or subtract one and add an extra power of m_π^2) to $V^{(0)}$; but this is actually a three-momentum, because we eliminated time derivatives, so we end up with an odd number of three-momenta and no parity conserving terms can be constructed.

There are many corrections of second-order ($\nu = 0$), though.

First, we could have still no loops and one factor from $\mathcal{L}^{(2)}$ (or two from $\mathcal{L}^{(1)}$, but we just saw that there are no suitable vertices in (20)). This means that the graphs are still the ones in Fig.1, and either i)one vertex comes from the interactions in (21), the nucleons still static, or ii)vertices come from (19), but now we include recoil in the intermediate state. We get

$$\begin{aligned} V^{(2)}_{tree} &= -\frac{2g_A}{F_\pi^2} t_1 \cdot t_2 \frac{\vec{\sigma}_1 \cdot \vec{q}\,\vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^2 + m_\pi^2} \times \\ &\quad \times (A_1 q^2 + A_2 k^2 - 2g_A \frac{E - \frac{1}{4m_N}(4\vec{k}^2 + \vec{q}^2)}{\sqrt{\vec{q}^2 + m_\pi^2}}) \\ &\quad + C_1 \vec{q}^2 + C_2 \vec{k}^2 + (C_3 \vec{q}^2 + C_4 \vec{k}^2) \vec{\sigma}_1 \cdot \vec{\sigma}_2 \\ &\quad + i C_5 \frac{\vec{\sigma}_1 + \vec{\sigma}_2}{2} \cdot (\vec{q} \times \vec{k}) + C_6 \vec{q} \cdot \vec{\sigma}_1 \vec{q} \cdot \vec{\sigma}_2 \\ &\quad + C_7 \vec{k} \cdot \vec{\sigma}_1 \vec{k} \cdot \vec{\sigma}_2 \end{aligned} \tag{25}$$

where the A_i's and C_i's are combinations (listed in Appendix B) of the A_i''s and C_i''s of (21).

Second, there are contributions from graphs of Fig.2 with one loop and all factors coming from (19). (Other one loop graphs only contribute to the renormalization of parameters in the lagrangian.) Now one can have, besides nucleons, also one or two isobars in intermediate states. Denoting

$$\omega_\pm \equiv \sqrt{(\vec{q} \pm \vec{l})^2 + 4m_\pi^2}, \tag{26}$$

$$\Delta \equiv m_\Delta - m_N \tag{27}$$

straightforward calculation gives

$$\begin{aligned} V^{(2)}_{loop,no\Delta} &= -\frac{1}{2F_\pi^4} t_1 \cdot t_2 \int \frac{d^3 l}{(2\pi)^3} \frac{1}{\omega_+ \omega_-} \frac{(\omega_+ - \omega_-)^2}{\omega_+ + \omega_-} \\ &\quad - 4\left(\frac{g_A}{F_\pi^2}\right)^2 t_1 \cdot t_2 \int \frac{d^3 l}{(2\pi)^3} \frac{1}{\omega_+ \omega_-} \frac{\vec{q}^2 - \vec{l}^2}{\omega_+ - \omega_-} \end{aligned}$$

$$-\frac{1}{4}\left(\frac{g_A}{F_\pi}\right)^4 \int \frac{d^3l}{(2\pi)^3}\frac{1}{\omega_+^3\omega_-}\left\{\left(\frac{3}{\omega_-}+\frac{8\boldsymbol{t}_1\cdot\boldsymbol{t}_2}{\omega_++\omega_-}\right)(\vec{q}^{\,2}-\vec{l}^{\,2})^2 + 4\left(\frac{3}{\omega_++\omega_-}+\frac{8\boldsymbol{t}_1\cdot\boldsymbol{t}_2}{\omega_-}\right)\vec{\sigma}_1\cdot(\vec{q}\times\vec{l})\vec{\sigma}_2\cdot(\vec{q}\times\vec{l})\right\} \quad (28)$$

for the diagrams of Fig.2a,b,c,d that do not include isobars in intermediate states,

$$\begin{aligned}
V^{(2)}_{loop,one\Delta} = \; & \frac{8}{9}\frac{h_A^2}{F_\pi^4}\boldsymbol{t}_1\cdot\boldsymbol{t}_2\int\frac{d^3l}{(2\pi)^3}\frac{1}{\omega_++\omega_-}\frac{\vec{q}^{\,2}-\vec{l}^{\,2}}{(\omega_++2\Delta)(\omega_-+2\Delta)} \\
& -\frac{1}{18}\frac{g_A^2h_A^2}{F_\pi^4}\{(3+4\boldsymbol{t}_1\cdot\boldsymbol{t}_2) \\
& \times\int\frac{d^3l}{(2\pi)^3}[(\vec{q}^{\,2}-\vec{l}^{\,2})^2+2\vec{\sigma}_1\cdot(\vec{q}\times\vec{l})\vec{\sigma}_2\cdot(\vec{q}\times\vec{l})]\times \\
& \times\left[\frac{1}{\omega_+\omega_-(\omega_++\omega_-)}\left(\frac{1}{\omega_+(\omega_-+2\Delta)}+\frac{1}{\omega_-(\omega_++2\Delta)}\right)\right. \\
& +\frac{1}{2}\frac{1}{\Delta\omega_+\omega_-}\left(\frac{1}{\omega_+\omega_-}+\frac{1}{\omega_+(\omega_-+2\Delta)}+\frac{1}{\omega_-(\omega_++2\Delta)}+\right. \\
& \left.\left.+\frac{1}{(\omega_++2\Delta)(\omega_-+2\Delta)}\right)\right] \\
& +(3-4\boldsymbol{t}_1\cdot\boldsymbol{t}_2)\int\frac{d^3\vec{l}}{(2\pi)^3}[(\vec{q}^{\,2}-\vec{l}^{\,2})^2-2\vec{\sigma}_1(\vec{q}\times\vec{l})\vec{\sigma}_2\cdot(\vec{q}\times\vec{l})]\times \\
& \times\frac{1}{\omega_+\omega_-}\left[\frac{1}{\omega_++\omega_-+2\Delta}\left(\frac{1}{\omega_+\omega_-}+\frac{1}{(\omega_++2\Delta)(\omega_-+2\Delta)}\right)\right. \\
& +\left(\frac{1}{\omega_+\omega_-}+\frac{1}{\omega_++\omega_-+2\Delta}\right)\times \\
& \left.\left.\times\left(\frac{1}{\omega_-(\omega_-+2\Delta)}+\frac{1}{\omega_+(\omega_-+2\Delta)}\right)\right]\right\} \qquad (29)
\end{aligned}$$

for diagrams of Fig.2b,c,d,e with one intermediate isobar, and

$$\begin{aligned}
V^{(2)}_{loop,two\Delta} = \; & -\frac{2h_A^4}{81F_\pi^4}\{(3-2\boldsymbol{t}_1\cdot\boldsymbol{t}_2)\cdot \\
& \cdot\int\frac{d^3l}{(2\pi)^3}[(\vec{q}^{\,2}-\vec{l}^{\,2})^2-\vec{\sigma}_1\cdot(\vec{q}\times\vec{l})\vec{\sigma}_2\cdot(\vec{q}\times\vec{l})]\times \\
& \times\frac{1}{\omega_+\omega_-}\frac{1}{\omega_++2\Delta}\frac{1}{\omega_-+2\Delta}\left[\frac{1}{\omega_++\omega_-}+\frac{1}{2\Delta}\right]
\end{aligned}$$

$$+(3+2\boldsymbol{t}_1\cdot\boldsymbol{t}_2)\int\frac{d^3l}{(2\pi)^3}[(\vec{q}^{\,2}-\vec{l}^2)^2+\vec{\sigma}_1\cdot(\vec{q}\times\vec{l})\vec{\sigma}_2\cdot(\vec{q}\times\vec{l})]\times$$
$$\times\frac{1}{\omega_+\omega_-(\omega_++\omega_-+4\Delta)}\left[\frac{1}{\omega_++2\Delta}\frac{1}{\omega_-+2\Delta}+\right.$$
$$\left.\left.\frac{\omega_++\omega_-+2\Delta}{\omega_++\omega_-}\left(\frac{1}{(\omega_-+2\Delta)^2}+\frac{1}{(\omega_++2\Delta)^2}\right)\right]\right\}\qquad(30)$$

for diagrams of Fig. 2c,d,e that have two intermediate Δ's.

Finally, still at one loop level we also have corrections of third order ($\nu=1$).

Again, some would come from the tree graphs of Fig.1 with one vertex from $\mathcal{L}^{(3)}$, but the same argument as for (24) quarantees that

$$V^{(3)}_{tree}=0\qquad(31)$$

Others would come from the loop graphs of Fig.2 with one vertex from (20). Parity says the contribution from Fig.2a vanishes, and that can be confirmed by explicit calculation; there are no $\boldsymbol{\pi}\bar{N}N$ coupling in $\mathcal{L}^{(1)}$, so Fig.2c,d,e do not contribute; and Fig.2b gives

$$V^{(3)}_{loop,no\Delta}=-\frac{1}{4}\left(\frac{g_A}{F_\pi^2}\right)^2\int\frac{d^3l}{(2\pi)^3}\frac{1}{\omega_+^2\omega_-^2}\left\{3(\vec{q}^{\,2}-\vec{l}^2)[-B_1(\vec{q}^{\,2}-\vec{l}^2)+4m_\pi^2B_3]\right.$$
$$\left.+16B_2\vec{\sigma}_1\cdot(\vec{q}\times\vec{l})\vec{\sigma}_2\cdot(\vec{q}\times\vec{l})\boldsymbol{t}_1\cdot\boldsymbol{t}_2\right\}\qquad(32)$$

for no Δ in the intermediate state, and

$$V^{(3)}_{loop,one\Delta}=-\frac{1}{9}\left(\frac{h_A}{F_\pi^2}\right)^2\int\frac{d^3l}{(2\pi)^3}\frac{1}{\omega_+\omega_-}\frac{1}{\omega_++\omega_-}\frac{1}{(\omega_++2\Delta)(\omega_-+2\Delta)}$$
$$\times\left\{(\omega_++\omega_-+2\Delta)[3(\vec{q}^{\,2}-\vec{l}^2)(-B_1(\vec{q}^{\,2}-\vec{l}^2)+4m_\pi^2B_3)+\right.$$
$$+4B_2\vec{\sigma}_1\cdot(\vec{q}\times\vec{l})\vec{\sigma}_2\cdot(\vec{q}\times\vec{l})\boldsymbol{t}_1\cdot\boldsymbol{t}_2]$$
$$\left.+6B_1\Delta\omega_+\omega_-(\vec{q}^{\,2}-\vec{l}^2)\right\}\qquad(33)$$

when there is one.

Further corrections are of higher order ($\nu \geq 2$). They include i) two-loop graphs, like the ones in Fig.3, that are numerous and harder to calculate; ii) tree graphs with a vertex from $\mathcal{L}^{(4)}$, which would bring *many* new undetermined coefficients. We do not attempt to include them here.

It is in this momentum space form that we can more easily summarize the structure of our potential and compare it with other approaches.

As usual, the longest range part is given by one pion exchange (OPE). It is dominated by the classic static OPE in (23), first obtained by Yukawa [38]. Then there are corrections. The A_1, A_2 terms in (25) can be viewed as coming from the first corrections to the $\pi\bar{N}N$ vertex in an expansion of its form factor in powers of momenta over the form factor parameter. A dependence on q^2 is usual (see, e.g. [19] where mono and dipole forms are used), but k^2 dependence has also been considered more recently (see, e.g. the Williamsburg model [39]). The other correction to OPE is the energy dependent term in (25), which arises from the recoil of the nucleon upon pion emission.

The intermediate range piece is due to two pion exchange (TPE). It is determined in terms of few parameters: g_A, h_A, $m_\Delta - m_N$, B_1, B_2, B_3 (and of course F_π). The contributions from box and crossed box diagrams (Fig.2c,d,e) are standard. The one in (28) (g_A^4 term) was first considered by Brueckner and Watson [40], while those with Δ's in (29) ($g_A^2 h_A^2$ term) and (30) (h_A^4 term) are due to Sugawara and von Hippel [41]. (As a check, our results agree with the appropriate limit of the expressions listed in Ref. [44]). But we would like to emphasize that there also exist TPE contributions from the "pair" diagrams of Fig.2a,b that are less commom. Those in (28) and the B_3–term in (32) have also been suggested before by Sugawara and Okubo [42], but with generic coefficients. Here the terms in (28) are fixed by chiral symmetry in terms of g_A and F_π while the B_3–term is nothing but the σ–term; and, to the same order, we also have in (32) two new terms (B_1, B_2). Finally the corresponding terms with Δ in (29) and (33) are also new. We would like to stress that these contributions from the non-linear coupling of the pion to the nucleon are a consequence of chiral symmetry that is *not* usually included in meson exchange potentials (e.g. [18, 19]). On the other hand, they are the only form of "correlated" pion exchange in our potential. The more traditional s–wave correlated TPE (Fig.3a) is formally of higher order.

Finally, the short range part is given in terms of several contact terms (the C_i's in (23) and (25)). They contain the effect of exchange of higher energy modes and are not constrained by chiral symmetry: all combinations of momenta (up to second power) that satisfy parity and time-reversal go, including spin-orbit (C_5), tensor (C_6, C_3) and spin independent central (C_S, C_1, C_2) forces. In order to compare with other approaches it will be convenient to "undo" our previous Fierz reordering and rewrite the coefficients C_i as

$$C_i = C_i^{(0)} + C_i^{(1)} \boldsymbol{t}_1 \cdot \boldsymbol{t}_2. \tag{34}$$

2.5 The two-nucleon potential in coordinate - space

We now go to coordinate space, where it is sometimes more useful to handle a potential.

In order to deal with our potential (23)—(33) we first have to face the fact that the loop integrals in (28), (29), (30), (32), (33) diverge, and so require regularization. One could use, for example, dimensional regularization, but the evaluation gets complicated due to the non-covariant nature of our graphs. Because it is conceptually and mathematically simpler, and also used in other nuclear potentials, we choose to work with a momentum space cut-off $\Lambda \lesssim M$. The form of the cut-off function and its value are somewhat arbitrary and presumably not very important, modifications being compensated to some extent by a redefinition of the free parameters in the theory. Again for simplicity, we follow the Nijmegen group [18] and take a gaussian cut-off $\exp(-\vec{l}^2/\Lambda^2)$. Furthermore, in our approach all momenta are smaller than M, so we can also cut-off the transferred momentum q with the same cut-off function $\exp(-\vec{q}^{\,2}/\Lambda^2)$.

Now all integrals over $\vec{q}$ and $\vec{l}$ can be worked out in terms of one dimensional integrals that can easily be evaluated numerically. We use the formulas and tricks presented in [43, 44]—see Appendix C for some details. Here we present only the final form.

We first define, as usual, the tensor operator, the total spin and the relative angular momentum,

$$
\begin{aligned}
S_{12} &= 3\frac{\vec{\sigma}_1\cdot\vec{r}\vec{\sigma}_2\cdot\vec{r}}{r^2} - \vec{\sigma}_1\cdot\vec{\sigma}_2 \\
\vec{S} &= \frac{1}{2}(\vec{\sigma}_1+\vec{\sigma}_2) \\
\vec{L} &= -i\vec{r}\times\vec{\nabla}
\end{aligned}
\tag{35}
$$

respectively. In terms of those and the Pauli matrices $\boldsymbol{\tau}$ in isospin space, we consider the 20 operators

$$
\begin{aligned}
\mathcal{O}^{p=1,\ldots,20} = \; & 1, \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2, \vec{\sigma}_1\cdot\vec{\sigma}_2, \vec{\sigma}_1\cdot\vec{\sigma}_2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2, S_{12}, S_{12}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2, \vec{L}\cdot\vec{S}, \\
& \vec{L}\cdot\vec{S}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2, \vec{L}^2, \vec{L}^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2, \vec{L}^2\vec{\sigma}_1\cdot\vec{\sigma}_2, \vec{L}^2\vec{\sigma}_1\cdot\vec{\sigma}_2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2, \\
& (\vec{L}\cdot\vec{S})^2, (\vec{L}\cdot\vec{S})^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2, S_{12}\vec{L}\cdot\vec{S}, S_{12}\vec{L}\cdot\vec{S}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2, S_{12}\vec{L}^2, \\
& S_{12}\vec{L}^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2, S_{12}(\vec{L}\cdot\vec{S})^2, S_{12}(\vec{L}\cdot\vec{S})^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2
\end{aligned}
\tag{36}
$$

Our potential in coordinate space can now be written as

$$
V = \sum_{p=1}^{20} V_p(r, \frac{\partial}{\partial r}, \frac{\partial^2}{\partial r^2}; E)\mathcal{O}^p \tag{37}
$$

where

$$
V_p(r, \frac{\partial}{\partial r}, \frac{\partial^2}{\partial r^2}; E) = V_p^0(r;E) + V_p^1(r;E)\frac{\partial}{\partial r} + V_p^2(r;E)\frac{\partial^2}{\partial r^2} \tag{38}
$$

is an energy dependent radial operator determined by the radial functions $V_p^0(r;E)$, $V_p^1(r;E)$ and $V_p^2(r;E)$. These sixty functions (some vanish) are listed in Appendix D: each one consists of a sum of terms having parameters of the lagrangian as coefficients and written in terms of at most one one-dimensional integral of the functions from Appendix C; energy dependence is linear.

The first eight operators, $\mathcal{O}^{p=1,\ldots,8}$, are standard, and are accompanied in most potentials by radial functions with no derivatives. They receive contributions from pion exchanges and contact terms. The next six operators, $\mathcal{O}^{p=9,\ldots,14}$, complete the set used in the phenomenological Urbana v14 potential [45]. Here, as there, $V_p^1 = V_p^2 = 0$, $p = 9, \ldots, 14$. What is characteristic of the structure (36)–(38) of our potential is the presence of the first and second derivative terms in the other terms, and of the six new operators $\mathcal{O}^{p=15,\ldots,20}$. They all arise from the dependence on the momentum operator, which comes from the k^2 dependence of the A_2, C_2, C_4 and C_7 terms.

2.6 Phase shifts and deuteron properties

The next step is to solve the Schrödinger equation with the potential (36)–(38). The procedure is standard, but some care has to be exercized with the derivative terms.

As usual, one works with basis functions of definite isospin I, total orbital angular momentum L, total spin S, total angular momentum J, and its third component, m, and decomposes the wave function into a sum of products of radial and angular functions. Upon projection on an angular function, and angular integration, the Schrödinger equation can be written schematically as

$$\left[X^{(2)}\frac{\partial^2}{\partial r^2} + X^{(1)}\frac{\partial}{\partial r} + B\right] R = 0 \tag{39}$$

where $B = \frac{1}{2\mu r^2}L^2 + \sum_p V_p^{(0)} < \mathcal{O}^p > -E$, $X^{(2)} = -\frac{1}{2\mu} + \sum_p V_p^{(2)} < \mathcal{O}^p >$ and

$X^{(1)} = -\frac{1}{\mu r} + \sum_p V_p(1) < \mathcal{O}^p >$, with μ the reduced mass and $<>$ denoting matrix elements between angular basis functions. Spin singlet and triplet $L = J$ channels are uncoupled, so for them R is a single radial function. On the other hand, the tensor operator couples triplet states of $L = J \pm 1$, so in this case R has two components, and B, $X^{(2)}$ and $X^{(1)}$ are 2×2 matrices. In order to eliminate the first derivative term, we redefine $R = K\phi$, with K a matrix chosen so that ϕ satisfies an equation with no first derivative. This imposes a differential equation on K that depends on the X's, and the boundary condition on K is fixed by further requiring that the two ϕ components be linearly independent as $r \to \infty$; K at finite r can then be obtained by Runge-Kutta integration. Meanwhile, the equation for ϕ became of the form $\phi'' = -C(E)\phi$, with C a 2×2 matrix function of B, $X^{(2)}$ and $X^{(1)}$ with the usual boundary conditions: ϕ vanishes at $r = 0$ and $\frac{\phi}{r}$ for large r is related to the scattering phase shifts.

The ϕ equation is finally solved in a standard numerical way for several scattering energies to obtain phase shifts, and for the bound state to obtain the binding energy and other properties. These are all functions of the undetermined parameters in the lagrangian. We fix a cut-off and fit the other parameters to Arndt's np phase shifts [46] (with error bars from [47]), and to known deuteron quantities.

A sample of some results with a cut-off equal to the ρ mass can be found, for phase shifts, in the graphs of Fig.4. Agreement is reasonable, up to laboratory energies around 100MeV. We get a single bound state, with $I = 0$ and properties as listed in Table 1. The binding energy B and the asymptotic d/s ratio η are within 10% of the experimental results, while agreement is worse for the

	Experimental [48]	Our fit
B(MeV)	2.22457	2.31807
$Q(fm^2)$	0.2859(3)	0.2216
η	0.0271(4)	0.0251
μ_d(nuclear magnetons)	0.85741	0.85765
$P_D(\%)$	—	3.9

Table 1: Deuteron properties: binding energy B, quadrupole moment Q, asymptotic d/s ratio η, magnetic moment μ_d, D-state probability P_D.

quadrupole moment Q and much better for the magnetic moment μ_d. (B could be decreased with a simultaneous increase in Q, at the expense of quality in the fitting of phase shifts.) The values of the parameters corresponding to this fitting are listed in Table 2.

2.7 Conclusion

We derived a nucleon-nucleon potential, solved the corresponding Schrödinger equation and fitted scattering phase shifts and deuteron properties.

In spirit, our approach is similar to that of the Paris group [21]: information on pion dynamics is used to construct the longer range pieces of the potential, while more complicated dynamics is buried in an unconstrained, short range part. The fundamental difference resides in our use of effective field theory, as opposed to dispersion relations. This not only ensures that our results are consistent with other aspects of pion phenomenology (chiral lagrangians to the order we use generally agree with data at the 20% level), but more importantly,

parameter(units)	value	parameter(10^{-9}MeV^{-4})	value
F_π(MeV)	192	$C_1^{(0)}$	0.662
g_A	1.33	$C_1^{(1)}$	0.391
h_A	2.03	$C_2^{(0)}$	3.40
$A_1(10^{-6}\text{MeV}^{-2})$	-1.38	$C_2^{(1)}$	3.12
$A_2(10^{-6}\text{MeV}^{-2})$	2.44	$C_3^{(0)}$	-0.331
$B_1(10^{-2}\text{MeV}^{-1})$	0.342	$C_3^{(1)}$	-0.0303
$B_2(10^{-2}\text{MeV}^{-1})$	0.855	$C_4^{(0)}$	-0.144
$B_3(10^{-2}\text{MeV}^{-1})$	1.77	$C_4^{(1)}$	0.431
$C_S^{(0)}(10^{-4}\text{MeV}^{-2})$	1.12	$C_5^{(0)}$	2.10
$C_S^{(1)}(10^{-4}\text{MeV}^{-2})$	0.130	$C_5^{(1)}$	-0.904
$C_T^{(0)}(10^{-4}\text{MeV}^{-2})$	-0.266	$C_6^{(0)}$	0.281
$C_T^{(1)}(10^{-4}\text{MeV}^{-2})$	-0.672	$C_6^{(1)}$	0.112
		$C_7^{(0)}$	0.582
		$C_7^{(1)}$	1.25

Table 2: Parameters in our potential as obtained by the fitting to phase shifts and deuteron properties.

explicitly incorporates the symmetries of QCD and provides a natural perturbative expansion. In this way, we, like the Nijmegen group [17, 18], build on a theoretical basis, but unlike them, can carry out a controlled expansion. Our use of field theory and old-fashioned perturbation theory, on the other hand, brings our potential closer to a low-energy version of Bonn's [19].

Our potential in momentum space shares several features with these and other potentials. The short range part has all the necessary spin- isospin structure; pion exchange has contributions that have been considered before, but also exhibits some new terms related to chiral symmetry; and energy dependence (which has implications to few-body forces) arises naturally.

We used a gaussian cut-off to transform to coordinate space, and there some of the particularities of our approach show up as some unusual operators and derivative terms. After some care was taken with the latter, the Schrödinger equation was solved by standard numerical methods.

We end up with a single bound state with roughly the correct deuteron properties and phase shifts that do not look too bad up to around 100 MeV. This shows that our approach is also quantitatively successful to some extent and therefore, that the gross features of the nucleon-nucleon potential can be naturally understood on the basis of the symmetries of QCD. But it also makes clear that it is unpractical to try to compete with those other, more phenomenological approaches in providing a numerically useful fitting: both their range of energies and quality of fitting could only be reproduced in our approach, presumably, by exploring higher orders in chiral perturbation theory.

Figure Captions

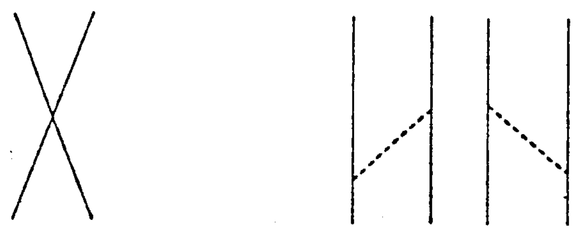
Figure (1) : Tree graphs contributing to the two-nucleon potential (solid lines are nucleons, dashed lines pions)

Figure (2) : One loop graphs contributing to the two-nucleon potential (double lines represent nucleons or isobars). Only one time ordering is shown for each type of graph. In (d) and (e) we only consider those orderings that have at least one pion or one isobar in intermediate states.

Figure (3) : Examples of two-loop graphs that are *not* included in our potential.

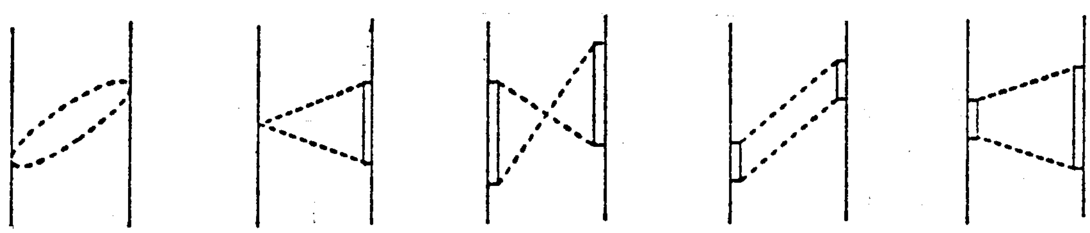
Figure (4) : Phase shifts for several channels. Dots are experimental results and lines, the results of our fitting.

Fig. 1



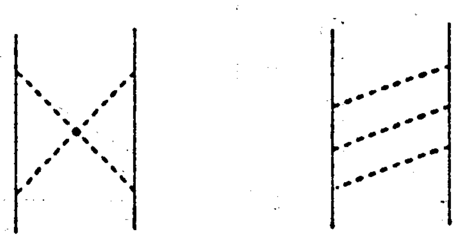
(a) (b)

Fig. 2



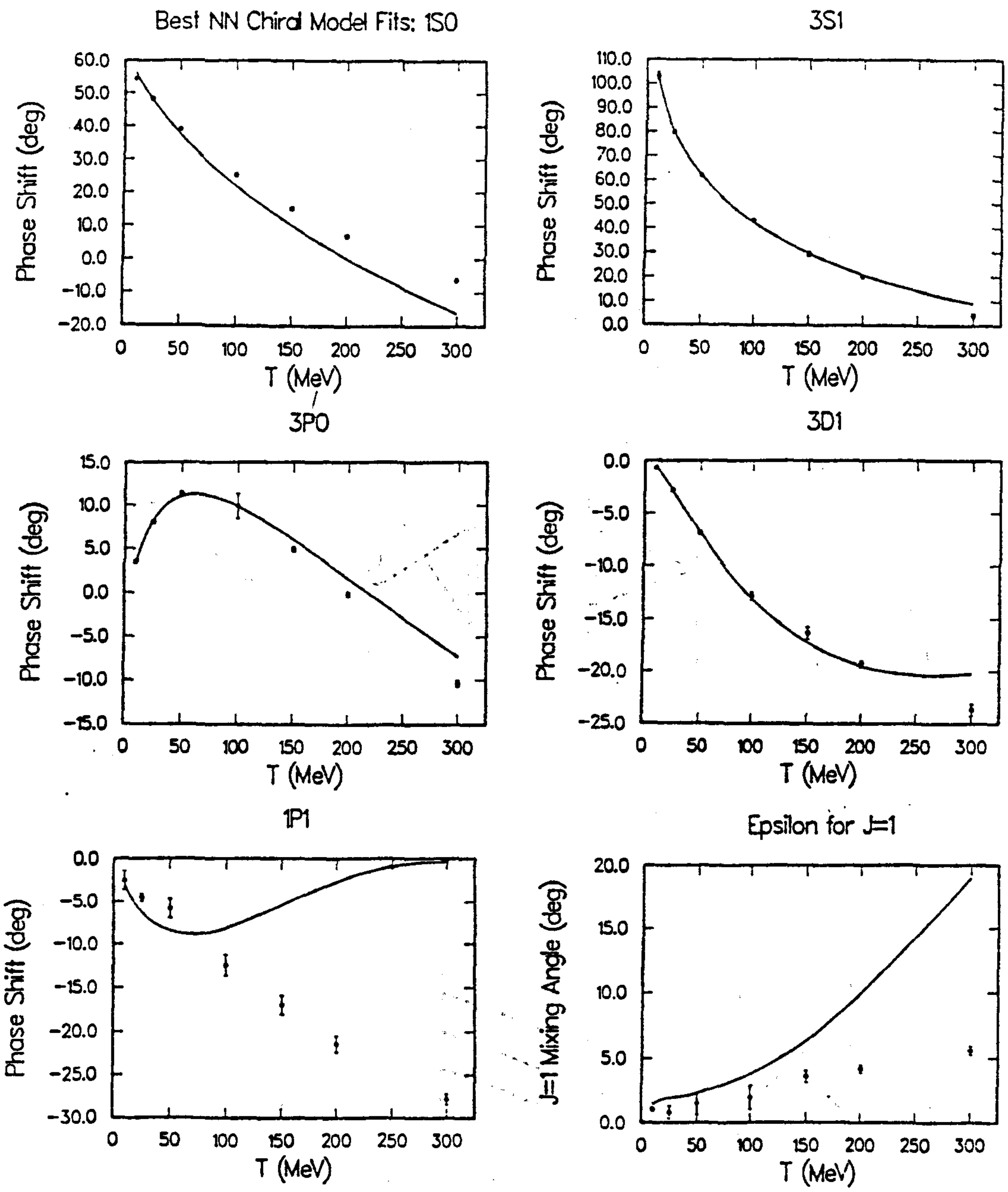
(a) (b) (c) (d) (e)

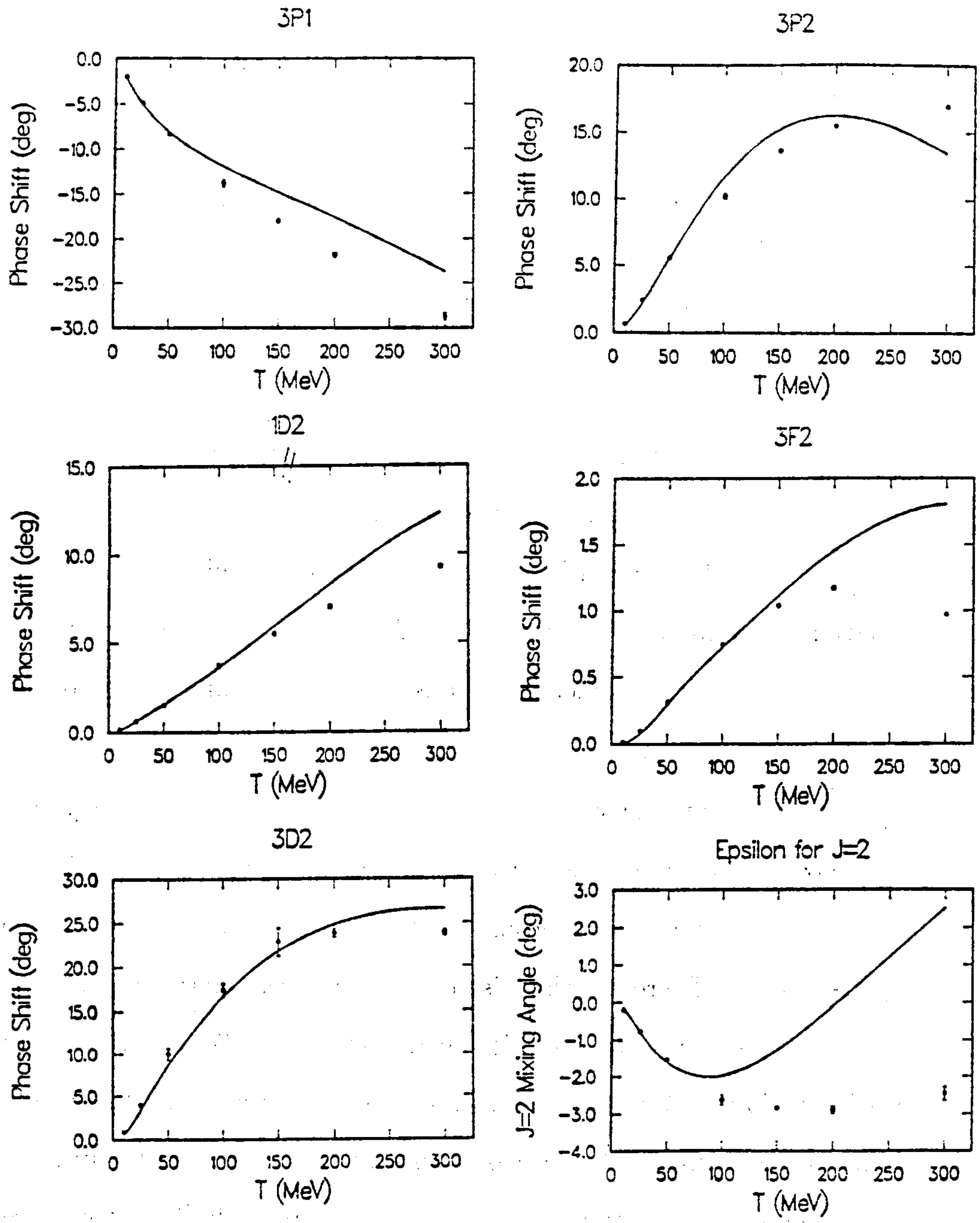
Fig. 3



(a) (b)

Fig. 4


Best NN Chiral Model Fits: 1S0
Phase Shift (deg)
60.0
50.0
40.0
30.0
20.0
10.0
0.0
−10.0
−20.0
0
50
100
150
200
250
300
T (MeV)
3S1
110.0
100.0
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0.0
3P0
15.0
10.0
5.0
−5.0
−10.0
−15.0
3D1
−15.0
−20.0
−25.0
1P1
−30.0
Epsilon for J=1
J=1 Mixing Angle (deg)


3P1
Phase Shift (deg)
0.0
-5.0
-10.0
-15.0
-20.0
-25.0
-30.0
0
50
100
150
200
250
300
T (MeV)
3P2
Phase Shift (deg)
20.0
15.0
10.0
5.0
0.0
0
50
100
150
200
250
300
T (MeV)
1D2
Phase Shift (deg)
15.0
10.0
5.0
0.0
0
50
100
150
200
250
300
T (MeV)
3F2
Phase Shift (deg)
2.0
1.5
1.0
0.5
0.0
0
50
100
150
200
250
300
T (MeV)
3D2
Phase Shift (deg)
30.0
25.0
20.0
15.0
10.0
5.0
0.0
0
50
100
150
200
250
300
T (MeV)
Epsilon for J=2
J=2 Mixing Angle (deg)
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
-4.0
0
50
100
150
200
250
300
T (MeV)

3. The few-nucleon potential

3.1 Introduction

Getting the correct binding energy of light nuclei from the underlying dynamics has been a longstanding problem in nuclear physics (for a clear introduction, see [49]). Such remarkable progress has been achieved on few-nucleon calculations that nowadays they undoubtedly provide important information about their input, two- and sometimes three-nucleon (NN and $3N$) potentials. *What* information, however, is still debatable

The traditional view [49] is that there is already some evidence for the existence of $3N$ forces. The strongest indication comes from the fact that most realistic NN potentials underbind the triton by $0.5-1.1MeV$ and the α-particle by $4-5MeV$. There are also discrepancies between data and calculations with NN potentials only for the $^3H/^3He$ rms charge radii, the asymptotic normalization ratio C_2/C_0 and the nd spin doublet scattering length, which can all be improved by including $3N$ forces adjusted to reproduce ^3H binding.

This interpretation has been challenged recently [50]. It has been argued that certain $3N$ observables display a much larger sensitivity to some NN potential parameters than NN data do. As a consequence, a fine-tuning of the NN potential is possible that is of little effect in the NN system but improves $3N$ results considerably. An example is the large uncertainties in the NN ε_1 mixing parameter, which allow a static, one-boson exchange version of the Bonn potential with a particularly low deuteron D-state probability P_D to yield almost the correct triton binding without resorting to $3N$ forces.

Here I want to discuss what our theoretical prejudices are, from the viewpoint of chiral symmetry. I will argue that a general chiral lagrangian naturally explains many of the features of nuclear systems, so it could be used as a guide for what ingredients we should expect to need. In particular, it is found that $3N$ forces arise at the same level as some important features of the NN force (such as the short range tensor force and the spin-isospin independent central attraction). Clearly this line of reasoning is no substitute for the above debate concerning what data are actually saying, but it suggests that its best framework is one in which both NN and $3N$ forces are included simultaneously and consistently from the start. (From this standpoint, getting the correct binding from NN forces alone can only be considered a success after $3N$ forces calculated with the same assumptions —e.g. same mesons exchanged, same couplings and cut-offs— are shown to be irrelevant.)

After first reviewing some of the consequences of chiral symmetry to nuclear forces [51], I consider few-body potentials in more detail, and derive the momentum space $3N$ force from the chiral lagrangian (some of these points have already been mentioned in [52] and [53]). The coordinate space $3N$ force is presented in Appendix E.

3.2 Generalities

Most of traditional nuclear physics concerns processes involving momenta up to a few hundred MeV. (Approximate) chiral symmetry is the single most important ingredient in an effective theory of hadronic processes (at such low energies) that is compatible with the theory of strong interactions, QCD. The pion is the (pseudo-)Goldstone boson of the spontaneous breaking of $SU(2)_L \times SU(2)_R$ to $SU(2)_V$, a fact that has (literally, too) far-reaching consequences. One is the lightness of the pion. It is essential to include it in the effective lagrangian with the nucleon and possibly the delta isobar, while heavier mesons can be integrated out of the theory. Not surprisingly, one finds a dominance of pion exchange in few-nucleon systems [54]. The other consequence is that the symmetry restricts the form of the interaction terms in the effective lagrangian, while the details of QCD dynamics are buried in the coupling constants (not fixed by symmetry).

With the assumption of naturalness—the notion that when expressed in the appropriate scale, these coupling constants are of $O(1)$—one can show that chiral symmetry turns a simple power counting argument into a classification scheme for the strength of interactions. An arbitrary diagram contributing to a given process can be obtained by sewing together irreducible diagrams, which are those that cannot be separated in two by cutting the lines of initial or final particles in an intermediate state. For processes where all momenta Q are of

$O(m_\pi)$, an irreducible diagram with E_n nucleon (and any number of pion) external lines, L loops, C separately connected pieces and V_i vertices of type i which contain d_i derivatives (or powers of pion mass) and f_i baryon fields, is of $O(Q^\nu)$ where

$$\nu = 4 - E_n + 2(L - C) + \sum_i V_i \Delta_i \tag{40}$$

with

$$\Delta_i = d_i + \frac{n_i}{2} - 2 \geq 0 \tag{41}$$

referred to as the index of the interaction i.

Therefore, at energies small compared to some characteristic QCD scale M (of the order of a typical hadronic energy scale, say the rho mass), the most important interactions are those with smaller indices. Say $\boldsymbol{\pi}$ denotes the pion field of mass m_π and decay constant $F_\pi (\sim 190 MeV)$, $N(\Delta)$ is a two-spinor (four-component spinor) in both spin and isospin spaces that represents the nucleon (delta) of mass $m_N(m_\Delta)$, $\frac{1}{2}\vec{\sigma}(\boldsymbol{t})$ is the generator of spin (isospin) transformations in this representation, $\vec{S}$ $(\boldsymbol{T})$ is the transition operator that satisfy $S_i S_j^+ = \frac{1}{3}(2\delta_{ij} - i\varepsilon_{ijk}\sigma_k)$ $(T_a T_b^+ = \frac{1}{6}(\delta_{ab} - i\varepsilon_{abc}t_c))$, $\boldsymbol{t}^{(3/2)}$ is the isospin generator in the $\frac{3}{2}$ representation, and D is a shorthand for

$$D = 1 + \frac{\boldsymbol{\pi}^2}{F_\pi^2},$$

then the lagrangian to lowest order ($\Delta_i = 0$) is

$$\begin{aligned} \mathcal{L}_{(0)} &= -\frac{1}{2}D^{-2}\partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi} - \frac{1}{2}m_\pi^2 D^{-1}\boldsymbol{\pi}^2 \\ &\quad + \bar{N}(i\partial_0 - \frac{2D^{-1}}{F_\pi^2}\boldsymbol{t} \cdot (\boldsymbol{\pi} \times \dot{\boldsymbol{\pi}}) - m_N - \frac{2g_A D^{-1}}{F_\pi}\boldsymbol{t} \cdot \vec{\sigma} \cdot \vec{\nabla}\boldsymbol{\pi})N \end{aligned}$$

$$
\begin{aligned}
&+\bar{\Delta}(i\partial_0 - \frac{2D^{-1}}{F_\pi^2}\boldsymbol{t}^{(3/2)} \cdot (\boldsymbol{\pi} \times \dot{\boldsymbol{\pi}}) - m_\Delta)\Delta \\
&-\frac{2h_A}{F_\pi}D^{-1}\left[\bar{N}(\boldsymbol{T} \cdot \vec{S} \cdot \vec{\nabla}\boldsymbol{\pi})\Delta + h.c.\right] \\
&-\frac{1}{2}C_S\bar{N}N\bar{N}N - \frac{1}{2}C_T\bar{N}\vec{\sigma}N \cdot \bar{N}\vec{\sigma}N \\
&-D_T\bar{N}\boldsymbol{t}\vec{\sigma}N \cdot \cdot (\bar{N}\boldsymbol{T}\vec{S}\Delta + h.c.)
\end{aligned} \tag{42}
$$

and the first order ($\Delta_i = 1$) terms are

$$
\begin{aligned}
\mathcal{L}_{(1)} \;=\; & \frac{-B_1}{F_\pi^2}D^{-2}\bar{N}N((\vec{\nabla}\boldsymbol{\pi})^2 - \dot{\boldsymbol{\pi}}^2) \\
&-\frac{B_2}{F_\pi^2}D^{-2}\bar{N}\boldsymbol{t}\vec{\sigma}N \cdot \cdot (\vec{\nabla}\boldsymbol{\pi} \times \times \vec{\nabla}\boldsymbol{\pi}) - \frac{B_3 m_\pi^2}{F_\pi^2}D^{-1}\bar{N}N\boldsymbol{\pi}^2 \\
&-\frac{D_1}{F_\pi}D^{-1}\bar{N}N\bar{N}(\boldsymbol{t} \cdot \vec{\sigma} \cdot \vec{\nabla}\boldsymbol{\pi})N - \frac{D_2}{F_\pi}D^{-1}(\bar{N}\boldsymbol{t}\vec{\sigma}N \times \times \bar{N}\boldsymbol{t}\vec{\sigma}N) \cdot \cdot \vec{\nabla}\boldsymbol{\pi} \\
&-\frac{1}{2}E_1\bar{N}N\bar{N}\boldsymbol{t}N \cdot \bar{N}\boldsymbol{t}N - \frac{1}{2}E_2\bar{N}N\bar{N}\boldsymbol{t}\vec{\sigma}N \cdot \cdot \bar{N}\boldsymbol{t}\vec{\sigma}N \\
&-\frac{E_3}{2}(\bar{N}\boldsymbol{t}\vec{\sigma}N \times \times \bar{N}\boldsymbol{t}\vec{\sigma}N) \cdot \cdot \bar{N}\boldsymbol{t}\vec{\sigma}N.
\end{aligned} \tag{43}
$$

Here $g_A, h_A, C_S, C_T, D_T, B_{1,2,3}, D_{1,2}$ and $E_{1,2,3}$ are undetermined constants, to be obtained either by solving QCD or by fitting data. Note that I i) only show those terms relevant for what follows, and omit others that have more isobars, and ii) have applied Fierz reordering to terms with four and six nucleons in order to rewrite six other possible combinations of $\vec{\sigma}$, t in terms of those shown above.

For systems with at most one nucleon, (51) and (52) tell us that dominant contributions are due to tree graphs from (53) and (54), which just reproduce the time-honored current algebra results. Consideration of higher order lagrangians and loops allow a systematic accounting of corrections (for a review see [55]).

For systems with many nucleons, the same power counting yields the main features of traditional nuclear physics.

The first feature is that a nucleus is basically made out of nucleons. At the level of nuclear dynamics itself, that is because the nucleon mass is large compared to M so that reducible diagrams have small energy denominators. This leads to a picture where nucleons interact non-perturbatively through a nuclear potential, consisting of the perturbative contributions of pions and deltas (and, indirectly, everything else). When external probes (pions and photons) are brought into play, the leading contributions come from diagrams with the maximum number of connected pieces: the probe interacts with each nucleon separately—that is the impulse approximation. First corrections to such an approximation, which are of pion-exchange type, have one less connected piece, and so are expected to be of order $(\frac{m_\pi}{M})^2 \sim 5\%$. Meson exchange currents are actually a little larger $(10-15\%)$ than this estimate; a systematic chiral lagragian analysis of them is being carried out by Rho and collaborators [56]. Moreover, Weinberg [53] has considered pion-deuteron scattering, and pion photo/electroproduction on light nuclei is being studied [57].

The second feature is that nucleons (and isobars) are non-relativistic. This is again because the massive nucleon is not much disturbed by the little kicks it receives from other particles. At the level of the potential, this is why one-pion exchange is essentially static, corrections having $\Delta_i = 2$ and so being again a few percent. I will return shortly to their energy dependence, and their effect on the iteration of the potential.

The third aspect is that nucleons interact mainly via pairwise forces. This is again a consequence of decreasing the number of connected pieces in diagrams with $E_n = 2A$, and so should also be of order $(\frac{m_\pi}{M})^2$. Notice that this is a non-trivial result, since models without chiral symmetry exist (e.g. that in [58]) where two-pion exchange is large and, therefore, yield large $3N$ forces. I now turn to the multi-nucleon dynamics in more detail.

3.3 Leading forces and energy dependence

For a nucleus with A nucleons, we see from (51), (52) that the smallest possible power of the small momentum Q is

$$\nu_{min} = 6 - 4A. \tag{44}$$

It corresponds to tree ($L = 0$) diagrams constructed out of the lowest order lagrangian ($\Delta_i = 0$) with the maximum number of separated connected pieces ($C = A - 1$). To this order, then, the nuclear potential V is simply a sum over all pairs (ij),

$$V^{(0)}(\vec{r}_1, \ldots, \vec{r}_A) = \sum_{(ij)} V_2^{(0)}(\vec{r}_i - \vec{r}_j) \tag{45}$$

of two-body potentials $V_2^{(0)}(r_i - r_j)$ consisting of static one-pion exchange (OPE) plus two contact terms [51].

This is obviously a very crude approximation to the NN potential. Corrections ($L = 0, \sum_i V_i\Delta_i = 1, 2, 3; L = 1, \sum_i V_i\Delta_i = 0, 1$) have already been calculated [52] up to

$$\nu = 9 - 4A = \nu_{min} + 3, \tag{46}$$

and used to fit NN scattering and bound state data [59]. I refer the reader to ref.[60] for the complete expressions. I just mention some of the results. Of particular importance here is that at $\nu_{min}+2$, recoil has to be accounted for in OPE, which leads to a dependence on the energy $2m_N+E$ of the incoming nucleons. Denoting by $\vec{p}_i(\vec{p}_i{}')$ the cm initial (final) momentum of nucleon i, $\vec{q}_{ij}\equiv\vec{p}_i-\vec{p}_i{}'$, $\vec{k}_{ij}\equiv\frac{1}{2}(\vec{p}_i+\vec{p}_i{}')$ and $w_{ij}\equiv\sqrt{\vec{q}_{ij}{}^2+m_\pi^2}$, we get to this order (57), in momentum space,

$$\sum_{n=0}^{3} V_2^{(n)}(\vec{q}_{ij},\vec{k}_{ij};E) = \\ -\left(\frac{2g_A}{F_\pi}\right)^2 \boldsymbol{t}_i\cdot\boldsymbol{t}_j \frac{\vec{q}_{ij}\cdot\vec{\sigma}_i\vec{q}_{ij}\cdot\vec{\sigma}_j}{w_{ij}^2}\left[1+\frac{1}{w_{ij}}\left(E-\frac{1}{m_N}(\vec{k}_{ij}^2+\frac{1}{4}\vec{q}_{ij}{}^2)\right)\right] \\ +\dots \tag{47}$$

Here I displayed the OPE piece with recoil but hid in the dots the most interesting parts, namely, two-pion exchange (TPE) and contact terms. They are necessary for a reasonable fitting of phase shifts, which we indeed achieve, up to around $100MeV$. (To go further in energy presumably requires even higher order corrections. They can be included, but involve the more complicated calculation of two-loop diagrams and the introduction of many more undetermined parameters due to a host of new contact terms.)

Since these corrections contain such important information in the case of the two-body system, it is just natural to consider their effects in $A\geq 3$ nuclei. No calculation of the triton has been carried out with our two-body potential, so we cannot be quantitative as to how much room it leaves to three-body forces. If the D-state admixture can serve as a guide, one might not expect much underbinding since our $P_D\sim 4\%$ is lower than most realistic NN potentials. In any case, and this is the important point, consistency of the approach requires that we evaluate *all* corrections to a certain order. To the order given by (57), we also encounter forces involving irreducibly three nucleons V_3, and two pairs of nucleons $V_{2,2}$, for both of which $C = A - 2$. We then write

$$\sum_{n=0}^{3} V^{(n)}(\vec{r}_1, \ldots, \vec{r}_A) = \sum_{(ij)} \sum_{n=0}^{3} V_2^{(n)}(\vec{r}_i, \vec{r}_j) + \sum_{(ijk)} \sum_{n=2}^{3} V_3^{(n)}(\vec{r}_i, \vec{r}_j, \vec{r}_k) + \sum_{(ij;kl)} \sum_{n=2}^{3} V_{2,2}^{(n)}(\vec{r}_i - \vec{r}_j; \vec{r}_k - \vec{r}_l), \tag{48}$$

where the second sum extends over all triplets (ijk) and the third over all pairs of pairs $(ij;kl)$.

I now move to touch on these few-body parts.

The largest contribution is expected to come at $\nu = \nu_{min} + 2$, being due to tree ($L = 0$) diagrams given by $\mathcal{L}_{(0)}$ in (53) ($\sum_i V_i \Delta_i = 0$). If we ignore the isobar for a while, the corresponding diagrams for the three-body potential are given in Fig.1. One finds [51] that the various orderings of Fig.1c add to zero, while Figs.1a,1b yield

$$\begin{aligned} V_3^{(2)no\Delta}(\vec{q}_{ij}, \vec{q}_{jk}) &= 2\left(\frac{g_A}{F_\pi}\right)^2 \frac{1}{w_{jk}^3} \vec{\sigma}_k \cdot \vec{q}_{jk} \, [\boldsymbol{t}_i \cdot \boldsymbol{t}_k (C_S \vec{\sigma}_i + C_T \vec{\sigma}_j) \cdot \vec{q}_{jk} + \\ &\qquad + \boldsymbol{t}_j \cdot \boldsymbol{t}_k (C_S \vec{\sigma}_j + C_T \vec{\sigma}_i) \cdot \vec{q}_{jk}] \\ &\quad -4\left(\frac{g_A}{F_\pi}\right)^4 \frac{w_{ij} + w_{jk}}{w_{ij}^3 + w_{jk}^3} \vec{\sigma}_i \cdot \vec{q}_{ij} \vec{\sigma}_k \cdot \vec{q}_{jk} \, [\boldsymbol{t}_i \cdot \boldsymbol{t}_k \vec{q}_{ij} \cdot \vec{q}_{jk} - \\ &\qquad - 2\boldsymbol{t}_j \cdot (\boldsymbol{t}_i \times \boldsymbol{t}_k) \vec{\sigma}_j \cdot (\vec{q}_{ij} \times \vec{q}_{jk}) \,] \\ &\quad + \text{two cyclic permutations of } (ijk). \end{aligned} \tag{49}$$

The second term is just TPE (Fig.1b) and has been calculated long ago by Brueckner et al. [61]. The first term comes in part from the contact terms in (53), first considered by Weinberg [51]. (Note, however, that I correct here the corresponding result in [51].) The double-pair potential, in turn, comes from the diagrams in Fig.2 and is

$$V_{2,2}^{(2)}(\vec{q}_{ij},\vec{q}_{kl}) = \left(\frac{2g_A}{F_\pi}\right)^2 \left\{C_S + C_T\vec{\sigma}_i\cdot\vec{\sigma}_j + \left(\frac{2g_A}{F_\pi}\right)^2 \boldsymbol{t}_i\cdot\boldsymbol{t}_j\frac{\vec{\sigma}_i\cdot\vec{q}_{ij}\vec{\sigma}_j\cdot\vec{q}_{ij}}{w_{ij}^2}\right\}\times \times\boldsymbol{t}_k\cdot\boldsymbol{t}_l\frac{\vec{\sigma}_k\cdot\vec{q}_{kl}\vec{\sigma}_l\cdot\vec{q}_{kl}}{w_{kl}^3}. \tag{50}$$

What is the effect of these leading few-body forces? The remarkable fact is that they are canceled by the energy dependence of the two-body potential (58) when the latter is iterated in the Lippmann-Schwinger equation.

To see this, consider the diagrams of Fig.3, which are all orderings where nucleon j emits or absorbs a pion (that flies to or from nucleon k), before getting in touch with nucleon i. According to our power counting, Figs.3a,3b are the most important, because they are iterations of the NN potential, which is given by (58). They will be proportional to

$$\frac{1}{E(\vec{p}_j-\vec{q}_{jk})+E(\vec{p}_k+\vec{q}_{jk})-E(\vec{p}_j)-E(\vec{p}_k)}\times \times\left[\frac{1}{E(\vec{p}_j-\vec{q}_{jk})+w_{jk}-E(\vec{p}_j)}+\frac{1}{E(\vec{p}_k-\vec{q}_{jk})+w_{jk}-E(\vec{p}_k)}\right] = = \frac{2}{w_{jk}[E(\vec{p}_j-\vec{q}_{jk})+E(\vec{p}_k+\vec{q}_{jk})-E(\vec{p}_j)-E(\vec{q}_{jk})} -\frac{1}{w_{jk}^2}\left(1+O\left(\frac{E}{w}\right)\right). \tag{51}$$

The first term is just the iteration of the leading order potential $V_2^{(0)}$; it is big because the difference in nucleon energies is small, $O(\frac{m_\pi^2}{m_N})$, while w is $O(m_\pi)$. The second term is the iteration of the recoil correction to OPE shown in (58); the small energy denominator is canceled by the small recoil energy. The leading

$3N$ force is expected to be of the same order; it is given by Fig.3c (which is just Fig.1a), which is proportional to

$$\frac{1}{w_{jk} + E(\vec{p}_j - \vec{q}_{jk}) - E(\vec{p}_j)} \frac{1}{w_{jk} + E(\vec{p}_i') + E(\vec{p}_j') - E(\vec{p}_i) - E(\vec{p}_j)} = \frac{1}{w_{jk}^2}\left(1 + O\left(\frac{E}{w}\right)\right). \tag{52}$$

Here the first term is already the leading $3N$ force for which nucleons are static. The important point now is that, because the diagrams in Fig.3 differ *only* in their energy denominators, the $\frac{1}{w^2}$ terms cancel when we compute the T-matrix or, equivalently, when we solve the Faddeev equations.

It is not difficult to show in the same way that a similar cancelation happens also in the $3N$ TPE piece (Fig.4) and double-pair force (Fig.5). This cancelation has been noted before in the case of the TPE $3N$ force [62], but its model independence is particularly clear in our context. As a result, in a few-body calculation both V_3 of (60) and $V_{2,2}$ of (61) can be omitted as long as we do the same to the recoil term in (58). (This is nicely exemplified [63] by comparing the triton binding energies, one from the full Bonn potential plus the above TPE $3N$ forces and the other from its energy independent version obtained using the folded diagram technique.) More generally, it is clear that it makes no sense to use an energy dependent NN potential in a few-body calculation without at the same time including $3N$ and double-$2N$ forces calculated in the respective framework.

3.4 Remaining three-body forces

I now go to next order, $\nu = \nu_{min} + 3$, which still comes from tree ($L = 0$) diagrams, but now have one vertex from (54) ($\sum_i V_i \Delta_i = 1$). In the case of the double-pair force, the diagrams are still the same as in Fig.2, but there are no corresponding $\bar{N}N\pi$ or $\bar{N}N\bar{N}N$ vertices in (54), so that

$$V_{2,2}^{(3)}(\vec{q}_{ij}, \vec{q}_{kl}) = 0. \tag{53}$$

As for the $3N$ force, the same remark applies to Figs.1a,1b; there are contributions though, from Figs.1c,6, that are readily calculated,

$$\begin{aligned} V_3^{(3)}(\vec{q}_{ij}, \vec{q}_{jk}) \; = \; & E_1 \boldsymbol{t}_i \cdot \boldsymbol{t}_k + E_2 \vec{\sigma}_i \cdot \vec{\sigma}_k \boldsymbol{t}_i \cdot \boldsymbol{t}_k + E_3 \vec{\sigma}_j \cdot (\vec{\sigma}_i \times \vec{\sigma}_k) \boldsymbol{t}_j \cdot (\boldsymbol{t}_i \times \boldsymbol{t}_k) \\ & - \frac{2g_A}{F_\pi^2} \frac{1}{w_{jk}^2} \vec{\sigma}_k \cdot \vec{q}_{jk} \, [D_1 (\boldsymbol{t}_i \cdot \boldsymbol{t}_k \vec{\sigma}_i + \boldsymbol{t}_j \cdot \boldsymbol{t}_k \vec{\sigma}_j) \\ & \qquad - D_2 \boldsymbol{t}_j \cdot (\boldsymbol{t}_i \times \boldsymbol{t}_k) \vec{\sigma}_i \times \vec{\sigma}_j \,] \cdot \vec{q}_{jk} \\ & + 2 \left(\frac{2g_A}{F_\pi} \right)^2 \frac{1}{w_{ij}^2 w_{jk}^2} \vec{\sigma}_i \cdot \vec{q}_{ij} \vec{\sigma}_k \cdot \vec{q}_{jk} \left[\boldsymbol{t}_i \cdot \boldsymbol{t}_k (B_1 \vec{q}_{ij} \cdot \vec{q}_{jk} + B_3 m_\pi^2) \right. \\ & \qquad \left. - B_2 \boldsymbol{t}_j \cdot (\boldsymbol{t}_i \times \boldsymbol{t}_k) \vec{\sigma}_j \cdot (\vec{q}_{ij} \times \vec{q}_{jk}) \,\right] \\ & + \text{ two cyclic permutations of } (ijk) \end{aligned} \tag{54}$$

(see Appendix E for the coordinate space version). Hence, this $3N$ force has eight undetermined parameters. Of course, three of them (the B_i's) can be fixed once a systematic chiral lagrangian analysis of πN scattering is carried out. Two others (the D_i's) can in principle be determined by processes like π-deuteron scattering, or π production/absortion on NN systems, but it is unlikely that this could be done without much more accurate data than currently available. More importantly, the three remaining parameters (the E_i's) can only be determined from data involving $3N$ systems, so we do not have great predictive power. There are, of course, more than three measured quantities in these systems, and again in principle the above force is testable. The problem is, it seems that all current data can be fitted by appending to realistic NN potentials a "reasonable" $3N$ force with just *one* parameter that is fixed by the triton binding energy [49].

The situation is not completely hopeless, though, because I have been ignoring the Δ isobar. If the Δ is integrated out of the theory, its contributions appear only indirectly, in the coefficients of the general chiral lagrangian, and are suppressed by powers of the mass difference to the nucleon. If this difference were of order M or larger, no changes in the power counting arguments given above would be necessary. As it happens, though, $m_\Delta - m_N$ is only $\sim 2m_\pi$, which is closer to m_π than to M. It is more convenient to keep the Δ explicitly in the lagrangian, and treat it as the nucleon field, as far as power counting goes. That is what was done in our study of the NN potential [59, 60]; here it implies an additional $3N$ force of order $\nu = \nu_{min} + 2$, obtained from the graphs in Fig.7 where all vertices are from (53). Not surprisingly, it has the form (65), but it is suppressed by $m_\Delta - m_N$:

$$V_3^{(2)one\Delta}(\vec{q}_{ij},\vec{q}_{jk}) = V_3^{(3)}(\vec{q}_{ij},\vec{q}_{jk}) \text{ with } \begin{cases} E_1 & \rightarrow & 0 \\ E_2 & \rightarrow & \frac{D_T^2}{9(m_\Delta - m_N)} \\ E_3 & \rightarrow & -\frac{D_T^2}{18(m_\Delta - m_N)} \\ D_1 & \rightarrow & -\frac{4D_T h_A}{9(m_\Delta - m_N)} \\ D_2 & \rightarrow & \frac{2}{9}\frac{D_T h_A}{m_\Delta - m_N} \\ B_1 & \rightarrow & -\frac{4}{9}\frac{h_A^2}{m_\Delta - m_N} \\ B_2 & \rightarrow & -\frac{2}{9}\frac{h_A^2}{m_\Delta - m_N} \\ B_3 & \rightarrow & 0 \end{cases} \tag{55}$$

Again, these forces (65) and (66) have some known elements, corresponding to the TPE pieces, because they are obviously related to the πN scattering amplitude. The importance of the Δ was recognized early [64], and so the TPE piece in (66) (h_A^2 terms, Fig.7c) is simply the old Fujita-Miyazawa force. Similarly, the relevance of current algebra was noted in the 60's [65]. Chiral symmetry has been implemented in this context by the Brazil group [66] using a chiral lagrangian involving the ρ and the Δ in conjunction with a parametrization of the isoscalar amplitude. The TPE in (65) is the same as theirs, but hopefully it is clear that its derivation here is as model independent as possible (it does not involve any explicit assumptions about QCD dynamics in the form of the ρ), and comes from a perturbative expansion. A similar force was obtained by the Tucson-Melbourne group [67] by extrapolating amplitudes off mass-shell using dispersion relations. The connection between these two approaches was examined in [68], the main difference arising from a term present in the isoscalar amplitude of the latter that generates a contact term similar to those in Fig.6a.

It was also pointed out that this and other short range terms that arise from the TPE in (65) are responsible for an extreme sensitivity of triton quantities on the cut-off parameter, which is introduced to regularize the coordinate space potential, but should otherwise not affect the potential much.

This problem can now be reinterpreted. First, I note that these troublesome short range terms (except for the one from the parametrization of the isoscalar amplitude) have exactly the same structure as our new contact terms of Fig.6, and so cannot be distinguished from them in a calculation using the complete $3N$ potential, (65) and (66). The mentioned sensitivity shows, therefore, only a dependence on terms that contain bona fide parameters (the D_i's and E_i's) of the general chiral lagrangian. It is no more surprising than the sensitivity to, say, the $\pi N\Delta$ coupling h_A. Second, in the approach presented in this paper, the cut-off is not an independent parameter anyway: it is the same parameter that was used in the NN potential, the fitting of which yielded the values of some of the other parameters appearing in (65) and (66) (g_A, h_A and the B_i's). Changes of the cut-off parameter are compensated to a certain extent by changes in all the parameters of the complete potential.

I stop here. I have discussed all there is to the order given by (57). As mentioned before, the inclusion of higher orders in the NN potential involves two-loop diagrams and lots of new contact terms. At the level of the $3N$ potential, one is also required to consider three-pion exchange. And then there are $4N$ forces.

3.5 Conclusion

In conclusion, I hope to have argued convincingly that we should look at two- and more-body forces consistently, and that the general chiral lagrangian provides the appropriate framework. We see then that $3N$ forces appear at the same level as some important features of the NN potential. In particular, the leading (static) $3N$ force is canceled by the leading energy dependence of the iterated NN force. The remaining $3N$ force has some important terms related to πN scattering, but also shorter range components. It is expected to be dominated by the Fujita-Miyazawa force plus a shorter range term that depends on only one undetermined parameter (D_T); they should be $O(\frac{m_\pi^2}{M^2})$ and so some $5-10\%$ of the NN contribution. Finally, $4N$ forces are expected to be $O(\frac{m_\pi^4}{M^4})$, more like 1%, so that $4N$ systems are expected to be underbound by NN forces by roughly four times the triton underbinding.

Figure Captions

Figure (1) :Tree graphs contributing to the $3N$ potential. All other time orderings and permutations are to be considered, as long as there is at least one pion in intermediate states. (Solid lines are nucleons, dashed lines pions.)

Figure (2) :Tree graphs contributing to the double-pair potential. All other time orderings and permutations are to be included, as long as there is at least one pion in intermediate states.

Figure (3) :Diagrams representing part of the iteration of the NN potential (a,b) whose energy dependence partially cancels the contribution from part of the $3N$ force (c). Same cancelation applies to other time orderings.

Figure (4) :Example of cancelation analogous to the one in Fig.3 for the TPE sector. In (a), (b) recoil is considered in the pion line to the right. The same cancelation occur in three other sets of four TPE diagrams corresponding to different orderings.

Figure (5) :Same cancelations as in Fig.3 and 4, but for the double-pair potential. Analogous result holds for other orderings, and for TPE diagrams.

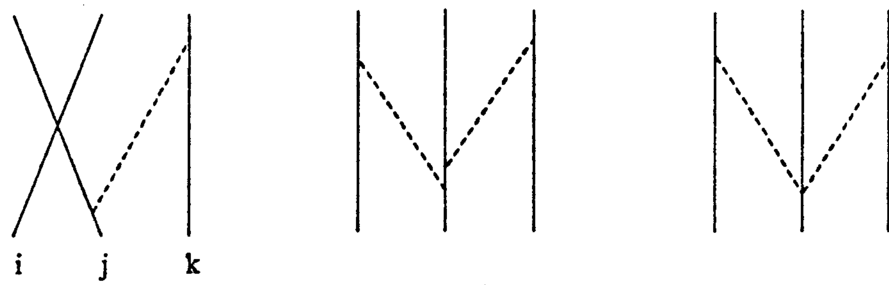
Figure (6) :Other tree diagrams contributing to the $3N$ potential. Both ordering of (a) must considered, as well as permutations.

Figure (7) :Tree graphs with isobar contributing to the $3N$ potential.

All other time orderings and permutations are to be considered.

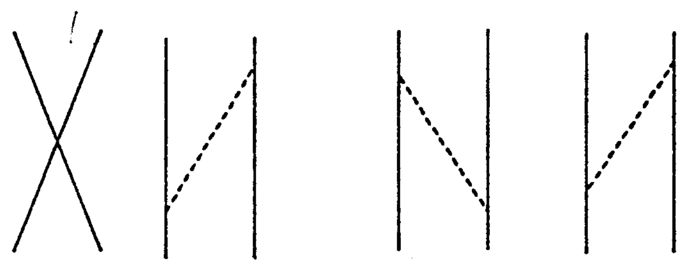
(A double line represent the Δ isobar.)

Fig. 1



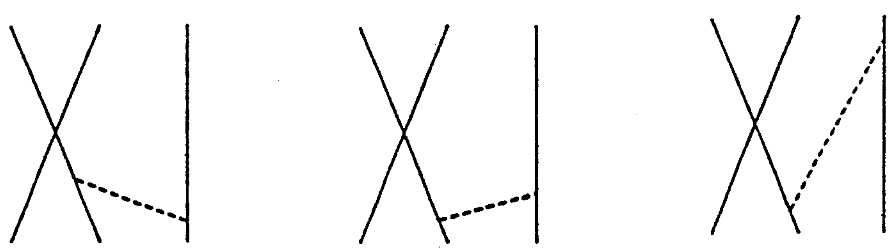
(a) (b) (c)

Fig. 2



(a) (b)

Fig. 3



(a) (b) (c)

Fig. 4

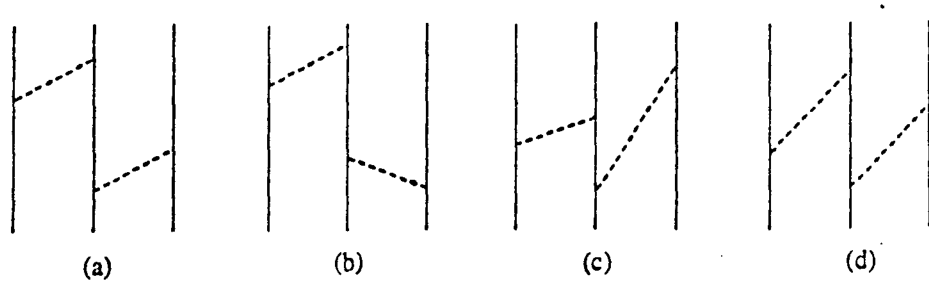


Fig. 5

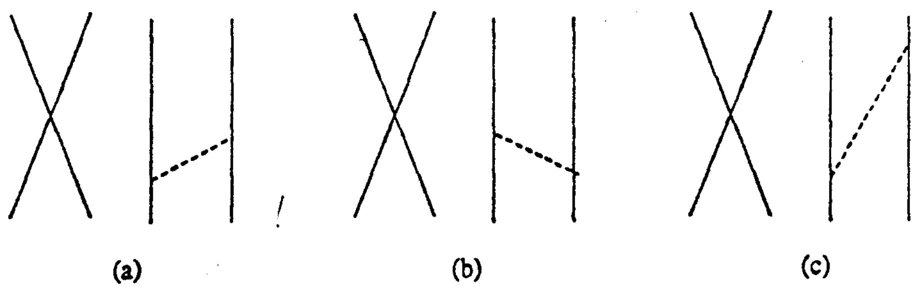


Fig. 6

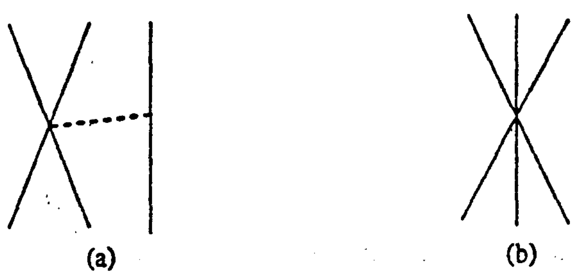
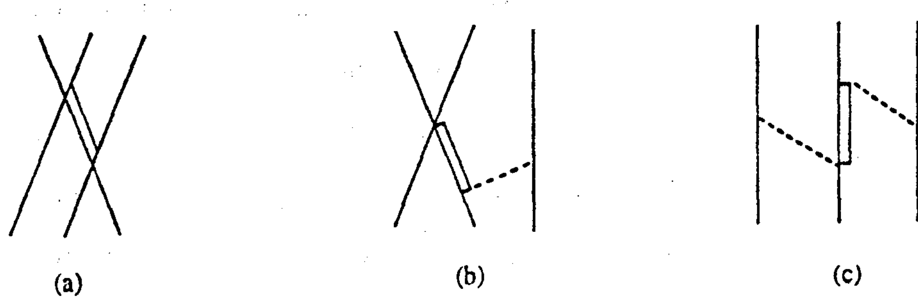


Fig. 7



4. Isospin Violation

4.1 Introduction

Why is isospin such a good symmetry of low-energy hadronic physics? Since the early days of nuclear physics, strong interactions were found to be nearly invariant under isospin transformations. Any differences between particles of the same isospin multiplet, be them in masses or interactions, were attributed to electromagnetic interactions. Hard to understand seemed only to be the fact that the neutron is heavier than the proton.

The advent of QCD as the underlying theory of strong interactions did not make immediately obvious the explanation for the lack of strong isospin violation. Quarks have current masses that break explicitly both chiral and isospin symmetries. Estimates of the masses of the lighter quarks, up and down, that are the most relevant at low energies, indicate that they are nowhere near equal. The favored values are essentially those found by Weinberg long ago [69], $m_u \cong 4MeV$ and $m_d \cong 7MeV$. (There is some controversy—see, e.g. [70]— surrounding these values, pushed forward by the attempt to make room for a massless u quark and the resulting explanation for the absence of strong

CP violation, but that would only make isospin violation in the quark masses larger.)

The standard answer to the opening question was presented by Weinberg in the same paper [69]. The parameters of the explicit breaking of those global symmetries in the QCD lagrangian, namely the current masses, are small compared to the scale that governs the non-perturbative regime of the theory, which we guess is the confinement scale $\Lambda_{QCD} \sim 300 MeV$.

This nicely explains why chiral symmetry is a good approximate symmetry, as discovered in the current algebra days, but cannot for exactly this reason be the whole answer. Isospin seems to be a much better symmetry indeed than chiral symmetry. A measure of isospin compared to chiral breaking in the QCD lagrangian is the ratio of quark mass difference to sum, and using the values given above, that is $\sim 1/3$, or 30%. However, the pion masses, for example, are due to this explicit symmetry breaking plus electromagnetic effects and still, the pion mass difference is only 3% of the average pion mass.

In this paper I want to argue that the rest of the answer is due i) generically to the constraints imposed on operators by the chiral transformation properties of the relevant fields, and ii) specifically to experimental limitations. At a scale of a few GeV, the dynamics of quarks is essentially governed by a renormalizable lagrangian where quarks interact minimally with gluons and photons; isospin breaking arises from the differences in quark masses and charges. No attempt of exploring the QCD dynamics is carried out; I look only at the symmetry structure of the general chiral lagrangian (which is the effective theory of hadrons at energies smaller than a typical QCD mass), and assume that its undetermined coefficients are natural, i.e. of order $O(1)$ when written in the appropriate scale. Then we find that the leading isospin violation, apart from the pion mass difference which is mainly electromagnetic, happens to be related to the nucleon splitting, involve a π^0 and be hard to observe. As a result, we see why the NN scattering lenghts are essentially the only dynamical quantities really displaying isospin violation, and that the isospin violating nuclear potential is mostly due to the pion mass difference. I should note that some of these points have been made before by Weinberg either using current algebra [69] or looking at the pion chiral lagrangian [71], but I believe this is the first time the subject is treated systematically with chiral lagrangians (involving nucleons).

The chapter is organized as follows. In Sect.2, I list the leading operators that break isospin in the same way the quark mass difference does, and in Sect.3 those that are due to the high energy photons integrated out of the low-energy theory. In Sect.4 I finally discuss where the effects of these operators might be manifest. Nowhere do I explicit calculate the contributions of soft photons remaining in the theory, but I also note that the general chiral lagrangian provides some guidance as to which diagrams should be included to a given order in specific problems, like the isospin violating nuclear potential.

4.2 Operators due to the quark mass difference

QCD got to the spotlight thanks to its dimensional transvestism. Quantization of a non-abelian gauge theory with the appropriate number of minimally coupled fermions introduces a scale, in this case of some 300MeV, below which, we suppose, confinement is manifest. Compared to this scale the u and d quark masses that appear in the QCD Lagrangian may be considered small. It makes sense to look at the limit where the masses are zero and the Lagrangian has a global invariance under the full chiral group $SU(2)_L \times SU(2)_R \sim SO(4)$. The absence of parity doublets but presence of (approximate) isospin multiplets in the hadron spectrum indicates that this symmetry is broken by the vacuum down to the isospin subgroup $SU(2)_V \sim SO(3)$. It results in the delivery of a multiplet of isospin 1 Goldstone bosons, the pions $\boldsymbol{\pi}$, which will inhabit the three-sphere $S^3 \sim \frac{SO(4)}{SO(3)}$ of radius we call F_π (found to coincide with the pion decay constant $\simeq 190MeV$). If we embed this sphere in the euclidean E^4 space of coordinates $\varphi = (\varphi_\alpha) = (\boldsymbol{\varphi}, \varphi_4 = \sigma)$, it is defined by

$$\sum_{\alpha=1}^{4} \varphi_\alpha^2 = F_\pi^2. \tag{56}$$

Three pions $\boldsymbol{\pi}$ can be represented by any coordinates we choose to parametrize this sphere with. Any point there can be obtained by applying a 4-rotation $R(\boldsymbol{\pi})$,

$$R(\boldsymbol{\pi})R^T(\boldsymbol{\pi}) = 1, \tag{57}$$

to the north pole $(0, F_\pi)$:

$$\varphi_\alpha(\boldsymbol{\pi}) = R_{\alpha 4}(\boldsymbol{\pi})F_\pi. \tag{58}$$

It is convenient to use stereographic coordinates by taking

$$R[\boldsymbol{\pi}] = (R_{\alpha\beta}[\boldsymbol{\pi}]) = \begin{pmatrix} \delta_{ij} - 2D^{-1}\frac{\pi_i\pi_j}{F_\pi^2} & 2D^{-1}\frac{\pi_i}{F_\pi} \\ -2D^{-1}\frac{\pi_j}{F_\pi} & D^{-1}\left(1 - \frac{\boldsymbol{\pi}^2}{F_\pi^2}\right) \end{pmatrix} \tag{59}$$

where

$$D \equiv 1 + \frac{\boldsymbol{\pi}^2}{F_\pi^2}. \tag{60}$$

The derivative nature of the Goldstone boson interactions is then implemented in the effective low energy lagrangian through its covariant derivative

$$\boldsymbol{D}_\mu = D^{-1}\frac{\partial_\mu \boldsymbol{\pi}}{F_\pi}. \tag{61}$$

On the other hand, fermions, lacking the protection of chiral symmetry, can now be massive. It is therefore equally convenient to use for the nucleon, too, a non-linear realization N,

$$N = D^{-\frac{1}{2}}\left(1 + 2i\gamma_5 \frac{\boldsymbol{t}\cdot\boldsymbol{\pi}}{F_\pi}\right)\psi, \tag{62}$$

where ψ transforms linearly under the chiral group and $\boldsymbol{t}$ is half the Pauli matrices in isospin space. Its covariant derivative is

$$\mathcal{D}_\mu N = \left(\partial_\mu + \frac{2i}{F_\pi}\boldsymbol{t}\cdot(\boldsymbol{\pi}\times\boldsymbol{D}_\mu)\right)N. \tag{63}$$

A similar construction can be carried out for nucleon resonances, but I will, for simplicity, restrict myself here to the nucleon.

The effective lagrangian is then made out of all the isoscalar, parity and time reversal invariant operators that we can build with $\boldsymbol{D}_\mu$ and N, and their covariant derivatives. Note that, because we are interested in low energies, nucleons are non-relativistic, so we can use Pauli spinors and also substitute their time derivatives in interaction terms by their values given by the equations of motion. Furthermore, we can order the operators in the lagrangian according to their indices Δ defined by

$$\Delta \equiv d + \frac{n}{2} - 2. \tag{64}$$

where $d(n)$ is the number of derivatives (nucleon fields). The larger the indices of the interactions of a diagram, the more powers of soft momenta will be involved, and thus, under an assumption of naturalness—that any coupling of mass dimension $-\delta$ is of order $M^{-\delta}$, where M is a characteristic QCD mass scale (like the rho mass)—,the smaller the contribution of that diagram to a given process.

This resembles the world we see around us, light pions and heavy nucleons, except that the pions do have masses and the neutron is heavier than the proton. We then turn back to the neglected mass terms in the QCD Lagrangian,

$$\mathcal{L}_{mass} = -\frac{1}{2}(m_u + m_d)\bar{q}q - \frac{1}{2}(m_d - m_u)(-2\bar{q}t_3 q) \tag{65}$$

where

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \tag{66}$$

is the quark doublet and $m_u(m_d)$ is the $u(d)$ quark mass. Eq. (65) was written as a sum of two terms with definite transformation properties under the full group $SO(4)$. The first is the fourth component of an $SO(4)$ vector S,

$$S = (2\bar{q}i\gamma_5 \boldsymbol{t}q, \bar{q}q), \tag{67}$$

and the second, the third component of another $SO(4)$ vector P (with opposite transformation properties under parity and time reversal),

$$P = (-2\bar{q}\boldsymbol{t}q, \bar{q}i\gamma_5 q). \tag{68}$$

Because a vector does transform under the full group, both terms break chiral symmetry, but because there is an $SO(3)$ subgroup that does not affect the fourth component, only the second term breaks isospin.

What is the effect of these symmetry breaking terms at low energies? Clearly they produce S-matrix elements that transform under chiral symmetry as tensor products of such vectors, what in turn demands terms with the same transformation properties in the effective lagrangian. We should thus construct all tensors $T_{\alpha\beta\dots}[\boldsymbol{\pi};\boldsymbol{D}_\mu,N]$ out of $\boldsymbol{\pi}$, $\boldsymbol{D}_\mu$ and N (and their covariant derivatives) and then select third and fourth components (again paying due respect to parity and time reversal invariance, because I neglect here strong CP violation). They will appear in the chiral lagrangian with coefficients proportional to the corresponding powers of $(m_u - m_d)$ and $(m_d + m_u)$.

This task is made easier if we recall (58): an $SO(4)$ vector involving the pion field $\boldsymbol{\pi}$ can be constructed by applying a chiral rotation on another that does not. It follows that we just need to search for those tensors $T_{\alpha\beta\dots}[0;\boldsymbol{D}_\mu,N]$ written in terms of covariant objects only, and then rotate them,

$$T_{\alpha\beta\dots}[\boldsymbol{\pi};\boldsymbol{D}_\mu,N] = \sum_{\alpha'\beta'\dots} R_{\alpha\alpha'}[\boldsymbol{\pi}]R_{\beta\beta'}[\boldsymbol{\pi}]\dots\ T_{\alpha'\beta'\dots}[0;\boldsymbol{D}_\mu,N], \tag{69}$$

with $R(\boldsymbol{\pi})$ given by (59).

The simplest case is the one that is obtained when we do not include either N or $\boldsymbol{D}_\mu$. Having only a number available we cannot construct a pseudoscalar isoscalar nor an isovector, so we only have

$$S_1[0;0,0] = (0,1). \tag{70}$$

From (69) and (59),

$$S_1[\boldsymbol{\pi};0,0] = \left(2D^{-1}\frac{\boldsymbol{\pi}}{F_\pi}, 1 - 2D^{-1}\frac{\boldsymbol{\pi}^2}{F_\pi^2}\right); \tag{71}$$

$(S_1)_3$ cannot appear in the lagrangian because it is a pseudoscalar, but we can incorporate $(S_1)_4$,

$$\mathcal{L}^{(0)}_{\pi,qm} = \frac{1}{2D}m_\pi^2\boldsymbol{\pi}^2 + \text{constant}, \tag{72}$$

which introduces another scale in the problem, and gives the pion a mass m_π whose square is proportional to $(m_u + m_d)$. Now, this operator generates interactions with no derivatives or fermion fields, that might be relatively big when momenta are very small. But we are tipically interested in nuclear physics processes were particles have, as nucleons in nuclei, momenta of the order of the pion mass itself, in which case this operator is comparable with the term responsible for the pion kinetic energy, that has $\Delta = 0$.

We then can and should incorporate this and other chiral symmetry breaking terms in the classification provided by the index (64), by extending the definition of d_i to the number of derivatives *and* powers of pion mass. An operator that breaks chiral symmetry (but not isospin) like a tensor of rank k, and has therefore a coefficient proportional to $(m_u + m_d)^k$, has index $2k + \frac{n}{2} - 2$, if it has no derivatives. An operator that further breaks isospin symmetry like a tensor of rank l has an index $2(k+l) + \frac{n}{2} - 2$ and an additional suppression of l powers of the parameter

$$\varepsilon = \frac{m_d - m_u}{m_d + m_u}. \tag{73}$$

In the following I will book isospin breaking operators according to their (generalized) index (64), but will keep track of factors of ε.

I will now go on to list the most important isospin violating operators; the procedure I follow is exactly the same as above. I use a superscript in the lagrangian to denote the index of the interactions, and will go only up to index 2, because they already become numerous at this point.

First, let us continue to look at operators involving no nucleon fields. There is no way to increase the index by only 1: Lorentz invariance requires two derivatives, and one power more of the chiral breaking parameters means two extra powers of the pion mass. The first case arise from a vector like (70),

$$S_2[0;\boldsymbol{D}_\mu,0] = \boldsymbol{D}_\mu \cdot \boldsymbol{D}^\mu\, S_1[0;0,0], \tag{74}$$

and again contributes only an isospin symmetric term. The second case comes from the tensors

$$T_{1,2} = ((T_{1,2})_{\alpha\beta})[0;0,0] = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} \delta_{ij} & 0 \\ 0 & 0 \end{pmatrix} \tag{75}$$

which give rise to the same operators: after the rotation (69) the T_{34}'s are pseudoscalar and so disregarded, while the T_{44}'s are allowed but isospin conserving; I exhibit only the isospin violating piece from the T_{33}'s,

$$\mathcal{L}^{(2)}_{\pi,qm} = \frac{1}{2D^2}\delta m_\pi^2 \pi_3^2 + \text{constant}, \tag{76}$$

where δm_π^2 is a contribution to the pion mass splitting of order $O(\varepsilon^2 \frac{m_\pi^4}{M^2})$. Other isospin breaking operators involving just pions come only with $\Delta = 4$.

I consider now operators involving a pair of nucleon fields. Here we can have $\Delta = 1$ if we build a vector with no derivatives. Again there is a vector

$$S_3[0;0,N] = \bar{N}N\, S_1[0;0,0] \tag{77}$$

that contributes only an isospin symmetric term

$$\mathcal{L}^{(1)}_{N,qm} = \frac{2\sigma}{F_\pi^2}\boldsymbol{\pi}^2 \bar{N}N \tag{78}$$

with σ of order $O(\frac{m_\pi^2}{M})$. But there is another,

$$P_1[0;0,N] = (\bar{N}\boldsymbol{t}N, 0), \tag{79}$$

that after the rotation (69) generates

$$\mathcal{L}^{(1)}_{N,qm} = \delta m_N(-\bar{N}t_3N + 2D^{-1}\frac{\pi_3\boldsymbol{\pi}}{F_\pi^2}\cdot\bar{N}\boldsymbol{t}N), \tag{80}$$

where δm_N is a contribution to the nucleon mass splitting of order $O(\varepsilon\frac{m_\pi^2}{M})$. Δ can be increased by one if we add one derivative. Once more, there is a couple of vectors

$$S_{4,5}[0;0,N] = (0,\bar{N}\boldsymbol{t}\vec{\sigma}N\cdot\cdot\vec{\boldsymbol{D}}),(\bar{N}\boldsymbol{t}\vec{\sigma}\cdot\vec{\mathcal{D}}N + \text{h.c.},0) \tag{81}$$

that does not contribute to isospin violation, and another

$$P_{2,3}[0;0,N] = (\bar{N}\vec{\sigma}N\cdot\vec{\boldsymbol{D}},0),(0,\bar{N}\vec{\sigma}\cdot\vec{\mathcal{D}}N + \text{h.c.}) \tag{82}$$

which does, leading (after integration by parts) to just one term

$$\mathcal{L}^{(2)}_{N,qm} = \beta_1(\vec{D}_3 - \frac{2D^{-1}}{F_\pi^2}\pi_3\boldsymbol{\pi}\cdot\vec{\boldsymbol{D}})\cdot\bar{N}\vec{\sigma}N, \tag{83}$$

where β_1 is of order $O(\varepsilon\frac{m_\pi^2}{M^2})$. Other operators with a pair of nucleon fields have $\Delta = 3$ or higher.

Next are four-nucleon terms. The only ones to order $\Delta = 2$ have no derivatives and only one power of quark masses. There are two vectors of the type

$$S_6^\Gamma[0;0,N] = \bar{N}\Gamma N\,\bar{N}\Gamma N\,S_1[0;0,0] \tag{84}$$

where $\Gamma = 1$, $\vec{\sigma}$, and two more with $\boldsymbol{t}$'s inserted, but they will contribute only isospin conserving terms. And there are two others,

$$P_4^\Gamma[0;0,N] = (\bar{N}\boldsymbol{t}\Gamma N\bar{N}\Gamma N,0) \tag{85}$$

that will lead to isospin violation,

$$\begin{aligned}\mathcal{L}^{(2)}_{NN,qm} &= \gamma_s[\bar{N}t_3N - 2D^{-1}\frac{\pi_3\boldsymbol{\pi}}{F_\pi^2}\cdot N\boldsymbol{t}N]NN \\ &\quad +\gamma_\sigma[\bar{N}t_3\vec{\sigma}N - 2D^{-1}\frac{\pi_3\boldsymbol{\pi}}{F_\pi^2}\cdot\bar{N}\boldsymbol{t}\vec{\sigma}N]\cdot\bar{N}\vec{\sigma}N, \end{aligned}\tag{86}$$

where γ_s and γ_σ are of order $O(\varepsilon\frac{m_\pi^2}{M^4})$.

Finally, operators with more nucleon fields have $\Delta \geq 3$.

In summary, the leading terms in the chiral lagrangian that violate isospin and originate from the quark mass difference are given by (80), followed by those in (76), (83) and (86).

4.3 Operators due to electromagnetic interactions

The quark mass difference is not the only source of isospin violation in the renormalizable QCD Lagrangian: u and d have also different charges. The quark doublet (66) couples to the photon field A_μ

$$\mathcal{L}_{em} = ie\bar{q}Q\not{A}q \tag{87}$$

via its charge matrix eQ,

$$Q = \frac{1}{6} + t_3, \tag{88}$$

whose t_3 piece evidently destroys isospin invariance. Exchange of photons between quarks will generate two classes of interactions in the effective low-energy theory. Soft photons (those with momenta$< M$) remain and couple to pions and nucleons in the most general way that respects gauge invariance; their contributions are evaluated calculating Feynman diagrams as usual. Hard photons, on the other hand, can be integrated out and produce operators that do not involve the electromagnetic field explicitly. It is to these that I now turn my attention to.

Integrating out photons in (87) will produce four-quark "effective" interactions. It is useful to introduce three quantities

$$c_\mu \equiv \bar{q} i \gamma_\mu q \tag{89}$$

$$\boldsymbol{i}_\mu \equiv \bar{q} i \gamma_\mu \boldsymbol{t} q \tag{90}$$

$$\boldsymbol{j}_\mu \equiv \bar{q} i \gamma_\mu \gamma_5 \boldsymbol{t} q. \tag{91}$$

It is easy to verify that c_μ is invariant under both axial and vector transformations, while $\boldsymbol{i}_\mu$ and $\boldsymbol{j}_\mu$ transform as vectors under isospin, but go into each other under an axial transformation. In other words, $\boldsymbol{i}_\mu$ is a sum, and $\boldsymbol{j}_\mu$ a difference, of two isovectors, one which changes only under $SU(2)_L$, the other only under $SU(2)_R$: they form an $(1,0)+(0,1)$ representation of $SU(2)_L \times SU(2)_R$. As in the case of electric and magnetic fields with respect to the Lorentz group, it is convenient to put $\boldsymbol{i}_\mu$ and $\boldsymbol{j}^\mu$ together in an $SO(4)$ antisymmetric tensor

$$F^\mu = (F^\mu_{\alpha\beta}) = \begin{pmatrix} \varepsilon_{ijk} j^\mu_k & i^\mu_i \\ -i^\mu_j & 0 \end{pmatrix}. \tag{92}$$

The "effective" four-fermion interaction (from which we can read off the way electromagnetic interactions break isospin) can now be written schematically as

$$\mathcal{L}_{em,eff} = -e^2 \left\{ \frac{1}{36} c^\mu D_{\mu\nu} c^\nu + \frac{1}{3} c^\mu D_{\mu\nu} i^\nu_3 + i^\mu_3 D_{\mu\nu} i^\nu_3 \right\}, \tag{93}$$

where $D_{\mu\nu}$ stands for the photon propagator. The first term is chiral, isospin invariant and of no further consequence. The second and third, however, transform under $SO(4)$ as F_{34} and $T_{[34][34]} \sim F_{34} F_{34}$, respectively.

In order to construct objects that break isospin in the same way in the chiral lagrangian, I follow the same strategy as in the previous section: I start with antisymmetric rank-two tensors (and their products) built out of covariant quantities ($\boldsymbol{D}_\mu$, N, and their covariant derivatives), apply (69) and then select 34-components. This will produce operators that do not vanish in the chiral limit ($m_{u,d} \to 0$).

Once again, the simplest case is when we consider no nucleon fields. With no covariant derivative, there is no antisymmetric rank-two tensor, and two rank-four tensors with pairs of antisymmetric indices,

$$(T_{1,2})_{[\alpha\beta][\gamma\delta]}[0;0,0] = \varepsilon_{\alpha\beta\gamma\delta} \ , \ \delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma}, \tag{94}$$

have trivial [34][34] components after the rotation (69). The only interesting object is

$$(T_3)_{[\alpha\beta][\gamma\delta]}[0;0,0] = \begin{cases} \delta_{km}\delta_{ln} - \delta_{kn}\delta_{lm} & \text{for} \alpha = k, \beta = l, \gamma = m, \delta = n; \\ 0 & \text{if any index} = 4, \end{cases} \tag{95}$$

from which (69) and (59) give

$$(T_3)_{[34][34]}[\boldsymbol{\pi};0,0] = \frac{4D^{-2}}{F_\pi^2}(\boldsymbol{\pi}^2 - \boldsymbol{\pi}_3^2) \tag{96}$$

There is then a term in the effective theory of the form

$$\mathcal{L}^{(-2)}_{\pi,em} = -\frac{1}{2D^2}\bar{\delta} m_\pi^2(\boldsymbol{\pi}^2 - \boldsymbol{\pi}_3^2) \tag{97}$$

which contributes $\bar{\delta} m_\pi^2$ to the pion mass splitting. This term has index -2 according to (64), but has of course also a suppression compared to a hypothetical $\Delta = -2$ strong interaction due to the feeble electromagnetic coupling constant. In the following I will keep labeling lagrangian terms by their index (64) and will relegate to the next section a discussion of how these terms compare to the ones obtained from the quark mass difference. Because they soon become numerous (and thus uninteresting) as the index and the power of charge increase, and also in antecipation of the forecoming discussion, I will list here only interactions up to $\Delta = 0$ and to e^4 (for $\Delta = -2$ only).

Still without nucleon fields there is a term proportional to e^4, which comes from the tensor product of two of the tensors (95) and leads to

$$\mathcal{L}^{(-2)}_{\pi,em} = \bar{\alpha}_1 D^{-4}(\boldsymbol{\pi}^2 - \pi_3^2)^2 \tag{98}$$

with $\bar{\alpha}_1$ proportional to e^4. Two rank-two tensors can be introduced using $\boldsymbol{D}_\mu$,

$$F_1^\mu[0; \boldsymbol{D}_\mu, 0] = \begin{pmatrix} \varepsilon_{ijk} D_k^\mu & 0 \\ 0 & 0 \end{pmatrix} \tag{99}$$

and

$$F_2^\mu[0; \boldsymbol{D}_\mu, 0] = \begin{pmatrix} 0 & D_i^\mu \\ -D_j^\mu & 0 \end{pmatrix}. \tag{100}$$

They cannot be used directly because they are four-vectors, but then three rank-four tensors exist: one comes from the obvious

$$(T_4)_{[\alpha\beta][\gamma\delta]}[0; \boldsymbol{D}_\mu, 0] = \boldsymbol{D}_\mu \cdot \boldsymbol{D}^\mu (T_3)_{[\alpha\beta][\gamma\delta]}[[0; 0, 0] \tag{101}$$

and the others from the more interesting

$$(T_5)_{[\alpha\beta][\gamma\delta]}[0; \boldsymbol{D}_\mu, 0] = (F_1^\mu)_{\alpha\beta}[0; \boldsymbol{D}_\mu, 0](F_{1\mu})_{\gamma\delta}[0; \boldsymbol{D}_\mu, 0] \tag{102}$$

and

$$(T_6)_{[\alpha\beta][\gamma\delta]}[0; \boldsymbol{D}_\mu, 0] = (F_2^\mu)_{\alpha\beta}[0; \boldsymbol{D}_\mu, 0](F_{2\mu})_{\gamma\delta}[0; \boldsymbol{D}_\mu, 0] \tag{103}$$

(the other combination $F_1^\mu F_{2\mu}$ leads to a term of wrong parity). They give us

$$\begin{aligned} \mathcal{L}^{(0)}_{\pi,em} &= \bar{\alpha}_2 D^{-2} F_\pi^2 \left[\left(1 - \frac{\boldsymbol{\pi}^2}{F_\pi^2}\right) D_{\mu 3} + \frac{2}{F_\pi^2} \pi_3 \boldsymbol{\pi} \cdot \boldsymbol{D}_\mu \right]^2 \\ &\quad + \bar{\alpha}_3 D^{-2}(\boldsymbol{\pi}^2 - \pi_3^2) \boldsymbol{D}_\mu \cdot \boldsymbol{D}^\mu + \bar{\alpha}_4 D^{-2} (\boldsymbol{\pi} \times \boldsymbol{D}_\mu)_3 (\boldsymbol{\pi} \times \boldsymbol{D}^\mu)_3 \end{aligned} \tag{104}$$

with $\bar{\alpha}_2$,$\bar{\alpha}_3$ and $\bar{\alpha}_4$ all of order $O(\frac{\bar{\delta} m_\pi^2}{M^2})$.

When a couple of nucleon fields are brought into play we get a new building block, the isospin generator $\boldsymbol{t}$. Begin with no derivatives. Now rank-two tensors can be constructed. One,

$$F_3^\Gamma[0;0,N] = \begin{pmatrix} \varepsilon_{ijk}\bar{N}t_k\Gamma N & 0 \\ 0 & 0 \end{pmatrix} \tag{105}$$

with $\Gamma = 1$, leads nowhere since the 34-component of its rotated version is a pseudoscalar, while the other,

$$F_4^\Gamma[0;0,N] = \begin{pmatrix} 0 & Nt_i\Gamma N \\ -\bar{N}t_j\Gamma N & 0 \end{pmatrix} \tag{106}$$

with $\Gamma = 1$, is more interesting. As for rank-four tensors, nothing really new appears, only one that is simply $\bar{N}N$ times (95),

$$(T_7)_{[\alpha\beta][\gamma\delta]}[0;0,N] = \bar{N}N(T_3)_{[\alpha\beta][\gamma\delta]}[0;0,0]. \tag{107}$$

(106) and (107) give

$$\begin{aligned} \mathcal{L}_{N,em}^{(-1)} &= \bar{\delta} m_N \left[-\bar{N}t_3N + \frac{2D^{-1}}{F_\pi^2}\bar{N}(\boldsymbol{\pi}^2 t_3 - \pi_3\boldsymbol{\pi}\cdot\boldsymbol{t})N\right] \\ &\quad +\bar{\beta}_1 F_\pi^{-2}D^{-2}(\boldsymbol{\pi}^2 - \pi_3^2)\bar{N}N \end{aligned} \tag{108}$$

with $\bar{\delta} m_N$ and $\bar{\beta}_1$ of order $O(\frac{\bar{\delta} m_\pi^2}{M})$. With one derivative we get to $\Delta = 0$ and a large number of terms. First there are four rank-two tensors: one that is similar to F_3,

$$F_5[0;0,N] = \begin{pmatrix} \varepsilon_{ijk}(\bar{N}t_k\vec{\sigma}\cdot\vec{\mathcal{D}}N + \text{h.c.}) & 0 \\ 0 & 0 \end{pmatrix}, \tag{109}$$

two that are obtained by multiplying F_1, F_2 with nucleon bilinears,

$$(F_6)_{\alpha\beta}[0;0,N] = \bar{N}N(F_1^0)_{\alpha\beta}[0;0,N] \tag{110}$$

and

$$(F_7)_{\alpha\beta}[0;0,N] = \bar{N}\vec{\sigma}N \cdot (\vec{F}_2)_{\alpha\beta}[0;0,N], \tag{111}$$

and one with a new structure,

$$(F_8)_{\alpha\beta}[0;\boldsymbol{D}_\mu,N] = \begin{pmatrix} 0 & \varepsilon_{ijk}\vec{D}_j \cdot \bar{N}t_k\vec{\sigma}N \\ -\varepsilon_{ijk}\vec{D}_j \cdot \bar{N}t_k\vec{\sigma}N & 0 \end{pmatrix}. \tag{112}$$

Then, there are five rank-four tensors: one that is just T_3 times an invariant bilinear,

$$(T_8)_{[\alpha\beta][\gamma\delta]}[0;\boldsymbol{D}_\mu,N] = \bar{N}\boldsymbol{t}\vec{\sigma}N \cdot \cdot \vec{\boldsymbol{D}}\ (T_3)_{[\alpha\beta][\gamma\delta]}[0;0,0] \tag{113}$$

and the four combinations of F_1 and F_2 with F_3 and F_4:

$$\begin{aligned}
(T_9)_{[\alpha\beta][\gamma\delta]}[0;\boldsymbol{D}_\mu,N] &= (\vec{F}_1)_{\alpha\beta}[0;\boldsymbol{D}_\mu,0] \cdot (F_3^{\vec{\sigma}})_{\gamma\delta}[0;0,N], &(114)\\
(T_{10})_{[\alpha\beta][\gamma\delta]}[0;\boldsymbol{D}_\mu,N] &= (F_1^0)_{\alpha\beta}[0;\boldsymbol{D}_\mu,0]\ (F_4^1)_{\gamma\delta}[0;0,N], &(115)\\
(T_{11})_{[\alpha\beta][\gamma\delta]}[0;\boldsymbol{D}_\mu,N] &= (F_2^0)_{\alpha\beta}[0;\boldsymbol{D}_\mu,0]\ (F_3^1)_{\gamma\delta}[0;0,N] &(116)
\end{aligned}$$

and

$$(T_{12})_{[\alpha\beta][\gamma\delta]}[0;\boldsymbol{D}_\mu,N] = (\vec{F}_2)_{\alpha\beta}[0;\boldsymbol{D}_\mu,0] \cdot (F_4^{\vec{\sigma}})_{\gamma\delta}[0;0,N]. \tag{117}$$

Putting all together (and integrating by pairs when convenient),

$$\begin{aligned}
\mathcal{L}^{(0)}_{N,em} = \ & \bar{\beta}_2\frac{D^{-1}}{F_\pi}(\boldsymbol{\pi}\times\boldsymbol{D}_0)\bar{N}N \\
&+\bar{\beta}_3 D^{-1}\left[\left(1-\frac{\boldsymbol{\pi}^2}{F_\pi^2}\right)\vec{D}_3 + \frac{2}{F_\pi^2}\pi_3\boldsymbol{\pi}\cdot\vec{\boldsymbol{D}}\right]\cdot\bar{N}\vec{\sigma}N \\
&+\bar{\beta}_4[(\boldsymbol{D}_i\times\bar{N}\boldsymbol{t}\sigma_i N)_3 - \frac{2D^{-1}}{F_\pi^2}(\boldsymbol{\pi}\cdot\vec{\boldsymbol{D}})\cdot(\boldsymbol{\pi}\times\bar{N}\boldsymbol{t}\vec{\sigma}N)_3] \\
&+\bar{\beta}_5 D^{-1}\left[\left(1-\frac{\boldsymbol{\pi}^2}{F_\pi^2}\right)(\boldsymbol{D}_i\times\bar{N}\boldsymbol{t}\sigma_i N)_3 + \frac{2}{F_\pi^2}\pi_3\boldsymbol{\pi}\cdot(\boldsymbol{D}_i\times\bar{N}\boldsymbol{t}\sigma_i N)\right] \\
&+\bar{\beta}_6\frac{D^{-2}}{F_\pi^2}(\boldsymbol{\pi}^2-\pi_3^2)\vec{\boldsymbol{D}}\cdot\cdot\bar{N}\boldsymbol{t}\vec{\sigma}N + \bar{\beta}_7\frac{D^{-2}}{F_\pi^2}(\boldsymbol{\pi}\times\vec{\boldsymbol{D}})_3\cdot(\boldsymbol{\pi}\times\bar{N}\boldsymbol{t}\vec{\sigma}N)_3
\end{aligned}$$

$$
\begin{aligned}
&+\bar{\beta}_8 \frac{D^{-2}}{F_\pi}(\boldsymbol{\pi} \times \boldsymbol{D}_0)_3 \left[\left(1-\frac{\boldsymbol{\pi}^2}{F_\pi^2}\right)\bar{N}t_3N + \frac{2}{F_\pi^2}\pi_3\boldsymbol{\pi}\cdot\bar{N}\boldsymbol{t}N\right] \\
&+\bar{\beta}_9 \frac{D^{-2}}{F_\pi}\left[\left(1-\frac{\boldsymbol{\pi}^2}{F_\pi^2}\right)D_{30} + \frac{2}{F_\pi^2}\pi_3\boldsymbol{\pi}\cdot\boldsymbol{D}_0\right](\boldsymbol{\pi}\times\bar{N}\boldsymbol{t}N)_3 \\
&+\bar{\beta}_{10}\left[\left(1-\frac{\boldsymbol{\pi}^2}{F_\pi^2}\right)\vec{D}_3 + \frac{2}{F_\pi^2}\pi_3\boldsymbol{\pi}\cdot\vec{\boldsymbol{D}}\right]\cdot \\
&\qquad\qquad \cdot\left[\left(1-\frac{\boldsymbol{\pi}^2}{F_\pi^2}\right)\bar{N}t_3\vec{\sigma}N + \frac{2}{F_\pi^2}\pi_3\boldsymbol{\pi}\cdot\bar{N}\boldsymbol{t}\vec{\sigma}N\right] \qquad (118)
\end{aligned}
$$

where the $\bar{\beta}_i$'s are all of order $O(\frac{\bar{\delta}m_\pi^2}{M^2})$.

We finally arrive at two pairs of nucleon fields, which with no derivatives have $\Delta = 0$ already. As for rank-two tensors, there are the ones corresponding to (105) and (106) times another nucleon bilinear; only the latter,

$$(F_9)_{\alpha\beta}[0;0,N] = \bar{N}\Gamma N(F_4^\Gamma)_{\alpha\beta}[0;0,N] \qquad (119)$$

will contribute. A few more possibilities arise from rank-four tensors. There is (95) times the four spin-isospin combinations of $\bar{N}N\bar{N}N$; by Fierz reordering we can eliminate two of them, so we take only

$$(T_{13})_{[\alpha\beta][\gamma\delta]}[0;0,N] = \bar{N}\Gamma N\bar{N}\Gamma N(T_3)_{[\alpha\beta][\gamma\delta]}[0;0,0] \qquad (120)$$

And then there are the products of (105) and (106):

$$
\begin{aligned}
(T_{14})_{[\alpha\beta][\gamma\delta]}[0;0,N] &= (F_3^\Gamma)_{\alpha\beta}[0;0,N](F_3^\Gamma)_{\gamma\delta}[0;0,N], \qquad (121) \\
(T_{15})_{[\alpha\delta][\gamma\delta]}[0;0,N] &= (F_4^\Gamma)_{\alpha\beta}[0;0,N](F_4^\Gamma)_{\gamma\delta}[0;0,N] \qquad (122)
\end{aligned}
$$

while the crossed F_3F_4 will again have parity problems. Putting the resulting four-nucleon terms together,

$$\mathcal{L}^{(0)}_{NN,em} = \bar{\gamma}_{1s}\left[\bar{N}t_3N + \frac{2D^{-1}}{F_\pi^2}(\pi_3\boldsymbol{\pi}\cdot\bar{N}\boldsymbol{t}N - \boldsymbol{\pi}^2\bar{N}t_3N\right]^2$$

$$+\bar{\gamma}_{1\sigma}\left[\bar{N}t_3\vec{\sigma}N+\frac{2D^{-1}}{F_\pi^2}(\pi_3\boldsymbol{\pi}\cdot\bar{N}\boldsymbol{t}\vec{\sigma}N-\boldsymbol{\pi}^2\bar{N}t_3\vec{\sigma}N)\right]^2$$
$$+\frac{D^{-2}}{F_\pi^2}(\boldsymbol{\pi}^2-\pi_3^2)[\bar{\gamma}_{2s}\bar{N}N\bar{N}N+\bar{\gamma}_{2\sigma}\bar{N}\vec{\sigma}N\cdot\bar{N}\vec{\sigma}N]$$
$$+\bar{\gamma}_{3s}\frac{D^{-2}}{F_\pi^2}(\boldsymbol{\pi}\times\bar{N}\boldsymbol{t}N)_3(\boldsymbol{\pi}\times\bar{N}\boldsymbol{t}N)_3$$
$$+\bar{\gamma}_{3\sigma}\frac{D^{-2}}{F_\pi^2}(\boldsymbol{\pi}\times\bar{N}\boldsymbol{t}\vec{\sigma}N)_3\cdot(\boldsymbol{\pi}\times\bar{N}\boldsymbol{t}\vec{\sigma}N)_3 \tag{123}$$

where the $\bar{\gamma}$'s are of order $O(\frac{\bar{\delta}m_\pi^2}{M^4})$.

These are all the operators up to $\Delta = 0$ and proportional to e^2 (and including the leading e^4 term (98)) that do not vanish when the quark masses go to zero.

Further, "mixed" operators arise when we look at tensor products of $SO(4)$ four-vectors and rank-two antisymmetric tensors. Again to $\Delta = 0$ and e^2, there is just one such operator, obtained by the product of (70) and (95),

$$\mathcal{L}^{(0)}_{\pi,em} = \bar{\alpha}_5 D^{-3}\frac{\boldsymbol{\pi}^2}{F_\pi^2}(\boldsymbol{\pi}^2-\pi_3^2) \tag{124}$$

with $\bar{\alpha}_5$ proportional to $e^2 m_\pi^2$.

In summary, the most important isospin breaking operator arising from hard photon exchange is given in (97), followed by those in (108), (104), (98), (124), (118) and (123).

4.4 Discussion

In order to learn something from the multitude of terms considered in the previous sections, I have to resort (due to my incompetence in better dealing with strong interactions) to dimensional analysis. Of course, this is only good insofar as we look for an order-of-magnitude understanding of things. One cannot keep track of factors like 2, since one does not even know for sure which QCD scale really sets the suppression of higher-derivative operators. (The pion decay constant $F_\pi \simeq 190$MeV, the confinement scale $\Lambda_{QCD} \sim 300$MeV, the ρ mass $m_\rho \simeq 770$MeV and the chiral symmetry breaking scale $\Lambda_{\chi SB} \sim 1.2$GeV are all supposedly related but differ by almost an order of magnitude.) I will take $M \sim m_\rho$ as a measure of higher energy effects, so the expansion parameter in the chiral lagrangian is $\sim \frac{m_\pi}{m_\rho} \simeq 0.2$.

The above point can be illustrated with the pion masses. We might guess that the square of the bulk of the pion mass, (72), is $O(M(m_u + m_d))$. With M in the GeV range, the sum of quark masses have indeed to be of the order of tens of MeV to give the correct pion mass; but using m_ρ as an estimate for M and the Weinberg values for m_u, m_d, we are off by a factor 2.

Despite this, we can have an idea of how electromagnetic and quark mass difference effects compare by looking at the pion mass difference. Eqs. (76) and (97) add to

$$\mathcal{L}_{\pi,\Delta mass} = -\frac{1}{2}\bar{\delta} m_\pi^2 D^{-2} \boldsymbol{\pi}^2 + \frac{1}{2}\Delta m_\pi^2 D^{-2} \pi_3^2 \tag{125}$$

where

$$\Delta m_\pi^2 = m_{\pi^+}^2 - m_{\pi^0}^2 = \bar{\delta} m_\pi^2 + \delta m_\pi^2 \tag{126}$$

is the pion mass difference, experimentally $\sim (35\text{MeV})^2$. The important fact is, the contribution from the quark mass difference δm_π^2 comes only at index $\Delta = 2$, being thus suppressed compared to the pion mass squared by factors of both $(\frac{m_\pi}{M})^2$ and ε^2. It will contribute only some $(7\text{MeV})^2$ to (126). The electromagnetic contribution $\bar{\delta}m_\pi^2$ is typically bigger. It is proportional to e^2, so one could guess that it is of order $\alpha m_\rho^2 \sim (66MeV)^2$; even if an extra $\frac{1}{4\pi}$ appears, we get more than the quark mass share. We can thus conclude that the pion mass difference is mainly electromagnetic, a result that is supported when more input is added: for example, using the Weinberg spectral sum rules in a ρ and $A1$ dominance model where the KSFR relation holds, one finds [72] $\Delta m_\pi^2 = \frac{3 ln 2}{2\pi}\alpha m_\rho^2 \sim (38\text{MeV})^2$.

In order to further estimate the size of the electromagnetic terms, I will neglect δm_π^2 in (126). Now, *numerically*, Δm_π^2 is approximately $\varepsilon \frac{m_\pi}{m_\rho} m_\pi^2$, so (97) would correspond to a hypothetical quark mass term in $\mathcal{L}^{(1)}_{\pi,qm}$ with one factor of the quark mass difference. One could then expect that for (97), (104), (108), (118) and (123), $\mathcal{L}^{(n)}_{em} \sim \mathcal{L}^{(n+3)}_{qm}$. For (98) and (123), there is at least one extra factor of α, so they would correspond to $\mathcal{L}^{(4)}_{qm}$. All this is to justify the following bookeeping: $\mathcal{L}^{(-2)}_{\pi,em}$ of (97) should be paired to $\mathcal{L}^{(1)}_{N,qm}$ of (80) as the leading isospin violating operators at low-energy, followed by $\mathcal{L}^{(-1)}_{N,em}$ of (108) lumped with $\mathcal{L}^{(2)}_{\pi,qm}$ of (76), $\mathcal{L}^{(2)}_{N,qm}$ of (83) and $\mathcal{L}^{(2)}_{NN,qm}$ of (86), as the more important corrections. The other electromagnetic terms of (104), (118), (123), (98) and(123) are probably smaller and I will neglect them. (They can be easily included in case I have subestimated their importance.)

I now discuss the implications of this dimensional analysis to various process.

Start with pion-pion scattering. The scattering amplitude $M_{ab,cd}$ for pions of isospin indices a, b (and momenta p_a, p_b) going into c, d (p_c, p_d) can be expressed, *assuming isospin symmetry*, as

$$M_{ab,cd} = A(s,t,u)\delta_{ab}\delta_{cd} + A(t,s,u)\delta_{ac}\delta_{bd} + A(u,t,s)\delta_{ad}\delta_{bc}, \qquad (127)$$

in terms of a single function $A(s,t,u)$ of the Mandelstam variables

$$s = -(p_a + p_b)^2, t = -(p_a - p_c)^2, u = -(p_a - p_d)^2. \qquad (128)$$

$A(s,t,u)$ was obtained in lowest order in the chiral expansion long ago by Weinberg [73]: it comes from the pion kinetic term plus (72) and is

$$A(s,t,u) = s - m_\pi^2. \qquad (129)$$

It is trivial to use (125) (which includes (97)) to calculate the leading, tree-level isospin violation. It is interesting that it does *not* change the result (127) plus (129). That is, at fixed s, t and u, there is no isospin violation [2] of order $O(\frac{\Delta m_\pi^2}{m_\pi^2}) \sim 5\%$, contrary to what one might have expected. This happens because of a cancelation between mass differences coming from the kinetic term expressed in Mandelstam variables, and from the mass term; this cancelation arises in this case because the isospin breaking interactions come from the expansion of D^{-2} around 1, as opposed to D^{-1} as in the case of the isospin symmetric mass term.

As a consequence, isospin violation from high-energy effects will arise at two orders higher in the perturbative expansion, which includes one loop. This means that an extra suppression of $(\frac{m_\pi}{m_\rho})^2$ will render the amount of breaking to a hopelessly small $\frac{\Delta m_\pi^2}{m_\rho^2} \sim 0.2\%$. On the other hand, soft photon exchange among the pion legs can be expected to be $O(\alpha m_\pi^2)$; they have been calculated in

[2] Except for a "kinematic" breaking hidden in s, t and u, that appears, for example, in the scattering lengths, due to different thresholds; but this is a trivial effect from the pion masses.

a chiral lagrangian approach [74] and found to contribute indeed at the percent level, after Coulomb corrections have been subtracted. I am forced to conclude that purely pionic reactions are not a good place to see isospin violation other than from photon exchange.

I then search for a signal of isospin breaking when nucleons are present. The leading term involving a nucleon is, according to the above discussion, given by (80). What effects does it have?

First, there is a nucleon mass splitting

$$\Delta m_N \;=\; m_p - m_n = \delta m_N + \bar{\delta} m_N .$$

Experimentally, it is -1.3MeV. That is of a magnitude that could be explained on basis of dimensional analysis for the electromagnetic contribution $\bar{\delta} m_N$ from (108), which is of order $\frac{\Delta m_\pi^2}{m_\rho} \sim 1.5$MeV. However, the electromagnetic $O(\alpha)$ correction has the wrong sign, making the proton heavier than the neutron, and this was in pre-QCD years a great puzzle. Now the same dimensional analysis gives for the contribution from (80) $\varepsilon \frac{m_\pi^2}{m_\rho} \sim m_u - m_d \sim -$(a few MeV), with the sign given by the flavor content of proton and neutron. That is, naive dimensional analysis suggests a nucleon mass difference mainly due to a quark mass difference of the same order of magnitute. And it is not that small; it just looks tiny compared to the nucleon mass because the latter is mainly due to the spontaneous breaking of chiral symmetry and, therefore, big.

Then, there are the interactions of an even number of pions with a nucleon, proportional to δm_N. To ilustrate its effects, I concentrate on pion-nucleon scattering close to threshold, neglecting the fact that the thresholds themselves depend on the mass differences. In the isospin symmetric limit, the various scattering lengths $a(\pi_a N \to \pi_b N)$ can be written as

$$\begin{aligned} a(\pi^0 N \to \pi^0 N) &= a_0 \\ a(\pi^+ n \to \pi^0 p) = a(\pi^- p \to \pi^0 n) &= \sqrt{2} a_1 \\ a(\pi^\pm N \to \pi^\pm N) &= a_0 \pm 2 a_1 t_3^{(N)} \end{aligned} \tag{130}$$

in terms of the scattering lengths a_0 and a_1 corresponding to isospin zero and one exchanged in the t channel. The chiral lagrangian to lowest order yields the famous Tomozawa-Weinberg relations [75, 73],

$$a_0 = 0, \tag{131}$$

$$a_1 = \frac{1}{1+\frac{m_\pi}{m_N}} \frac{m_\pi}{2\pi F_\pi^2}, \tag{132}$$

because the pseudo-vector coupling of the pion vanishes (for non-relativistic nucleons) at threshold, and the two-pion coupling (seagull) is isovector. The isospin zero amplitude is therefore sensitive to corrections, and so is the $\pi^0 N \to \pi^0 N$ scattering length. They can be calculated from the first order lagrangian; it contain two terms that do not vanish at threshold: the sigma term (77) that comes from explicit chiral breaking, plus another, chirally symmetric term $\frac{1}{F_\pi^2} D^{-2} \dot{\pi}^2 \bar{N} N$. They contribute to a_0,

$$a_0 = \frac{1}{1+\frac{m_\pi}{m_N}} \frac{\tilde{\sigma}}{\pi F_\pi^2}, \tag{133}$$

where $\tilde{\sigma}$ is $O(\frac{m_\pi^2}{m_\rho})$, and found to be around $60 MeV$.

In the same fashion we can use the isospin breaking interaction (80) to obtain extra contributions to (130),

$$\Delta a(\pi^0 N \to \pi^0 N) = \frac{1}{1+\frac{m_\pi}{m_N}} \frac{1}{\pi F_\pi^2} \delta m_N t_3^{(N)},$$

$$\begin{aligned}\Delta(a(\pi^+ n \to \pi^0 p) - a(\pi^- p \to \pi^0 n)) &= \frac{1}{1+\frac{m_\pi}{m_N}} \frac{\sqrt{2}}{\pi F_\pi^2} \delta m_N, \\ \Delta a(\pi^\pm N \to \pi^\pm N) &= 0. \end{aligned} \quad (134)$$

For processes that involve only one π^0, the contribution is $O(\varepsilon \frac{m_\pi}{m_\rho}) \sim 5\%$ of the isosymmetric result, but for $\pi^0 N \to \pi^0 N$ it should be $O(\varepsilon) \sim 30\%$ which reveals the full breaking in the QCD lagrangian and is huge! (This result was first pointed out using current algebra by Weinberg [69].)

Unfortunately, this one case of large isospin violation cannot be measured easily due to the lack of π^0 beams. So we are led to consider the possibility of at least one virtual π^0, which brings us to systems with several nucleons, i.e. nuclear physics.

The basic quantity in nuclear physics is the nuclear potential, and I first ask myself what the leading contributions are to two-body forces.

From (97) there is a somewhat trivial breaking effect, which is simply due to the pion mass difference in the leading, static one pion exchange (OPE).

It is easy to see that the effect of the other leading term, (80), is small. Indeed, the nucleon interacts with two pions, which then require a loop, and thus an extra suppression of $(\frac{m_\pi}{m_\rho})^2$. As for the dependence on the nucleon mass difference, that comes from an expansion of the energy denominador, it is quadratic (since the linear term has to vanish due to charge conservation) and is therefore of order $(\frac{\delta m_N}{m_\pi})^2 \sim \varepsilon^2 (\frac{m_\pi}{m_\rho})^2$ compared to the leading, static one-pion exchange. It is then formally of second order, but small.

These mass splitting terms are there, but are somewhat trivial, so I look further in second order. My grouping of terms according to dimensional analysis tells that I should take terms with one pion coupling to a nucleon, and with a nucleon contact interaction, and use them at tree level. Eq. (108) does not contribute, while (83) amounts to a coupling constant breaking in OPE, and (86) brings two contact interactions.

Putting all these first and second order contributions together, we arrive at the leading isospin violating potential

$$V = V_{II}[t_3^{(1)}t_3^{(2)} - \mathbf{t}^{(1)} \cdot \mathbf{t}^{(2)}] + V_{III}[t_3^{(1)} + t_3^{(2)}], \tag{135}$$

where the superscripts label the two nucleons and, in momentum space, $\vec{q}$ being the momentum exchanged and g_A the axial vector coupling of the nucleon,

$$V_{II} = -\left(\frac{2g_A}{F_\pi}\right)^2 \frac{\vec{q} \cdot \vec{\sigma}^{(1)}\vec{q} \cdot \vec{\sigma}^{(2)}}{(\vec{q}^{\,2} + m_{\pi^0}^2)(\vec{q}^{\,2} + m_{\pi\pm}^2)}(\Delta m_\pi^2 + \Delta m_N^2) \tag{136}$$

and

$$V_{III} = \frac{2g_A\beta_1}{F_\pi^2}\frac{\vec{q} \cdot \vec{\sigma}^{(1)}\vec{q} \cdot \vec{\sigma}^{(2)}}{\vec{q}^{\,2} + m_\pi^2} - (\gamma_s + \gamma_\sigma \vec{\sigma}^{(1)} \cdot \vec{\sigma}^{(2)}). \tag{137}$$

(To write the pion mass splitting contribution as in (136), I took the mass appearing in static OPE as the π^0 mass.)

To this potential we should add the potential coming from one photon and one photon-one pion exchange. Unfortunately, while the former has been well studied and its effects can be removed from most observables, the contributions of the latter have not been calculated reliably [76]. But (135)–(137) allow us to understand some of the evidence for isospin violation in two-nucleon systems. It contains a larger piece (V_{II}) dominated by the pion mass difference, that breaks isospin but still respects charge symmetry, which is defined as an isospin rotation of π around the 2-axis. Charge symmetry is broken more slightly by the other piece (V_{III}). It is natural, then, to follow the classification found in the literature

[77], where nuclear forces are divided in class I (isosymmetric), II ($\propto t_3^{(1)}t_3^{(2)}$), III ($\propto (t_3^{(1)}+t_3^{(2)})$) and IV ($\propto (t_3^{(1)}-t_3^{(2)})$ or $(\vec{t}^{(1)}\times\vec{t}^{(2)})_3$).

What are the signals of isospin violation here? Well, after purely electromagnetic effects are removed, there are large differences between the ${}^1S_0$ nn, np and pp scattering lengths [78],

$$\Delta a_{CD} = \frac{1}{2}(a_{nn}+a_{pp}) - a_{np} = (5.7\pm 0.3)fm, \tag{138}$$

$$\Delta a_{CSB} = a_{pp} - a_{nn} = (1.5\pm 0.5)fm \tag{139}$$

(for comparison, $a_{np} = (-23.748\pm 0.009)fm$). These differences are big because a difference between potentials corresponds to a difference between the inverses of the respective scattering lengths [79]. In the ${}^1S_0$ state, and in coordinate space, (135)–(137) give

$$\frac{1}{2}(\langle V\rangle_{pp}+\langle V\rangle_{nn}) - \langle V\rangle_{np} = -\left(\frac{g_A}{F_\pi}\right)^2 \frac{e^{-m_{\pi^0}r}}{2\pi r}\left\{m_{\pi^0}^2 + m_{\pi^\pm}^2 e^{-(m_{\pi^\pm}-m_{\pi^0})r} + \delta m_N^2(-2+m_\pi r)\right\} \tag{140}$$

$$\langle V\rangle_{pp} - \langle V\rangle_{nn} = \frac{4g_A\beta_1}{F_\pi^2}\left[m_\pi^2\frac{e^{-m_\pi r}}{4\pi r} - \delta^3(\vec{r})\right] + [\gamma_s - 3\gamma_\sigma]\delta^3(\vec{r}) \tag{141}$$

Here we see that, indeed, the dominant pion mass difference term contributes only to charge dependence (138). This agrees with the result of [80], obtained with pseudoscalar pion coupling. (The pseudo-vector coupling was once thought to scale as $\frac{1}{m_\pi}$—not as $\frac{1}{F_\pi}$ as in the chiral lagrangian—and therefore to imply a different contribution than in (140)). It was found to account for half of (138), the rest being attributed to pion-photon exchange and multi-pion exchange. Charge symmetry breaking, on the other hand, is naturally smaller. It is thought to be due mainly to $\rho-\omega$ mixing, and, to a smaller extent, to $\pi-\eta-\eta'$ mixing and, again, pion-photon exchange. Here the mixings do not appear explicitly, but are buried in the coefficients β_1 and γ_s, γ_σ. It can easily be shown from the explicit expression of the $\pi - \eta - \eta'$ mixing potential that it reduces to my β_1 term in the limit of momenta much smaller than the η mass; using the values for couplings and mixing element in [81], we find $\beta_1 \simeq 10^{-3}$, which is the same as my estimate $\varepsilon(\frac{m_\pi}{m_\rho})^2$. Similarly, the $\rho - \omega$ mixing potential reduces, to the lowest order in momenta much smaller than $m_\rho \sim m_\omega$, to the γ_s (and no γ_σ) term; one then finds [81] a much larger value for $\gamma_s (\sim m_\rho^{-2})$ than my estimate $\varepsilon(\frac{m_\pi}{m_\rho})^2$, but recently it has been argued by several authors [82] that the value of the $\rho - \omega$ mixing element in the spacelike region is much smaller, and of different sign, than the previous value, obtained at the ω mass shell.

There is evidence of isospin violation in other quantities, but it is either not definite or needs pion-photon exchange. An exception is the measurements of some observables in $n - p$ scattering that seem to indicate the presence of class IV forces [78]. Here, they would appear only at higher order.

One could now ask whether new effects would arise in systems of more than two nucleons. Power counting suggests that three-body forces, including one from (80), will be of higher order. Indeed, one finds [78] that all isospin violation that is not due to one-photon exchange (Nolen-Schiffer anomaly) is compatible with the information from the scattering lengths (138) and (139).

One can finally consider processes involving nuclei and external pions. The problem here is, generically, the same as with three-nucleon forces. Power counting in the chiral lagrangian tells us that the impulse approximation (in which external pions interact with each nucleon separately) is dominant, corrections due to two-nucleon terms being $(\frac{m_\pi}{m_\rho})^2$ smaller. These are already hard to see when they do not involve isospin violation (see e.g. [83] for pion-deuteron scattering at threshold), so it is very unlikely that one can find anything new here.

I can think of just one possible exception: $pp \to pp\pi^0$ at threshold. This is a very special process [3], for reasons that surfaced in the discussion of πN scattering: i) the leading one-nucleon term (in which the pion is emitted by one of the nucleons via the g_A term in the lowest order lagrangian) vanishes at threshold; ii) the leading two-nucleon term (in which the pion is emitted by one nucleon, and then interacts with the other nucleon via the seagull term in the lowest order lagrangian, before flying away) also vanishes, because the virtual pion is also neutral. The cross section has been calculated [84] using the first relativistic correction to the g_A term (in the one-nucleon graph), and data from s-wave πN scattering (in the two-nucleon case). A recent measurement at Indiana [85] found a cross-section five times bigger than the theoretical prediction. A full chiral lagrangian calculation is not trivial, because to the same order as the contributions considered in [84] there are other $\pi \bar{N} N$ vertices and one-nucleon loop graphs also have to be considered. Here I just want to point out the relevance of the isospin violating interaction (80). In [84] isospin relations (that reduce to (130) at threshold) are employed, together with charged pion-nucleon scattering data, to obtain the amplitude for $\pi^0 N$ scattering, which is then used in the calculation of the two-nucleon graph. This amounts to taking $\tilde{\sigma}$ of (133) into

[3]A note of warning: absorption or production of pions involve a change in the energy of the nuclear system of order m_π, which is shared between nucleons by exchanged momenta of order $\sqrt{2m_N m_\pi} \sim 500\text{MeV} \sim 4m_\pi$; this might be a little too heavy for chiral perturbation to handle, so what follows is even more qualitative than what preceeded it.

account, but neglecting the isospin violating contribution from (80), which, as we saw, is relatively large in this case.

4.5 Conclusion

A procedure was presented to derive the isospin violating lagrangian governing pions and nucleons at low energies. Using dimensional analysis the interactions were ordered and shown to reproduce most of the empirical observations. The structure (matter content plus symmetries) of the chiral lagrangian—which is a consequence of the pattern of symmetry breaking in the QCD lagrangian is such that:

i) there is no lowest order violation;

ii) a first correction is due to the quark mass difference, gives rise to the nucleon mass splitting and would be relatively big for a process that is hard to measure ($\pi^0 N \rightarrow \pi^0 N$);

iii) the other first correction is mainly electromagnetic, gives rise to the pion mass splitting, cancels in leading order in $\pi - \pi$ scattering, and dominates the observed breaking in the two-nucleon scattering lengths;

iv) other corrections are smaller, although there is some evidence for them in the two-nucleon system.

As a result of this conspiration of symmetries and experimental limitations, isospin violation in nuclear and pion physics is essentially due to trivial mass splitting effects, and is relatively much smaller than it appears from the looks of the QCD lagrangian.

5. The axial vector coupling and magnetic moment of the quark

5.1 Introduction

Some time ago Weinberg [86] argued that in the leading order in $1/N$, where N is the number of colors, the axial vector coupling of the constituent quark is equal to one and its anomalous magnetic moment is zero. This justifies the usual treatments of the constituent quark and bag models, where the quark is treated as a bare Dirac particle, provided the corrections in $1/N$ are shown to be small, especially for the magnetic moments. More recently Weinberg [87] (see also ref. [88]) has given an estimate of the corrections of order $1/N$ to the axial vector coupling of the constituent quark. His calculation was done using the chiral quark model Lagrangian [89] in the chiral limit and the limit of large number of colors. The essential input was the analogue of the famous Adler-Weisberger sum rule for pion-quark scattering. First order corrections in $1/N$ to the leading result were shown to come from tree-level pion-quark scattering and quark-antiquark pair production diagrams. The latter contribution turned out to be logarithmically divergent but relatively small even for the values of a cutoff as large as 5 GeV.

It is our aim in this chapter to elaborate on this last result, already quoted in [87]. Following the same pattern of reasoning we also analyze the magnetic moment of the constituent quark using the same chiral Lagrangian and the analogue of the Drell-Hearn-Gerasimov sum rule for photon-quark scattering. Our results seem to indicate that the chiral quark model with coefficients obtained by sum rules works well. (These results have been published in [90].)

5.2 The axial vector coupling

The lowest order terms in the chiral Lagrangian density in which the relevant degrees of freedom are the constituent u- and d-quarks, treated as massive particles subject to color force only at large separations, and pions treated as pseudo-Goldstone bosons, have the following form[4]:

$$\begin{aligned} \mathcal{L} &= -\frac{1}{2(1+\frac{\vec{\pi}^2}{F_\pi^2})^2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - \frac{1}{2(1+\frac{\vec{\pi}^2}{F_\pi^2})}m_\pi^2\vec{\pi}^2 \\ & -\bar{\psi}[\partial\!\!\!/ + m + \frac{2i}{F_\pi^2}\frac{1}{1+\frac{\vec{\pi}^2}{F_\pi^2}}\vec{t}\cdot(\vec{\pi}\times\partial\!\!\!/\vec{\pi}) \\ & +\frac{2ig_A}{F_\pi}\frac{1}{1+\frac{\vec{\pi}^2}{F_\pi^2}}\gamma_5\vec{t}\cdot\partial\!\!\!/\vec{\pi}]\psi \end{aligned} \tag{142}$$

where $\vec{\pi}$ denotes the pion field, ψ is the quark field, m_π ($\approx$ 135 MeV) is the pion mass, m stands for the mass of the constituent quark ($\approx$ 360 MeV), F_π is the

[4]Here we neglect isospin breaking due either to a quark mass difference in the QCD Lagrangian or to electromagnetic interactions. Accordingly, we do not include purely electromagnetic contributions to the sum rules.

pion decay constant (≈ 190 MeV), g_A is the axial vector coupling in the leading order in $1/N$, hence is set to one, and $\vec{t} = \frac{\vec{\sigma}}{2}$ where $\vec{\sigma}$ are the usual Pauli matrices in isospin space.

In order to estimate corrections to g_A to first order in $1/N$ we use the Adler-Weisberger [91] sum rule for pion-quark scattering in the form used by Weinberg [87] in the chiral limit ($m_\pi = 0$):

$$g_A^2 \;=\; 1 - \frac{F_\pi^2}{2\pi}\int_0^\infty \frac{d\omega}{\omega}[\sigma_-(\omega) - \sigma_+(\omega)] \tag{143}$$

where σ_+ and σ_- stand for the total cross-sections for scattering of π^+ and π^- respectively on a constituent u-quark at rest. The incoming energy of the pion is denoted by ω. In the large N limit F_π goes as $\sqrt{N}$. Taking this fact into account it is easy to conclude that the only relevant processes making contributions of order $1/N^2$ in the total cross-sections, or equivalently of order $1/N$ in g_A are tree-level pion-quark elastic scattering (Figure (1)) and the quark-antiquark pair production (Figure (2)) [87]. (In the latter case the extra $1/\sqrt{N}$ in the amplitude, coming from the coupling to the produced pairs, is canceled in the expression for the total rate, by the sum over the colors of the produced pair). Using the Lagrangian density given by (153) one finds that the only relevant contribution for pion-quark scattering comes from $\pi^- u \to \pi^0 d$ (contributions coming from $\pi^+ u \to \pi^+ u$ and $\pi^- u \to \pi^- u$ cancel). The corresponding differential cross section is [87] :

$$\left(\frac{d\sigma}{d\Omega}\right)_{\pi^- u \to \pi^0 d} \;=\; \frac{m^2}{2\pi^2 F_\pi^4}\frac{u^2(1-\cos\theta)^2}{[1+u(1-\cos\theta)]^3} \tag{144}$$

where θ is the angle between $\vec{p'}_\pi$ and $\vec{p_\pi}$ (see Figure (1)) and $u = \frac{\omega}{m}$.

The situation is a bit more involved for quark-antiquark pair production.

Three processes contribute: $\pi^- u \to dd\bar{d}$, $\pi^+ u \to uu\bar{d}$ and $\pi^- u \to du\bar{u}$. The last process can be broken into two non-interfering parts (to leading order in $1/N$). The first comes from diagrams of Figure (2) where the produced pair with momenta p_1 and p_2 consists of a d-quark and a $\bar{u}$-quark. The contribution from these diagrams to the sum rule (154) is exactly canceled by the corresponding contribution from the process $\pi^+ u \to uu\bar{d}$. The second part of the process $\pi^- u \to du\bar{u}$ comes from diagrams of Figure (2) where the produced pair is $u\bar{u}$. The corresponding contribution to g_A^2 is exactly equal to that of $\pi^- u \to dd\bar{d}$. Because of the complication coming from the three particle phase space we only quote the expression for the amplitude squared for $\pi^- u \to dd\bar{d}$:

$$| M |^2 \;=\; 2^9 \frac{m^6[(p-p\prime)\cdot p_\pi]^2}{F_\pi^6(p_1+p_2)^2(p\cdot p_\pi)(p\prime\cdot p_\pi)} + 2^9 \frac{m^4(p_1+p_2)^2}{F_\pi^6(p\prime - p)^2} \qquad (145)$$

Doing the three particle phase space integrals and making use of the Adler-Weisberger sum rule (154) we find a logarithmically divergent contribution to g_A^2. The final expression for g_A^2 up to first order in $1/N$ is:

$$g_A^2 \;=\; 1 - \frac{m^2}{2\pi^2 F_\pi^2} - 2N\left(\frac{m^2}{2\pi^2 F_\pi^2}\right)^2 \mathcal{I}. \qquad (146)$$

The second term in (157) comes from pion-quark elastic scattering and the last term comes from quark-antiquark pair production. $\mathcal{I}$ denotes a numerical integral evaluated from $4m$ to a cutoff Λ.

In the following table we summarize the dependence of $\mathcal{I}$ on different values of the cutoff Λ:

Λ (MeV)	$\mathcal{I}$
2000	0.022
2800	0.135
5000	0.634
10000	1.76
20000	3.42

As pointed out by Weinberg [87], the validity of the described estimate of the axial vector coupling of the constituent quark is based on the assumption that the integral over ω in (154) is dominated by energies less than or of the order of a typical N-independent QCD energy scale, such as the mass of the ρ-meson ($m_\rho \approx 770$ MeV). This condition is satisfied for the tree-level pion-quark scattering. On the other hand the contribution for the pair production process is logarithmically divergent and it obviously does not meet the above condition[5]. In particular the threshold for the pair production process is at $4m \approx 1400$ MeV which is not small compared to the above scale. Thus, although this process is formally of the same order in $1/N$ as the tree-level pion-quark elastic scattering, one could think that its inclusion would be problematic even if it did not diverge. In any case its contribution turns out to be small for any reasonable cutoff.

We have also checked how the second term in (157) changes if the mass of the pion is taken into account. Numerical calculation shows that the change in the final result is only three percent even if the mass of the pion is taken to be half of the constituent quark mass. Also, because the energy threshold is large the pion mass can be neglected in calculating $\mathcal{I}$.

[5]Because we work only to leading order in $1/N$ certain interference terms have been neglected in (157). An example is the diagrams of Figure (2) for $\pi^- u \rightarrow dd\bar{d}$ multiplied by those with the identical d-quarks interchanged. To check the numerical validity of the $1/N$ approximation, and to check that the neglect of these terms was not the cause of the divergence of the sum rule integral, we have calculated them for $\pi^- u \rightarrow dd\bar{d}$. They change the values of $\mathcal{I}$ given in the previous table by only about ten percent and do not improve the convergence of the integral.

5.3 The anomalous magnetic moment

Following the same logic we calculate the corrections of order $1/N$ to the magnetic moment of the constituent quark. Here we have to take into account the lowest order terms in the chiral Lagrangian density describing the electromagnetic interaction of quarks and pions. They are given by the following expression:

$$\begin{aligned} \Delta\mathcal{L} &= -\frac{1}{(1+\frac{\vec{\pi}^2}{F_\pi^2})^2}(eA^\mu(\vec{\pi}\times\partial_\mu\vec{\pi})_3 + \frac{e^2}{2}A_\mu A^\mu(\vec{\pi}^2-\pi_3^2)) \\ &+ieA_\mu\bar{\psi}\gamma^\mu[(\frac{(z_u+z_d)}{2}+(z_u-z_d)t_3)+\frac{2}{F_\pi^2}\frac{1}{1+\frac{\vec{\pi}^2}{F_\pi^2}}(\vec{\pi}^2 t_3-\vec{t}\cdot\vec{\pi}\pi_3) \\ &-\frac{2g_A}{F_\pi}\frac{1}{1+\frac{\vec{\pi}^2}{F_\pi^2}}\gamma_5(\vec{t}\times\vec{\pi})_3]\psi \end{aligned} \tag{147}$$

where A_μ denotes the photon field, $-e$ stands for the charge of the electron and $z_u e$ and $z_d e$ stand for the charges of the up and down quark respectively. Again we set $g_A = 1$ in the leading order in $1/N$.

In order to estimate the anomalous magnetic moment of the constituent quark, we use the analogue of the Drell-Hearn-Gerasimov sum rule [92] for photon-quark interaction, given by the following formula:

$$\kappa^2 = \frac{m^2}{2\pi^2\alpha z^2}\int_t^\infty \frac{d\omega}{\omega}[\sigma_P(\omega)-\sigma_A(\omega)] \tag{148}$$

where σ_P and σ_A represent the cross-sections for parallel and antiparallel photon and quark spins and ω denotes the incoming energy of the photon in the frame where the target quark is at rest. Also, ze denotes the charge of the target, $\alpha = \frac{e^2}{4\pi}$, t is the threshold for the relevant process and κ stands for the anomalous magnetic moment of the constituent quark.

Counting the powers of N is done as before. Only two types of processes make contributions of order $1/N$ to the cross-sections σ_P and σ_A and correspondingly of the same order to κ^2. These are: pion-photoproduction (Figure (3)) and quark-antiquark pair photoproduction (Figure (4)). The relevant calculations have to be performed while keeping the pion mass finite and then letting it to zero. Taking this fact into account,we obtain the following expression for the cross-sections coming from pion-photoproduction diagrams (with $\Delta\sigma = \sigma_P - \sigma_A$):

$$\begin{aligned}\Delta\sigma &= -\frac{m\alpha}{2F_\pi^2\omega}\{(z+z_d)[z(\frac{m_\pi^2}{m}-m-\omega)+z_d(\frac{m_\pi^2}{m}-3m-\omega)]\frac{1}{\omega}\ln\frac{A_+}{A_-} \\ &+\frac{z^2m_\pi^2}{m\omega}\ln\frac{A_+-m^2+m_\pi^2}{A_--m^2+m_\pi^2}+\frac{2}{m\omega}B[\frac{1}{m+2\omega}z_d(zm+z_d(2m-\frac{m_\pi^2}{m})) \\ &+(z+z_d)^2+z_d(z+z_d)-z_d^2\frac{(m+\omega)(m^2+m\omega-\frac{m_\pi^2}{2})}{m(m+2\omega)^2}]\}\end{aligned} \qquad (149)$$

where $z = 1 (z = 0)$ for the case where the charged(neutral) pion is in the final state (the up quark charge has been eliminated by $z_u = z + z_d$). Also $A_\pm = m(m+\omega) - \frac{m_\pi^2}{2} \pm B$ with $B = [(m\omega - \frac{m_\pi^2}{2})^2 - m_\pi^2 m^2]^{1/2}$. The pion mass is important only in the second term which contributes to the sum rule as $\frac{2\alpha z^2 m_\pi^2}{F_\pi^2}\int_{m_\pi}^{\infty}\frac{d\omega}{\omega^3}\ln\frac{2\omega}{m_\pi}$ as m_π goes to zero. This term, which is lost if one naively sets m_π to zero before calculating $\Delta\sigma$, gives a large contribution to κ^2 but is exactly canceled by the other terms[6]. Numerical evaluation of the integral in (159) gives $\kappa^2 = 0$ for both the charged and neutral pion, independent of the value of z_d.

Again, the contribution coming from quark-antiquark pair photoproduction is slightly more complicated, but in this case the sum rule integral is finite. The corresponding expressions for the magnetic moments of the constituent u

[6]We thank S. Drell for suggesting this to us.

and d quarks are:

$$\kappa_u^2 = \frac{2}{{z_u}^2} N \left(\frac{m^2}{2\pi^2 F_\pi^2} \right)^2 \mathcal{I}_1(z_u, z_d) \tag{150}$$

$$\kappa_d^2 = \frac{2}{{z_d}^2} N \left(\frac{m^2}{2\pi^2 F_\pi^2} \right)^2 \mathcal{I}_2(z_u, z_d) \tag{151}$$

where $\mathcal{I}_1, \mathcal{I}_2$ stand for numerical integrals whose values [7] are less than 0.001. Because the energy threshold is large the pion mass can be neglected in calculating $\mathcal{I}_1$ and $\mathcal{I}_2$. Of course the same objections as before can be raised to contributions from the pair production processes which obviously do not come from low energies but again, their contributions turn out to be numerically negligible[8].

Thus the $1/N$ corrections to κ^2 are essentially zero. Just by dimensional analysis we expect contributions to κ^2 of $O(1/N^2)$ to be of the following form:

$$\kappa_{u,d}{}^2 \sim \frac{1}{z_{u,d}^2} \left(\frac{m^2}{2\pi^2 F_\pi^2} \right)^2 . \tag{152}$$

5.4 Discussion

In order to discuss the meaning of the equations (157),(161) and (162) we argue that for consistency of our calculation the $N = 3$ limit should be taken after the axial vector coupling and the magnetic moments are obtained for the

[7] In fact as the number of sampling points in our Monte Carlo routine is increased the values of the above numerical integrals seem to converge to zero.

[8] The same comment would also apply to diagrams with possible four-quark vertices[94] which we have not considered.

nucleons in the framework of the large N constituent quark model[9].(In the large N limit of the naive quark model[93] the proton is built from $k+1$ up and k down quarks and analogously the neutron consists of k up and $k+1$ down quarks, where $N = 2k+1$). We also note that an extension of the usual power counting argument [94] indicates that the dominant contribution to both pion-nucleon and photon-nucleon interactions is the impulse approximation, in which the pion or photon interacts independently with each quark in the nucleon. All other processes are of higher order in the chiral expansion. On the other hand, all these processes are of the same order, N, in the $1/N$ expansion.

In the case of the axial vector coupling of the nucleon, we find in the nonrelativistic limit:

$$\begin{aligned}(g_A)_{nucleon} &= \frac{N+2}{3} g_A \\ &= \frac{1}{3}(N+2+N(g_A-1)) + O(1/N). \end{aligned} \tag{153}$$

Taking $\Lambda = 3500$ MeV for definiteness, which corresponds to the momenta of the incoming particles taken as m_ρ in the center of mass, $g_A = 0.87$ and $(g_A)_{nucleon} = 1.54$, which should be compared to the experimental value 1.25.

Analogously, we find the following expressions for the magnetic moments of the proton and neutron:

$$\mu_P = \frac{1}{6}(\mu_u(N+5) - \mu_d(N-1)) \tag{154}$$

$$\mu_N = \frac{1}{6}(\mu_d(N+5) - \mu_u(N-1)) \tag{155}$$

[9]If this limit is taken first the values for the magnetic moments of the nucleons are identical to the ones that follow.The value of the axial vector coupling, on the other hand, decreases a bit.

where $\mu_{u,d} = (1+\kappa_{u,d})\frac{z_{u,d}e}{2m}$ are the magnetic moments of the constituent up and down quarks. Using the usual quark charges in (165) and (166) we obtain:

$$\mu_P = \frac{e}{12m}(N+3+\frac{N}{3}(2\kappa_u+\kappa_d)) + O(1/\sqrt{N}) \qquad (156)$$

$$\mu_N = -\frac{e}{12m}(N+1+\frac{N}{3}(2\kappa_u+\kappa_d)) + O(1/\sqrt{N}). \qquad (157)$$

Since κ_u and κ_d are consistent with being zero the values for the magnetic moments are $\mu_P = 2.6\frac{e}{2m_N}$ and $\mu_N = -1.7\frac{e}{2m_N}$, where m_N is the nucleon mass. These values should be compared with the experimental results, $\mu_P = 2.79\frac{e}{2m_N}$ and $\mu_N = -1.91\frac{e}{2m_N}$. Alternatively one could use the values of the quark charges obtained from the quark model under the requirement that the nucleons have their usual charges for any N. Explicitly, $z_u = \frac{N+1}{2N}$ and $z_d = \frac{1-N}{2N}$ which in the large N limit goes to $z_u = \frac{1}{2}$ and $z_d = -\frac{1}{2}$. Then the corresponding values would be $\mu_P = 2.2\frac{e}{2m_N}$ and $\mu_N = -\mu_P$.

The value obtained for the axial vector coupling agrees within 20% with the experimental value. Perhaps the agreement could be improved by taking into account relativistic corrections [95]. Concerning the anomalous magnetic moment, we have shown explicitly that the potentially dangerous $O(1/\sqrt{N})$ corrections are negligible. We consider these results to be evidence that the constituent quark model can be understood in the large N limit in terms of chiral symmetry (in the form of the chiral quark model) and reasonable assumptions on the high-energy behavior of amplitudes (embodied in sum rules).

Figure Captions

Figure (1) : Feynman diagrams for pion-quark scattering that contribute to the Adler-Weisberger sum rule.

Figure (2) : Quark-antiquark pair production diagrams that contribute to the Adler-Weisberger sum rule to the same order in $1/N$ as those of Figure (1).

Figure (3) : Feynman diagrams for pion photoproduction that contribute to the Drell-Hearn-Gerasimov sum rule.

Figure (4) : Quark-antiquark pair photoproduction diagrams that contribute to the Drell-Hearn-Gerasimov sum rule to the same order in $1/N$ as those of Figure (3).

Figure 1

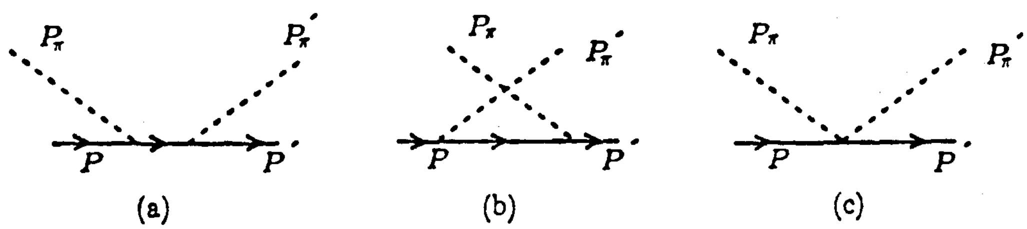


Figure 2

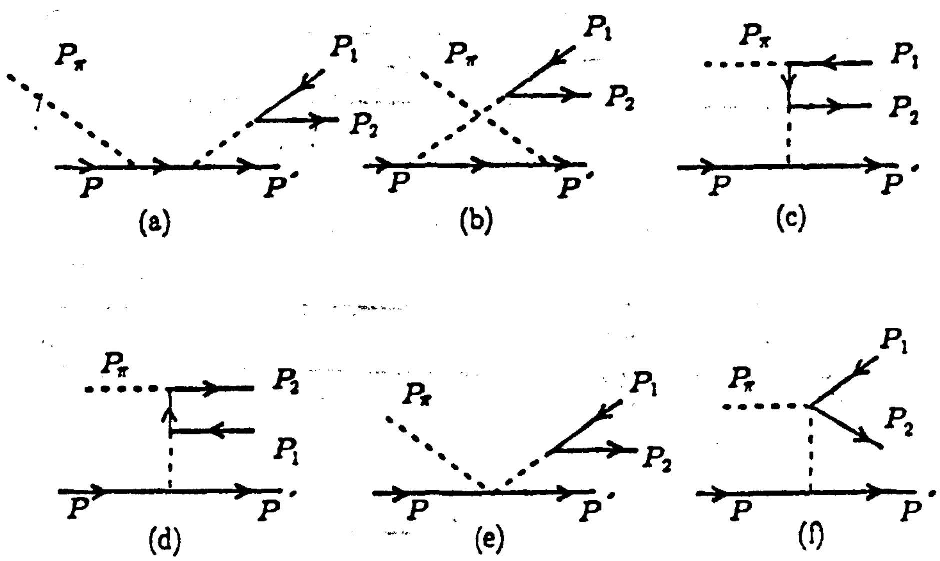


Figure 3

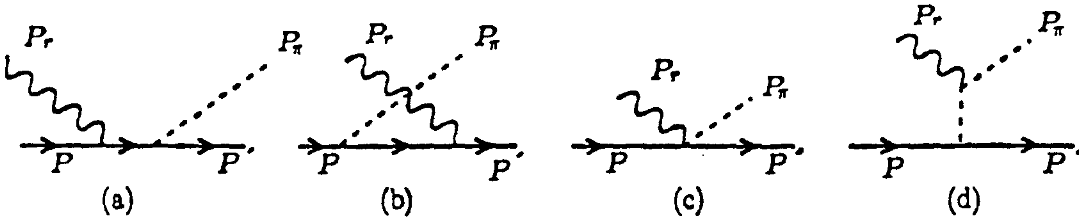
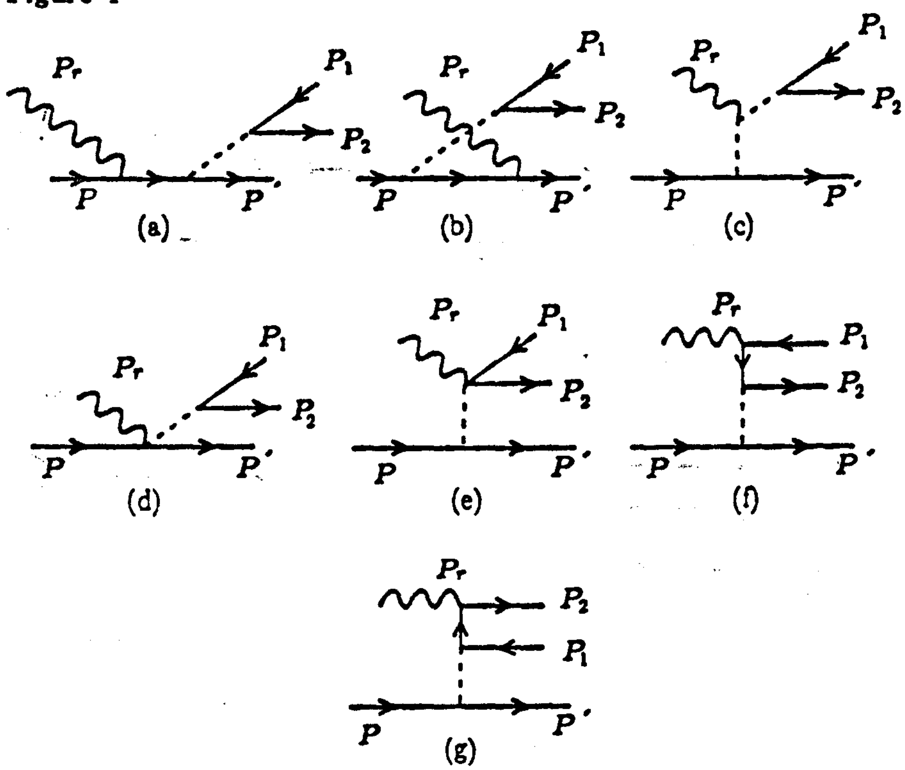


Figure 4



Appendices

Appendix A

Pions are (pseudo)Goldstone bosons of the spontaneous breaking $SO(4) \to SO(3)$. They are associate with the broken generators of $SO(4)$ and therefore live in the sphere $SO(4)/SO(3) \sim S^3$. If we embed it in the euclidean E^4 space, $SO(4)$ transformations can be viewed as rotations of S^3 in E^4 planes. For example, $SU(2)_V$ of isospin consists of rotations in planes orthogonal to the fourth axis, while axial $SU(2)_A$ are rotations through planes that contain the fourth axis.

The sphere can be parametrized any way you want, say with four cartesian coordinates $\{\boldsymbol{\varphi}, \varphi_4 \equiv \sigma\}$ subject to the constraint,

$$\sigma^2 + \boldsymbol{\varphi}^2 = F_\pi^2. \tag{158}$$

It is more convenient, however, to work with three unconstrained coordinates. We use stereographic coordinates

$$\boldsymbol{\pi} \equiv \frac{\boldsymbol{\varphi}}{1 + \frac{\sigma}{F_\pi}}. \tag{159}$$

Under an $SU(2)_V$ transformation with parameter $\boldsymbol{\varepsilon}$, they just rotate

$$\delta\boldsymbol{\pi} = \boldsymbol{\varepsilon} \times \boldsymbol{\pi}, \tag{160}$$

but they transform non-linearly under $SU(2)_A$ with parameter $\tilde{\boldsymbol{\varepsilon}}$

$$\delta\boldsymbol{\pi} = F_\pi \left(1 - \frac{\boldsymbol{\pi}^2}{F_\pi^2}\right) \frac{\tilde{\boldsymbol{\varepsilon}}}{2} + \frac{1}{F_\pi} \tilde{\boldsymbol{\varepsilon}} \cdot \boldsymbol{\pi}\boldsymbol{\pi} \tag{161}$$

A covariant derivative (12) can be constructed, which is an isospin 1 object,

$$\delta \boldsymbol{D}_\mu = \boldsymbol{\varepsilon} \times \boldsymbol{D}_\mu, \tag{162}$$

and transforms under axial rotations as if under $SU(2)_V$ with a field-dependent parameter,

$$\delta \boldsymbol{D}_\mu = (\tilde{\boldsymbol{\varepsilon}} \times \frac{\boldsymbol{\pi}}{F_\pi}) \times \boldsymbol{D}_\mu. \tag{163}$$

Fermions also transform linearly under the unbroken subgroup

$$\delta N = i\boldsymbol{\varepsilon} \cdot \boldsymbol{t} N \tag{164}$$

$$\delta \Delta = i\boldsymbol{\varepsilon} \cdot \boldsymbol{t}^{(3/2)} \Delta. \tag{165}$$

In this case, too, it is simplest to work with fields that realize the whole group non-linearly, i.e., that transform under axial transformations as if under isospin with the same field-dependent parameter as in (163),

$$\delta N = i(\tilde{\boldsymbol{\varepsilon}} \times \frac{\boldsymbol{\pi}}{F_\pi}) \cdot \boldsymbol{t} N \tag{166}$$

$$\delta \Delta = i(\tilde{\boldsymbol{\varepsilon}} \times \frac{\boldsymbol{\pi}}{F_\pi}) \cdot \boldsymbol{t}^{(3/2)} \Delta \tag{167}$$

One can easily verify that the covariant derivatives of the pion (15), nucleon (16) and isobar (17) are indeed covariant, that is, transform under $SU(2) \times SU(2)$ in the same way the fields $\boldsymbol{D}_\mu$, N and Δ do ((162)—(167)).

As a consequence, an isoscalar built out of $\boldsymbol{D}_\mu$, N, Δ and their covariant derivatives will automatically be invariant under whole $SU(2) \times SU(2)$. On the other hand, objects that transform under the full group as tensors involve also the field $\boldsymbol{\pi}$ itself. For example, an $SO(4)$ vector can be constructed as

$$\left(\frac{2\frac{\boldsymbol{\pi}}{F_\pi}}{1 + \frac{\boldsymbol{\pi}^2}{F_\pi^2}}, \frac{1 - \frac{\boldsymbol{\pi}^2}{F_\pi^2}}{1 + \frac{\boldsymbol{\pi}^2}{F_\pi^2}} \right), \tag{168}$$

and its fourth component gives rise to the pion mass term in (19).

Appendix B

Here we list the relations between the A_i's, C_i's of (25) and the A_i''s, and C_i''s of (21):

$$
\begin{aligned}
A_1 &= -(A_1' - \frac{1}{2}A_2') \\
A_2 &= -(A_1' + \frac{1}{2}A_2') \\
C_1 &= -C_1' + C_3' - \frac{1}{2}C_2' \\
C_2 &= 4(-C_1' + C_3' + \frac{1}{2}C_2') \\
C_3 &= -C_9' - \frac{1}{2}(C_{12}' + C_{14}') \\
C_4 &= 4(-C_9' + \frac{1}{2}(C_{12}' + C_{14}')) \\
C_5 &= -(2C_4' + C_5' - C_6') \\
C_6 &= -(C_7' + C_8' + \frac{1}{2}C_{10}' - C_{11}' - C_{13}') \\
C_7 &= -4(C_7' + C_8' - \frac{1}{2}C_{10}' + C_{11}' + C_{13}').
\end{aligned}
$$

Appendix C

In order to obtain a potential in coordinate space we take Fourier transforms with a gaussian cut-off of parameter Λ (see [43, 44] for details). With

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty dt e^{-t^2}$$

denoting the error function, we need the following functions

$$\begin{aligned}
I_0(r) &= \frac{1}{8\pi\sqrt{\pi}} \Lambda^3 e^{-(\frac{r\Lambda}{2})^2} \\
I_2(r, m_\pi) &= \frac{1}{8\pi r} e^{(\frac{m_\pi}{\Lambda})^2} \left[e^{-m_\pi r} \mathrm{erfc}\left(-\frac{\Lambda r}{2} + \frac{m_\pi}{\Lambda}\right) - e^{m_\pi r} \mathrm{erfc}\left(\frac{\Lambda r}{2} + \frac{m_\pi}{2}\right)\right] \\
I_3(r, m_\pi) &= \frac{1}{8\pi r^3} e^{(\frac{m_\pi}{\Lambda})^2} \left[\left(1 + m_\pi r + \frac{1}{3}(m_\pi r)^2\right) e^{-m_\pi r} \mathrm{erfc}\left(-\frac{\Lambda r}{2} + \frac{m_\pi}{\Lambda}\right) \right. \\
&\qquad \left. - \left(1 - m_\pi r + \frac{1}{3}(m_\pi r)^2\right) e^{m_\pi r} \mathrm{erfc}\left(\frac{\Lambda r}{2} + \frac{m_\pi}{\Lambda}\right) \right] \\
&\quad - \frac{1}{3}\left(1 + \frac{6}{\Lambda^2 r^2}\right) I_0(r) \\
I_4(r, m_\pi) &= m_\pi^2 I_2(r, m_\pi) - I_0(r) \\
I_5(r, m_\pi) &= m_\pi^2 I_4(r, m_\pi) + \Lambda^2 \left(\frac{3}{2} - \left(\frac{\Lambda r}{2}\right)^2\right) I_0(r) \\
I_6(r, m_\pi) &= m_\pi^2 I_3(r, m_\pi) - \frac{1}{3}\Lambda^2 \left(\frac{\Lambda r}{2}\right)^2 I_0(r) \\
G_n(\lambda, r) &= e^{-\frac{\lambda^2}{\Lambda^2}} I_n(r, \sqrt{m_\pi^2 + \lambda^2}) \\
F_n(\lambda, r) &= I_n(r, m_\pi) - G_n(\lambda, r)
\end{aligned}$$

plus the integrals

$$\begin{aligned}
A_\Delta^{(n,m)}(f) &= \frac{2}{\pi} \int_0^\infty d\lambda \frac{\lambda^{2m}}{(\lambda^2 + \Delta^2)^n} f(\lambda) \\
B(r) &= \Delta A_\Delta^{(1,0)}(G) \\
C(r) &= \frac{1}{\Delta}(I_2(r) - B(r))
\end{aligned}$$

where f is any function of λ, and $\Delta = m_\Delta - m_N$.

Appendix D

Here we give the explicit forms of the 60 functions $V_p^i(r)$, $p = 1, \ldots, 20$; $i = 0, 1, 2$, that appear in the coordinate space version of the potential.

In order not to clutter the formulas too much, we define the following combinations of functions and their derivatives:

$$
\begin{aligned}
d_1(f) &\equiv \frac{f'}{r}\left(2f'' + \frac{1}{r}f'\right) \\
d_2(f) &\equiv \frac{f'}{r}\left(f'' - \frac{1}{r}f'\right) \\
e_1(f) &\equiv f' + \frac{2}{r}f \\
e_2(f) &\equiv f' - \frac{1}{r}f \\
(f, g) &\equiv f''g'' + \frac{2}{r^2}f'g' \\
[f, g] &\equiv \frac{1}{r}(f''g' + f'g'') + \frac{1}{r^2}f'g' \\
\{f, g\} &\equiv \frac{2}{r^2}f'g' - \frac{1}{r}(f'g'' + f''g') \\
< f, g > &\equiv \left(-\frac{4}{r}f' - 2f'' + 2m_\pi^2 f - I_0\right) g
\end{aligned}
$$

where $f = f(r)$ and $g = g(r)$ are any of the functions defined in Appendix C, and a prime denotes differentiation with respect to r.

We then have:

$$
\begin{aligned}
V_1^0 = & \frac{1}{F_\pi^4}\{-3g_A^4 A_0^{(1,0)}((I_2, F_2)) + 6g_A^2[B_1(I_2, I_2) + B_3 m_\pi^2 I_2'^2] \\
& + \frac{2g_A^2 h_A^2}{3}\left[\Delta(C, C) - 4(I_2, C) - \frac{1}{\Delta}(I_2 + B_1, I_2 + B)\right] \\
& - \frac{16}{27}h_A^4 \Delta^2 A_\Delta^{(2,0)}((G_2, G_2)) \\
& + \frac{8}{3}h_A^2 \Delta[B_1(A_\Delta^{(1,0)}((G_2, G_2)) + A_\Delta^{(1,1)}(G_2'^2)) + B_3 m_\pi^2 A_\Delta^{(1,0)}(G_2'^2)]
\end{aligned}
$$

$$+C_S^{(0)} I_0 - (C_1^{(0)} + \frac{1}{4} C_2^{(0)}) e_1(I_0')\}$$

$$V_1^1 = -C_2^{(0)} e_1(I_0)$$

$$V_1^2 = -C_2^{(0)} I_0$$

$$\begin{aligned}
V_2^0 = & \frac{1}{F_\pi^4} \Big\{ -2g_A^4 A_0^{(1,1)}((G_2, F_2)) - A_0^{(0,0)}(< G_2, G_2 >) + 4g_A^2 A_0^{(0,0)}(G_2'^{\,2}) \\
& + \frac{2}{9} g_A^2 h_A^2 \Big[-3\Delta(C,C) + 4(I_2, C) + \frac{4}{\Delta}(B,B) - 4A_\Delta^{(1,0)}((G_2, G_2)) \\
& \qquad - \frac{3}{\Delta}(I_2 + B, I_2 + B) \Big] \\
& + \frac{8}{81} h_A^4 \Delta^2 A_\Delta^{(2,0)}((G_2, G_2)) - \frac{8}{9} h_A^2 A_\Delta^{(1,1)}(G_2'^{\,2}) \Big\} \\
& + \frac{1}{4} C_S^{(1)} I_0 - \frac{1}{4}(C_1^{(1)} + \frac{1}{4} C_2^{(1)}) e_1(I_0')
\end{aligned}$$

$$V_2^1 = -\frac{1}{4} C_2^{(1)} e_1(I_0)$$

$$V_2^2 = -\frac{1}{4} C_2^{(1)} I_0$$

$$\begin{aligned}
V_3^0 = & \frac{1}{F_\pi^4} \Big\{ -2g_A^4 A_0^{(1,0)}([G_2, F_2]) \\
& + \frac{2}{9} g_A^2 h_A^2 \Big(-3\Delta d_1(C) + 4[I_2, C] + \frac{4}{\Delta} d_1(B) - 4A_\Delta^{(1,0)}(d_1(G_2)) \\
& \qquad - \frac{1}{\Delta} d_1(I_2 + B) \Big) \\
& + \frac{8}{81} h_A^4 A_\Delta^{(2,1)}(d_1(G_2)) \Big\} \\
& + C_T^{(0)} - \left(C_3^{(0)} + \frac{1}{4} C_4^{(0)} + \frac{1}{3} C_6^{(0)} + \frac{1}{12} C_7^{(0)} \right) e_1(I_0')
\end{aligned}$$

$$V_3^1 = -(C_4^{(0)} + \frac{1}{3} C_7^{(0)}) e_1(I_0)$$

$$V_3^2 = -(C_4^{(0)} + \frac{1}{3} C_7^{(0)}) I_0$$

$$\begin{aligned}
V_4^0 = & \frac{1}{F_\pi^4} \Big\{ -\frac{4}{3} g_A^4 A_0^{(1,0)}([I_2, F_2]) - \frac{4}{3} g_A^2 B_2 d_1(I_2) \\
& + \frac{2}{27} g_A^2 h_A^2 \Big[\Delta d_1(C) - 4[I_2, C] - \frac{1}{\Delta} d_1(I_2 + B) \Big] \\
& - \frac{4}{243} h_A^4 \Delta^2 A_\Delta^{(2,0)}(d_1(G_2))
\end{aligned}$$

$$
\begin{aligned}
& \qquad - h_A^2 B_2 \Delta A_\Delta^{(1,0)}(d_1(G_2))\Big\} \\
& + \frac{g_A}{3F_\pi^2}\left\{ g_A I_4 - \frac{1}{2}\frac{A_1}{m_\pi^2} I_5 - \frac{A_2}{8m_\pi^2} e_1(I_4') \right. \\
& \qquad \left. + \frac{g_A}{4} A_0^{(1,0)} \left(48 E F_4 + \frac{1}{m_N}(12 F_5 + e_1(F_4')) \right) \right\} \\
& + \frac{1}{4} C_T^{(1)} I_0 - \frac{1}{4}\left(C_3^{(1)} + \frac{1}{4} C_4^{(1)} + \frac{1}{3} C_6^{(1)} + \frac{1}{12} C_7^{(0)} \right) e_1(I_0') \\
V_4^1 &= \frac{g_A}{6F_\pi^2}\left\{ -\frac{A_2}{m_\pi^2} e_1(I_4) + \frac{2g_A}{m_N} A_0^{(1,0)}(e_1(F_4)) \right\} \\
& - \frac{1}{4}(C_4^{(1)} + \frac{1}{3} C_7^{(1)}) e_1(I_0) \\
V_4^2 &= \frac{g_A}{6F_\pi^2}\left\{ -\frac{A_2}{m_\pi^2} I_4 + \frac{2g_A}{m_N} A_0^{(1,0)}(F_4) \right\} \\
& - \frac{1}{4}(C_4^{(1)} + \frac{1}{3} C_7^{(1)}) I_0 \\
V_5^0 &= \frac{1}{F_\pi^4}\{ - g_A^4 A_0^{(1,0)}(\{G_2, F_2\}) \\
& \qquad + \frac{2}{9} g_A^2 h_A^2 (-3\Delta d_2(G) + 2\{I_2, C\} + \frac{4}{\Delta} d_2(B) - 4 A_\Delta^{(1,0)}(d_2(G_2)) \\
& \qquad\qquad - \frac{1}{\Delta} d_2(I_2 + B)) \\
& \qquad + \frac{8}{81} h_A^4 A_\Delta^{(2,1)}(d_2(G_2)) \} \\
& - \frac{1}{3}(C_6^{(0)} + \frac{1}{4} C_7^{(0)}) e_2(I_0') \\
V_5^1 &= -\frac{1}{3} C_7^{(0)} e_2(I_0) \\
V_5^2 &= -\frac{1}{3} C_7^{(0)} I_0 \\
V_6^0 &= \frac{1}{F_\pi^4}\{ -\frac{2}{3} g_A^4 A_0^{(1,0)}(\{G_2, F_2\}) + \frac{4}{3} g_A^2 B_2 d_2(I_2) \\
& \qquad + \frac{2}{27} g_A^2 h_A^2 \left(\Delta d_2(C) - 2\{I_2, C\} - \frac{1}{\Delta} d_2(I_2 + B) \right) \\
& \qquad - \frac{4}{243} h_A^4 \Delta^2 A_\Delta^{(2,0)}(d_2(G_2)) - \frac{8}{27} h_A^2 B_2 \Delta A_\Delta^{(1,0)}(d_2(G_2)) \Big\} \\
& + \frac{g_A}{F_\pi^2}\{ g_A I_3 - \frac{1}{2}\frac{A_1}{m_\pi^2} I_6 - \frac{A_2}{8m_\pi^2}\left(e_1(I_3') - \frac{6}{r^2} I_3 \right)
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{g_A}{4}A_0^{(1,0)}\left(16EG_3+\frac{1}{m_N}(4F_6+e_1(G_3')-\frac{6}{r^2})\right)\ \} \\
& -\frac{1}{12}(C_6^{(1)}+\frac{1}{4}C_7^{(1)})e_2(I_0') \\
V_6^1 &= \frac{g_A}{2F_\pi^2}\left\{-\frac{A_2}{m_\pi^2}(e_1(I_3)+\frac{2}{r}I_3)+\frac{2g_A}{m_N}A_0^{(1,0)}(e_1(G_3)+\frac{2}{r}G_3)\right\} \\
& -\frac{1}{12}C_7^{(1)}e_1(I_0) \\
V_6^2 &= \frac{g_A}{F_\pi^2}\left\{-\frac{A_2}{m_\pi^2}I_3+2g_AA_0^{(1,0)}(G_3)\right\} \\
& -\frac{1}{12}C_7^{(1)}I_0 \\
V_7^0 &= \frac{1}{2r}\left[-C_5^{(0)}I_0'+\frac{1}{3}C_7^{(0)}e_2(I_0)\right] \\
V_7^1 &= \frac{1}{3r}C_7^{(0)}I_0 \\
V_7^2 &= 0 \\
V_8^0 &= \frac{1}{8r}\left[-C_5^{(1)}I_0'+\frac{1}{3}C_7^{(1)}e_2(I_0)\right] \\
V_8^1 &= \frac{1}{12r}C_7^{(1)}I_0 \\
V_8^2 &= 0 \\
V_9^0 &= C_2^{(0)}\frac{I_0}{r^2} \\
V_9^1 &= V_9^2=0 \\
V_{10}^0 &= \frac{1}{4}C_2^{(1)}\frac{I_0}{r^2} \\
V_{10}^1 &= V_{10}^2=0 \\
V_{11}^0 &= (C_4^{(0)}-\frac{1}{3}C_7^{(0)})\frac{I_0}{r^2} \\
V_{11}^1 &= V_{11}^2=0 \\
V_{12}^0 &= \frac{g_A}{6F_\pi^2}\left(\frac{A_2}{m_\pi^2}I_4-\frac{2g_A}{m_N}A_0^{(1,0)}(F_4)\right)+\frac{1}{4}\left(C_4^{(1)}-\frac{1}{3}C_7^{(1)}\right)\frac{I_0}{r^2} \\
V_{12}^1 &= V_{12}^2=0
\end{aligned}
$$

$$
\begin{aligned}
V_{13}^{0} &= -\frac{2}{3}C_7^{(0)}\frac{I_0}{r^2} \\
V_{13}^{1} &= V_{13}^{2} = 0 \\
V_{14}^{0} &= -\frac{1}{6}C_7^{(1)}\frac{I_0}{r^2} \\
V_{14}^{1} &= V_{14}^{2} = 0 \\
V_{15}^{0} &= \frac{1}{6}C_7^{(0)}\frac{I_0'}{r} \\
V_{15}^{1} &= \frac{1}{3}C_7^{(0)}\frac{I_0}{r} \\
V_{15}^{2} &= 0 \\
V_{16}^{0} &= \frac{g_A}{F_\pi^2}\frac{1}{r^2}\left(\frac{A_2}{m_\pi^2}I_3 - \frac{2g_A}{m_N}A_0^{(1,0)}(G_3)\right) + \frac{1}{24}C_7^{(1)}\frac{I_0}{r} \\
V_{16}^{1} &= \frac{1}{12}C_7^{(1)}\frac{I_0'}{r} \\
V_{16}^{2} &= 0 \\
V_{17}^{0} &= \frac{1}{3}C_7^{(0)}\frac{I_0}{r^2} \\
V_{17}^{1} &= V_{17}^{2} = 0 \\
V_{18}^{0} &= \frac{g_A}{F_\pi^2}\frac{1}{r^2}\left(\frac{A_2}{m_\pi^2}I_3 - \frac{2g_A}{m_N}A_0^{(1,0)}(G_3)\right) + \frac{1}{12}C_7^{(1)}\frac{I_0}{r^2} \\
V_{18}^{1} &= V_{18}^{2} = 0 \\
V_{19}^{0} &= -\frac{2}{3}C_7^{(0)}\frac{I_0}{r^2} \\
V_{19}^{1} &= V_{19}^{2} = 0 \\
V_{20}^{0} &= -\frac{1}{6}C_7^{(1)}\frac{I_0}{r^2} \\
V_{20}^{1} &= V_{20}^{2} = 0.
\end{aligned}
$$

Appendix E

Here I present the potential (65) in coordinate space. Denoting

$$\begin{aligned}\vec{r}_{ij} &= \vec{r}_i - \vec{r}_j \ , \ \hat{r}_{ij} = \frac{\vec{r}_{ij}}{r_{ij}} \\ \cos\theta_j &\equiv \hat{r}_{ij}\cdot\hat{r}_{jk}\end{aligned}$$

$$S_{nm}(\hat{r}_1,\hat{r}_2) \equiv 3\vec{\sigma}_n\cdot\hat{r}_1\vec{\sigma}_m\cdot\hat{r}_2 - \hat{r}_1\cdot\hat{r}_2\vec{\sigma}_n\cdot\vec{\sigma}_n$$

we find

$$\begin{aligned}V_3^{(3)}(\vec{r}_{ij},\vec{r}_{jk}) &= \boldsymbol{t}_i\cdot\boldsymbol{t}_k\Big\{V_S(\vec{r}_{ij},\vec{r}_{jk}) + \vec{\sigma}_i\cdot\vec{\sigma}_k V_\sigma^1(\vec{r}_{ij},\vec{r}_{jk}) \\ &\quad + S_{ik}(\hat{r}_{ij},\hat{r}_{ij})V_S^1(\vec{r}_{ij},\vec{r}_{jk}) + S_{ik}(\hat{r}_{jk},\hat{r}_{jk})V_S^1(\vec{r}_{jk},\vec{r}_{ij}) \\ &\quad + S_{ik}(\hat{r}_{ij},\hat{r}_{jk})V_{SS}^1(\vec{r}_{ij},\vec{r}_{jk})\ \Big\} \\ &\quad + i\boldsymbol{t}_j\cdot(\boldsymbol{t}_i\times\boldsymbol{t}_k)\Big\{i\vec{\sigma}_j\cdot(\vec{\sigma}_i\times\vec{\sigma}_k)V_\sigma^2(\vec{r}_{ij},\vec{r}_{jk}) \\ &\quad + [S_{ij}(\hat{r}_{ij},\hat{r}_{ij}),\vec{\sigma}_j\cdot\vec{\sigma}_k]V_S^2(\vec{r}_{ij},\vec{r}_{jk}) \\ &\quad - [S_{jk}(\hat{r}_{jk},\hat{r}_{jk}),\vec{\sigma}_i\cdot\vec{\sigma}_j]V_S^2(\vec{r}_{jk},\vec{r}_{ij}) \\ &\quad + [S_{ij}(\hat{r}_{ij},\hat{r}_{ij}),S_{jk}(\hat{r}_{jk},\hat{r}_{jk})]V_{SS}^2(\vec{r}_{ij},\vec{r}_{jk})\ \Big\}\end{aligned}$$

with

$$\begin{aligned}V_S(\vec{r}_{ij},\vec{r}_{jk}) &= E_1F_{-2}(r_{ij})F_{-2}(r_{jk}) \\ V_\sigma^1(\vec{r}_{ij},\vec{r}_{jk}) &= \frac{2}{9}\left(\frac{2g_A}{F_\pi^2}\right)^2\Big\{B_1\left(m_\pi^4F_0(r_{ij})F_0(r_{jk})\right. \\ &\quad \left. + (3\cos^2\theta_j - 1)F_2(r_{ij})F_2(r_{jk})\ \right) \\ &\quad - 3B_3m_\pi^2\cos\theta_jF_1(r_{ij})F_1(r_{jk})\ \Big\}\end{aligned}$$

$$
\begin{aligned}
& +\frac{2g_A}{3F_\pi^2} D_1' m_\pi^2 (F_0(r_{ij})F_{-2}(r_{jk}) + F_{-2}(r_{ij})F_0(r_{jk})) \\
& +E_2' F_{-2}(r_{ij})F_{-2}(r_{jk}) \\
V_S^1(\vec{r}_{ij},\vec{r}_{jk}) &= \frac{2}{9}\left(\frac{2g_A}{F_\pi^2}\right)^2 B_1 F_2(r_{ij})(m_\pi^2 F_0(r_{jk}) - F_2(r_{jk})) \\
& +\frac{2}{3}\frac{g_A}{F_\pi^2} D_1' F_2(r_{ij})F_{-2}(r_{jk}) \\
V_{SS}^1(\vec{r}_{ij},\vec{r}_{jk}) &= \frac{2}{3}\left(\frac{2g_A}{F_\pi^2}\right)^2 \left\{B_1 \cos\theta_j F_2(r_{ij})F_2(r_{jk}) - B_3 m_\pi^2 F_1(r_{ij})F_1(r_{jk})\right\} \\
V_\sigma^2(\vec{r}_{ij},\vec{r}_{jk}) &= \frac{2}{9}\left(\frac{2g_A}{F_\pi^2}\right)^2 B_2 m_\pi^4 F_0(r_{ij})F_0(r_{jk}) \\
& -\frac{2}{3}\frac{g_A}{F_\pi^2} D_2' m_\pi^2 (F_0(r_{ij})F_{-2}(r_{jk}) + F_{-2}(r_{ij})F_0(r_{jk})) \\
& -E_3' F_{-2}(r_{ij})F_{-2}(r_{jk}) \\
V_S^2(\vec{r}_{ij},\vec{r}_{jk}) &= \frac{2}{9}\left(\frac{2g_A}{F_\pi^2}\right)^2 B_2 m_\pi^2 F_2(r_{ij})F_0(r_{jk}) \\
& -\frac{2g_A}{3F_\pi^2} D_2' F_2(r_{ij})F_{-2}(r_{jk}) \\
V_{SS}^2(\vec{r}_{ij},\vec{r}_{jk}) &= \frac{2}{9}\left(\frac{2g_A}{F_\pi^2}\right)^2 B_2 F_2(r_{ij})F_2(r_{jk})
\end{aligned}
$$

Here

$$
\begin{aligned}
D_1' &\equiv D_1 - \frac{4g_A B_1}{3F_\pi^2} \\
D_2' &\equiv D_2 + \frac{4g_A B_2}{3F_\pi^2} \\
E_2' &\equiv E_2 - \frac{4}{3}\frac{g_A}{F_\pi^2}\left(D_1 - \frac{2}{3}\frac{g_A B_1}{F_\pi^2}\right) \\
E_3' &\equiv E_3 - \frac{4}{3}\frac{g_A}{F_\pi^2}\left(D_2 + \frac{2}{3}\frac{g_A B_2}{F_\pi^2}\right)
\end{aligned}
$$

(for Δ

$$
D_1' \longrightarrow -\frac{4}{9}\frac{h_A}{m_\Delta - m_N}\left(D_T - \frac{4}{3}\frac{g_A h_A}{F_\pi^2}\right)
$$

$$
\begin{aligned}
D_2' &\longrightarrow \frac{2}{9}\frac{h_A}{m_\Delta - m_N}\left(D_T - \frac{4}{3}\frac{g_A h_A}{F_\pi^2}\right) \\
E_2' &\longrightarrow \frac{1}{9(m_\Delta - m_N)}\left(D_T^2 + \frac{16}{3}\frac{g_A h_A}{F_\pi^2} - \frac{32}{9}\left(\frac{g_A h_A}{F_\pi^2}\right)^2\right) \\
E_3' &\longrightarrow -\frac{1}{18(m_\Delta - m_N)}\left(D_T^2 + \frac{16}{3}\frac{g_A h_A}{F_\pi^2} - \frac{32}{9}\left(\frac{g_A h_A}{F_\pi^2}\right)^2\right) \qquad)
\end{aligned}
$$

and the F's are Fourier-transforms with a cut-off function $F(\vec{q}^{\,2};\Lambda)$ of parameter $\Lambda < M$:

$$
\begin{aligned}
F_{-2}(r) &= \int \frac{d^3q}{(2\pi)^3} \exp(i\vec{q}\cdot\vec{r})\, F(\vec{q}^{\,2};\Lambda) \\
F_0(r) &= \int \frac{d^3q}{(2\pi)^3} \frac{\exp(i\vec{q}\cdot\vec{r}) F(\vec{q}^{\,2};\Lambda)}{\vec{q}^{\,2}+m_\pi^2} \\
F_1(r) &= F_0'(r) \\
F_2(r) &= F_0''(r) - \frac{1}{r}F_0'(r)
\end{aligned}
$$

If we define

$$
\begin{aligned}
a_0(r) &\equiv 1 \\
a_1(r) &\equiv 1 + \frac{1}{m_\pi r} \\
a_2(r) &\equiv 1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}
\end{aligned}
$$

then when the cut-off is removed

$$
\begin{aligned}
F_{-2}(r) &\xrightarrow{\Lambda\to\infty} \delta(r) \\
F_{n\geq 0}(r) &\xrightarrow{\Lambda\to\infty} \frac{(-m_\pi)^n}{4\pi r} \exp(-m_\pi) a_n(r)
\end{aligned}
$$

The cut-off function is the same used in the NN potential [59, 60],

$$
F(\vec{q}^{\,2};\Lambda) = \exp(-\vec{q}^{\,2}/\Lambda^2)
$$

in which case

$$
\begin{aligned}
F_{-2}(r) &= \left(\frac{\Lambda}{4\pi}\right)^3 \frac{\sqrt{\pi}}{2\Lambda r} g(r) \\
F_{n\geq 0}(x) &= \frac{m_\pi^n}{8\pi r}\left[a_n(r)f(r) - (-1)^n a_n(-r)f(-r) - b_n(r)g(r)\right]
\end{aligned}
$$

where

$$
\begin{aligned}
g(r) &\equiv \frac{2\Lambda r}{\sqrt{\pi}} \exp\left(-\frac{\Lambda^2 r^2}{4}\right) \\
f(r) &\equiv \exp\left(-m_\pi r + \frac{m_\pi^2}{\Lambda^2}\right)\mathrm{erfc}\left(-\frac{\Lambda r}{2} + \frac{m_\pi}{\Lambda}\right) \\
\mathrm{erfc}(x) &= \frac{2}{\sqrt{\pi}} \int_x^\infty dt \exp(-t^2)
\end{aligned}
$$

and

$$\begin{aligned} b_0(r) &= 0 \\ b_1(r) &= \frac{1}{m_\pi r} \\ b_2(r) &= \frac{3}{(m_\pi r)^2}\left(1 + \frac{1}{6}\Lambda^2 r^2\right). \end{aligned}$$

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