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Soft separation axioms in soft topological spaces

Sabir Hussain * and Bashir Ahmad †

Abstract

Shabir et. al [27] and D. N. Georgiou et. al [7], defined and studied some soft separation axioms, soft θ-continuity and soft connectedness in soft spaces using (ordinary) points of a topological space X. In this paper, we redefine and explore several properties of soft T_i, $i = 0, 1, 2$, soft regular, soft T_3, soft normal and soft T_4 axioms using soft points defined by I. Zorlutuna [30]. We also discuss some soft invariance properties namely soft topological property and soft hereditary property. We hope that these results will be useful for the future study on soft topology to carry out general framework for the practical applications and to solve the complicated problems containing uncertainties in economics, engineering, medical, environment and in general man-machine systems of various types.

Keywords: Soft topology, Soft open(closed) sets, Soft interior(closure), Soft T_i; ($i = 0, 1, 2, 3, 4$) spaces, Soft regular spaces, Soft normal spaces and Invariance properties.

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1. Introduction

In 1999, Molodtsov [22] initiated the theory of soft sets as a new mathematical tool to deal with uncertainties while modelling the problems with incomplete information in engineering, physics, computer science, economics, social sciences and medical sciences. Soft set theory does not require the specification of parameters. Instead, it accommodates approximate description of an object as its starting point which makes it a natural mathematical formalism for approximate reasoning. So the application of soft set theory in other disciplines and real life problems are now catching momentum. In [23], Molodtsov applied soft sets successfully in directions such as smoothness of functions,

*Department of Mathematics, College of Science, Qassim University, P. O. Box 6644, Buraydah 51482, Saudi Arabia.
Email: sabiriub@yahoo.com, sh.hussain@qu.edu.sa
†Department of Mathematics, King Abdul Aziz University, P. O. Box 80203, Jeddah 21589, Saudi Arabia.
Email:drbashir9@gmail.com

game theory, operations research, Riemann-integration, Perron integration, probability and theory of measurement. Maji et. al [20] applied soft sets in a multicriteria decision making problems. It is based on the notion of knowledge reduction of rough sets. They applied the technique of knowledge reduction to the information table induced by the soft set. In [21], they defined and studied several basic notions of soft set theory. In 2005, Pei and Miao [25] and Chen [6] improved the work of Maji et.al [20-21]. A. Kharal and B. Ahmad [19] defined and discussed the several properties of soft images and soft inverse images of soft sets. They also applied these notions to the problem of medical diagnosis in medical systems. Many researchers have contributed towards the algebraic structure of soft set theory [1-2],[5], [7], [9-19], [24], [27-28].
In 2011, Shabir and Naz [27] initiated the study of soft topological spaces. Also in 2011, S. Hussain and B. Ahmad [9] continued investigating the properties of soft open(closed), soft neighbourhood and soft closure. They also defined and discussed the properties of soft interior, soft exterior and soft boundary.
Shabir et. al [27] and D. N. Georgiou et. al [7], defined and studied some soft separation axioms, soft θ-continuity and soft connectedness in soft spaces using (ordinary) points of a topological space X. In this paper, we redefine and explore several properties of soft T_i, $i = 0, 1, 2$, soft regular, soft T_3, soft normal and soft T_4 axioms using soft points defined by I. Zorlutuna [30]. We also discuss some soft invariance properties namely soft topological property and soft hereditary property. We hope that these results will be useful for the future study on soft topology to carry out general framework for the practical applications and to solve the complicated problems containing uncertainties in economics, engineering, medical, environment and in general man-machine systems of various types.

2. Preliminaries

For the definitions and results on soft set theory, we refer to [1-2],[5], [7], [9-19], [24], [27-28]. However, we recall some definitions and results on soft set theory and soft topology.

Definition 1 [22]. Let X be an initial universe and E a set of parameters. Let $P(X)$ denote the power set of X and A a non-empty subset of E. A pair (F, A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A). Clearly, a soft set is not a set.
Definition 2 [22]. The complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where, $F^c : A \to P(U)$ is a mapping given by $F^c(\alpha) = U - F(\alpha)$, for all $\alpha \in A$.
Let us call F^c to be the soft complement function of F. Clearly $(F^c)^c$ is the same as F and $((F, A)^c)^c = (F, A)$.
Definition 3 [30]. A soft set (F, A) over X is said to be a null soft set, denoted by Φ_A , if for all $e \in A$, $F(e) = \phi$. Clearly, $(\Phi_A^c)^c = \Phi_A$.
Definition 4 [30]. A soft set (F, A) over X is said to be an absolute soft set, denoted by X_A, if for all $e \in A$, $F(e) = X$. Clearly, $X_A^c = \Phi_A$.
Definition 5 [27]. Let τ be the collection of soft sets over X with the fixed set of parameters A. Then τ is said to be a soft topology on X, if
(1) Φ_A, X_A belong to τ,
(2) the union of any number of soft sets in τ belongs to τ,
(3) the intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, A) is called a soft topological space over X. The members of τ are called soft open sets. The soft complement of a soft open set A is called the soft closed set in (X, τ, A). If (F, A) belongs to τ, we write $(F, A)\tilde{\in}\tau$.
Proposition 1 [27]. Let (X, τ, A) be a soft topological space over X. Then the collection $\tau_\alpha = \{F(e) : (F, A)\tilde{\in}\tau\}$, for each $e \in A$ defines a topology on X.
Remark 1. It is known that the intersection of two soft topologies over the same universe X is a soft topology, whereas the union may or may not be a soft topology as given in [27].

Hereafter, $SS(X)_A$ denotes the family of soft sets over X with the set of parameters A.
Definition 6 [30]. The soft set $(F, A)\tilde{\in}SS(X)_A$ is called a soft point in X_A, denoted by e_F, if for the element $e \in A$, $F(e) \neq \phi$ and $F(e^{'}) = \phi$, for all $e^{'} \in A - \{e\}$.
Definition 7 [30]. The soft point e_F is said to be in the soft set (G, A), denoted by $e_F\tilde{\in}(G, A)$, if for the element $e \in A$, $F(e) \subseteq G(e)$.
Proposition 2 [30]. Let $e_F\tilde{\in}X_A$ and $(G, A)\tilde{\in}SS(X)_A$. If $e_F\tilde{\in}(G, A)$, then $e_F\tilde{\notin}(G, A)^c$.
Definition 8 [30]. Let (X, τ, A) be a soft topological space over X and (F, A) a soft set in $SS(X)_A$. The soft point $e_F\tilde{\in}X_A$ is called a soft interior point of a soft set (F, A), if there exists a soft open set (H, A) such that $e_F\tilde{\in}(H, A)\tilde{\subseteq}(F, A)$. The soft interior of a soft set (F, A) is denoted by $(F, A)^\circ$ and is defined as the union of all soft open sets contained in (F, A). Clearly $(F, A)^\circ$ is the largest soft open set contained in (F, A).
Definition 9 [30]. Let (X, τ, A) be a soft topological space. Then a soft set (G, A) in $SS(X)_A$ is called a soft neighborhood (briefly: soft nbd) of the soft point $e_F\tilde{\in}X_A$, if there exists a soft open set (H, A) such that $e_F\tilde{\in}(H, A)\tilde{\subseteq}(G, A)$.
The soft neighborhood system of a soft point e_F, denoted by $N_\tau(e_F)$, is the family of all its soft neighborhoods.
Definition 10 [27]. Let (X, τ, A) be a soft topological space over X. Then a soft set (G, A) in $SS(X)_A$ is called a soft neighborhood (briefly: soft nbd) of the soft set (F, A), if there exists a soft open set (H, A) such that $(F, A)\tilde{\subseteq}(H, A)\tilde{\subseteq}$
(G, A).
Definition 11 [27]. Let (X, τ, A) be a soft topological space over X and (F, A) a soft set over X. Then the soft closure of (F, A), denoted by $\overline{(F, A)}$, is the intersection of all soft closed supersets of (F, A). Clearly $\overline{(F, A)}$ is the smallest soft closed set in (X, τ, A) which contains (F, A).
Definition 12 [27]. Let (X, τ, A) be a soft topological space over X and $Y \subseteq X$. Then $\tau_Y = \{(F_Y, A) = Y_A\tilde{\cap}(F, A) | (F, A)\tilde{\in}\tau\}$ is said to be the soft relative topology on Y, where $F_Y(e) = Y \cap F(e)$, for all $e \in A$. (Y, τ_Y, A) is called a soft subspace of (X, τ, A). We can easily verify that τ_Y is, in fact, a soft topology on Y.
Proposition 3 [27]. Let (X, τ, A) be a soft topological space over X and $Y \subseteq X$. Then (Y, τ_{eY}) is a subspace of (X, τ_e), for each $e \in A$.
Proposition 4 [27]. Let (Y, τ_Y, A) be a soft subspace of a soft topological space (X, τ, A) and (F, A) a soft open set in (Y, τ_Y, A). If $Y_A\tilde{\in}\tau$, then $(F, A)\tilde{\in}\tau$.
Theorem 1 [27]. Let (Y, τ_Y, A) be a soft subspace of a soft topological space (X, τ, A) and (F, A) a soft set over X. Then
(1) (F, A) is soft open in (Y, τ_Y, A) if and only if $(F, A) = Y_A\tilde{\cap}(G, A)$, for some soft open set (G, A) in (X, τ, A).
(2) (F, A) is soft closed in (Y, τ_Y, A) if and only if $(F, A) = Y_A\tilde{\cap}(G, A)$, for some soft closed set (G, A) in (X, τ, A).

3. Soft T_i; $(i = 0, 1, 2)$ Spaces

In this section, we redefine soft separation axioms namely soft T_i axioms, for $(i = 0, 1, 2)$ using soft points and discuss several properties and their relationship with the help of examples. Note that some authors ([7],[27]) defined soft separation axioms using ordinary points of a topological space.
Now we define:
Definition 13. Two soft sets (G, A), (H, A) in $SS(X)_A$ are said to be soft disjoint, written $(G, A)\tilde{\cap}(H, A) = \Phi_A$, if $G(e) \cap H(e) = \phi$, for all $e \in A$.

Definition 14. Two soft points e_G, e_H in X_A are distinct, written $e_G \neq e_H$, if there corresponding soft sets (G, A) and (H, A) are disjoint.

Definition 15. Let (X, τ, A) be a soft topological space over X and $e_G, e_H \tilde{\in} X_A$ such that $e_G \neq e_H$. If there exist at least one soft open set (F_1, A) or (F_2, A) such that $e_G \tilde{\in} (F_1, A)$, $e_H \tilde{\notin} (F_1, A)$ or $e_H \tilde{\in} (F_2, A)$, $e_G \tilde{\notin} (F_2, A)$, then (X, τ, A) is called a soft T_0-space.

Definition 16. Let (X, τ, A) be a soft topological space over X and $e_G, e_H \tilde{\in} X_A$ such that $e_G \neq e_H$. If there exist soft open sets (F_1, A) and (F_2, A) such that $e_G \tilde{\in} (F_1, A)$, $e_H \tilde{\notin} (F_1, A)$ and $e_H \tilde{\in} (F_2, A)$, $e_G \tilde{\notin} (F_2, A)$, then (X, τ, A) is called a soft T_1-space.

Definition 17. Let (X, τ, A) be a soft topological space over X and $e_G, e_H \tilde{\in} X_A$ such that $e_G \neq e_H$. If there exist soft open sets (F_1, A) and (F_2, A) such that $e_G \tilde{\in} (F_1, A)$, $e_H \tilde{\in} (F_2, A)$ and $(F_1, A)\tilde{\cap}(F_2, A) = \Phi_A$, then (X, τ, A) is called a soft T_2-space.

Proposition 5. (1) Every soft T_1-space is a soft T_0-space.
(2) Every soft T_2-space is a soft T_1-space.
Proof. (1) Obvious.
(2) If (X, τ, A) is a soft T_2-space, then by definition of soft T_2-space, for $e_G, e_H \tilde{\in} X_A$, $e_G \neq e_H$, there exist soft open sets (F_1, A) and (F_2, A) such that $e_G \tilde{\in} (F_1, A)$, $e_H \tilde{\in} (F_2, A)$ and $(F_1, A)\tilde{\cap}(F_2, A) = \Phi_A$. Since $(F_1, A)\tilde{\cap}(F_2, A) = \Phi_A$, $e_G \tilde{\notin} (F_2, A)$ and $e_H \tilde{\notin} (F_1, A)$. Thus it follows that (X, τ, A) is a soft T_1-space. □

Note that every soft T_1-space is a soft T_0-space. Every soft T_2-space is a soft T_1-space. The converses do not hold in general.
Example 1. Let $X = \{x_1, x_2\}$, $A = \{e_1, e_2\}$ and $\tau = \{\Phi_A, X_A, (F, A)\}$, where $(F, A) = \{(e_1, \{x_1\}), (e_2, \{x_2\})\}$. Then (X, τ, A) is a soft topological space over X. There are two pairs of soft points namely $e_{1(G_1)} = (e_1, \{x_2\})$, $e_{1(H_1)} = (e_1, \{x_1\})$ and $e_{2(G_2)} = (e_2, \{x_1\})$, $e_{2(H_2)} = (e_2, \{x_2\})$. Since $e_{1(G_1)} \neq e_{1(H_1)}$, then there is soft open set (F, A) such that $e_{1(G_1)} \tilde{\notin} (F, A)$, $e_{1(H_1)} \tilde{\in} (F, A)$. Similarly for the pair $e_{2(G_2)} \neq e_{2(H_2)}$, there is soft open set (F, A) such that $e_{2(H_2)} \tilde{\in} (F, A)$, $e_{2(G_2)} \tilde{\notin} (F, A)$. This shows that (X, τ, A) is a soft T_0-space. Clearly (X, τ, A) is not a soft T_1-space.

Example 2. Let $X = \{x_1, x_2\}$, $A = \{e_1, e_2\}$ and $\tau = \{\Phi_A, X_A, (F_1, A), (F_2, A), (F_3, A), (F_4, A)\}$, where $(F_1, A) = \{(e_1, \{x_2\}), (e_2, \{x_1\})\}$, $(F_2, A) = \{(e_1, \{x_1\}), (e_2, \{x_2\})\}$, $(F_3, A) = \{(e_1, \{x_1\})\}$, $(F_4, A) = \{(e_1, X), (e_2, \{x_1\})\}$ Then (X, τ, A) is a soft topological space over X. Note that $\tau_{e_1} = \{\phi, X, \{x_1\}, \{x_2\}\}$ and $\tau_{e_2} = \{\phi, X, \{x_1\}, \{x_2\}\}$ are topologies on X. Clearly (X, τ_{e_1}) and (X, τ_{e_2}) are T_i-spaces (for $i = 0, 1$). There are two pairs of distinct soft points namely, $e_{1(G_1)} = (e_1, \{x_2\})$, $e_{1(H_1)} = (e_1, \{x_1\})$ and $e_{2(G_2)} = (e_2, \{x_1\})$, $e_{2(H_2)} = (e_2, \{x_2\})$. Then for the soft pair $e_{1(G_1)} \neq e_{1(H_1)}$ of points, there are soft open sets (F_1, A) and (F_2, A) such that $e_{1(G_1)}\tilde{\in}(F_1, A)$, $e_{1(H_1)}\tilde{\notin}(F_1, A)$ and $e_{1(H_1)}\tilde{\in}(F_2, A)$, $e_{1(G_1)}\tilde{\notin}(F_2, A)$. Similarly for the pair $e_{2(G_2)} \neq e_{2(H_2)}$, there are soft open sets (F_2, A) and (F_1, A) such that $e_{2(G_2)}\tilde{\notin}(F_2, A)$, $e_{2(H_2)}\tilde{\in}(F_2, A)$ and $e_{2(H_2)}\tilde{\notin}(F_1, A)$, $e_{2(G_2)}\tilde{\in}(F_1, A)$. This shows that (X, τ, A) is a soft T_1-space and hence a soft T_0-space. Note that (X, τ, A) is a soft T_2-space.

Example 3. Let $X = \{x_1, x_2\}$, $A = \{e_1, e_2\}$ and $\tau = \{\Phi_A, X_A, (F_1, A),$ $(F_2, A), (F_3, A)\}$, where $(F_1, A) = \{(e_1, \{x_1\})\}$, $(F_2, A) = \{(e_2, \{x_2\})\}$, $(F_3, A) = \{(e_1, \{x_1\}), (e_2, \{x_2\})\}$. Then (X, τ, A) is a soft topological space over X. There are two pairs of distinct soft points namely, $e_{1(G_1)} = (e_1, \{x_2\})$, $e_{1(H_1)} = (e_1, \{x_1\})$ and $e_{2(G_2)} = (e_2, \{x_1\})$, $e_{2(H_2)} = (e_2, \{x_2\})$. Then for the soft pair $e_{1(G_1)} \neq e_{1(H_1)}$ of points, there does not exist soft disjoint soft open sets (F, A) and (G, A) such that $e_{1(G_1)}\tilde{\in}(F, A)$, $e_{1(H_1)}\tilde{\notin}(F, A)$ and $e_{1(H_1)}\tilde{\in}(G, A)$, $e_{1(G_1)}\tilde{\notin}(G, A)$. Thus (X, τ, A) is not a soft T_2-space. Clearly (X, τ, A) is a soft T_1-space and hence a soft T_0-space.

Theorem 2. Let (X, τ, A) be a soft topological space over X. Then each soft point is soft closed if and only if (X, τ, A) is a soft T_1-space.

Proof. Suppose soft points $e_F = (F, A)$, $e_G = (G, A)$ are soft closed and $e_F \neq e_G$. Then $(F, A)^c$ and $(G, A)^c$ are soft open in (X, τ, A). Then by definition $(F, A)^c = (F^c, A)$, where $F^c(e) = X - F(e)$ and $(G, A)^c = (G^c, A)$, where $G^c(e) = X - G(e)$. Since $F(e) \cap G(e) = \Phi$. This implies $F(e) \subseteq X - G(e) = G^c(e)$, for all e. This implies $e_F = (F, A)\tilde{\in}(G, A)^c$. Similarly $e_G = (G, A)\tilde{\in}(F, A)^c$. Thus we have $e_F\tilde{\in}(G, A)^c$, $e_G\tilde{\notin}(G, A)^c$ and $e_F\tilde{\notin}(F, A)^c$, $e_G\tilde{\in}(F, A)^c$. This proves that (X, τ, A) is soft T_1-space.

Conversely, let (X, τ, A) is soft T_1-space. To prove that $e_F = (F, A) \in \tilde{X}_A$ is soft closed, we show that $(F, A)^c$ is soft open in (X, τ, A). Let $e_G = (G, A)\tilde{\in}(F, A)^c$. Then $e_F \notin e_G$. Since (X, τ, A) is soft T_1-space, there exists a soft open set (H, A) such that $e_G\tilde{\in}(H, A)$ and $e_F\tilde{\notin}(H, A)$. Thus $e_G\tilde{\in}(H, A)\tilde{\subseteq}(F, A)^c$ and hence $\tilde{\bigcup}_{e_G}\{(H, A), e_G\tilde{\in}(F, A)^c\} = (F, A)^c$. This proves that $(F, A)^c$ is soft open in (X, τ, A), that is $e_F = (F, A)$ is soft closed in (X, τ, A). □

Remark 2. In general, if (X, τ, A) is a soft T_1-space, then (X, τ_e) is not necessarily a T_1-space for $e \in A$. The following propositions give conditions for (X, τ_e) to be a T_1 space. We use Proposition 3 to prove this.

Proposition 6. Let (X, τ, A) be a soft topological space over X and $e_G, e_H\tilde{\in}X_A$ such that $e_G \neq e_H$. If there exist soft open sets (F_1, A) and (F_2, A) such that $e_G\tilde{\in}(F_1, A)$ and $e_H\tilde{\in}(F_1, A)^c$ or $e_H\tilde{\in}(F_2, A)$ and $e_G\tilde{\in}(F_2, A)^c$, then (X, τ, A) is a soft T_0-space and (X, τ_e) is a T_0-space, for each $e \in A$.

Proof. Clearly $e_H\tilde{\in}(F_1, A)^c = (F_1^c, A)$ implies $e_H\tilde{\notin}(F_1, A)$. Similarly $e_G\tilde{\in}(F_2, A)^c = (F_2^c, A)$ implies $e_G\tilde{\notin}(F_2, A)$. Thus we have $e_G\tilde{\in}(F_1, A)$, $e_H\tilde{\notin}(F_1, A)$ or $e_H\tilde{\in}(F_2, A)$, $e_G\tilde{\notin}(F_2, A)$. This proves (X, τ, A) is a soft T_0-space. Now for any $e \in A$, (X, τ_e) is a topological space and $e_G\tilde{\in}(F_1, A)$ and $e_H\tilde{\in}(F_1, A)^c$ or $e_H\tilde{\in}(F_2, A)$ and $e_G\tilde{\notin}(F_2, A)^c$. So that $G(e)\tilde{\in}F_1(e)$, $H(e)\tilde{\notin}F_1(e)$ or $H(e)\tilde{\in}F_2(e)$, $G(e)\tilde{\notin}F_2(e)$. Thus (X, τ_e) is a T_0-space. □

Proposition 7. Let (X, τ, A) be a soft topological space over X and e_G, e_H $\tilde{\in}X_A$ such that $e_G \neq e_H$. If there exist soft open sets (F_1, A) and (F_2, A) such that $e_G\tilde{\in}(F_1, A)$, $e_H\tilde{\in}(F_1, A)^c$ and $e_H\tilde{\in}(F_2, A)$, $e_G\tilde{\in}(F_2, A)^c$ then (X, τ, A) is a soft T_1-space and (X, τ_e) is a T_1-space, for each $e \in A$.

Proof. The proof is similar to the proof of Proposition 6. □

The following propositions 8, 9 and 11 show that the each soft T_i, $(i = 0, 1)$ property is a soft hereditary property.
Proposition 8. Let (X, τ, A) be a soft topological space over X and $Y \subseteq X$. If (X, τ, A) is a soft T_0-space, then (Y, τ_Y, A) is a soft T_0-space.
Proof. Let $e_G, e_H \tilde{\in} Y_A$ be such that $e_G \neq e_H$. Then $e_G, e_H \tilde{\in} X_A$. Since (X, τ, A) is a soft T_0 space, therefore there exist soft open sets (F, A) and (G, A) in (X, τ, A) such that $e_G \tilde{\in} (F, A)$ and $e_H \tilde{\notin} (F, A)$ or $e_H \tilde{\in} (G, A)$ and $e_G \tilde{\notin} (G, A)$. Therefore $e_G \tilde{\in} Y_A \tilde{\cap} (F, A) = (F_Y, A)$. Similarly it can be proved that if $e_H \tilde{\in} (G, A)$ and $e_G \tilde{\notin} (G, A)$, then $e_H \tilde{\in} (G_Y, A)$ and $e_G \tilde{\notin} (G_Y, A)$. Thus (Y, τ_Y, A) is a soft T_0-space. □

Proposition 9. Let (X, τ, A) be a soft topological space over X and Y be a non-empty subset of X. If (X, τ, A) is a soft T_1-space, then (Y, τ_Y, A) is a soft T_1-space.
Proof. The proof is similar to the proof of Proposition 8. □

Proposition 10. Let (X, τ, A) be a soft topological space over X. If (X, τ, A) is a soft T_2-space over X, then (X, τ_e) is a T_2-space, for each $e \in A$.
Proof. Let (X, τ, A) be a soft topological space over X. For any $e \in A$,
$\tau_e = \{F(e) : (F, A) \tilde{\in} \tau\}$ is a topology on X. Let $x, y \in X$ such that $x \neq y$. Since (X, τ, A) is a soft-T_2 space, therefore soft points $e_G, e_H \tilde{\in} X_A$ such that $e_G \neq e_H$ and $x \in G(e)$, $y \in H(e)$, there exist soft open sets (F_1, A) and (F_2, A) such that $e_G \tilde{\in} (F_1, A)$, $e_H \tilde{\in} (F_2, A)$ and $(F_1, A) \tilde{\cap} (F_2, A) = \Phi_A$. This imply that $x \in G(e) \subseteq F_1(e)$, $y \in H(e) \subseteq F_2(e)$ and $F_1(e) \cap F_2(e) = \phi$. This proves that (X, τ_e) is a T_2-space. □

Proposition 11. Let (X, τ, A) be a soft topological space over X and $Y \subseteq X$. If (X, τ, A) is a soft T_2-space, then (Y, τ_Y, A) is a soft T_2-space and (X, τ_e) is a T_2-space, for each $e \in A$.
Proof. Let $e_G, e_H \tilde{\in} Y_A$ such that $e_G \neq e_H$. Then $e_G, e_H \tilde{\in} X_A$. Since (X, τ, A) is a soft-T_2-space, therefore there exist soft open sets (F_1, A) and (F_2, A) such that $e_G \tilde{\in} (F_1, A)$, $e_H \tilde{\in} (F_2, A)$ and $(F_1, A) \tilde{\cap} (F_2, A) = \Phi_A$. Thus $e_G \tilde{\in} Y_A \tilde{\cap} (F_1, A) = (F_{1_Y}, A)$, $e_H \tilde{\in} Y_A \tilde{\cap} (F_2, A) = (F_{2_Y}, A)$ and $(F_{1_Y}, A) \tilde{\cap} (F_{2_Y}, A) = \Phi_A$. This proves that (Y, τ_Y, A) is a soft T_2-space. □

Theorem 3. Let (X, τ, A) be a soft topological space over X. If (X, τ, A) is a soft T_2-space and for any $e_G, e_H \tilde{\in} X_A$ such that $e_G \neq e_H$, then there exist soft closed sets (F_1, A) and (F_2, A) such that $e_G \tilde{\in} (F_1, A)$, $e_H \tilde{\notin} (F_1, A)$ and $e_G \tilde{\notin} (F_2, A)$, $e_H \tilde{\in} (F_2, A)$, and $(F_1, A) \tilde{\cup} (F_2, A) = X_A$.
Proof. Since (X, τ, A) is a soft T_2-space and $e_G, e_H \tilde{\in} X_A$ such that $e_G \neq e_H$, there exist soft open sets (G_1, A) and (G_2, A) such that $e_G \tilde{\in} (G_1, A)$ and $e_H \tilde{\in} (G_2, A)$ and $(G_1, A) \tilde{\cap} (G_2, A) = \Phi_A$. Clearly $(G_1, A) \tilde{\subseteq} (G_2, A)^c$ and $(G_2, A) \tilde{\subseteq}$
$(G_1, A)^c$. Hence $e_G \tilde{\in} (G_2, A)^c$. Put $(G_2, A)^c = (F_1, A)$. This gives $e_G \tilde{\in} (F_1, A)$ and $e_H \tilde{\notin} (F_1, A)$. Also $e_H \tilde{\in} (G_1, A)^c$. Put $(G_1, A)^c = (F_2, A)$. Therefore $e_G \tilde{\in} (F_1, A)$ and $e_H \tilde{\in} (F_2, A)$. Moreover
$(F_1, A) \tilde{\cup} (F_2, A) = (G_2, A)^c \tilde{\cup} (G_1, A)^c = X_A$. □

4. Soft Regular, Soft Normal and Soft T_i; $(i = 4, 3)$ Spaces

In this section, we redefine soft regular and soft T_3 spaces using soft points and characterize soft regular and soft normal spaces. Moreover, we prove that soft regular and soft T_3 properties are soft hereditary, whereas soft normal and soft T_4 are soft closed

hereditary properties.
Now we define soft regular space as:
Definition 18. Let (X, τ, A) be a soft topological space over X, (G, A) a soft closed set in (X, τ, A) and $e_F \tilde{\in} X_A$ such that $e_F \tilde{\notin} (G, A)$. If there exist soft open sets (F_1, A) and (F_2, A) such that $e_F \tilde{\in} (F_1, A)$, $(G, A) \tilde{\subseteq} (F_2, A)$ and $(F_1, A) \tilde{\cap} (F_2, A) = \Phi_A$, then (X, τ, A) is called a soft regular space. □

In the following theorem, we give the characterizations of soft regular spaces.
Theorem 4. Let (X, τ, A) be a soft topological space over X. Then the following statements are equivalent:
(1) (X, τ, A) is soft regular.
(2) For any soft open set (F, A) in (X, τ, A) and $e_G \tilde{\in} (F, A)$, there is a soft open set (G, A) containing e_G such that $e_G \tilde{\in} \overline{(G, A)} \tilde{\subseteq} (F, A)$.
(3) Each soft point in (X, τ, A) has a soft nbd base consisting of soft closed sets.
Proof. (1) $\Rightarrow$ (2) Let (F, A) be a soft open set in (X, τ, A) and $e_G \tilde{\in} (F, A)$. Then $(F, A)^c$ is a soft closed set such that $e_G \tilde{\notin} (F, A)^c$. By the soft regularity of (X, τ, A), there are soft open sets (F_1, A), (F_2, A) such that $e_G \tilde{\in} (F_1, A)$, $(F, A)^c \tilde{\subseteq} (F_2, A)$ and $(F_1, A) \tilde{\cap} (F_2, A) = \Phi_A$. Clearly $(F_2, A)^c$ is a soft closed set contained in (F, A). Thus $(F_1, A) \tilde{\subseteq} (F_2, A)^c \tilde{\subseteq} (F, A)$. This gives $\overline{(F_1, A)} \tilde{\subseteq} (F_2, A)^c$
$\tilde{\subseteq} (F, A)$. Put $(F_1, A) = (G, A)$. Consequently, $e_G \tilde{\in} (G, A)$ and $\overline{(G, A)} \tilde{\subseteq} (F, A)$. This proves (2).
(2) $\Rightarrow$ (3) Let $e_G \tilde{\in} X_A$. For soft open set (F, A) in (X, τ, A), there is a soft open set (G, A) containing e_G such that $e_G \tilde{\in} (G, A)$, $\overline{(G, A)} \tilde{\subseteq} (F, A)$. Thus for each $e_G \tilde{\in} X_A$, the sets $\overline{(G, A)}$ form a soft nbd base consisting of soft closed sets of (X, τ, A). This proves (3).
(3) $\Rightarrow$ (1). Let (F, A) be a soft closed set such that $e_G \tilde{\notin} (F, A)$. Then $(F, A)^c$ is a soft open nbd of e_G. By (3), there is a soft closed set (F_1, A) which contains e_G and is a soft nbd of e_G with $(F_1, A) \tilde{\subseteq} (F, A)^c$. Then $e_G \tilde{\notin} (F_1, A)^c$, $(F, A) \tilde{\subseteq} (F_1, A)^c = (F_2, A)$ and $(F_1, A) \tilde{\cap} (F_2, A) = \Phi_A$. Therefore (X, τ, A) is soft regular. □.

The following theorem shows that soft regularity is a soft hereditary property:
Theorem 5. Let (X, τ, A) be a soft regular space over X. Then every soft subspace of (X, τ, A) is soft regular.
Proof . Let (Y, τ_Y, A) be a soft subspace of a soft regular space (X, τ, A). Suppose (H, A) is a soft closed set in (Y, τ_Y, A) and $e_F \tilde{\in} Y_A$ such that $e_F \tilde{\notin} (H, A)$. Then $(H, A) = (G, A) \tilde{\cap} Y_A$, where (G, A) is soft closed in (X, τ, A). Then $e_F \tilde{\notin} (G, A)$. Since (X, τ, A) is soft regular, there exist soft disjoint soft open sets (F_1, A), (F_2, A) in (X, τ, A) such that $e_F \tilde{\in} (F_1, A)$, $(G, A) \tilde{\subseteq} (F_2, A)$. Clearly $e_F \tilde{\in} (F_1, A) \tilde{\cap} Y_A = (F_{1_Y}, A)$ and $(H, A) \tilde{\subseteq} (F_2, A) \tilde{\cap} Y_A = (F_{2_Y}, A)$ such that $(F_{1_Y}, A) \tilde{\cap} (F_{2_Y}, A) = \Phi_A$. This proves that (Y, τ_Y, A) is a soft regular subspace of (X, τ, A). □

Next we give another characterization of soft regular spaces.:
Theorem 6. Let (X, τ, A) be a soft topological space over X. A space (X, τ, A) is soft regular if and only if for each $e_H \tilde{\in} X_A$ and a soft closed set (F, A) in (X, τ, A) such that $e_H \tilde{\notin} (F, A)$, there exist soft open sets (F_1, A), (F_2, A) in (X, τ, A) such that $e_H \tilde{\in} (F_1, A)$ and $(F, A) \tilde{\subseteq} (F_2, A)$ and $\overline{(F_1, A)} \tilde{\cap} \overline{(F_2, A)} = \Phi_A$.
Proof. For each $e_H \tilde{\in} X_A$ and a soft closed set (F, A) such that $e_H \tilde{\notin} (F, A)$, by Theorem 4(2), there is a soft open set (G, A) such that $e_H \tilde{\in} (G, A)$, $\overline{(G, A)} \tilde{\subseteq} (F, A)^c$. Again by Theorem 4(2), there is a soft open set (F_1, A) containing e_H such that $\overline{(F_1, A)} \tilde{\subseteq} (G, A)$. Let

$(F_2, A) = (\overline{(G,A)})^c$. Then $\overline{(F_1,A)}\tilde{\subseteq}(G,A)\tilde{\subseteq}\overline{(G,A)}\tilde{\subseteq}(F,A)^c$ implies $(F,A)\tilde{\subseteq}(\overline{(G,A)})^c = (F_2, A)$ or $(F,A)\tilde{\subseteq}(F_2,A)$. Also $\overline{(F_1,A)}\tilde{\cap}\overline{(F_2,A)} = \overline{(F_1,A)\tilde{\cap}(\overline{(G,A)})^c} \tilde{\subseteq} \overline{(G,A)\tilde{\cap}(\overline{(G,A)})^c} \tilde{\subseteq} \overline{(G,A)\tilde{\cap}(\overline{(G,A)})^c} = \overline{\Phi_A} = \Phi_A$.
Thus (F_1, A), (F_2, A) are the required soft open sets in (X, τ, A). This proves the necessity. The sufficiency is immediate. □

Definition 19. Let (X, τ, A) be a soft topological space over X. Then (X, τ, A) is said to be a soft T_3-space, if it is a soft regular and a soft T_1-space.

Remark 3. (1) A soft T_3-space may not be a soft T_2-space.
(2) If (X, τ, A) is a soft T_3-space, then (X, τ_e) may not be a T_3-space for each parameter $e \in A$.

The following proposition follows form Proposition 9 and Theorem 5.
Proposition 12. Let (X, τ, A) be a soft topological space over X and $Y \subseteq X$. If (X, τ, A) is a soft T_3-space then (Y, τ_Y, A) is a soft T_3-space.

The notions of soft normal and soft T_4 spaces have been introduced in [25]as:
Definition 20[25]. Let (X, τ, A) be a soft topological space over X, (F, A) and (G, A) soft closed sets over X such that $(F,A)\tilde{\cap}(G,A) = \Phi_A$. If there exist soft open sets (F_1, A) and (F_2, A) such that $(F,A)\tilde{\subseteq}(F_1,A)$, $(G,A)\tilde{\subseteq}(F_2,A)$ and $(F_1,A)\tilde{\cap}(F_2,A) = \Phi_A$, then (X, τ, A) is called a soft normal space.

Definition 21[25]. Let (X, τ, A) be a soft topological space over X. Then (X, τ, A) is said to be a soft T_4-space, if it is soft normal and soft T_1-space.

Now we prove the following theorem which characterizes soft normal spaces.:
Theorem 7. A soft topological space (X, τ, A) is soft normal if and only if for any soft closed set (F, A) and soft open set (G, A) such that $(F,A)\tilde{\subset}(G,A)$, there exists at least one soft open set (H, A)containing (F, A) such that

$$(F,A)\tilde{\subseteq}(H,A)\tilde{\subseteq}\overline{(H,A)}\tilde{\subseteq}(G,A).$$

Proof. Suppose that (X, τ, A) is a soft normal space and (F, A) is any soft closed subset of (X, τ, A) and (G, A) a soft open set such that $(F,A)\tilde{\subseteq}(G,A)$. Then $(G,A)^c$ is soft closed and $(F,A)\tilde{\cap}(G,A)^c = \Phi_A$. So by supposition, there are soft open sets (H, A) and (K, A) such that $(F,A)\tilde{\subseteq}(H,A)$, $(G,A)^c\tilde{\subseteq}(K,A)$ and $(H,A)\tilde{\cap}(K,A)$
$= \Phi_A$. Since $(H,A)\tilde{\cap}(K,A) = \Phi_A$, $(H,A)\tilde{\subseteq}(K,A)^c$. But $(K,A)^c$ is soft closed, so that $(F,A)\tilde{\subseteq}(H,A)\tilde{\subseteq}\overline{(H,A)}\tilde{\subseteq}(K,A)^c\tilde{\subseteq}(G,A)$. Hence $(F,A)\tilde{\subseteq}(H,A)\tilde{\subseteq}\overline{(H,A)}$
$\tilde{\subseteq}(G,A)$.
Conversely, suppose that for every soft closed set (F, A) and a soft open set (G, A) such that $(F,A)\tilde{\subseteq}(G,A)$, there is a soft open set (H, A) such that $(F,A)\tilde{\subseteq}(H,A)\tilde{\subseteq}\overline{(H,A)}\tilde{\subseteq}(G,A)$.
Let (F_1, A), (F_2, A) be any two soft disjoint soft closed sets. Then $(F_1,A)\tilde{\subseteq}(F_2,A)^c$, where $(F_2,A)^c$ is soft open. Hence there is a soft open set (H, A) such that $(F_1,A)\tilde{\subseteq}(H,A)\tilde{\subseteq}\overline{(H,A)}\tilde{\subseteq}(F_2,A)^c$. But then $(F_2,A)\tilde{\subseteq}(\overline{(H,A)})^c$ and $(H,A)\tilde{\cap}(\overline{(H,A)})^c \neq \Phi$. Hence $(F_1,A)\tilde{\subseteq}(H,A)$, $(F_2,A)\tilde{\subseteq}(\overline{(H,A)})^c$ with $(H,A)\tilde{\cap}(\overline{(H,A)})^c = \Phi_A$. Hence (X, τ, A) is soft normal. This completes the proof.

The following proposition is easy to proof.
Proposition 13. Let (Y, τ_Y, A) be a soft subspace of a soft topological space (X, τ, A) and (F, A) be a soft open (closed) in (Y, τ_Y, A). If Y_A is soft open(closed) in (X, τ, A), then (F, A) is soft open(closed) in (X, τ, A).

Soft normality is a soft closed hereditary property as is proved in the following:
Theorem 8. A soft closed subspace of a soft normal space is soft normal.
Proof. Let (Y, τ_Y, A) be soft subspace of soft normal space (X, τ, A) such that $Y_A \in \tau^c$. Let (F_1, A), (F_2, A) be two disjoint soft closed subsets of (Y, τ_Y, A). Then there exists soft closed sets (F, A), (G, A) in (X, τ, A) such that $(F_1, A) = Y_A \tilde{\cap} (F, A)$ and $(F_2, A) = Y_A \tilde{\cap} (G, A)$. Since Y_A is soft closed in (X, τ, A), therefore (F_1, A), (F_2, A) are soft disjoint soft closed in (X, τ, A). Then (X, τ, A) is soft normal implies that there exist soft open sets (F_3, A), (F_4, A) in (X, τ, A) such that $(F_1, A) \tilde{\subseteq} (F_3, A)$, $(F_2, A) \tilde{\subseteq} (F_4, A)$ and $(F_3, A) \tilde{\cap} (F_4, A) = \Phi_A$. But then $(F_1, A) \tilde{\subseteq} Y_A \tilde{\cap} (F_3, A)$, $(F_2, A) \tilde{\subseteq} Y_A \tilde{\cap} (F_4, A)$, where $Y_A \tilde{\cap} (F_3, A)$, $Y_A \tilde{\cap} (F_4, A)$ are soft disjoint soft open subsets of (Y, τ_Y, A). This proves that (Y, τ_Y, A) is soft normal. Hence the proof.

The following corollary directly follows from proposition 9 and Theorem 8. **Corollary 1.** Every soft closed subspace of a soft T_4-space is a soft T_4-space.

Conclusion : The study of soft sets and soft topology is very important in the study of possible applications in classical and non classical logic. We redefined and explored soft separation axioms, namely soft T_i, $i = 0, 1, 2$, soft regular, soft T_3, soft normal and soft soft T_4 axioms using soft point defined by I. Zorlutuna [30]. We also discussed some soft invariance properties namely soft topological property and soft hereditary property . These soft separation axioms would be useful for the development of the theory of soft topology to solve the complicated problems containing uncertainties in economics, engineering, medical, environment and in general man-machine systems of various types. These findings are the addition for strengthening the toolbox of soft topology.

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