Soft-Switching Adaptive Technique of Impulsive Noise Removal in Color Images

Bogdan Smolka[1,⋆] and Konstantinos N. Plataniotis[2]

[1] Silesian University of Technology, Department of Automatic Control, Akademicka 16 Str, 44-100 Gliwice, Poland
bsmolka@ia.polsl.gliwice.pl

[2] The Edward S. Rogers Sr. Department of Electrical and Computer Engineering, University of Toronto, 10 King's College Road, Toronto ON, M5S 3G4, Canada

Abstract. In this paper a novel class of filters designed for the removal of impulsive noise in color images is presented. The proposed filter family is based on the kernel function which regulates the noise suppression properties of the proposed filtering scheme. The comparison of the new filtering method with standard techniques used for impulsive noise removal indicates superior noise removal capabilities and excellent structure preserving properties.

1 Introduction

During image *formation*, *acquisition*, *storage* and *transmission* many types of distorsions limit the quality of digital images. Transmission errors, periodic or random motion of the camera system during exposure, electronic instability of the image signal, electromagnetic interferences from natural or man-made sources, sensor malfunctions, optic imperfections, electronics interference or aging of the storage material all disturb the image quality.

In many practical situations, images are corrupted by the so called *impulsive noise* caused mainly either by faulty image sensors or due to transmission errors. In this paper we address the problem of impulsive noise removal in color images and propose an efficient technique capable of removing the impulsive noise and preserving important image features.

2 Vector Median Based Filters

Mathematically, a $N_1 \times N_2$ multichannel image is a mapping $\mathbb{Z}^l \rightarrow \mathbb{Z}^m$ representing a two-dimensional matrix of three-component samples (pixels), $\mathbf{x}_i = (x_{i1}, x_{i2}, \ldots, x_{im}) \in \mathbb{Z}^l$, where l is the image domain dimension and m denotes the number of channels, (in the case of standard color images, parameters l and m are equal to 2 and 3, respectively). Components x_{ik}, for $k = 1, 2, \ldots, m$ and $i = 1, 2, \ldots, N$, $N = N_1 \cdot N_2$, represent the color channel values quantified into the integer domain, [1].

⋆ This research has been supported by the KBN grant 4T11F01824.

M. Kamel and A. Campilho (Eds.): ICIAR 2005, LNCS 3656, pp. 686–693, 2005.

The majority of the nonlinear, multichannel filters are based on the ordering of vectors in a sliding filter window. The output of these filters is defined as the lowest ranked vector according to a specific vector ordering technique, [2,3].

Let the color images be represented in the commonly used RGB color space and let $\mathbf{x}_1$, $\mathbf{x}_2$, ..., $\mathbf{x}_n$ be n samples from the sliding filter window W, with $\mathbf{x}_1$ being the central pixel in W. Each of the $\mathbf{x}_i$ is an m-dimensional vector. The goal of the vector ordering is to arrange the set of n vectors $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n\}$ belonging to W using some sorting criterion.

In [3,4] the ordering based on the cumulative distance function has been proposed: $R(\mathbf{x}_i) = \sum_{j=1}^{n} \rho(\mathbf{x}_i, \mathbf{x}_j)$, where $\rho(\mathbf{x}_i, \mathbf{x}_j)$ is a function of the distance among $\mathbf{x}_i$ and $\mathbf{x}_j$. The increasing ordering of the scalar quantities $\{R_1, \ldots, R_n\}$ generates the ordered set of vectors $\{\mathbf{x}_{(1)}, \mathbf{x}_{(2)}, \ldots, \mathbf{x}_{(n)}\}$.

One of the most important noise reduction filter is the vector median, [2]. Given a set W of n vectors, the vector median of the set is defined as $\mathbf{x}_{(1)} \in W$ satisfying $\sum_j \left\|\mathbf{x}_{(1)} - x_j\right\| \leq \sum_j \left\|\mathbf{x}_i - \mathbf{x}_j\right\|$.

The orientation difference between two vectors can also be used as their dissimilarity measure. This so-called vector angle criterion is used by the *Basic Directional Filter* (BDF), to remove vectors with atypical directions, [5]. Other techniques like the *Directional Distance Filter* DDF, [5,6,7,8] and their modifications, [9,10,16] combine the distance and angular criteria to achieve better noise suppression results,.

3 Proposed Filtering Design

The well known local statistic filters constitute a class of linear minimum mean squared error estimators, based on the non-stationarity of the signal and the noise model, [11,12]. These filters make use of the local mean and the variance of the input set W and define the filter output for the gray-scale images as

$$y_i = \hat{x}_i + \alpha\left(x_i - \hat{x}_i\right) = \alpha x_i + (1-\alpha)\hat{x}_i\,, \tag{1}$$

where $\hat{x}_i$ is the arithmetic mean of the image pixels belonging to the filter window W centered at pixel position i and α is a filter parameter usually estimated through, [13]

$$\alpha = \frac{\sigma_x^2}{\sigma_n^2 + \sigma_x^2}\,,\ \hat{x}_i = \frac{1}{n}\sum\nolimits_{k=1}^{n} x_k\,,\ \nu^2 = \frac{1}{n}\sum\nolimits_{k=1}^{n}\left(x_k - \hat{x}_i\right)^2, x_k \in W, \tag{2}$$

$$\sigma_x^2 = \max\left\{0, \nu^2 - \sigma_n^2\right\},\ \alpha = \max\left\{0, 1 - \sigma_n^2/\nu^2\right\}\,, \tag{3}$$

where ν^2 is the local variance calculated from the samples in the filter window and σ_n^2 is the estimate of the variance of the noise process. If $\nu \gg \sigma_n$, then $\alpha \approx 1$ and practically no changes are introduced. When $v < \sigma_n$, then $\alpha = 0$ and the central pixel is replaced with the local mean. In this way, the filter smooths with the local mean, when the noise is not very intensive and leaves the pixel value unchanged when a strong signal activity is detected. The major drawback of this filter is that it **fails to remove impulses** and leaves noise in the vicinity of high gradient image features.

Equation (1) can be rewritten using the notation $x_i = x_1$, [13] as

$$y_1 = \alpha x_i + (1-\alpha)\hat{x}_i = \alpha x_1 + (1-\alpha)\hat{x}_1 = (1-\alpha)\left(\psi_1 x_1 + x_2 + \ldots + x_n\right)/n, \quad (4)$$

with $\psi_1 = (1-\alpha+n\alpha)/(1-\alpha)$ and in this way the local statistic filter (1) is reduced to the *central weighted average*, with a weighting coefficient ψ_1.

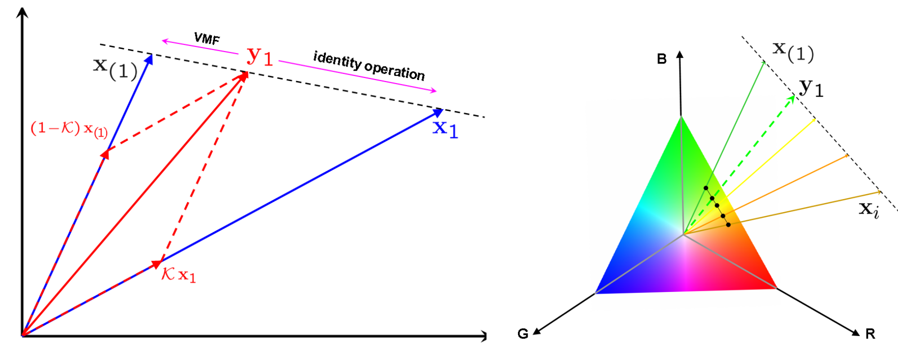

Fig. 1. Vector $\mathbf{y}_i$ lies on the line connecting the vector $\mathbf{x}_i$ and $\mathbf{x}_{(1)}$ in the RGB space

Table 1. Kernel functions, $(x = \langle -1, 1\rangle, h = \langle 0, \infty), [f(x)]^+ = f(x)$ for $x \geq 0$ and 0 if $x < 0)$ used for the construction of the proposed filter (**a**) and its efficiency in comparison with VMF, BDF and DDF, (**b**)

a)

Kernel	$K(x)$	$\mathcal{K}(x) = \gamma_h K(x)$
(L)	$e^{-\left\|\frac{x}{h}\right\|}$	$\frac{1}{2h} e^{-\left\|\frac{x}{h}\right\|}$
(G)	$e^{-\frac{x^2}{2h^2}}$	$\frac{1}{\sqrt{2\pi}h} e^{-\frac{x^2}{2h^2}}$
(C)	$\frac{1}{1+\frac{x^2}{h^2}}$	$\frac{1}{\pi h} \frac{1}{1+\frac{x^2}{h^2}}$
(T)	$\left[1 - \left\|\frac{x}{h}\right\|\right]^+$	$\left[\frac{h(1-\left\|\frac{x}{h}\right\|)}{2h-1}\right]^+$
(E)	$\left[1 - \frac{x^2}{h^2}\right]^+$	$\left[\frac{3h^2\left(1-\frac{x^2}{h^2}\right)}{6h^2-2}\right]^+$

b)

Filtering efficiency, (PSNR, [dB] LENA)						
Noise	$p = 1\%$		$p = 3\%$		$p = 5\%$	
Kernel	h_{opt}	h_{est}	h_{opt}	h_{est}	h_{opt}	h_{est}
L	40.75	40.70	37.92	37.90	36.38	36.35
G	39.22	39.22	36.96	36.95	35.68	35.67
C	39.65	39.39	37.11	37.03	35.72	35.67
T	40.46	40.45	37.76	37.76	36.27	36.27
E	40.87	40.81	37.96	37.94	36.39	36.34
VMF	33.33		32.94		32.58	
DDF	32.90		32.72		32.25	
BDF	32.04		31.81		31.14	

The structure of the new filter called *Kernel based VMF* (KVMF) is similar to the presented above approach. However, as our aim is to construct a filter capable of removing impulsive noise, instead of the mean value, the VMF output is utilized and the noise intensity estimation mechanism is accomplished through the similarity function, which can be viewed as kernel function, known from the nonparametric probability density estimation, (Tab. 1a).

In this way, the proposed technique is a **compromise** between the VMF and the identity operation. When an impulse is present, then it is detected by the kernel $\mathcal{K} = f(\|\mathbf{x}_1 - \mathbf{x}_{(1)}\|)$, which is a function of the distance between the central pixel $\mathbf{x}_i = \mathbf{x}_1$ and the vector median $\mathbf{x}_{(1)}$, and the output $\mathbf{y}_i$ is close to the VMF. If the central pixel is not disturbed by the noise process then the kernel function is close to 1 and the output is near to the original value $\mathbf{x}_1$. If the central pixel in W $\mathbf{x}_i$ is denoted as $\mathbf{x}_1$ and the vector norm as $\|\cdot\|$, then

$$\boxed{\mathbf{y}_i = \mathbf{x}_{(1)} + \mathcal{K}\left(\mathbf{x}_1, \mathbf{x}_{(1)}\right) \cdot \left(\mathbf{x}_1 - \mathbf{x}_{(1)}\right) = \mathcal{K}\mathbf{x}_1 + (1-\mathcal{K})\mathbf{x}_{(1)} ,} \tag{5}$$

where $\mathcal{K} = f\left(\|\mathbf{x}_1 - \mathbf{x}_{(1)}\|\right)$, which is quite similar to (1).

If $\left\{\mathbf{x}_{(1)}, \mathbf{x}_{(2)}, \ldots, \mathbf{x}_i, \ldots, \mathbf{x}_{(n)}\right\}$ denotes the ordered set of pixels in W, then the weighted structure corresponding to (4) is $\left\{(1-\mathcal{K})\mathbf{x}_{(1)}, \ldots, \mathcal{K}\mathbf{x}_1, \ldots, \mathbf{x}_{(n)}\right\}$.

It is interesting to observe that the filter output $\mathbf{y}_i$ lies on the line joining the vectors $\mathbf{x}_i$ $(\mathbf{x}_1)$ and $\mathbf{x}_{(1)}$ and depending on the value of the kernel $\mathcal{K}$ it slides from the identity operation and the vector median, (Fig. 1).

The proposed structure can be seen as a modification of the known techniques used for the suppression of the Gaussian noise. In the proposed technique we replace the mean of the pixels in W with the vector median and such an approach proves to be capable of removing strong impulsive noise while preserving important image features like edges, corners and texture.

4 Experimental Results

The noise modelling and evaluation of the efficiency of noise removal methods using the widely used test images allows the objective comparison of the noisy, restored and original images.

In this paper we assume a simple *salt & pepper* noise model, [3,6,14]

$$\mathbf{x}_i = \begin{cases} \{v_{i_1}, o_{i_2}, o_{i_3}\}, \text{ with probability } p, \\ \{o_{i_1}, v_{i_2}, o_{i_3}\}, \text{ with probability } p, \\ \{o_{i_1}, o_{i_2}, v_{i_3}\}, \text{ with probability } p, \end{cases} \tag{6}$$

where $\mathbf{x}_i$ represents the pixel in the corrupted image, $\mathbf{o}_i = \{o_{i_1}, o_{i_2}, o_{i_3}\}$ represents the original sample and $v_{i_1}, v_{i_2}, v_{i_3}$ are random, uncorrelated variables taking the value 0 or 250, with equal probability. The impulsive noise suppression efficiency was measured using the commonly used PSNR image quality measure

$$PSNR = 20 \log_{10}\left(\frac{255}{\sqrt{MSE}}\right), \quad MSE = \frac{\sum_{i=1}^{N}\sum_{k=1}^{m}\left(x_{ik} - o_{ik}\right)^2}{Nm} . \tag{7}$$

The efficiency of the proposed filtering approach is summarized in Tab. 1 and also presented in Fig. 2. As can be seen the dependence on the kind of the kernel function is not, as expected, very strong. However, the main problem is to find an adaptive optimal bandwidth parameter h, as the proper setting of the bandwidth guarantees good performance of the proposed filtering design.

The experimentally found *rule of thumb* for the value of h called h_{est} is: $h_{est} = \gamma_1/\sqrt{\hat{\sigma}}$, where $\hat{\sigma}$ is the mean value of the approximation of variance, [15] calculated using the whole image: $\hat{\sigma}^2 = \sum_{i=1}^{N}(x_i - \hat{x}_i)^2/8N^2$ or randomly selected image pixels and γ_1 is the coefficient taken from Tab. 1.

The comparison of the efficiency of the proposed scheme in terms of PSNR for the optimal values of h and estimated by the developed *rule of thumb* is shown in Tab. 1 and Fig. 4. In the corner of the Fig. 4 the magnified part of the plot shows the excellent performance of the proposed bandwidth estimator. The dotted lines represent the best possible PSNR values and the continuous line show the PSNR obtained with the proposed estimation of the kernel bandwidth. Practically the h_{est} yields the best possible impulsive noise attenuation, (see also the comparison in Tab. 1).

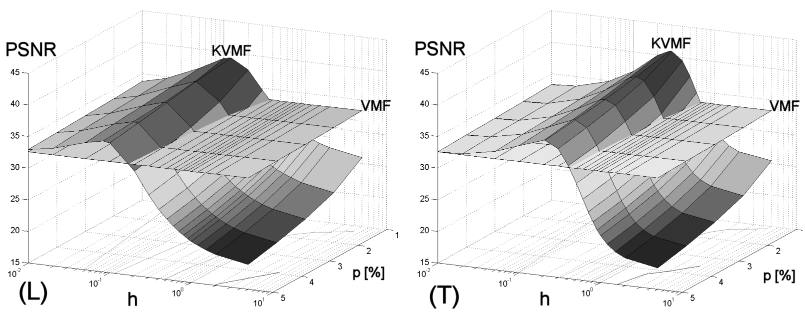

Fig. 2. Dependence of the PSNR on the h parameter for the KVMF with the L and T kernels in comparison with the VMV for p ranging from 1% to 5%, (*LENA* image)

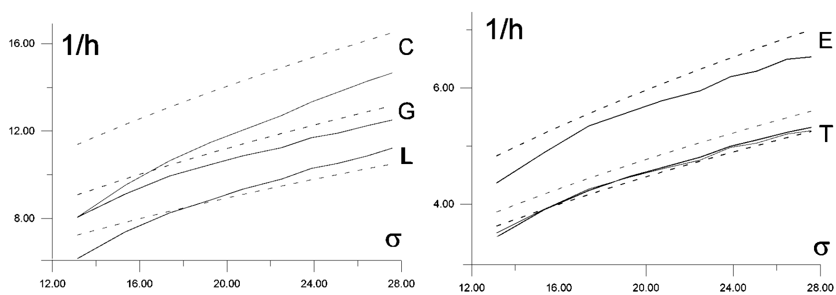

Fig. 3. Comparison of the estimated, (dashed line) and optimal bandwidth, (solid line) as functions of the noise intensity expressed through σ for the *LENA* image

The illustrative examples depicted in Fig. 6 show that the proposed filter efficiently removes the impulses and preserves edges and small image details. Additionally due to its smoothing nature it is also able to suppress slightly the Gaussian noise present in natural images, (see Fig. 5).

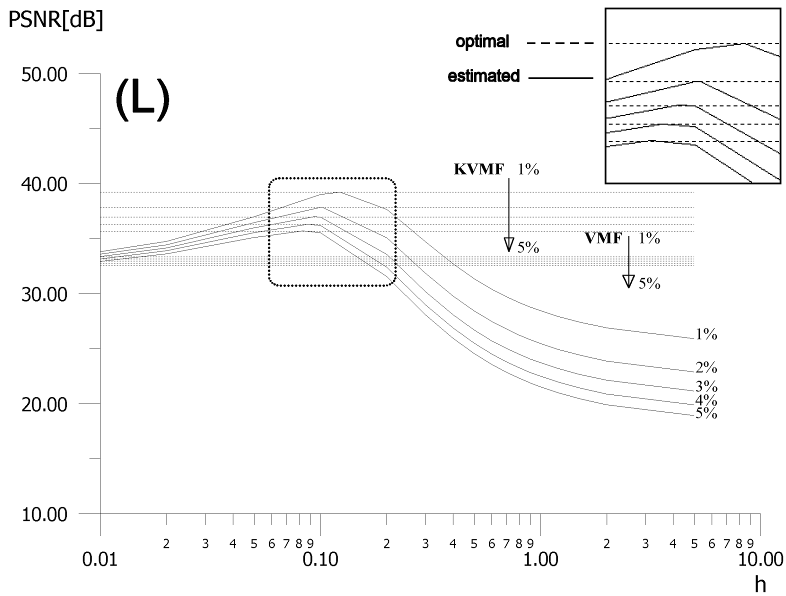

Fig. 4. Dependence of the PSNR on the h parameter of the L kernel, for $p = 1 - 5\%$ in comparison with the standard VMF, (*LENA* image). The dotted lines indicate the optimal, (best possible) values of PSNR achievable by the KVMF filter and the VMF and the continuous line presents the achieved PSNR using the h_{est} bandwidth.

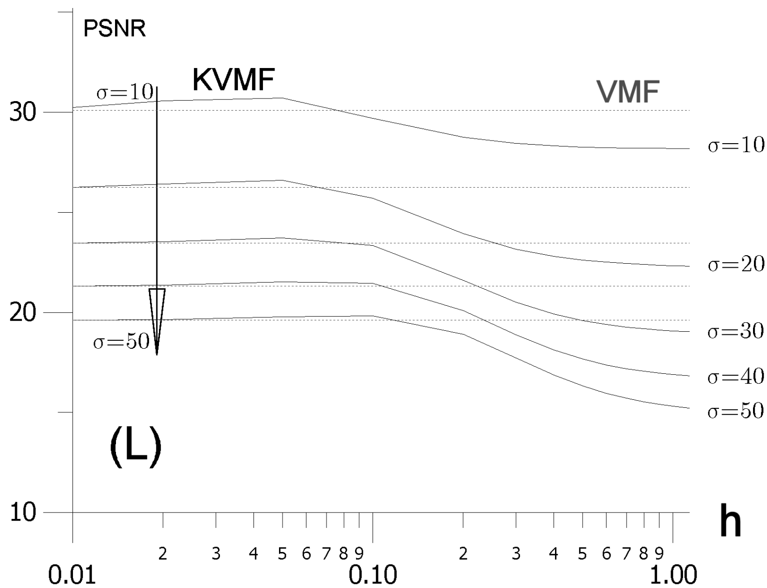

Fig. 5. Dependence of PSNR on the h parameter of the L kernel, for the Gaussian noise of $\sigma = 10 - 50$, (solid line) in comparison with the VMF, (dotted line)

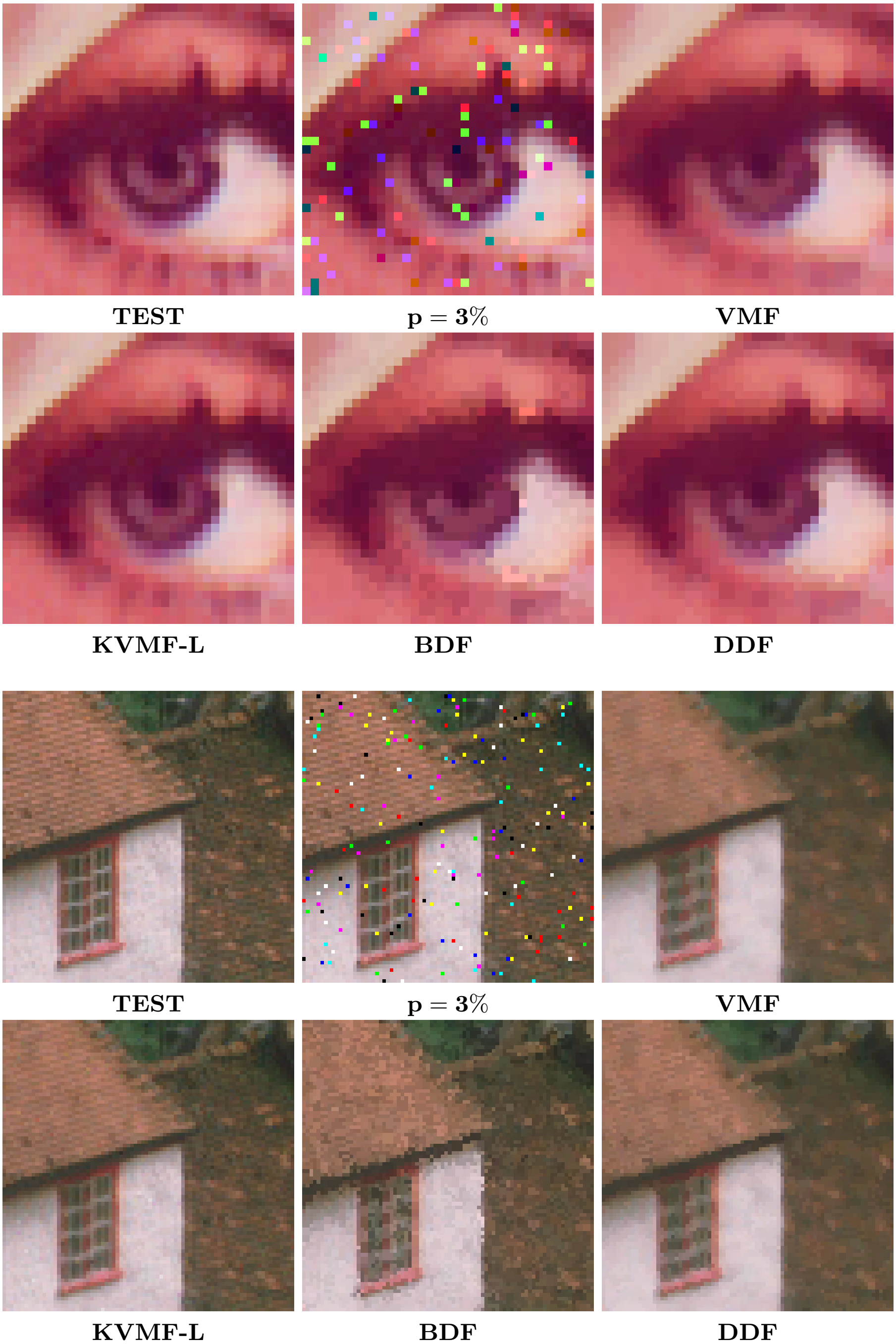

Fig. 6. Comparison of the filtering efficiency of the proposed filter with the Laplace kernel (KVMF-L) with the VMF, BDF and DDF methods

5 Conclusion

In the paper an adaptive soft-switching scheme based on the vector median and similarity function has been presented. The proposed filtering structure is superior to the standard filtering schemes and can be applied for the removal of impulsive noise in natural images. It is relatively fast and the proposed bandwidth estimator enables automatic filtering independent of noise intensity.

References

1. Lukac R.: Adaptive Color Image Filtering Based on Center-Weighted Vector Directional Filters. Multidimensional Systems and Signal Processing, Vol. 15, No. 2, (2004) 169-196
2. Astola, J., Haavisto, P., Neuvo, Y.: Vector Median Filters. Proc. of IEEE, Vol. 78, No. 4, (1990) 678-689,
3. Pitas, I., Tsakalides, P.: Multivariate Ordering in Color Image Processing. IEEE Trans. on Circuits and Systems for Video Technology, Vol. 1, No. 3, (1991) 247-256
4. Tang, K., Astola, J., Neuvo, Y.: Nonlinear Multivariate Image Filtering Techniques. IEEE Trans. on Image Processing, Vol. 4, (1995) 788-798
5. Trahanias, P.E., Venetsanopoulos, A.N.: Vector Directional Filters: a New Class of Multichannel Image Processing Filters. IEEE Trans. on Image Processing, Vol. 2, No. 4, (1993) 528-534
6. Plataniotis, K.N., Venetsanopoulos, A.N.: Color Image Processing and Applications. Springer Verlag, (2000)
7. Lukac, R.: Optimised Directional Distance Filter. Machine Graphics and Vision, Vol. 11, No. 2-3, (2002) 311-326
8. Lukac R.: Vector LUM Smoothers as Impulse Detector for Color Images. Proc. of European Conference on Circuit Theory and Design, (ECCTD'01) in Espoo, Finland, August 28-31, III, (2001) 137-140
9. Smolka, B., Plataniotis, K.N., Chydzinski, A., Szczepanski, M., Venetsanopulos, A.N., Wojciechowski, K.: Self-Adaptive Algorithm of Impulsive Noise Reduction in Color Images. Pattern Recognition, Vol. 35, (2002) 1771-1784
10. Smolka, B., Lukac, R., Chydzinski, A., Plataniotis, K.N., Wojciechowski, K.: Fast Adaptive Similarity Based Impulsive Noise Reduction Filter. Real-Time Imaging, Vol. 9, No. 4, (2003) 261-276
11. Lee, J.S.: Digital Image Enhancement and Noise Filtering by use of Local Statistics. IEEE Trans. on PAMI, Vol. 2, No. 2, (1980) 165-168
12. Kuan, D.T., Sawchuk, A.A., Strand, T.C., Chavel, P.: Adaptive Noise Smoothing Filter for Images with Signal-Dependent Noise. IEEE Trans. on PAMI, Vol. 7, No. 2, (1985) 165-177
13. Sun, T., Gabbouj, M., Neuvo, Y.: Adaptive L-Filters with Applications in Signal and Image Processing. Signal Processing, Vol. 38, (1994) 331-344
14. Smolka, B., Plataniotis, K.N., Venetsanopoulos, A.N., Nonlinear Techniques for Color Image Processing. In: Nonlinear Signal and Image Processing: Theory, Methods, and Applications, CRC Press, (2004) 445-505
15. Lukac, R.: Adaptive Vector Median Filtering. Pattern Recognition Letters, Vol. 24, No. 12, (2003) 1889-1899
16. Lukac, R., Fischer, V., Motyl, G., Drutarovsky, M.: Adaptive Video Filtering Framework. International Journal of Imaging Systems and Technology, Vol. 14, No. 6, (2004) 223-237