Soil Damping and Its Use in Dynamic Analyses

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Proceedings: Third International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, April 2-7, 1995, Volume I, St. Louis, Missouri

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Paper No. 1.13

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SYNOPSIS: Soil response under dynamic loading has been modeled using linear-viscoelasticity for many decades. The definitions of various damping-related quantities are reviewed and the implications of their use in the analysis of continuous masses are given. The selection of an equivalent damping ratio as a parameter for modeling damping in most geotechnical applications is discussed together with the assumptions underlying the selection procedure. The techniques for damping determination using laboratory testing are summarized, emphasizing the influence that factors such as apparatus type, loading path, rate of loading, and strain level have on the measurement. Disturbance effects in samples recovered from the ground are discussed and contrasted with the advantages and disadvantages of emerging techniques for in-situ determination of damping. Finally, the paper addresses the importance of selecting damping values for different types of analyses in earthquake geotechnical engineering and of correctly accounting for radiation damping.

1. INTRODUCTION

Damping studies on materials in general, and metals in particular started more than two centuries ago. Coulomb, in his "Memoir on Torsion" in 1784, recognized the fact that energy dissipates within a metal when loaded cyclically. He also realized that energy loss is not only due to air friction, but also to internal damping within the material (Lazan, 1965). In the nineteenth century, many scientists examined the phenomenon in more detail, and developed a theoretical basis for later works. The research has been supported in the last few decades by the fast advances in applied fields such as aircraft industry, rotating machinery, and construction of large structures. Major progress in the soil dynamics field was achieved during the last three decades, mainly motivated by the increasing interest in the areas of earthquake engineering, foundation vibration, blasting and wave loading on off-shore structures. Although most of the work dealt with laboratory measurement of dynamic soil properties and earthquake engineering applications, less attention has been given to the fundamental understanding of such properties. Energy dissipation phenomena, in particular, still need more study.

Various measures of damping or energy dissipation are used in different fields. In geotechnical engineering, the damping ratio is widely used. In order to represent the constitutive behavior of the soil, an equivalent Kelvin-Voigt model (also called the complex stiffness model) is implemented in all but a few studies. The purpose of this study is to review current practices in the geotechnical engineering field, and to evaluate the models and analysis methods implemented. Laboratory and in-situ techniques for damping measurement are compared, and sampling disturbance effects on damping measurement are investigated.

2. MEASURES OF DAMPING

Damping can be defined as the loss of energy within a vibrating or a cyclically loaded system, usually dissipated in the form of heat. The damping ratio is commonly used in geotechnical engineering as a measure for energy dissipation during dynamic or cyclic loading. As will be described later, the term "damping ratio" only applies to SDOF systems, and is used in this context as an equivalent parameter. In the following subsections, the general classification terminology used in classic thermodynamics is given. Various measures of damping commonly employed in the literature are introduced, and the relationship among them is presented.

2.1. Classification of Damping Types

Damping can be subdivided into two general categories: internal and external. Internal damping denotes the energy dissipation within the material itself, mainly due to microstructural mechanisms. In soils, this is attributed to many factors including inter-particle sliding and friction, structure rearrangement, and pore fluid viscosity. Internal damping is an inherent material property and is therefore commonly referred to as "material damping". External damping indicates energy losses within a structure or a structural member due to factors other than internal friction. This type of damping is therefore not an inherent property of the material and is commonly called "system damping".

Internal damping can be subdivided, in turn, into two categories: intrinsic damping and extrinsic damping. While intrinsic damping describes the energy losses at a specific point within the material, extrinsic damping characterizes the global energy loss within a finite volume. For linear materials, where damping is independent of the strain (ϵ) and the strain rate ($\partial\epsilon/\partial t$), intrinsic and extrinsic damping are equal. If, however, damping is a function of strain level and rate, then the measured (extrinsic) damping will be an average of the intrinsic material damping over the volume (V) of the specimen:

$$D_{measured} = \frac{1}{V} \int_V D(\epsilon, \frac{\partial\epsilon}{\partial t})\, dV \qquad (1)$$

where $D_{measured}$ is the global (extrinsic) damping ratio and $D(\epsilon,\partial\epsilon/\partial t)$ is the intrinsic damping ratio as a function of strain level and rate. It can be seen from Equation (1) that $D(\epsilon,\partial\epsilon/\partial t)$ cannot be easily obtained for a given $D_{measured}$. This shows the difficulty of damping measurement for highly non-linear materials, such as soils.

2.2. Basic Definitions of Damping Measures

2.2.1. The Specific Damping Capacity (Ψ)

The specific damping capacity, also called the specific loss, is considered the most fundamental measure of damping. It is defined as the ratio ($\Delta W/W$) where ΔW is the energy dissipated during a loading cycle, and W is the maximum elastic energy stored during the cycle. The specific damping capacity, Ψ, for the ideal stress-strain path shown in Figure 1, is calculated by:

$$\Psi = \frac{A_{loop}}{A_{triangle}} \qquad (2)$$

where A_{loop} ($=\Delta W$) is the area of the hysteresis loop, and $A_{triangle}$ ($=W$) is the area of the triangle ABC.

2.2.2. The Tangent of the Phase Lag (tan ϕ)

Due to damping, the response (displacement or strain) within an inelastic material lags the input (force or stress) by a phase angle (ϕ). Depending on the type and behavior of the material, this phase angle may or may not be a function of the applied frequency. For a single-degree-of-freedom (SDOF) Kelvin-Voigt (KV) solid with no inertia (Figure 2a), the phase lag is given by:

$$\tan\phi = \frac{c\omega}{k} \qquad (3)$$

where c is the coefficient of damping, ω is the circular frequency, and k is the spring

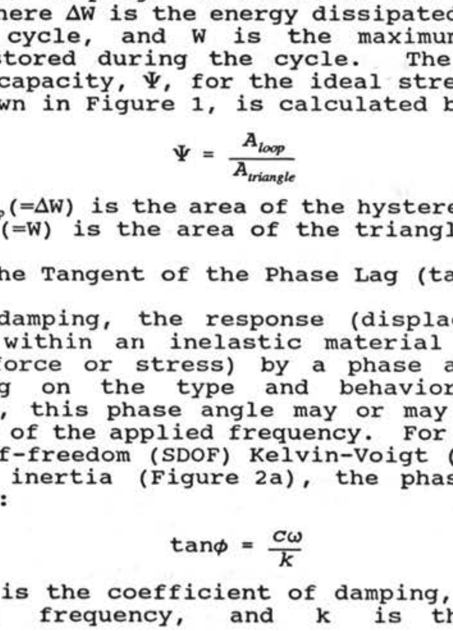

Figure 1. Hysteresis stress-strain loop for viscously-damped material.

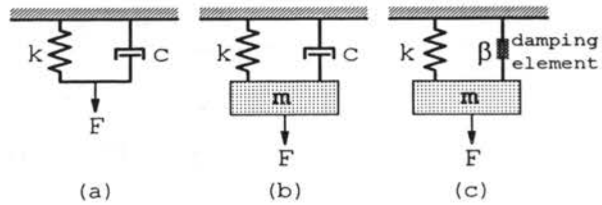

Figure 2. (a) KV model, (b) KV model with inertia, and (c) CS model with inertia.

stiffness. For an inertia-less SDOF complex stiffness (CS) system, which can model hysteretic (frequency independent) damping, the complex stiffness can be written as ($k^*=k+i\beta$) where β is the imaginary component of the complex stiffness, sometimes referred to as the hysteretic damping coefficient. The phase lag for a CS solid is independent of the frequency of vibration, and is given by:

$$\tan\phi = \frac{\beta}{k} \qquad (4)$$

In soil dynamics, the CS model is implemented in most studies, although it is frequently referred to as the KV model. The parameter (β), for the CS solid, is equivalent to the KV quantity ($c\omega$), hence the confusion. The basic difference between both models is that (c) and (β) are constants for KV and CS models, respectively. Consequently, (tan ϕ) varies linearly with frequency for the first model, and is independent of frequency for the latter.

For KV or CS systems with inertia (Figure 2b, 2c), caution should be taken with respect to the definition of the phase angle. Let us introduce a new quantity, θ, which denotes the phase angle by which the force (F) applied on the mass leads the displacement of the mass. For both materials, the tangent of the phase angle (θ) is obtained from the following relation:

$$\tan\theta = \frac{\tan\phi}{1 - (\omega/\omega_n)^2} \qquad (5)$$

where ω_n is the undamped natural frequency of the system (Graesser and Wong, 1992). The angle (ϕ) still denotes the phase lag of the displacement (or strain) with respect to the force (or stress) within the material. In most laboratory tests, measurements are made at the mass, and only θ is measured.

2.2.3. The Damping Ratio (D)

The damping ratio is defined, for a KV single-degree-of-freedom system with inertia, as the ratio between the coefficient of damping (c) and the critical damping(c_{cr}). Since critical damping is a function of the mass (m) and the spring constant (k), the damping ratio can be expressed as follows:

$$D = \frac{c}{c_{cr}} = \frac{c}{2\sqrt{km}} \qquad (6)$$

By far, this is the most used measure of damping in soil mechanics. Although the damping ratio is used to characterize energy dissipation properties of soils, the classic definition of

the term "damping ratio" only applies to KV SDOF systems. In fact, what is referred to as the damping ratio (D) in most geotechnical engineering applications is an "equivalent damping ratio for a KV SDOF system at resonance."

2.2.4. The Logarithmic Decrement (δ)

For a SDOF system, the amplitude of motion under free vibration decays exponentially with time. The logarithm of the ratio of the amplitudes at subsequent cycles is therefore constant, and is referred to as the logarithmic decrement (δ):

$$\delta = \ln \frac{A_n}{A_{n+1}} \qquad (7)$$

where A_n and A_{n+1} are the amplitudes of motion at any two subsequent cycles.

2.2.5. The Inverse Quality Factor (Q^{-1})

The quality factor (Q) denotes the sharpness of resonance. The inverse quality factor is determined for SDOF systems with inertia under forced vibration. The method, also called the "half-power method," consists of measuring the two frequencies (ω_1 and ω_2) where the steady state energy (or power) is half of that at the resonant frequency (ω_n). The inverse quality factor is defined as

$$Q^{-1} = \frac{|\omega_2 - \omega_1|}{\omega_n} \qquad (8)$$

Higher Q^{-1} values indicate higher material damping. The half-power points correspond to the frequencies where $MF/MF_{max} = (1/2)^{½}$, where MF is the magnification factor, as shown on the frequency response plot in Figure 3.

2.3. Damping Relationships

It is important at this stage to introduce the relationships that interrelate the various damping measures, and to understand the limitations and ranges of application of each of these relations. For small damping, where roughly tan ϕ is less than 0.1 (or the equivalent damping ratio is less than 0.07), the following holds for CS materials:

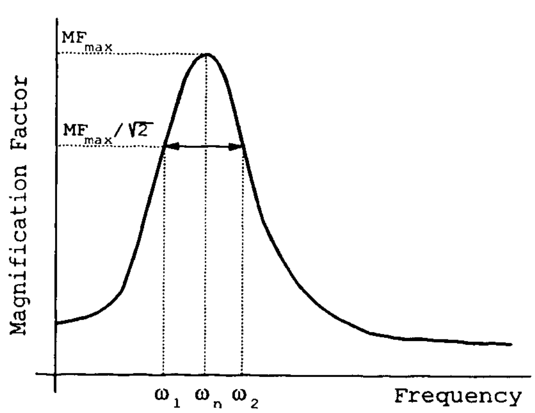

Figure 3. Half-power method for measuring damping

$$\Psi = 2\pi \tan\phi = 4\pi D = 2\delta = 2\pi Q^{-1} \qquad (9)$$

The same relationships hold for a SDOF Kelvin-Voigt viscoelastic solid, provided that the vibration occurs in the vicinity of the resonance frequency. A more general expression for KV solids can be written as:

$$\Psi = 2\pi \tan\phi = 4\pi D \frac{\omega}{\omega_n} = 2\delta \frac{\omega}{\omega_n} = 2\pi Q^{-1} \frac{\omega}{\omega_n} \qquad (10)$$

The quantity commonly referred to as the "damping ratio" in most geotechnical engineering literature, is, in fact, half the tangent of the phase lag (½ tan ϕ). From Equation (10), it can be concluded that this quantity equals the damping ratio of an equivalent KV SDOF system at resonance. For consistency purposes, the term damping ratio (D) will be used throughout the rest of the paper to denote (½ tan ϕ).

3. CURRENT STATE-OF-PRACTICE IN GEOTECHNICAL ENGINEERING

Measurements of material properties can be performed either in-situ or on laboratory specimens. In geotechnical engineering, in-situ measurements are usually preferred over laboratory tests because of the absence of sampling disturbance. In addition, in-situ measurements allow for the estimation of global properties and account for macro-scale effects, which cannot be studied on small specimens. Laboratory tests, on the other hand, allow for more control on various parameters, such as confining pressure, strain level, and boundary conditions. Three testing apparatus are commonly used for determining damping ratios during dynamic testing: the resonant column, the torsional shear, and the cyclic triaxial.

3.1. Laboratory Measurement of Damping

Among the various laboratory tests for damping measurement, the resonant column, the torsional shear, and the cyclic triaxial are most common. In the resonant column test, two methods of damping measurement are used: the logarithmic decrement, and the magnification factor. In the first method, the specimen vibrates freely after being given an initial condition. The amplitude decay with respect to time is recorded and the damping ratio is calculated accordingly. In the second method, the steady-state peak amplitude at resonance is used to establish the damping ratio.

Unlike the resonant column test, the torsional simple shear test involves quasi-static loading. Strains larger than those in a resonant column can be attained, and failure of the specimen is often possible. Stress-strain paths are plotted at different strain levels and damping is calculated from the area of the hysteresis loop. Similar to the torsional simple shear, cyclic triaxial devices allow for testing soil specimens under quasi-static conditions. Loading, however, is in the axial direction, and the specimen is not subjected to a pure-shear state of stress.

3.2. In-Situ Measurement of Damping

Although in-situ testing is being increasingly used as a tool for measuring various soil parameters, very little work has been done so far in terms of in-situ measurement of material damping. Results obtained by various investigators and reported in Stewart and Campanella (1991) show significant deviation from lab measurements.

In-situ techniques are nearly always based on the amplitude attenuation equation for harmonic body wave propagation in an infinite elastic homogeneous medium

$$A = A_o \frac{1}{R} e^{-\alpha R} \qquad (11)$$

where A is the strain amplitude at the receiver, A_o is the amplitude at the source, R is the distance between the source and the receiver, and α is the attenuation coefficient. The parameter α is related to the damping ratio through the relation ($D = \alpha\lambda/2\pi$) where λ is the wavelength.

Mok, et al. (1988) directly make use of Equation (11) to calculate the damping ratio. Cross-hole tests are performed, and motion is recorded at both the source and the receiver. Because the signal is composed of a large range of frequencies, the signal is first windowed. Dispersion curves are then obtained and the damping ratio is averaged based on the amplitudes at different frequencies. Stewart and Campanella (1991) describe a more sophisticated analysis which eliminates frequency-independent geometric terms from Equation (11). The method basically consists of expressing the Equation in the form of a ratio between amplitudes at two points, and differentiating the Equation with respect to distance (depth) and frequency. Down-hole seismic cone tests are performed, and the damping ratio is again obtained from the dispersion curves. Further details on the method can be found in Stewart and Campanella (1991).

Other techniques for in-situ measurement of material damping include back-calculating the damping ratio using existing wave propagation analysis programs, such as SHAKE. Motion at both the source and the receiver is recorded, and the damping ratio is adjusted to match calculated and observed motions. Although in-situ techniques for damping measurement are promising, the results obtained so far do not correlate well with lab data. More research is still needed in order to account for all the variables that affect field measurements.

3.3. Effect of Sample Disturbance

Sample disturbance is mainly caused by the stress path associated with the sampling procedure, resulting in a change in the structure or fabric of the soil. In order to investigate the effects of sampling disturbance on damping, a kaolinite slurry was consolidated under a mean effective stress of approximately 60 kPa to form a cross-anisotropic homogeneous sample. A specimen was trimmed and consolidated isotropically under 210 kPa confining pressure in a fixed-free resonant column apparatus. The pressure was then reduced to 70 kPa, and damping ratio vs strain amplitude measurements were taken for the overconsolidated soil (OCR = 3).

In order to cause disturbance to the soil specimen, a freezing-thawing technique was implemented. After being frozen for 24 hours, the specimen was allowed to thaw inside the resonant column apparatus under a 70 kPa confining pressure. There was practically no net change in void ratio due to the freezing-thawing process. Measurements of damping ratio vs strain amplitude were, again, taken for this overconsolidated yet highly disturbed specimen.

From the results plotted on Figure 4, it can be seen that damping ratios at small strains were practically unaffected by disturbance. The same specimen showed a 50% reduction in small-strain shear modulus upon disturbance. At higher strain levels, the freezing-thawing action caused damping ratios to decrease significantly. Although this may be partly due to breakage of inter-particle bonding due to freezing, further research is needed in order to fully understand the phenomenon. Other aspects of sampling disturbance that were not investigated include aging and time effects, and high OCR. Also highly structured clays, such as bentonite, and coarse-grained materials may be more sensitive to disturbance than kaolinite.

4. EQUIVALENT VISCOELASTIC PARAMETERS

The Kelvin-Voigt viscoelastic model (Figure 2a) is commonly used in practice because of its mathematical tractability. The equation of motion associated with a viscously damped material is linear, and its closed form solution can often be found. Viscous damping in soils can be partially attributed to the pore fluid, but may not necessarily follow the KV constitutive behavior. Moreover, energy dissipation during cyclic loading in soils can be attributed to other phenomena such as inter-particle friction and sliding, and structure rearrangement. These mechanisms of damping may or may not be of viscous nature.

The Maxwell model, shown in Figure 5a is usually used to describe the behavior of viscous fluids and highly creeping materials. Under a constant load, the steady-state displacements increase linearly with time. The standard-linear-solid model (Figure 5b) consists of an elastic element

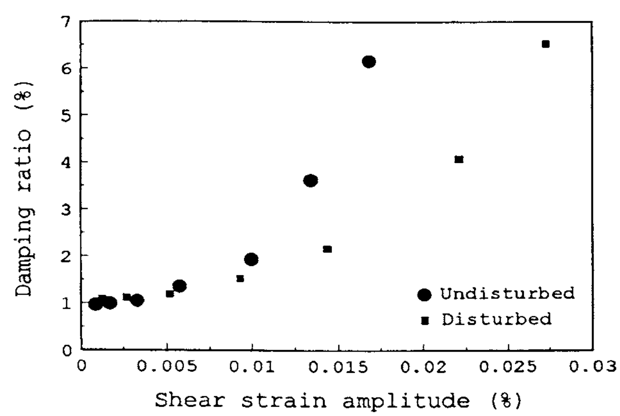

Figure 4. Effect of disturbance on damping ratio for kaolinite clay.

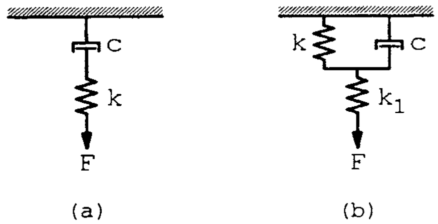

Figure 5.(a) Maxwell model, and (b) Standard linear solid.

(a spring) connected in series to a Kelvin-Voigt system. More sophisticated viscoelastic models, which can better predict materials behavior, may be developed through various combinations of springs and dashpots. Analytical solutions for such models are, however, difficult to develop.

Hardin (1965) performed resonant column and quasi-static tests on various sands, and showed that the soil exhibited little variation (±20% of average) in the damping ratio (or tan ϕ) with frequency. Since then, all soils, including clays, have been assumed to follow a CS model. More research is needed in order to investigate the validity of this assumption. Figure 6 illustrates a typical variation of the damping ratio (tan ϕ) with frequency of excitation for KV, CS, Maxwell, and standard-linear-solid materials. Actual materials may or may not follow any of these models. From Figure 6, it can be seen that, for different models, (tan ϕ) can be matched at one or, at most, two frequencies. Equivalent viscoelastic parameters from one model can be used to represent behaviors of other models, only within limited frequency ranges. For instance, evaluating equivalent KV parameters of a CS solid at a specific frequency, and extrapolating the use of those parameters at other frequencies can be highly misleading.

Bianchini (1985) describes the use of a non-linear model to capture the "true" behavior of soils. Model predictions, when compared to resonant column test results, showed significant variation. Two sets of parameters were required

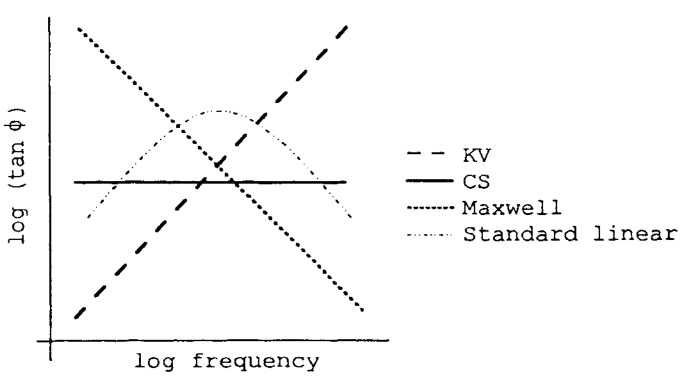

Figure 6. Variation of (tan ϕ) with frequency for different models. (after Kolsky, 1992)

in some cases to model the behavior over the full range of testing, and the model is mathematically difficult to handle. It seems, therefore, that the use of an equivalent viscoelastic model to characterize dynamic properties of soils including damping is the best solution available so far.

5. OTHER CONSIDERATIONS

5.1. Effect of Vibration Mode

The torsional simple shear test and the cyclic triaxial are similar in terms of loading rate, but different in terms of loading path. In the resonant column test, the specimen vibration can be either torsional or longitudinal, although torsional vibration is more common. While it is widely agreed upon that the moduli obtained from both vibration modes are different (Young's vs shear modulus), the damping ratio is commonly assumed to be the same. It is fully recognized in other fields of engineering and science that damping depends on the loading direction.

For a single-degree-of-freedom system, the damping ratio (D) is equal to (c/c_{cr}). If D_{axial} and D_{shear} are equal, it follows that

$$\frac{c_{axial}}{\sqrt{k_{axial}m}} = \frac{c_{shear}}{\sqrt{k_{shear}J}} \qquad (12)$$

where k is the spring stiffness, m is the mass, and J is the rotational inertia.
Similarly, in terms of a distributed mass system, we have

$$\frac{c_{axial}\omega}{E} = \frac{c_{shear}\omega}{G} \qquad (13)$$

where E is Young's modulus and G is the shear modulus. It is very unlikely that the relationships given by Equations (12) and (13) hold. Hardin (1965) showed that, for dry Ottawa sand (20-30), $D_{axial} \approx \frac{3}{4} D_{shear}$. Therefore, it should be recognized that damping ratios obtained from axial and torsional tests are different in nature, and could be very different in magnitude.

5.2. Location of the Resonant Peak

Since soils are non-linear, the resonant frequency depends on the amplitude of vibration. This can be seen in Figure 7a where the locus of the resonant peaks for a natural clay is not described by a vertical straight line in the frequency response plot. Theoretically, if the shear stress-shear strain relationship is hyperbolic, the locus of the resonant peaks should follow the path shown in Figure 7b.

More interesting is the shape of the frequency response curves shown in Figure 7a. Due to soil non-linearity, the curve is "skewed", and the magnification factor at some frequencies is not unique. This is characterized during testing by a sudden "jump" in the measured response at some frequencies. As a consequence, the half power method often yields inaccurate values due to the skewness of the frequency response curve for highly non-linear soils.

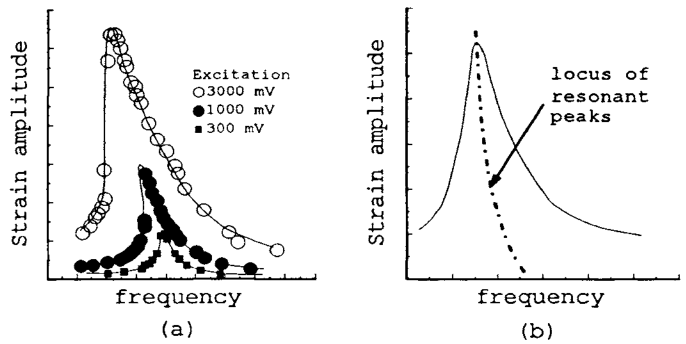

Figure 7.(a) Frequency response of kaolinite, and (b) locus of resonant peaks for hyperbolic materials. (after Bianchini, 1985)

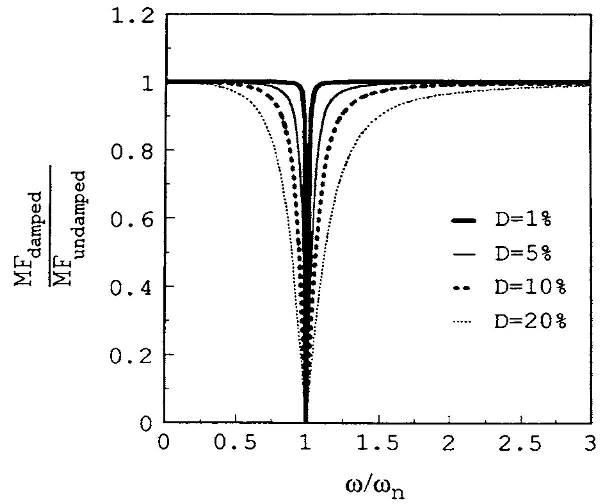

Figure 8. Effect of damping ratio on the magnification factor.

5.3. Shape of the Hysteresis Loop

Hysteresis loops for viscously damped solids are elliptical in shape. For rate-independent materials, the ends of the loop become more angular, and the shape of the loop may be distorted. Although equivalent viscous damping characterizes the area within the loop, it does not model the full shape of the stress-strain curve. Under these circumstances, it becomes more difficult to calculate the maximum elastic energy stored. It is a common practice to connect the tips of the hysteresis loop with a straight line to obtain the hypotenuse of the maximum strain energy triangle.

6. DAMPING IN DYNAMIC ANALYSES OF SOIL MASSES

6.1. Influence of Damping on Dynamic Analyses

The response of a system to dynamic loading is affected by several factors such as stiffness, damping, type of loading, geometry, boundary conditions. For a single-degree-of-freedom system, the magnification factor (MF) is controlled by the mass, stiffness, damping coefficient, and frequency ratio. Figure 8 shows the variation of ($MF_{damped}/MF_{undamped}$) with frequency ratio ($\omega/\omega_n$) over a range of damping ratios for a SDOF KV system. It can be seen that damping significantly influences the response for frequency ratios between 0.5 and 2 only.

It is therefore conceded that damping contributes to the total response of a dynamic system only in the vicinity of resonance. Damping is typically determined through laboratory measurement at frequencies between 100 and 200 Hz (resonant column) and between 0.1 and 1 Hz (cyclic triaxial or torsional shear). The predominant period of a soil deposit mainly depends on the soil depth and wave velocity, with typical values ranging between 1 and 10 Hz. Similar values are also typical for predominant earthquake frequencies. Since none of the laboratory tests is performed within this range, damping parameters are estimated away from the potential resonant frequency. This emphasizes the importance of verifying the validity of the CS model to describe soil behavior.

6.2. Radiation (Geometrical) Damping

For a homogeneous elastic infinite space, compression and shear (body) waves propagate radially from the source. If no material damping is present, the amplitude of the wave at any point is inversely proportional to the distance form the source. This can be easily deduced from Equation (11) by setting the attenuation coefficient $\alpha = 0$. In the case of a semi-infinite half-space, Rayleigh (surface waves) are also present. The far-field relative amplitude of the signal ($A_{receiver}/A_{source}$) is theoretically equal to $1/R^2$ for body (shear and compression) waves, and $1/R^{1/2}$ for surface waves.

Because soils are heterogeneous, non-linear and anisotropic in nature, wave propagation paths are much more complicated during earthquake events; hence the use of empirical attenuation relationships. The ideal relative amplitudes described are often used in the case of in-situ measurements (e.g. cross-hole tests) and foundation vibration problems. Accurate methods for correctly accounting for radiation damping become important in such cases. It is believed that in-situ damping measurements can be improved considerably if geometrical damping is modeled properly.

For machine vibrations, it is undesirable, if not unacceptable, to operate in the vicinity of resonance. Away from resonance, damping is not an important parameter since it does not contribute much to the response of the system. Different models or even inaccurate damping ratios will only affect the predictions to a small extent. Caution should be taken if the system passes through resonance during start up and shut down, not only during operation. Geometrical damping needs to be properly modeled if the analysis is based on wave propagation, or if vibration is potentially disturbing to adjacent structures.

7. CONCLUSIONS

A review of the current state-of-practice for damping measurement in geotechnical engineering was presented. The use of viscous damping in practice was justified by the fact that it is mathematically tractable but more work is required in the area of constitutive modeling to capture the non-linear characteristics of soils.

It is commonly presumed in geotechnical engineering that the quantity ($c\omega$) is constant, which implies that the "damping ratio" is independent of frequency (or strain rate). This assumption needs further validation, and frequency-dependent viscoelastic models may better describe the behavior of soils. The dependency of damping on the loading direction (or path) must also be considered when analyzing data obtained from different apparatus. It can be argued, however, that the current practice is adequate for practical purposes, since most applied soil dynamics disciplines, such as earthquake engineering, involve much uncertainty.

Preliminary test results on a kaolinite specimen showed that the effect of disturbance on damping ratio is insignificant at small strains. Disturbance seemed to cause a decrease in damping ratio with increasing strain level. More research is needed to verify whether or not this trend is specific to the soil used in this study.

In dynamic analyses of soil masses, damping becomes an important parameter near resonance. Therefore, it would be best if laboratory damping measurements are performed in the vicinity of the expected in-situ resonant frequency. This would minimize the influence that the selected model has on the total response.

ACKNOWLEDGEMENTS

The authors would like to thank Professors J.D. Bray of the University of California at Berkeley and R.J. Deschamps of Purdue University for valuable contributions.

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